PRACTICAL KNOWLEDGE FOR ALL

Volume 3

COURSES IN THIS VOLUME

On each of the pages indicated above is printed a detailed schedule of the Lessons making up that particular Course.

COLOUR PLATES IN THIS VOLUME

For ready reference to any item in text or illustrations consult the General Index at the end of Volume 5.

ART OF PERSIA. These archers of King Darius (reigned B.C. 521–486), on enamelled bricks from the Palace of Susa, are an excellent example of the cosmopolitan art of the Persian kings, to a great extent derived from Mesopotamia.

The Louvre

PRACTICAL KNOWLEDGE FOR ALL

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VOLUME 3

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ARCHAEOLOGY

THIS *Course is concerned with the fundamentals of History. Without the science of Archaeology, in exploring and examining the remains on ancient sites throughout the world, the history of its earliest cultures could not be written. After discussing the latest archaeological methods the Course proceeds to an evaluation of the facts of prehistory from evidences of primitive man to the early civilizations of Egypt, the Near East, and Europe. Accounts of the cultures of India, the Far East, and of South America conclude the survey. Associated Courses are* ANCIENT HISTORY, *in Vol. 1* ; ANTHROPOLOGY, *in Vol. 2* ; GEOLOGY, *in Vol. 4* ; *and* SOCIAL HISTORY, *in Vol. 4.*

30 LESSONS

LESSON 1

Recovering the Past

Archaeology began in antiquarianism, just as chemistry began in alchemy. The early antiquarian was the uncritical collector, a kind of human magpie, who collected objects because they were strange or were what he considered beautiful or interesting. Then came a stage when the antiquarian collected books, statuary, and coins, whose only interest to him was that they were old.

The modern archaeologist is interested in ancient things because they throw light on the past history of mankind. He does not merely collect ; he studies objects from two aspects.

He learns all he can from the object itself, its material, the method of its making, and the use to which it was put.

He then studies it in relation to the objects with which it was found, and compares it with similar objects found in other countries or periods, noting similarities and differences.

To put the matter shortly, the antiquarian values things for their own sake, while the archaeologist values them for what they *mean*.

The primitive conception of an antiquarian's field work is that it consists in digging up buried treasure of gold and jewels, all of fabulous value, or of unearthing beautiful statues which are the admiration of the whole artistic world, or of finding marvellous manuscripts on the interpretation of which the world's greatest scholars may expend their energies.

The earliest recorded antiquarians of this type were the ancient Egyptians, who held the wisdom of their ancestors in such respect that they ransacked temples in their search for documents. When their efforts were crowned with success, the document was considered to have a religious value so great as to be almost divine. In ancient Egypt medical remedies were recommended on the grounds that they had been in use for several centuries.

Ancient Manuscripts

When Diodorus the Greek visited the Egyptian city of Thebes in the 1st century B.C. he found an ancient library of manuscripts, over the doorway of which (so he recorded) were inscribed the words "The Medicine of the Mind." In a storehouse of the mortuary temple of Rameses the Great (*c.* 1300–1234 B.C.) a collection of papyri was uncovered of a much earlier date than his reign.

The library of the Assyrian king Ashurbanipal (668–626 B.C.) contained almost all the great works of Sumerian and Babylonian literature, collected from temple libraries through his vast realm and stored for study and consultation in the royal palace at Nineveh, where the archaeologists Layard and Rassam found them in 1850.

The kings of Babylonia and Assyria delighted in unearthing ancient inscriptions from the foundations of palaces and temples, and compiling from these records historical and chronological data which are sometimes of use to modern historians. Collections of antiquities, made perhaps rather for their magico-religious value than for their intrinsic interest as *objets d'art*, have been found in the temples of ancient Mesopotamia. And a princess of Ur took the trouble to label each piece in her "museum" with the name of the place where it was discovered.

Famous Archaeologists

The Renaissance gave a great stimulus to antiquarianism. In England one or two heads tower over the rest in this respect. The *Britannia* of the historian William Camden (1551–1623) was a survey of the antiquities of the British Isles which superseded the earlier work of the "king's antiquary" John Leland (1506–52), and aroused great interest. Elias Ashmole (1617–92) was a collector of art and antiques ; his collections, though unscientific in the modern sense, form the basis of one of the greatest museums of modern times, the Ashmolean Museum at Oxford. William Stukeley (1687–1765) achieved a new standard of accuracy in his field surveys at Stonehenge and Avebury, and helped to found the London Society of Antiquaries in 1709.

ELIAS ASHMOLE (1617–92), English antiquary ; founder of the Ashmolean Museum, Oxford.

Sir Richard Colt Hoare (1758–1838) was a recorder of ancient monuments whose measurements are still used by archaeologists on account of their accuracy. During the 18th century the discovery, in Italy, of the buried cities Herculaneum and Pompeii gave impetus to classical archaeology. Napoleon's expedition to Egypt in 1798 aroused widespread interest in Egyptian antiquity. By the middle of the 19th century the Middle East was yielding up its treasures : Layard and Rassam were bringing to light the sculptures of the Assyrian palaces, and Mariette had founded the Cairo Museum.

AN ANCIENT MUSEUM. In this mortuary temple at Thebes, Rameses II had a library of Egyptian MSS. of much earlier date than his reign.

The early excavators, guided by enthusiasm rather than caution, and with little idea beyond the digging out of buried treasures, destroyed as much as they recovered and had little inkling of what archaeology can teach to the meticulous observer. But towards the end of the 19th century new methods were evolving. The vision and energy of Heinrich Schliemann (1822-90) were tempered by the scientific experience of his colleague Dörpfeld at Troy.

In England, General Pitt-Rivers (1827-06) laid down the principles of stratified excavation and the importance of recording and preserving all finds however humble or fragmentary. He was closely followed by Sir W. M. Flinders Petrie (1853 – 1942), whose work in Palestine and Egypt revolutionised archaeological method in the Near East and laid the foundations upon which all later archaeologists have built. The first chair of archaeology was held by him at University College, London, for the training of students in practical work.

SIR W. M. FLINDERS PETRIE (1853-1942). Egyptologist and pioneer of practical archaeology.

Among the great discoveries made by archaeology during the last hundred years have been wonderfully preserved finds in the peat-bogs of Jutland; the brilliant civilizations of Troy and Mycenae unearthed by Schliemann and his successors; the revelation of the hitherto undreamed-of empire of the Hittites in Anatolia, and of the strange beauty of Minoan Crete; the excavation of Babylon by Koldewey; the discovery of the intact tomb of the pharaoh Tutankhamen, with its fabulous riches; and the royal tombs of Ur in Babylonia; and, in 1940, of the painted caves of Lascaux in France.

Such landmarks in the fascinating history of archaeology have kindled the public imagination. But year by year the less spectacular work of the archaeologist goes on—the painstaking methodical recovery of the past. New techniques are added to his equipment from time to time. Air photography now aids him in planning his site. With the invention of the aqualung, underwater archaeology has become a reality.

The science of geochronology links the methods of the geologist and palaeobotanist with those of the archaeologist; and for dating the remains of human habitation, pollen analysis and, more recently, the examination of radio-active carbon particles, have been called into co-operation. Antiquarianism, the hobby of the dilettante, has given way to the science of archaeology.

LESSON 2

Scientific Methods of Excavation

There are four main subjects in which archaeologists should be trained: (1) excavation; (2) study in museums of objects already excavated; (3) study of published material; (4) study of living races.

To many people archaeology means only the excavating of ancient cities and tombs, and, above all, the finding of buried treasure. Excavation, however, is not an end in itself: it is only the preliminary to the serious study, merely the means of obtaining the material on which the real study is based. It must be scientifically done.

First the site must be chosen. Accident may bring a site to light, as when the plough unearths a quantity of flint implements or when a

landslip shows stratified walls and habitations. But no archaeologist worthy of the name would depend on chance. Nowadays an aerial survey is usually made of areas which are judged likely to be archaeologically rich. By this means was discovered the Iron Age site at Woodbury, on the hill south of Salisbury, which in 1938 was selected by the Prehistoric Society as a training-ground for young archaeologists.

If an aerial survey is impossible, there are other means of learning from the surface of the ground what lies beneath. There are indications which even the untrained eye can see, as, for instance, circles of megalithic stones, burial barrows, ruined walls, broken columns, or mounds covered with potsherds. When there is nothing on the surface, and an air photograph is not obtainable, the method must necessarily be different.

Crop plants with deep roots may avoid the lines of buried walls and thus by their absence reveal the ground plan of a building beneath the soil. In Egypt the rare rain, which falls on the narrow strip of desert bordering the cultivated land of the Nile valley, is a help to the excavator. Under the hot sun the moisture evaporates more quickly from the sand which covers buried walls than from the deeper layers. Half an hour's sun after rain may show the whole plan of a building of which not a trace can be seen on the surface in dry weather.

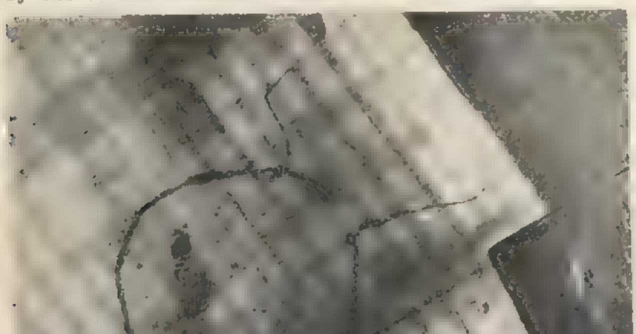
The remains of the palace of Knossos were laid bare because the tradition as to the site had been preserved by some of the classical authors, and when the topographical indications given by those authors were identified the find was made. Folk memory preserved by name the sites of the cities Nineveh and Babylon. Others, like Jericho near the crossing of the Jordan, revealed their identity by their geographical position.

Having chosen his site, the excavator decides on the method of digging. He will have a general idea of what he expects to find. In a town-site it will be houses and probably a temple, perhaps a palace ; in a tomb he may find bodies, or at least bones, and a certain number of objects. He must furnish himself with the necessary gear to preserve and transport what he expects to find, and engage workmen to excavate and, often, expert assistants to aid him. The site must first be surveyed and a datum fixed by which all levels can be measured ; the plan should be divided into squares, so that the position of everything found can be entered at once.

Excavating a Town-site

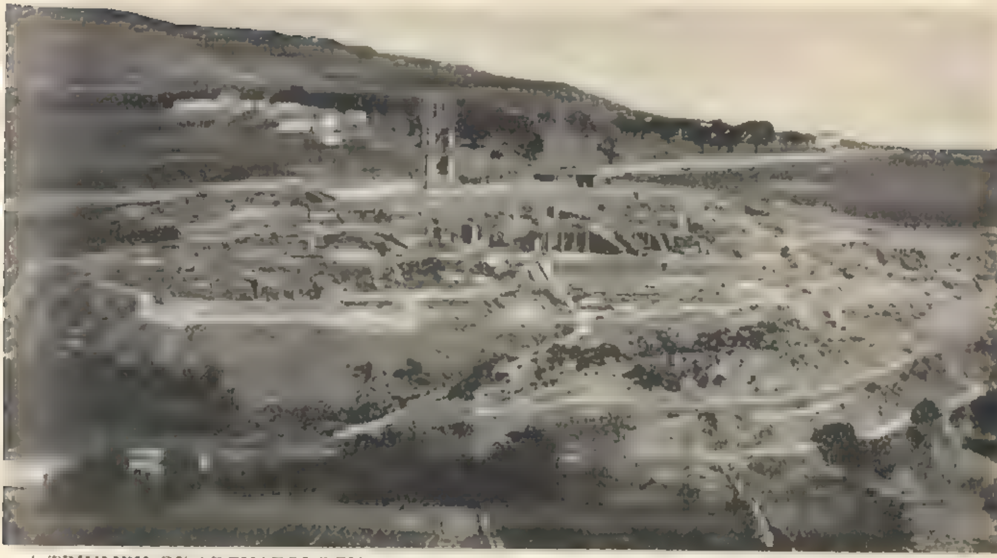
In excavating a town-site the digger must decide exactly what he wants to do. Then he will have to remove tons of earth or sand, and he must arrange that the dump-heaps are conveniently near the dig and yet not covering any part of it. If he chooses the position of the dump-heaps wisely, many of his difficulties are solved.

One method of excavating a town-site is to clear the whole surface, descending foot by foot till bed-rock or virgin soil is reached. The fullest records must be kept as to levels and objects found, besides plans of every building and wall, and cross-sections relating one level to another, so that the place could be reconstructed by any future archaeologist. But by this method of excavation all evidence on the spot is destroyed. It is also the costliest of all methods, for the digger must be prepared to spend fully half his time and half his money on an elaborate and careful publication.



ARCHAEOLOGICAL TRAINING-GROUND. At Woodbury, on the hill south of Salisbury, in Wiltshire, a site for training amateur archaeologists was acquired in 1938 by the Prehistoric Society. Aerial survey disclosed rings of dark colour in the corn ; excavation proved the site to be an Early Iron Age settlement.
Photo: The Times

On a large site it is better to attempt the full excavation of only a portion, working within strict limits and leaving the rest of the area as a "control" for those who come after. In excavating tombs, whether single graves or a whole cemetery, the survey must be made first as for a town-site. Each burial, as it comes to light, must be carefully cleared so that the position of each object can be planned and photographed. As objects are unearthed they must be so carefully marked that their provenance cannot be lost ; if broken, each piece must be marked. When the objects arrive in the camp they should be photographed and copied in facsimile by hand.



A TRIUMPH OF ARCHAEOLOGY. Long-buried Knossos, in Crete, as resurrected by Sir Arthur Evans (1851-1941). These remarkable remains of the once great Minoan capital lie on the slope of a hill about 3½ miles from Candia. The skeleton tower in the middle distance was built by the archaeologists so that the excavations could be seen as a whole.

The excavator's life is not always a happy one. In the Near East the language of the workmen is Arabic and he must learn to speak this fluently. He has to be up early in the mornings so as to begin work with the men. He has to oversee them continuously and assess their work if they work by the piece. He must assess the rewards for objects found and see that the men understand his system of payment, for it is essential to gain their confidence if he is to get the best out of them.

His evenings and spare time will be spent in drawing, photographing, planning, and all the miscellaneous jobs that occur in camp. He must be skilled in the repair and preservation of fragile objects. He will have to work hard and with great accuracy, for in archaeological excavation evidence once lost is lost for ever and can never be recovered. He must recognize the imperative duty of prompt publication of his results and realize that to hold back results for more than three years constitutes an archaeological crime. If his work is done well and scientifically he has his reward in having added something to the world's knowledge of the origin and progress of civilization.

LESSON 3

The Training of an Archaeologist

THOUGH excavation is the foundation of archaeology, the training of the archaeologist should begin with the study of the material already excavated by others. This is most conveniently done in a museum, where the student is generally in a position to handle the objects.

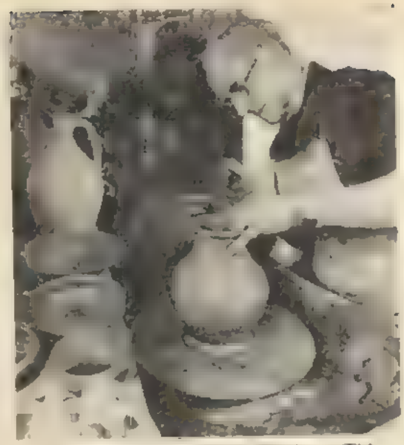
The first thing that every archaeologist must know and understand is pottery. Archaeologically speaking, a knowledge of pottery is essential. Pottery is the commonest of all objects and is virtually indestructible. Fire or water will not destroy it; and if a pot is broken the shards can be put together and the form identified. Pottery was always so common that even the poorest person would have at least one vessel buried with him. It is bulky, heavy, and fragile, and therefore travels badly, so that it is rarely carried any distance from the place of making. The ware of each country is distinctive, and the sudden appearance of a new form, decoration, or technique may indicate the advent of newcomers from elsewhere.

Art of the Potter

The ancient world knew three methods of making a pot—pinching, coiling, and the wheel. The modern potter has introduced a fourth-casting. Pinching is the most primitive of all the methods; the potter took a lump of

clay, hollowed it with his hands, smoothed the outside, and the pot was ready.

Coiling was done by rolling the clay into long "sausages," and then laying these round, one above the other, in the desired shape. The hollows between the coils were then filled by pressing the clay of each coil upwards and downwards till the hollow was hardly visible. This is a more elaborate method than pinching. A coiled pot can be distinguished from a pinched pot by the fact that there are slight horizontal ridges where the coils were set in place; these, when regular, make it difficult to be certain in many cases whether a vessel is made by coiling or on the wheel.



THE MODERN POTTER. This modern practitioner of an ancient art is "throwing a pot" on an improved form of wheel (a flat table which rotates on a pivot).

The potter's wheel is not only of great importance in itself but is remarkable as one of the earliest mechanical appliances. It is a flat table which rotates on a pivot. A lump of clay is placed in the middle, and as the table spins, the vase springs up under the potter's hands. The earliest wheel was merely a small board or thin slab of stone, pivoted probably on another stone and turned by an occasional push of the potter's hand or foot. Wheel-made pottery dating from 3000 B.C. is found in Egypt.

In making pottery, whether by hand or wheel, the vessel after being shaped was allowed to dry thoroughly. It was then fired in a kiln, and the clay when thoroughly baked became pottery. The skill of the potter is shown not only in shaping the vessel but also in his method of firing it.

There are three ways of decorating pottery—slip, paint, and incisions. Slip is perhaps the earliest. Finely ground clay is mixed with water, and the thick liquid is applied as a wash over the whole surface; very thick liquid can be laid on in a raised design. Paint can also be used to cover the whole surface or to make decorative patterns. Both slip and paint are applied when the vessel is dry and before firing.

Incised designs are made while the clay is still wet. After firing, the hollows were often filled with gypsum so that the pattern was in white on the darker background of the pottery. Glazed earthenware is not met with till comparatively recent times, but burnished pottery was not uncommon even in the prehistoric periods of Egypt. It is found in the Neolithic period of Malta and in the Bronze Age throughout Europe.

In studying pottery the points to be noted are material, shape, method of making, method of firing, colour, and decoration. Each point should serve as a guide and should tell its own story as to the date and place of origin.

Aids to Identification

A knowledge of other basic techniques will stand the archaeologist in good stead. He should be familiar with the simpler processes used by the metallurgist, and with the methods of quarrying and stone-working, weaving, and agriculture, practised by ancient peoples. A knowledge of chemistry will help him in the preservation of fragile objects.

PRE-DYNASTIC POTS OF EGYPT. The earliest Egyptian potters achieved symmetry without the use of the wheel. The pot on the left is crudely painted to imitate marbled stone. That on the right is painted with animal designs.
British Museum

A smattering of botany and geology will aid him in recognizing specimens found in an archaeological context, and anatomical knowledge will enable him to identify any human or animal remains he may find. He should be familiar with excavated material from other sites in his area, and with published finds from other places which may throw light on his own work.

If he is excavating a site of an historical age he must be familiar with its written history and literature; if he expects to find tablets, papyri, coins, or inscriptions written in an ancient language, he should have with him an expert who can interpret them.

An important item in his training is the study of the living branches

of mankind: archaeology has been well described as the "anthropology of the past." The modern anthropologist, by his work on the more backward peoples of the present day, has thrown a flood of light on the ancient civilizations in every part of the world. There are peoples still in existence to whom the use of metal is unknown, and whose tools and weapons are of stone. These peoples live, in many respects, the life led by Neolithic man.

Among less primitive peoples can be traced the use of mechanism applied to the making of various objects. A pointed stick rotated between the hands could be used to pierce small, smoothed pieces of dried clay to make beads; when fed with emery or sharp sand, the pointed stick would pierce stone.

HANDIWORK OF PREHISTORIC POTTERS. These vessels are typical primitive Greek pottery, and are assigned to the Stone Age or early Bronze Age.
British Museum

The stick could be turned more rapidly if a piece of string were twisted round it and the ends of the string each attached to a piece of wood, which was pierced in the middle to allow the stick to revolve in the hole. The movement of the stick up and down caused the drill to move much faster than if rotated by the hands alone. This device is called the pump drill. A similar device, the bow drill, involves horizontal movement of the wood.

With a copper point or copper tube attached to the cutting end of the drill, the capacity for working stone would be immensely increased. The tubular drill was commonly used by the sculptors of ancient Egypt for blocking out their statues; with its aid they were able to carve stones regarded as too hard by the modern sculptor.

Though the main work of the archaeologist is with the physical remains of ancient civilizations, there are other aspects of those civilizations which can also be studied in the light of anthropology. These include religion.

Systematised religions, such as those of Egypt, Babylonia, India, and Greece, had organized priesthoods, temples, and temple furniture. The more organized the priesthood, the greater is the number of material objects connected with the cult. To understand the use and the meaning of many of such objects it is necessary to study some of the religions which are still in existence.

A CARVED STONE MACE HEAD from Hierakonpolis. Far too heavy for anyone to wield, the mace was probably a ceremonial weapon. The figure with the hoe symbolises the beginning of the agricultural year.
Ashmolean Museum, Oxford

In Egyptian tombs have been found models of food and drink, and of the bakers, butchers, and brewers who were to provide these necessities in the future life; models, too, of the servants who were to minister to the dead man, and of his chariot and his ship. These symbols served instead of the actual persons and things; it was held that they would become what they represented.

In many instances social conditions can be inferred from burial customs. If the tomb of a woman is as elaborate, and the offerings buried with her are as rich, as those of a man, it can be concluded that the position of women in that community was high. Often, also, it is possible to learn from the objects in a grave what the dead person's trade was, and whether certain trades were regarded as belonging specifically to one sex. A change of burial custom, if it be introduced suddenly, may indicate the influx of a new population, for men have always maintained a considerable conservatism towards things religious and funerary.

If the archaeologist's period is documented by written records, it is then, of course, a much easier task to breathe life into the material he uncovers.

LESSON 4

When the Ice Sheet Covered Europe

The earliest phase of archaeology begins with geology. The great geological divisions are named Primary, Secondary, Tertiary, and Quaternary ; the last of these is the present one. With the first two divisions archaeology has nothing to do. At the end of the Tertiary, mammals first made their appearance, and it is here that the beginnings of man and his works must be sought.

The Tertiary Period has several subdivisions, of which only the last, the Pliocene, need concern us. The Quaternary Period has two subdivisions, the Pleistocene and the Holocene.

The first consideration in treating of these remote epochs is the condition of the land surfaces and the climate. The surface of the earth in Pliocene and Pleistocene times was altogether different from what it is now. The Mediterranean consisted of two comparatively small lakes entirely shut off from the Atlantic and from each other. African and European animals passed freely from one continent to the other across the land bridges.

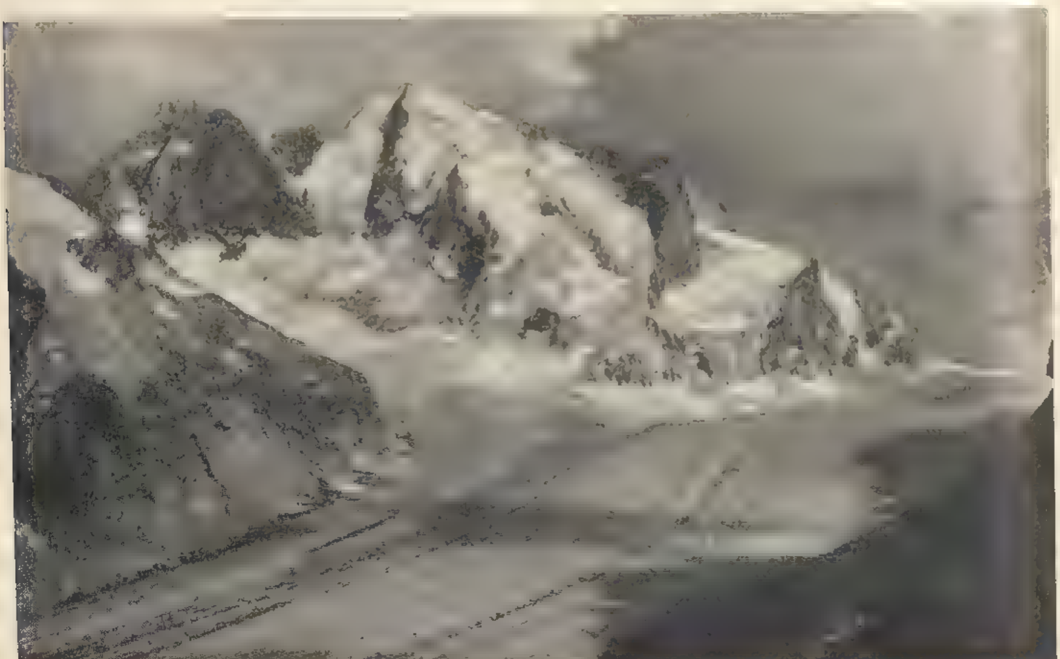
The British Isles were then part of the main land mass now called Europe, and that continent probably extended far into the Atlantic Ocean. Therefore in the Pleistocene period we are dealing with land surfaces in which the present barriers to intercourse did not exist. This is an important fact to remember when considering the spread of Palaeolithic culture.

Four Great Glaciations

At the end of the Pliocene period the climate of Europe appears to have been temperate or even warm, for animals which occur now only in tropical and sub-tropical climates lived freely as far north as Germany. The climate then changed and gradually became exceedingly cold, with a spread of polar ice over the whole of northern Europe, forming a cap like that at the polar regions of the present day. The ice forced itself as glaciers into the warmer parts, cutting out valleys in the underlying rock

The extent of the glaciers is shown by the "erratic" boulders which they picked up and carried southwards for many miles, and also by the great terminal moraines of earth and gravel deposited when forward movement ceased and they began to melt.

The polar ice advanced and retreated more than once ; in fact, geologists recognize four



EUROPE IN THE GRIP OF ICE. This photograph of the Forno glacier near St. Moritz, in the Engadine, Switzerland, shows the kind of bleakly majestic scenery that predominated throughout Europe during the long glacial periods of geological history.

great glaciations or ice ages. These are called by the names of the four Alpine valleys in which the evidence for their existence was first established.

The names were purposely selected from many others as they occur in order alphabetically, and there is thus no difficulty in remembering their relative chronological position. These names are: (1) Günz, (2) Mindel, (3) Riss, and (4) Würm. The warm periods which occurred between the four glaciations are known as Günz-Mindel, Mindel-Riss, and Riss-Würm.

The causes of these alterations in climate are not fully understood, but there seems to be some connexion with alterations in the amount of radiation from the sun.

Climatic Influence

The archaeologist who studies the earliest remains of man must understand the evidence of the various deposits which are the result of glaciation. These immense masses of snow and ice necessarily had an effect on the climate, which would be extremely cold when within range of the glaciers and excessively rainy in the tropical and sub-tropical regions. The climate would affect the vegetation, and the vegetation would in its turn affect the types of animal which could live on it and successfully withstand the rigours of an Arctic environment.

Attempts are being made by many archaeologists to correlate the pluvial or rainy periods of sub-tropical Africa with the glacial periods of Europe. This is one of the problems which an archaeologist is called on to solve, and perhaps one of the most fascinating.

It is difficulties such as this which give life and action to the subject, for a subject which is already thoroughly known and completely cut-and-dried is seldom inspiring, whereas archaeology is still in its infancy.

The actual appearance of man on the earth must be sought in these remote epochs of the world's history. In this quest there is nothing to guide the student but the archaeological method, assisted by geology and zoology. The position of the remains in, under, or above certain geological strata and the animal bones which accompany those remains must be thoroughly understood before any opinion can be formed as to the period to which the remains can belong.

LESSON 5

First Traces of Primitive Man

It must be remembered when dealing with the ages before history that man's knowledge is still fragmentary, and that the theories which are advanced to explain the various phenomena are not in any sense like "the law of the Medes and Persians which altereth not." On the contrary, though a working hypothesis is necessary, that hypothesis must be capable of modification in the light of such new evidence as may arise.

It is as yet impossible to fix with any accuracy the period at which man appeared on the earth. Actual human remains, such as bones and skulls, are not found till well into the quaternary era, but flints which some authorities believe to show artificial chipping have been discovered in the strata formed at the junction of the tertiary and quaternary epochs. These early stone implements are called eoliths (Greek *eos*, dawn; *lithos*, stone).

PREHISTORIC, BUT IN USE TO-DAY. Most backward of the world's peoples are the Australian aborigines, one of whom is seen here chipping flints much as did his ancestor of 20,000 or so years ago.
Spencer & Gillen, "Across Australia," Macmillan & Co., Ltd

The most interesting eoliths are the rostro-carinates, or keel-backed implements. The shape is characteristic, and it is not found at any other period. It is like an upturned boat with a keel running the whole length of the back, a downward-curving beak at the front, and with the chipping so arranged as to give a firm grip to the thumb and fingers of the right hand.

The making of tools and the discovery of fire mark two of the greatest steps in the evolution of mankind. The making of tools differentiates man from the animals, and though all the types of eolith are extremely primitive they are a definite advance on the unworked stones which must have been man's earliest tools.

As regards fire, the evidence of archaeology proves that it was one of the earliest of all the epoch-making inventions which have carried

man so far from his primitive beginnings and have differentiated him from his animal ancestors. It is not known how the making of fire first came about. Dry branches blown violently against each other by the wind will, as the result of continued friction, give rise to fire. And it may be that flames were "borrowed" from the chance-ignited timber to set fire to piles of brushwood to scare away enemies. Fire deliberately made may have resulted from the rapid rotation of a dry stick on a piece of tinder-dry wood, the rotation being maintained until the heated wood-dust thus created could be blown into flame. Or it may have come from the brisk clashing together of two flints or other stones to induce sparks. What *is* certain is that a fire so laboriously created would be kept going at all cost.

It is extremely doubtful, however, whether the creature who at that remote era invented stone tools and understood the use of fire could be called man, or whether he was some anthropoid animal evolving towards the human.

Types of Flint Tool

After the Eolithic period comes the Palaeolithic (Old Stone) Age, divided into lower, middle, and upper. All these are within the quaternary era ; they begin in the Günz-Mindel interglacial period and appear to end when the ice was retreating after the last great glaciation.

The types of flint tools which are characteristic of the Palaeolithic age are named after the places where they were first found :

Lower	**Abbevillean**	Abbeville (Somme)
	Clactonian	Clacton-on-Sea (Essex)
	Acheulean	St. Acheul (Somme)
	Levalloisian	Levallois-Perret (suburb of Paris)
Middle	**Mousterian**	Cave of Le Moustier (Dordogne)
Upper	**Aurignacian**	Aurignac (Haute Garonne)
	Gravettian	La Gravette (Dordogne)
	Solutrean	Le Solutré (Rhône)
	Magdalenian	Cave of La Madeleine (Dordogne)

At one time these names were considered to represent separate epochs rigidly divided from one another ; now it is recognized that they represent different traditions of tool-making and adaptations to different types of environment. It also seems that the people who made these tools did not follow one another in strict succession, but that in the Lower and Middle Palaeolithic at any rate two or three groups were living in Europe at the same time.

Flint-working Methods

The type of flint working associated with Abbevillean and Acheulean is a core industry. The core is worked on both sides ; the flint worker took a lump of flint from which he knocked off chips, large and small, until it was reduced to the desired shape and was thin

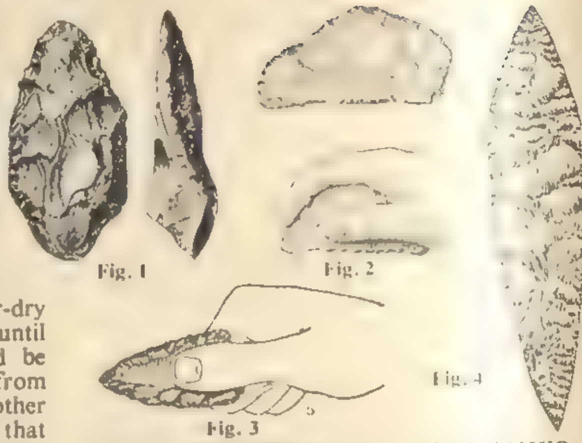


Fig. 1 Fig. 2 Fig. 3 Fig. 4

FLINT IMPLEMENTS OF PALAEOLITHIC MAN. Fig. 1, Acheulean type ; Fig. 2, Acheulean knife, showing method of use ; Fig. 3, Mousterian "point" ; Fig. 4, beautifully fashioned Solutrean type (Upper Palaeolithic).

enough for use. After this the edge was sharpened by bevelling it down with tiny chips.

The Clactonian and Levalloisian flake implement is worked on one side only ; the flint worker began as for a core implement, working the surface, then he struck off a flake which was the whole length of the core. A flake implement, therefore, has one worked side and one which shows the *bulb of percussion*, where the stone first fractured under the blow which struck the flake off ; the rest of the flake on that side shows only the natural fracture of the flint.

The characteristic Abbevillean tool is somewhat almond-shaped, and large, thick flakes have been removed from both faces, making an irregular edge all round. The typical tools are called *coups-de-poing* or hand axes, although their use is still a matter of conjecture.

Importance of Detail

The Acheulean tools appear to be a development of the Abbevillean ; they are thinner and flatter, the flakes were taken off in smaller chips, and particular attention was paid to keeping the edges straight. Flakes are also found among Acheulean implements and were certainly used by their makers, probably as knives. They were, in a sense, accidental, the maker of the implement merely striking them off in order to reduce the size of the flint nodule, whereas the Clactonian or Levalloisian flint worker required the flakes only. (*See also* Lesson 7.)

Students of these early epochs must be on the *qui vive* to note every detail connected with every object found. It is only by careful attention to the strata in which the implements are found, and to the position of the implements, and of the flakes, burnt stones, and bones found with them, that any accurate knowledge of our earliest ancestors can be attained.

LESSON 6

Earliest Types of the Human Race

THE origin of man is one of the most absorbingly interesting subjects of study. The area of investigation comprises the whole of the Old World, and it is always possible for a student to make epoch-making discoveries.

Human remains of the Pleistocene era are rare, for earth movements and climatic changes have destroyed them. Stone implements fashioned by the hand of man can survive long immersion in water or being dashed about in a torrent ; sudden and heavy falls of earth and rock will not break them or grind them to powder. But human bones are much more fragile than stone, and the wonder is that any have survived.

Prehistorians and anatomists are agreed that no human bones of the Tertiary period have survived ; the earliest appear to belong to the Lower Pleistocene. All authorities are of opinion that the earliest type of man was nearer to the ape than our present more developed species, but it is not believed that men are descended from apes. It seems rather that men and apes represent different branches of the same family and that both developed from an ancestral stock more primitive than either.

The apes which are nearest to man are the chimpanzee and the gorilla, but there is a marked difference in the skulls and the teeth of apes and men. The human skull has a much larger brain capacity than the ape's, so much so that the size of the brain cavity is one of man's distinguishing marks.

Pithecanthropus

The most primitive of all the early skulls discovered is one which was found in Java in 1891. The stratum in which it lay belongs to the earliest Quaternary period, almost to the Tertiary. Only the upper part of the skull was found, and with it were a thigh bone and three teeth.

The creature had several ape-like characters, such as the heavy bony ridges above the brows, the crest-like ridge up the back of the skull, and the size and shape of the teeth ; it had also some human characters. The brain cavity, though smaller than the human, was larger than in any known ape, whether fossil or living.

The straightness of the thigh bone showed that the animal stood erect in a position that apes cannot assume, and the anatomical evidence showed that in all probability the creature was capable of communicating with its fellows in a more complex manner than do the animals : the capacity for speech had already begun. This Java find was named *Pithecanthropus*, to indicate that it combined the characters of ape and man.

In 1937 the discovery in central Java of two new skulls of *Pithecanthropus*—of a female and of an infant—established that the so-called ape-man was a very primitive human being. In a cave near Peking have been found skulls of another type (*Sinanthropus*) related to *Pithecanthropus* ; these cave-dwellers built fires, and made simple flake tools.

Other early human remains are a jaw found at Mauer, near Heidelberg ; though not of the type of modern man it is certainly human. It can be dated to the early Pleistocene period by the animal bones which occur in the same stratum, and is therefore one of the earliest known specimens of the human race.

PREHISTORIC BURIAL. These skeletons of an old woman (left) and a young man, found in the Grimaldi caves at Mentone, France, suggest careful and reverent sepulture—if not an actual belief in a life beyond the grave—for the man's head was encircled with a chaplet of shells and there were bracelets on the woman's arm.

From Dr. R. Verneau, "Les Grottes de Grimaldi"

"Piltdown Man"

In 1954 a group of experts proved conclusively that "Piltdown man," claimed to be of Lower Pleistocene date and to combine a human skull with an ape-like jaw, was a fraud. The remains were discovered by Charles Dawson (died 1916) and others, from 1912 onwards, and the fragments gave rise to considerable discussion. They presented archaeologists with baffling problems. The fragment of skull appears to be a genuine fossil, but of much later (Upper Pleistocene) date. It was "planted" in the shallow gravel at Piltdown, Sussex, with the jaw of a

recent ape (stained to give it an ancient appearance), bones of Lower Pleistocene fauna brought from elsewhere, and flint implements (also stained).

In the Middle Pleistocene, *Homo sapiens* first appears, represented by part of a skull found in 1936 at Swanscombe, in Kent, in a gravel deposit belonging to the Mindel-Riss interglacial. Acheulean hand-axes and Clactonian flakes were also found in the gravel, but it is not certain which type of implement was made by Swanscombe man.

Much later, in the Riss-Würm interglacial, a type less similar to modern man appeared—*Homo neanderthalensis*, called after the place where in 1856 the first skull was found: Neanderthal, near Bonn, on the Rhine. It belongs essentially to the Mousterian culture, and was spread chiefly over south-west Europe, though it is found also in Germany and Croatia. The best early example of this type was found at La Chapelle-aux-Saints (1908), where much of the skeleton was also preserved.

This creature when living must have stood about five foot three or four inches in height, though it is clear from the shape of the skull that he did not stand quite erect, but carried the head well forward, if not so markedly as an ape. Though the heavy brow ridges and the strong, projecting jaws are reminiscent of the ape, the shapes of the jaw and the teeth are human.

NEANDERTHAL MAN. Reconstruction based on the well-preserved skull discovered in 1939 by Dr. A. C. Blanc in a cave at San Felice Circeo, Italy.

In 1939 the best-preserved Neanderthal skull hitherto found was discovered in the interior of a cave at San Felice Circeo, about half-way between Rome and Naples. The cave, also newly discovered, had evidently become blocked in Mousterian times, and its contents had remained untouched for at least 100,000 years. The skull, found lying among stones which appeared to have been disposed in a circle, is nearly complete. Under it were antlers and flints, some showing intentional flaking.

Probably Neanderthal man is not a direct ancestor of the present human race, but he is undoubtedly man, and on the road to civilization. He lived in caves, possibly because of the coldness of the climate, but this may also show the beginnings of settlement. He was a hunter, killing even large animals for food, and he was well acquainted with the use of fire, and he probably cooked his food.

An interesting point in regard to Neanderthal man is that here, it would seem, is the first indication of religion. The skeleton from La Chapelle-aux-Saints had received ceremonial burial, for with it were stone implements of Mousterian types, as well as bones of food animals. This suggests that already man had evolved the idea of a hereafter, and the tribesmen had sent their great chief to the next world well equipped. The stones and other objects placed round the mutilated Circeo skull suggest also that a cult involving the collection of heads or skulls was practised.

The time which elapsed between the first recorded appearance of tool-making creatures in Europe (if the eoliths are really tools) and the advent of Neanderthal man with his relatively developed culture has been estimated at about 450,000 years.

THE ICE AGE (PLEISTOCENE) IN WESTERN EUROPE

Period	*Climate*	*Typical fauna*	*Types of tools*	*Types of man*
Upper Pleistocene	End and middle Würm: cold	Reindeer Wild horse Mammoth	Magdalenian Solutrean Gravettian Aurignacian	Homo sapiens
	Beginning of Würm	Cave bear	Mousterian Levalloisian	Homo neanderthalensis
	Riss-Würm: warm	Elephant	Mousterian Levalloisian Acheulean	
Middle Pleistocene	Riss: cold	Rhinoceros	Levalloisian Acheulean	
	Mindel-Riss; warm	Elephant Rhinoceros	Acheulean Clactonian	Swanscombe man
Lower Pleistocene	Mindel: cold	Reindeer	No tools yet found	Homo heidelbergensis
	Günz-Mindel: warm	Elephant Cave lion Deer	Abbevillean	
	Günz cold	Beaver Rhinoceros	Eoliths	

LESSON 7

Life in the Old Stone Age

THE great advances in culture which mark the final phases of the Palaeolithic are due to the appearance in Europe of new races of *Homo sapiens*, the forerunners of modern man. An earlier race of *Homo sapiens* seems to have interbred with Neanderthal man in Palestine, but in the Upper Palaeolithic, Neanderthal man as such vanished from the European scene.

It is not known whether he was exterminated by the newcomers or simply failed to adapt his culture and equipment to meet changing conditions and more intense competition. For the last 100,000 years the history of the world has been the history of *Homo sapiens*.

Cave Homes

The Upper Palaeolithic period in Western Europe is divided into several parts, each represented by a different culture, of which the most important are the Aurignacian, Gravettian, Solutrean, and Magdalenian. But only in France do they follow each other in this order ; elsewhere the sequence differs and some cultures are not represented at all.

The people of the Upper Palaeolithic age lived in caves and rock shelters, where their tools and weapons and the remains of their food have been found. Usually their homes were close to streams, partly because the inhabitants were fishers, partly because they were hunters and the game would frequent a stream, and partly because they required water for their own needs. They were also traders, exchanging with other communities decorated objects and other luxury goods. Pierced shells were made up into necklaces and girdles and may have served as amulets as well as trinkets.

Their tools were far more varied than those of the earlier periods, showing that their requirements had increased as their civilization had advanced. The Upper Palaeolithic flint industries are called blade-and-burin industries, because their makers were chiefly interested in producing small blade-like tools and burins, or chisel-ended implements, specially adapted for working the bone, antler, and ivory which were now for the first time freely used.

It is deduced from their shape that many of the tools were hafted, probably with wood, though this has not survived. The weapons of the chase were smaller and lighter, showing that the large fierce animals of the Ice Age had disappeared with the approach of a more genial climate.

Upper Palaeolithic men made bows and sharp flint arrows with hafts of reed or wood, weapons suitable for the pursuit of swiftly moving animals, such as the herds of wild horses and reindeer which by this time were roaming the countryside.

Aurignacian Period

The Aurignacian period shows an increase in the number of forms of tools over the Mousterian. Bone pins were used, probably for fastening the skins worn as garments, and there are various implements which may have been used in dressing the skins, as well as in woodworking.

Gravettian Period

The Gravettian was the most widespread of the Upper Palaeolithic cultures. It is found nearly everywhere in Europe ; it flourished especially in Italy and in Russia and in those countries survived for a very long time. The photograph on p. 1131 shows the burial practices of the Italian form of the Gravettian culture.

HAUNT OF PALAEOLITHIC MEN. The cave whose entrance is shown here is at Font-de-Gaume in the Dordogne, Southern France. It contains one of the most famous picture galleries of prehistoric art. The studies of animals—elephants, reindeer, bison, and wolves—are remarkably realistic and brilliantly executed.

The typical tools and weapons are small flint blades, one edge left sharp for cutting purposes, the other blunted so that it could be fitted into a haft, and tanged arrowheads. In Russia the Gravettians hunted mammoths and used the bones and tusks to weight the roofs of hide tents which were set up over sunken floors. Here stylised figures, mostly of women, were carved in ivory.

Solutrean Civilization

The Solutrean civilization does not occur in every site, even in France, and the people seem to have made only a brief appearance. Their traditions of flint-working are quite different from those of all other Upper Palaeolithic groups, so that they seem to be intruders. Even the direction from which they came remains a subject of conjecture.

Their flint working is very fine ; the type of work is called pressure flaking. It was done by breaking off the flake from the nodule by pressing, instead of by a blow, a method which leaves a series of transverse ripples across the surface of the cleavage. The characteristic forms are called laurel leaves and willow leaves on account of their shape. So exquisitely fashioned are some that they are thought to have had a ceremonial use. These people also made barbed-and-tanged arrowheads exactly like those of the Late Neolithic of Western Europe many thousands of years later.

Magdalenian

The Magdalenian culture is as important for the Upper Palaeolithic of France as is the Mousterian for the Lower and Middle. There are few distinctive flint tools, and flint seems to have been used chiefly for working bone and reindeer antlers. Weapons and other implements were now made almost exclusively of bone, antler, or ivory. The most important weapons were spears and harpoons.

Eyed needle in reindeer horn, and bodkins of ivory and bone, used by Solutrean housewives.
Photo, Prof. D. K. Absolon

Harpoons show a definite sequence of development, beginning with a shaft with a single notch on one side and gradually changing into a more formidable weapon with barbs down both sides of the shaft. These objects could be used for fishing as well as for hunting. When the head of the harpoon was embedded in the body of a fish or a running animal, the shaft to which it was tied would come loose and hinder the efforts of the prey to escape.

Among the interesting developments of bone implements are needles, long and short, sharpened to a point at one end and pierced at the other. As the only use for a needle is for sewing, it is obvious that the Magdalenians were able to sew ; but as there is no evidence that they could spin or weave, the sewing must have been on leather, with sinews or fine strips of leather as the thread. Many of the small flint tools at this time may have been designed for the dressing and cutting of the leather and preparation of the sewing materials.

It is as yet uncertain whether the people of the Upper Palaeolithic had learned the use of pottery, for no pottery which can be dated to this period has been found. The meat must, therefore, have been roasted on some form of spit or stewed in leather containers.

LESSON 8

Prehistoric Paintings and Sculptures

THE most surprising survival of the Upper Palaeolithic Age, and one which most appeals to the modern mind, is its art. Even as far back as the Aurignacian period artists had begun to record their impressions in the form of sculpture, both in the round and in relief. Occasionally also they made line drawings with a sharp tool on bone, horn, or stone ; these artistic efforts become more frequent among the Magdalenians.

Palaeolithic Women

The Gravettian confined his efforts to the representation of the human being. The greater number of his statuettes are of women, and are carved in mammoth ivory, in limestone, and in steatite. This type of female beauty does not appeal to modern taste, but it must have been admired for many centuries in the ancient world, for statuettes of the same type are found in Egypt in the prehistoric (Amratian) period and in Malta in the Neolithic era, and it survives among the Bushman women of Africa.

The chief characteristic is a marked fatness of the thighs and buttocks. Many of the Palaeolithic women were depicted as immensely fat in the body as well, though the arms and lower part of the legs were slender, even according to modern standards. The faces of the statuettes and reliefs are never fully delineated and are often left blank. Traces of colour show that the figures were originally painted.

The engravings are often of human beings clothed in animal skins, with the animal's head

covering the face like a mask. Such figures occur in several sites of the Upper Palaeolithic era, even in Derbyshire. The motif is highly developed among the Magdalenians.

"Chattel Art"

An artist is always more or less limited by his material. The Gravettian worked in ivory, and in that almost ideal substance was able to produce really fine work in the round. The Magdalenian was not so fortunate, for the mammoth had deserted the warmer country and there was no more ivory to be had.

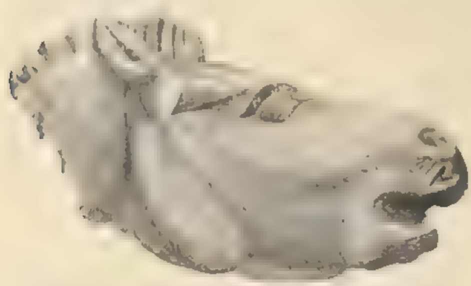
The artist was restricted to bone and horn, neither of which is well adapted for carving in the round, and he developed a technique called by the French *contour découpé*. In this the figures, though carved on both sides of the bone or horn, are really flat, the outlines being incised, but the roundness is that of the object and is not effected by modelling.

These decorated objects are numerous, but the uses to which they were put are uncertain. Occasionally some can be identified by a similarity with objects now in use among savage tribes. Among these are the spear-throwers that are used to lengthen the throw of the weapon.

There are also the handles of daggers or knives, but the long *bâtons de commandement*, which appear to have been of the utmost importance to the people who used them, are still unidentified. All these small carved objects are classed under the head of *art mobilier*, or chattel art, as they belong to the chattels of these primitive folk.

Splendid Art of the Caves

The human statuettes of Gravettian times were certainly painted, but there the colour was subsidiary to the modelling ; on a rock surface the artist discovered that paint could be used as a medium in itself. It is possible that the new method took its rise in the custom of



REALISM IN PALAEOLITHIC ART. To carve so virile and individual a horse's head as this from reindeer antler, the Palaeolithic artist must have had real genius, together with a very intelligently developed technique. The carving was found in the Magdalenian deposits at Mas d'Azil. France.

MAGDALENIAN CARVING. Left, chamois carved on spear-throwers of reindeer horn. Centre, the notch to hold the spear-butt is well defined in this broken thrower. Right, skilfully carved grouse, partly restored.

From Mas d'Azil, after Breuil and Piette

smearing the hand with colour and then making an impress on a smooth part of the rock ; or by placing the hand on a piece of rock previously prepared and then powdering the paint over it, leaving the form of the hand in the natural colour of the rock imprinted against a background of colour.

Sketches begin very tentatively, gradually becoming free and naturalistic. And there is a sudden advance when the painted outline was softened by rubbing to produce the effect of shading. Then came the full tide of Palaeolithic painting, the splendid art of the caves, which was never equalled by any ancient nation. The greater number of the subjects are animals. Not only is the drawing brilliant, but the pigments are so laid on as to produce the effect of light and shade as well as the natural colouring of the animal represented.

Rock-Artist at Work

The method was to cut on the rock-surface parts of the outline of the figure. The rock within those lines was then scraped till quite smooth, and the pigments were laid on, either as a paste or as a liquid, until the whole surface was covered. The artist proceeded, with flint or bone scrapers, or perhaps with a piece of hide from which the hair had not been removed, to wash and scrape away the paint until he had

obtained the result he desired; then with a little black paint he added the horns, hoofs, and other details.

The colours used were mineral products. They are forms of iron ore, such as the different ochres. The lumps were powdered with pestles in stone mortars, and both pestles and mortars have been found in the settlements. The powdered paints were kept in hollow tubes of bone. The outlines were drawn with sharpened pieces of iron ore.

Early Religious Beliefs

Besides the large number of remains of his cultural environment, Palaeolithic man has left some indications of his religious beliefs. These have to be interpreted by present knowledge of the religions of modern savages, for though the savage of the present day is farther advanced both intellectually and in the capacity for intellectual improvement than Palaeolithic man, yet he has retained certain beliefs and customs which date back to a more primitive stratum.

Of the earliest periods nothing remains to show what the beliefs were, if, indeed, any existed. But Neanderthal man has left several burials from which it is possible to learn something of the religious views of the Mousterian people. In addition to the burial at La Chapelle aux Saints described on p. 1132, there is another, found in a cave at La Ferrassie (Dordogne) excavated from about 1900 to 1933, which is still more interesting.

Here the body of a child had been laid in a carefully dug grave, accompanied by animal bones and Mousterian implements, and covered by a large stone slab, on the under side of which circular pits or "cup-marks" had been hollowed out. These are the earliest known efforts at "art," and obviously they had some magic or religious significance.

Preservation of the Skull

All this indicates that this early race had arrived at the belief that death was not the end, had conceived the idea of a hereafter. It should be noted that there are no signs of human sacrifice; the dead people do not appear to have been victims, and human beings were not killed to accompany them to the other world. Such rites were reserved for the more developed and systematised religions of a later date. *Homo sapiens* also buried his dead with ceremony, as is shown by the skeletons in the cave at Mentone.

In Franconia and in Switzerland other cave burials of a different type have been found. In these the skulls only had been preserved; they were arranged between the side of the cave and a little wall which had been built so as to form a kind of chamber.

The preservation of the skull only may perhaps be explained by the custom of some aboriginal tribes in India who, after the burial of a kinsman, dig up the skull and place it with those of his ancestors in a shrine, believing that the spirits of the dead survive and inhabit the skulls, and that, being treated with honour, they will protect and help their descendants.

In many instances the bones are found covered with red ochre for a reason which is obscure. It may have been done with some view of placating the spirit of the dead, who is often supposed by primitive minds to be incensed at being driven out of the body. Burials of women are as common as those of men, showing that the women were treated as being on the same level with men.

Hidden in Darkness

Though the burial customs are interesting, they are never so vivid as the beliefs of the living. Among the people of the Upper Palaeolithic such beliefs are indicated in their art, especially the art of the caves. The painted caves were never inhabited by man, who always lived in shallow caves with wide entrances, or in rock-shelters. Some of the rock-shelters were decorated with friezes of animals carved in stone.

The painted caves are nearly always long and very tortuous, and the decoration is in the innermost passages, where no light could penetrate. The artist worked by the dim glow of a sandstone lamp in which animal fat was

Advent of the Neolithic Age

changes but by cultural changes resulting from man's discovery that he could domesticate wild animals and cultivate wild plants and thus control his food supply.

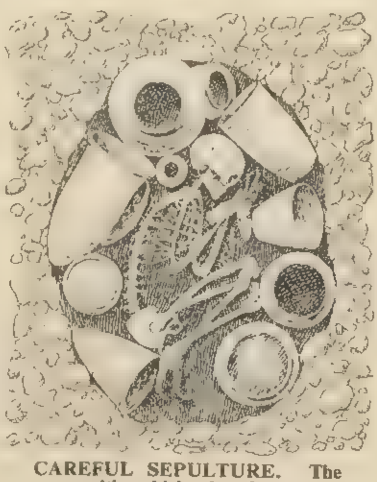
The Mesolithic inhabitants of Europe were probably descended in the main from the *Homo sapiens* population of Upper Palaeolithic times, though in Spain there may have been newcomers from North Africa as well. The old ways of life continued, but under greater difficulties. The vast herds of mammoth, wild horses, and reindeer had gone. The forests now helped to conceal smaller groups of red deer and wild boar, and to isolate human communities from one another.

One important forward step was taken: quite early in the Mesolithic period (though exactly when, where, or how is unknown) man domesticated the dog. To this time also may belong the invention of boats, skis, and sledges. In the peat bogs of northern Europe, where such wooden objects are well preserved, paddles, skis, and elaborately constructed sledges have been found in deposits of Mesolithic date. Nothing of this kind has yet been found in the Palaeolithic, so that these Mesolithic finds perhaps commemorate man's first innovations in transport.

Microliths

Over most of Europe the worked flints of this time are of extremely small size—mostly tiny blades, not more than an inch in length, hence the term microlithic (from the Greek *mikros*, small, *lithikos*, of stone). These may have been used chiefly as heads and barbs for arrows.

In most places these microliths are the only traces left behind by Mesolithic man, but on the coasts of Portugal, Scotland, and Denmark he also left monuments to his presence in the shape of great middens of sea-shells. In France the traditions of Palaeolithic art continued for a while in a degenerate form—just streaks of paint applied to pebbles. But in eastern Spain are many rock-shelters with paintings of temperate fauna and hunting scenes, and here the tradition survived into the succeeding (Neolithic) era.



CAREFUL SEPULTURE. The care with which the dead were interred in the Neolithic age, with some of their possessions grouped about them (as in the Egyptian burial seen here), is evidence of a belief in the continued existence of the disembodied soul.
From de Morgan, "Prehistoric Man"

Owing to the preservative effect of peat rather more is known about the Mesolithic cultures of northern Europe. Here the antlers of red deer, and bone, were still used by the Maglemoseans (from the Danish, meaning the dwellers by the great bog) for making barbed points and harpoons quite similar to those carved by the Magdalenians. But the Maglemoseans also appear to have used microlithic flints as arrow-heads and barbs.

Unlike other Mesolithic groups, who seem to have avoided heavily wooded areas and to have lived for preference on sand where trees grew less thickly, the Maglemoseans were forest folk. In order to make clearings and build their huts, boats, and sledges from the abundant timber, they invented axes and adzes of flint.

During the summers they frequented the edges of lakes, and here they built on the marshy ground platforms of logs and brushwood.

The earliest known Maglemosean settlement of this kind was discovered at Star Carr, near Scarborough, Yorks, in 1948. During the Mesolithic period the North Sea had not yet reached its present size, and eastern England was joined to the Continent by a great area of low-lying ground, probably full of streams and lakes. This could easily be traversed by canoe in summer, over the ice in winter.

The Mesolithic age came to an end first in those areas of the Near East where the great discoveries of Neolithic civilization were made, and not till some three to four thousand years later in Europe. There was a long period of development in Hither Asia before expansion of population sent groups of settlers off to colonise the Continent and Mediterranean islands, and the various Neolithic arts were carried northwards by slow degrees from these new centres.

Agriculture

It is thought that agriculture — beginning with the cultivation of wild plants for food — was first practised in the Near East, in a country such as Palestine, North Iraq, or Syria, where wheat grows wild and there are animals suitable for domestication (sheep, cattle, and goats).

The Natufian cave-dwellers of Mt. Carmel in Palestine seem to have milled grass-seeds and may perhaps claim to have been the earliest farmers. Their culture in other respects resembles the Mesolithic, and their flint industry is more properly classified as such.

Neolithic settlements are found in almost all regions of the Near East. In Palestine a long period of development must have elapsed between the time of the Natufians and that of the Neolithic peoples of Jericho, for their village was of considerable size and had a wall of great strength.

Sickle blades, saddle querns, and clay-lined silos, testify to a developed agriculture. Spindle-whorls and loom-weights show that fibres were spun for clothing. And the houses were built of mud-brick, with floors and walls of polished lime-plaster, sometimes painted red.

At a level which is tentatively dated to about 5000 B.C. plaster heads were found, with the features moulded over human skulls—which had its development in the later funerary portraiture of the Near East. Stone dishes were roughly carved. But pottery does not seem to have been known until later in the period, when vessels of gourd or skin or basketry were being replaced by vessels of a more durable nature, fashioned from clay.

On the Kurdistan Borders

In Northern Iraq, on the borders of Kurdistan, Neolithic settlements have also been found. At Qalaat Jarmo, between Kirkuk and Sulaimaniyeh, excavators in 1950 found remains of a village of some three acres in extent, which has been dated by radio-carbon tests to between 5000 and 4500 B.C. The people of this village lived in mud houses with several rooms.

Sickles with flint teeth set in bitumen, and querns, mortars, and clay ovens testify to the cultivation of grain. Their flint industry also was mainly microlithic. Portable pottery vessels were not made until the latest levels of occupation of the site, when a coarse, roughly made, ill-baked pottery appeared, obviously still in the experimental stage.

Sites of a later date, such as Tell Hassuna near Mosul, produce a more developed type of pottery, at first plain, later with incised or painted decoration. Grain was stored in large pottery silos. Bones of cattle, goats, and sheep show that animals were domesticated, and that gazelle and onager were hunted, probably with the sling.

In the Nile Valley

It is not yet possible to determine exactly which area of the Near East can claim to be "the cradle of civilization," but it is certain that in the Nile Valley the cultivation of cereals and the general speeding-up of human progress began at a very early date. Egypt is in some ways uniquely suited to such experiments, for her naturally fertile soil is annually enriched by the silt of the Nile flood. As the waters receded, man would soon notice pockets of mud where seed could be scattered to ensure the community's food supply.

To increase the irrigated area he would learn to build banks of mud to retain the ebbing flood a little longer, until the ground was saturated. Thus the beginnings of agriculture in Egypt must have been closely bound up with the beginning of irrigation.

Tasian Agriculturists

Neolithic communities practising the cultivation of cereals and the domestication of animals are known from a number of sites. In Upper Egypt the oldest agriculturists seem to have been those known to us as the Tasians, from the site, Deir Tasa, where they were first recognised. Grains of wheat and barley have been found in their settlements ; hunting and fishing also played their part in securing the daily food supply. Rough pottery, sometimes decorated to imitate basketry, was made.

Ornaments of shell, bangles, and beads of bone and ivory, and slate palettes for grinding cosmetics have been found in their graves. In the Fayum, flax was grown ; and the lake-side community harpooned fish and hippopotamus, and killed game with bows and arrows. Pigs, cattle, and sheep or goats were kept. Similar settlements, with considerable local differences in technique and equipment, have been found in the Delta and at Helwan.

Earliest Inhabitants of Crete

In Crete the Neolithic stratum extends to a depth of about 20 feet below the oldest layers of the Minoan culture at Knossos, and this must represent a considerable length of time. These earliest inhabitants of Crete were already advanced in the art of making pottery ; the vessels are provided with handles and spouts. Stone axes were ground down to an edge instead of being chipped, and grinding was also applied as a method of drilling holes in stone implements to facilitate hafting.

Spools and spindle whorls of clay show that they could spin and work in textiles. The Neolithic Cretan built himself a square house with stone foundations. Trade must have come already to the island, since obsidian, which does not occur there, was used for knives. Water traffic had already begun in the Eastern Mediterranean.

In Southern Anatolia the Neolithic period is represented by a deep level of occupation on some sites. The striking likenesses between some of the cult-figurines and early ceramics of Anatolia and those of Neolithic Crete have led some authorities to infer a colonisation of that island from the mainland of Asia Minor.

LESSON 10

Advancing Culture in Barbaric Europe

The new way of life, together with the necessary stocks of domesticated animals and seed corn, were carried from the Near East to Europe by two main routes—north-westwards by land, westwards by sea.

These colonising movements took place after the great centres of urban civilization were already well established in Mesopotamia, Anatolia, and Egypt, and were the result of expanding populations and their need for new sources of raw materials and fresh land to cultivate.

The culture of the earliest colonies reflects faithfully the traditions of the home-lands. As these in turn sent pioneers to the west or the north, into landscapes and climates which demanded radical adaptations of the familiar ways of living, cultures became progressively simplified and many of the refinements had to be abandoned.

Greece and the Balkans

In the fertile river valleys of Greece and the Balkans permanently inhabited villages grew up. New houses were built on the sites of old, so that with successive reconstructions small mounds or "tells" were formed, just like the larger tells of the Near East. Life was supported by farming and stock-breeding, and perhaps also by the cultivation of orchards. The finely made and sophisticated pottery, figurines, and clay seals reflect Asiatic origins. Axes and adzes were made of stone, but it is possible that copper was already known.

The Danubians

North of the Balkans, where the Continental climate, with its cold winters and evenly distributed rainfall, demanded drastic adjustments in the construction of houses and in farming methods, the origins of Neolithic cultures are less readily detected.

Nevertheless, Anatolian connexions are revealed by small, non-utilitarian objects such as clay seals. The fertile plains bordering the Danube were gradually populated by farmers who eventually spread as far as Belgium and Southern Holland. These people are known as the Danubians, and in this fresh and almost limitless territory they evolved a new method of farming.

Small areas of forest were cleared. The felled trees were burnt, and the ash served to fertilize the soil. Successive crops were then taken from the fields ; and when, after 25 to 50 years, the soil was exhausted, a new area was cleared and the process was repeated. As the villages were thus constantly being shifted, there are no "tells" to indicate their presence, and traces of them are discovered only by accident.

Danubian villages consisted in the early stages of long rectangular wooden dwellings inhabited by several families. The harvested grain was stored in large barns. Hunting seems to have played little part in the economy. Danubian pots look as though they were made to imitate gourds ; even the decoration seems to reflect the patterns of the slings by which gourds would be carried.

Adzes were made of polished stone, in shape resembling a cobbler's last—hence they are called "shoe-last" adzes. The Danubians were fond of ornaments and bracelets made of shell, and shells of a Mediterranean mussel were imported to provide the raw material.

Funnel-beaker Culture

The origins of the funnel-beaker culture are obscure. It may have resulted from the adoption by native Mesolithic people of a Neolithic economy through contact with the Danubians. The name comes from the characteristic funnel-shaped necks of the pots. This culture flourished in Poland and in Central and Northern Germany, and it was the first Neolithic culture of Denmark and Sweden.

South-eastern Italy

In the form of painted pottery Near Eastern traditions are still faintly reflected in the early settlements of South-eastern Italy. Such traditions failed to penetrate as far west as Spain and Southern France ; here a brief Neolithic period is represented chiefly by pottery decorated with impressions made with sea-shells in the moist clay before the vessels were fired, and by plain, round-bottomed pots which seem to imitate the forms of leather containers.

The cultures represented by these types of pottery are thought to have come from North Africa, perhaps originally from Egypt. Little is as yet known about this first phase of Neolithic settlement, which was quickly followed by the megalithic cultures.

Lake Dwellings

Here we meet again early settlers using plain "leathery-looking" pots, axes of flint and stone, and flint arrow-heads. The best-known settlements are those round the edges of the Swiss lakes and in the French Jura. Villages consisted of small groups of houses built on wooden platforms which were supported by piles driven into the lake bottom. The villages

NEOLITHIC LAKE DWELLINGS. Some early settlers in western Europe lived in small groups of houses built on wooden platforms which were supported by piles driven into the lake bottom. The best known of these settlements are on the Swiss lakes and in the French Jura.
After Prof. J. M. Tyler, "New Stone Age" (Scribners)

were reached from the shore by means of wooden causeways similarly supported on piles.

Remains of the wooden structures, pottery, implements, and even refuse, have been preserved in the mud of the lakes, so that a good deal is known about how these people lived. They made wooden receptacles as well as pots, and used wood and antler for the hafts of axes and knives. An adhesive was made from birch gum. Oxen, sheep, and pigs were kept. Fish were speared and hooked. Barley, wheat, and flax were grown. Wild crab-apples, cut in halves were preserved.

In central France related groups lived in hill-top camps surrounded by ditches and ramparts. From somewhere in the north of France set forth the first Neolithic people to reach the south of England.

Although there are minor differences in the individual cultures of each area, the group which shared the tradition of "leathery" pottery—in Spain, Switzerland, France, and Britain—is called the Western Neolithic complex.

Windmill Hill Culture

In England the Western Neolithic complex is represented by the Windmill Hill culture, which takes its name from a hill-top camp near Avebury in Wiltshire. About a dozen such camps are known in the south of England ; a roughly circular area is surrounded by from one to four shallow ditches and low banks which are interrupted by several entrances.

They were not lived in permanently, but may have served as corrals for cattle during autumn round-ups. Plain "leathery" pottery, flint axes, and leaf-shaped flint arrow-heads are found in the ditches. Querns for grinding the corn into flour were made of sandstone.

FOR THE NEOLITHIC REAPER. It is believed that these crescent-shaped flint implements discovered in Neolithic remains were used as sickles.
British Museum

The Windmill Hill people buried their dead in long barrows (up to 300 feet in length) built of chalk and earth. Usually more than one skeleton is found in each barrow.

The megalith-builders who colonised the western shores of Britain and nearly the whole of Ireland also belonged to the Western Neolithic group, but they probably came directly from Spain, Portugal, or the South of France .

It is in the Orkneys that the most interesting Neolithic settlements have been found. These belonged to a culture called Rinyo-Clacton, which does not seem to be related to the Western Neolithic ; its origins are still obscure. Here two villages, at Rinyo and at Skara Brae, consisted of seven or eight one-roomed houses. The walls and furniture were built of stone slabs, and each house was planned and furnished like the others.

In the centre of the floor was a hearth. A stone dresser stood against the wall opposite the doorway, and stone bedsteads against the other walls. Cupboards were constructed in the walls for storing personal possessions. Adjoining cells may have served as latrines.

Such well-equipped dwellings show that even in places as remote as the Orkneys, Neolithic people had a high standard of domestic comfort. Other groups must have had equally good arrangements, but as they built in wood all but the post-holes of their houses has vanished.

LESSON 11

Village Craftsmen of Prehistoric Egypt

THE progressive evolution of prehistoric man can be studied in the Nile valley perhaps better than anywhere else, partly because the area has been more thoroughly explored than any other in the Near East, partly because of the unique climatic conditions.

In Egypt early man lived for the most part on desert spurs riding above the still-swampy valley, and at all times he buried his dead in the dry sandy soil of the low desert, in which even normally perishable materials like wood, reeds, and leather are miraculously preserved. In Egypt, as in no other country, can be recovered the actual remains of each successive phase of civilization.

From these remains can be formed a fairly clear (though still incomplete) idea of how men lived, what they ate and wore, what their crafts, pastimes, and daily pursuits must have been, and even (to a small extent) what gods they worshipped and what beliefs they held. Egypt is, in fact, the archaeologist's earthly paradise.

Absolute dating is impossible for this early period, though attempts have been made to fit the sequence of pre-dynastic cultures into a chronological framework. The system of relative dating evolved by Flinders Petrie to establish a chronological relationship between groups of objects found in different tombs in a cemetery, based largely on the evolution of pottery types, is still used as a convenient method of dating prehistoric material.

But this Sequence Dating has to be used with caution, and it has considerable limitations. If we date one pot-form to S.D. 35, another to S.D. 36, and a third to S.D. 37, we know with some degree of assurance that the latter is later than the former, but we have no clue as to the length of time separating them, nor to the absolute dating of an object at any point along the typological series.

It is not until about 2000 B.C., far into the historical period, that there is anything like a fixed date for Egyptian history. In the earlier dynastic history the archaeologist is still dealing with approximations. For the period before written records begin, dating becomes the merest guesswork—and scholars differ widely.

BADARIAN WARE. The pottery of the Badarian civilization, so called because of the remains first found at Baradi, near Assiut, in Upper Egypt, is characterised by its rippled surface.

Badarian Potters

In Upper and Middle Egypt the Tasians were succeeded by people we know as the Badarians, anthropologically rather different but enjoying a similar culture in a more advanced form. The Badarians achieved real mastery in the potter's craft. Badarian pottery (dated S.D. 22-29) is delicately thin, fine grained, and well baked; it is often covered with a wash of haematite (iron ore), which burns to a fine dark red in an open kiln, or to a lustrous black when oxygen is excluded.

The Badarian potters seem to have discovered this property of haematite and they used it to good effect by placing the pot mouth-downwards in the kiln so that a few inches were covered by ash. When thus fired, the pot is dark red, with a lustrous band of black at the rim. The pot was burnished after firing.

Such pots are called "black-topped." The surface was decorated, in some instances, when the clay was still wet by rippling the surface with a tooth-comb. The shape peculiar to the period is a shallow bowl with a sharp keel, possibly a copy in pottery of a leather vessel then in everyday use.

The Chalcolithic Age

Imported materials are found among the grave-offerings of the Badarians: malachite from Sinai; shells from the Red Sea; and coniferous wood and resin, probably from the Lebanon in Syria. This must imply some sort of commerce, direct or indirect, along trade routes of considerable distance. Metal came into use towards the end of the period, though in such small quantities that stone was still the usual material for tools and weapons.

Pins of copper to fasten the cloaks of the living and the goatskin in which the body of the dead was wrapped were the first metal objects to be made. Harpoons for spearing fish came next, and then, soon after, chisels made their appearance. The metal was not cast but hammered, and at this point the student enters what the archaeologists call the Chalcolithic Age, a period during which the malleable properties of native copper are known, but not its propensity to fuse under heat. At S.D. 30 appear the people now called the Amratians, who introduced sudden changes in pottery

shapes, though the muffle-kiln technique of making black-topped vessels continued. Amratian potters introduced a new method of decoration by applying white slip to the burnished red ground in criss-cross patterns. The actual ware was not so good as in the previous period, but the decoration included human and animal figures as well as geometric designs ; the artist could give free play to his fancy. An open dish in the form of a boat has a slip decoration representing the oars and cabins—the first actual evidence of water transport at this early time.

Stone vases were carved from basalt, alabaster, and limestone, all obtainable in the Nile valley. Though writing was still unknown, the Amratians used signs as marks on property, scoring the signs on pottery vessels. Another advance in civilization is seen in the more decided views on religion. A definite ritual was followed in the burials, and the objects placed in the tomb for the use of the dead make it obvious that there was among these primitive people a firm belief in the hereafter.

HISTORY ON A KNIFE HANDLE. Both sides of the famous knife handle found at Gebel Araq are shown above. The pictorial details of pre-dynastic boats, costume, etc., are of very great interest. Note especially the battle (lower right) between Nile boats and square-prowed boats, and the Sumerian-looking group of men and lions at the top left.
The Louvre

Carved ivory figurines, decorated combs, and slate palettes for grinding eyepaint, in the shape of animals or fish, are evidence of an awakening interest in plastic art. Various characteristics link these people with the Libyans of the western desert as we know them from the historic period, and there may be some common tradition based on early migration from the oases into the Nile valley.

The Gerzean Civilization

Near the Nile valley lived a people, workers in stone, who penetrated into Egypt and introduced a new type of civilization, called the Gerzean. Instead of the clumsy unsteady stone vessels of the Amratians there are barrel-shaped vases with flat ledge-rims ; there is no foot to the vase, which stands firmly on a flat base.

The stones are porphyry and other hard kinds which do not occur in the Nile valley. Gold is first used in this period. It was beaten out into thin sheets, cut into strips, and then laid over the rims and handles of stone vases. Beads were made by overlaying a stone core with sheet gold. Gold was used only for ornamentation, being valued for its colour and sheen. It never rivalled copper, which was of use for tools.

In the Gerzean era a great advance was made by the building of houses and tombs with sun-dried bricks. This industry continued throughout the whole historical period of Egypt ; burnt bricks were introduced only when the Romans came. The houses of the Badarians and Amratians were probably reed huts smeared with mud and thatched with straw or reeds.

Gerzean boats, a common decorative theme on their red-painted buff pottery, were large rowing galleys steered by two long oars at the stern, and big enough to carry a considerable amount of cargo. In addition are found illustrations of a different kind of craft, perhaps a sea-going boat, with a high square prow and stern, very similar to boats depicted in Sumer and to the square-ended boats drawn in hundreds by travellers passing through the Wady Hammamet, the ancient caravan route between the Nile valley and the Red Sea.

Egypt's Extending Trade

At about the same time, and a little later at the beginning of the dynastic period, there are other indications that the civilization of Mesopotamia was not entirely unknown to the inhabitants of the Nile valley. But the actual Sumerian objects found in Egypt are few and small, and they could have travelled long distances, so that trade may have been indirect. It is possible that Egypt was already in touch with the coast of Arabia, via the Red Sea route starting at Coptos, a little north of Thebes.

The ancient Egyptians never forgot that once, in prehistoric times, their country had been two countries—southern or Upper Egypt, and northern or Lower Egypt—and they credited Menes, the first king in the dynastic lists, with the unification of the land under one rule.

To the end of their history the pharaohs of Egypt bore the double title, "Lord of the Two Lands, King of Upper and Lower Egypt." They wore the double crown of north and south, and they observed double rites at festivals. Traces of this duality are seen in the administration and in mythology. Thus the task of the historian-archaeologist is aided by legend and tradition.

EGYPT'S CONQUEROR. Slate palette showing a southern ruler who subdued northern Egypt.
Quibell "Hierakonpolis"

There are indications in the archaeological record of predynastic Egypt pointing to the existence of two kingdoms, and to a conquest of the king of the north by the king of the south.

Stories Told in Symbols

At Hierakonpolis in Upper Egypt, which seems to have been a royal centre in very early times, were found a number of large votive palettes and mace-heads carved with scenes in relief showing in a symbolic way the historic event they were made to commemorate. They are the direct forerunners of the hieroglyphic, or picture-writing, of the historic period, though the story is mostly told with the aid of symbols rather than of conventional signs.

The most important of the slate palettes records the conquest of northern Egypt from the south. On one side of the palette is the king wearing the crown of the south and in the act of slaying a northern chief, while his falcon totem leads the north captive. On the other side is the king wearing the crown of the north and going to see the sacrifice of ten human victims.

The few rudimentary hieroglyphs which explain the scene suggest that the king is taking possession of and sanctifying the Great Door (i.e. the chief port). The stone mace-heads are obviously votive offerings; they are too large for anyone to wield (*see* illus., p. 1127). The scenes on them record religious ceremonies in which the king plays a leading part. During this protodynastic period civilization made big strides. One of the greatest achievements was progress in the art of writing. Hieroglyphs for use on monuments, and hieratic cursive script for writing in ink, appear at this time and simultaneously, and with them there appears a whole system of numeration, with figures denoting numbers from one to a million. The method of numeration was by special signs for units, tens, hundreds, thousands, and so on, as in the Roman numerals.

Our present method of numeration comes from India, and it was not introduced into Egypt till after the Christian era. The early writing was at first little more than names of kings, but it soon developed into the careful record of events and even to the writing of books. From this time forward the archaeologist has in Egypt written documents—inscriptions or papyri—to assist his investigations.

Two other inventions introduced at about this time were a form of potter's wheel and some kind of mechanical means for drilling stone. Glass must have been invented elsewhere, since only a very few examples of this period have been found in Egypt, and always in places where the value put on it appears to have been very great. It was probably brought in through trade. Besides glass, Egypt also imported timber. The Nile valley has never produced any good building timber; all wood for that purpose had to be imported.

Royal Tombs of the First Dynasty

At the beginning of the historic period new types of building had been introduced. The royal tombs of the First Dynasty show the development in the construction of all parts of a building. The walls at first were of mud-brick, with which also the floors were paved, but the roofs were of wood, the beams being taken across the whole span of the roof without intermediate supports.

As in many instances the shortest span was 20 feet, a long piece of timber capable of standing the strain was required. The appropriate wood would be some kind of conifer, but no trees of the kind grow in Egypt, the nearest place from which they could be obtained being the Lebanon.

Later in the dynasty another advance was made in building, when stone was used for paving the floors of the royal burial chambers. The stone was granite, laboriously hammer-dressed. It was not until the Second Dynasty

that stone was used for building walls—and then only walls of tombs, not of houses.

Long before methods of agriculture had been evolved to suit the peculiar conditions of the Nile valley, a system of canal irrigation had developed which, with modern improvements, is in use to-day. The Nile itself was the highway for all traffic. The tradition, preserved by the Greek historian Herodotus (*c.* 484–424 B.C.), that King Menes built a large dyke across the Nile valley near Memphis reflects the large-scale projects which a centralised administration could now undertake. The unification of Egypt brought peace and prosperity to the land, and under one supreme ruler civilization developed with astonishing rapidity.

LESSON 12

Early Civilization in Mesopotamia

In the Land of the Two Rivers—the alluvial plain watered by the Tigris and the Euphrates—early man found conditions of life different from those of the Nile Valley, but no less favourable to his development.

The country consists essentially of an upland plateau, watered by the Tigris and Zab rivers, and bordered by hills to the north and east (the land which was to become Assyria), and a low-lying alluvium through which the rivers in their lower course meander towards the Persian Gulf—the lands of Sumer and Akkad.

Closely linked with Sumer was the plain of Susa, the capital of the ancient country Elam, with whose destinies those of Sumer were at all times linked. Susa lies on the eastern bank of the Kerkha river at the foot of the Zagros mountains. The Euphrates has shifted its course many times, even during the historic period, and the sites of several of the cities of Sumer, once flourishing riverine ports, are now miles from the river, in arid desert.

Until comparatively recent times the two rivers flowed separately into the sea ; now they join to form the Shatt el Arab, into which the rivers Karun and Kerkha also flow. To the west the great Syrian desert forms a barrier which in ancient times isolated Mesopotamia from the west, but a route was found up the Euphrates, which at Carchemish approaches to within 100 miles of the Mediterranean.

From very early times inland caravan routes linked Persia with Afghanistan and India, while sea-going vessels plied the Persian gulf, linking Sumer and Elam with Bahrain, the coasts of Arabia, and ultimately with India and with Africa.

Chalcolithic village settlements, larger than the Neolithic and with a more advanced culture, are found in Northern Iraq and at Susa. The people of the so-called Tell Halaf period kept cattle and sheep, goats and pigs. Their villages had cobbled streets, and the houses were comfortably designed. Curious buildings with low domed roofs may have been used for storage or may have been shrines.

Halafian Ceramics

The great achievement of the Halafians was in ceramics ; their pots are eggshell-fine, beautifully hand-made, and carefully fired, and they are decorated with polychrome designs of exquisite delicacy. A varied repertory of shapes includes carinated bowls and beakers that look like imitations of metal forms. At Susa, too, the Chalcolithic potter achieved vessels of great beauty, with bold designs in lustrous black paint on a grey ground.

The handsomest pottery, in the ancient Near East at any rate, is the early Chalcolithic. Later peoples, achieving the mastery of stone and metal, and expressing their need for colour in vessels of glass, faience, and gold, no longer lavished their skill on the painting of pots, and designs grew careless and decoration deteriorated.

HEAD-DRESS of a Sumerian queen, about 3000 B.C., from a stone tomb discovered at Ur.
Courtesy of Joint Expedition to Ur

Halafian has not been found in the south, where the earliest phase of settlement in the Euphrates valley is called Al Ubaid, after the site, Tell al Ubaid, where it was first noticed. Here, in fertile swamps about the head of the Persian Gulf, arable land had to be reclaimed from the river by a system of dykes and ditches, the beginnings of the irrigation system on which the Sumerians were to depend.

The problem was twofold, to protect one's land from flood, an ever-present danger during the spring when the rivers swelled and might overtop their banks, and to conserve the flood

water so that during the parched days of summer, after the retreat of the inundation, crops might be kept moist till the harvest. These two duties were paramount among the functions of a Sumerian ruler, and in later times each king boasts of the canals which he dug, as among the greatest of his achievements.

Such collective tasks as the digging of a dyke postulate an organized community, and the Ubaid people had a centre of worship in at least one of the sites excavated : the primitive temple at Eridu, a simple shrine which was successively rebuilt and enlarged, but which shows in its very earliest phase evidences of the worship of Enki, Sumerian god of the deep waters.

Sumerian Town Planning

As in Egypt, those three great inventions, metal working, writing, and the potter's wheel, made their appearance at approximately the same time. How and by what route, and in which direction, these new ideas travelled is not known. Certainly it was the idea of writing, and not the script itself, which was communicated, for there is no similarity at all between the systems of writing developed in Sumer and in Egypt.

Perhaps Sumer may claim the priority, for what evidence there is suggests that the Uruk period was rather earlier than the late predynastic or protodynastic period in Egypt perhaps in the middle of the fourth millennium B.C.

In this period people already congregated in large urban communities. The elaborate planning of their towns and the impressive size of their public buildings, such as the temple-complex round which their economic life revolved, give an idea of the great advances which had been made since the Ubaid period, in what must have been a relatively short space of time.

Cuneiform Writing

The earliest Sumerian script, simple pictographic signs scratched with a pointed stick on an oblong of clay, was used for short lists, dockets, and receipts. As writing came into greater use, the signs lost their pictorial character and were written with a square-ended metal stylus more suited to the soft material, and this when held at a slant produces a triangular sign. Hence the name " cuneiform " (wedge-shaped) for this writing.

The written tablets were sun-dried or baked. Contracts or letters might be enclosed in an envelope of clay and the whole sealed by rolling the owner's cylinder seal over it. Tablets and cylinder seals are henceforward typical features of Sumerian and Babylonian life, and the many thousands of tablets which are normally found on a Sumerian site form a wonderful and important rich source of the archaeologist's knowledge of ancient life and thought.

The government of Sumer at this time was quite unlike the contemporary monarchical unity achieved in Egypt. It more resembled that of the later city-states of classical Greece. Each city was governed, in theory, by the city deity, in practice by a priest-ruler who was regarded as the god's deputy. As time went on, these earthly rulers lost some of their priestly functions and, as the leaders of warring states, assumed the title of king over conquered territories.

Later generations preserved lists of early kings, ascribing fabulous lengths to the reigns of the first, semi-mythical, heroes. Later the lists become more plausible, and some names can be identified with those found on inscribed monuments by modern archaeologists. By 2800 B.C. comes the daylight of history (*see* Lesson 15).

Mastery of Metal Techniques

Early dynasties of rulers who held more than local sway are recorded from Ur, Uruk, Lagash, Umma, and Kish ; excavations on the sites of these cities afford evidence of flourishing city life during this early dynastic period. A series of ritual graves found by Sir Leonard Woolley at Ur contained gold and silver vessels, jewellery of gold and precious stones, harps decorated with shell inlay, gold, and lapis lazuli, and gold and bronze objects which show a mastery of metal techniques such as closed-mould casting, repoussé, and granulation. Caravans must have brought some of these materials from Persia, Arabia, Armenia, and even distant Badakhshan. A scene of warfare shows lancers in helmets and leather cloaks, and heavy ass-drawn chariots with solid wheels.

A Typical Sumerian City

In the centre of the typical Sumerian city was usually the great temple of the city god with its shrines, courtyards, and magazines, and the temple tower, or *ziggurat*, which would make it a landmark for miles around. The town was intersected by narrow streets and, on the river side, flanked by quay walls.

Houses were simple in plan, either a plain rectangle or built around a courtyard. The roof was flat and provided storage space and sleeping accommodation. The houses were built of mud-brick, cemented and plastered with mud, and when a house collapsed after fire or heavy rain, or was pulled down, the mud was levelled and a new house built on the resulting platform. After a time the surface of the street, having sunk below that of the houses on either side, would be raised.

The level of the city was thus constantly rising, and ancient cities which had been continuously occupied over a long period stood atop a considerable mound of débris. These mounds, today called *tell* in Arab countries and *tepe* in

Turkey, are sites which hold potential treasures for the archaeologist.

The excavation of them entails peeling off layer after layer, each older than the last, until the earliest levels of occupation are reached. The history of the town can thus be read in reverse. Some of the great cities of the Near East, such as Aleppo, Erbil, and Kirkuk, are still on top of their ancient *tells*, showing a continuous history of occupation for thousands of years.

Sumerian civilization spread throughout Mesopotamia and N. Syria, and the painted pottery characteristic of the early phase spread even to the Mediterranean coast of Cilicia.

LESSON 13

Earliest Workers in Metal

SOME archaeologists have claimed that four great stages in the development of human culture can be discerned : the Stone Age, the Bronze Age, the Iron Age, and the Mechanical Age, in the last of which we are now living. Each of these can be subdivided into smaller sections. In the Bronze Age, which is dealt with in this Lesson, the first section is called the Copper Age, because pure copper without any deliberate admixture of another ingredient was the metal in use.

Stone Hammer and Anvil

The use of metal began with the laborious method of hammering out nuggets of copper with one stone as a hammer and another as an anvil. The sheet of metal thus obtained was then cut into the desired shapes and the edges ground down to the required sharpness.

This was a slow process, slower than the manufacture of flint tools, but it was the precursor of the great development following on the discovery of the arts of smelting and casting. When this discovery was made is not known, but it must have been in the Old World, for the most advanced American Indians had not gone beyond the stage of hammering the metal at the period when European civilization was first introduced into the New World.

Working in Copper

The first metal to be used, for either hammering or smelting, was copper. The difference in hardness between different ores of copper must very soon have set inquiring minds at work, for before 2000 B.C. not only was the mixed metal, bronze, in use but the mixture of copper and tin was in the proportion of 9 to 1, a proportion which in modern times is regarded as the best. As it seems likely that the deliberate mixture of copper and tin would be used first in those countries where the metals occur together, it is worth while noting the countries in the Near East where this happens. They are surprisingly few: the Caucasus, Persia, and Anatolia.

Metal-workers soon found that pure copper was unsuitable for tools, being too soft and losing its edge quickly ; in this way it is inferior to stone. But it is not brittle, and many more tools can be made from one mould and one "pouring" than could be made in the same length of time from stone. With all its disadvantages it was so superior to stone that the workman's scope was greatly enlarged by using it.

Tools and Weapons of Bronze

Bronze very soon ousted copper for the manufacture of tools and weapons. Copper requires a very high temperature for melting, always a difficulty in primitive furnaces ; the addition of tin lowers the melting point appreciably. Copper can be cast only in open moulds, which limits the shapes of tools to the simplest forms, and the object has to be cast solid. Bronze can be cast in a closed mould or by the *cire perdue* (waste wax) process, a method which allows an object to be cast hollow and at the same time economises the metal.

The mixed metal is less likely to have bubbles, which cause flaws in the finished tool. Above all, bronze is a hard metal, can take and keep a finer edge than copper, and does not break like stone ; thus it has all the advantages and none of the disadvantages of stone and copper.

The introduction of smelted metal is important in the history of human culture. Those countries where both copper and tin occurred increased their trade relations by exporting either the raw material or the manufactured articles. Other peoples imported the metal and worked it up themselves or bought the tools ready-made.

The Expert Smith

Conditions of life were now changed. In Neolithic times each village, probably each family, was self-supporting. Every man could make his own flint tools, and every woman could make her own pots or weave her own baskets. But metal-working, especially smelting and smith's work, needed special training.

This was the beginning of specialising in work, the smith being one of the first to become an expert in his craft. In all probability it was kept a profound secret, to which only initiates

were admitted, for the metal-worker was an important person.

Bronze was not used for rough work, however; almost to the Iron Age, sickles and other farming tools were fashioned of flint which was set in wooden handles. Arrow-heads for hunting were also made of stone.

The increase in trade gave an impetus to methods of communication. Wheeled vehicles came into being for land transport, and vessels with sails for water traffic. More important still, writing was invented, and a simple system of arithmetic also came into use, by which barter could be effected.

LESSON 14

Megalithic Monuments of the Neolithic and Bronze Ages

THE Neolithic was an age of great voyages by sea. Setting forth from Spain and Portugal, adventurous seafarers made their way to Brittany, Ireland, and the whole west coast of Britain. Eventually they reached even the Orkney and Shetland islands. And they managed to transport not only themselves but sheep and probably cattle as well.

The origins of these people in the Iberian peninsula and the places where they settled are revealed by the durable monuments they built to inter their dead. These are called megalithic tombs. The pottery placed in the tombs at the time of interment normally belongs to the "leathery" Western Neolithic type.

For Collective Burial

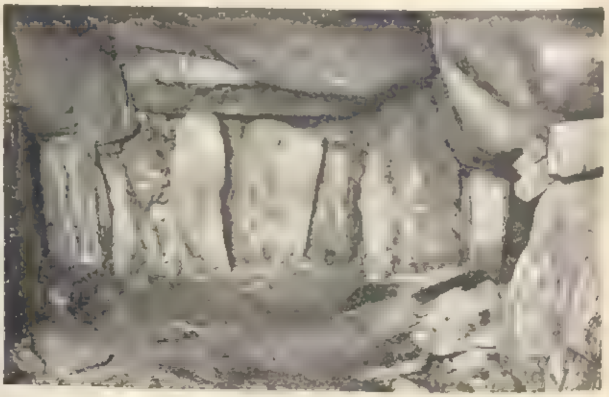
Strictly, the term megalithic should be reserved for tombs or other monuments built of large stones, but it is generally applied to all stone-built tombs which were used for collective burial. There are three main types of megalithic tomb, all of which are represented in Spain, Portugal, southern France, Brittany, and the British Isles. All types were originally covered by mounds of earth or cairns of stones.

Passage-graves have a more or less circular burial chamber, sometimes with small side-chambers opening off it, approached by a passage leading from the exterior of the mound. Burials are normally found only in the chambers.

Gallery-graves consist simply of a long passage in which the bodies were placed. Sometimes the passage is divided into segments by stone slabs set across the floor or projecting from the walls at intervals.

Both passage-graves and gallery-graves were built either of large slabs of stone or of small slabs set one upon another. In Portugal and southern France passage-graves were also—in limestone areas—cut out of the soft rock so that they were entirely underground. These were approached from the surface by a sloping ramp.

The name dolmens, probably of Cornish origin, should be applied only to the type of structure seen in the photograph of Kit's Coty House—two or three upright slabs supporting a capstone. They are often simply the remains of more elaborate tombs which have lost their mounds and been partially destroyed. But some intact specimens seem to be simplified



MEGALITHIC MONUMENTS. Typical of the less elaborate dolmen tombs is Kit's Coty House (left), at Aylesford, in Kent. Originally the whole structure was enclosed within an earthen barrow. The photo on the right shows the central chamber of the elaborate tomb at La Hougue Bie in Jersey, built of huge monoliths and situated in a vast tumulus composed of some 20,000 tons of earth.

Right photo, Société Jersaise

TOMB CONSTRUCTION IN BRITTANY. In the great dolmen called the "Table des Marchands," at Carnac, the immense monolithic roof was supported originally by several side-stones.

versions of passage- or gallery-graves. These are always closed chambers without entrances.

Megalithic tombs are also found in Denmark, Sweden, north-west Germany, and northern Holland. In Denmark and Sweden there are "dolmens" similar to those just described, but probably developed locally. They belong to the Early Neolithic of Scandinavia. In Middle Neolithic times passage-graves appeared in all areas previously mentioned ; although built on a somewhat different plan, they are thought to be imitations of the western European type.

Megalithic tombs were nearly always used for collective burial : they were vaults in which whole families or small clans were interred. As many as 100 skeletons have been found in one tomb. It was customary to include offerings of pots (which may have held food or drink) and sometimes axes, arrow-heads, and beads.

In the British Isles, Brittany, and some of the Mediterranean islands, monuments not intended to serve as burial chambers were erected in the Neolithic and Early Bronze Ages. Like the tombs, they were entirely or in part constructed of large stones. They were centres where people gathered for ritual or religious ceremonies.

Avebury Monument

The most famous stone circles are at Avebury, north Wiltshire, and at Stonehenge, south Wiltshire. The Avebury monument belongs to the late Neolithic period and there were two phases of construction. To the first phase belonged three small circles of upright stones. These circles were set in a line from north to south. Subsequently the northernmost was demolished to make way for the grandiose structure which is now to be seen.

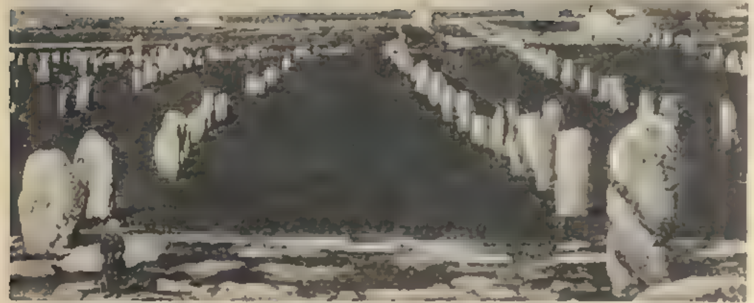
An area of 30 acres was surrounded by a deep ditch with a high bank outside, through which there were originally three entrances. At the inner edge of the ditch and following its line was erected a series of giant sarsen stones. From the south entrance of the circle extended an avenue of similar sarsens, set in pairs, and it is believed that this may have led to a smaller monument on Overton Hill, a mile away.

Similarly at Stonehenge there was more than one period of construction. The first phase is Neolithic and contemporary with Avebury. A much smaller area than at Avebury was enclosed by a shallow ditch and low bank, with a single entrance. Following round the inner edge of the ditch were dug a series of holes, now called the Aubrey holes (after the antiquary John Aubrey, who first observed them in 1666). These were not intended to receive stones or posts, and their significance is obscure.

The ditch and Aubrey holes were allowed to silt up and then they were used as a cemetery by a group of Neolithic people who practised cremation. The little packets of burnt human bones were buried in the filling.

Not very much later, but already in the Early Bronze Age, the stone circles were erected within the enclosed area. There are two concentric rings. The outer is of the local sarsen stone, and, when complete, consisted of uprights on which cross-bars or lintels were laid, the whole forming a complete circle with the

REMAINS OF THE AVEBURY MONUMENT. The late Neolithic period saw the building, in two phases, of the grandiose structure at Avebury, Wiltshire, where people gathered for ritual or religious ceremonies. Some of the giantic stones are seen above.



ALIGNMENTS OF MONOLITHS IN BRITTANY. At Carnac the big stones stand in rows, forming wide avenues which lead up to a cromlech, or small stone circle. The cromlechs are believed to be burial places, and the alignments of monoliths processional avenues. The stones diminish in size the farther they are from the cromlech.

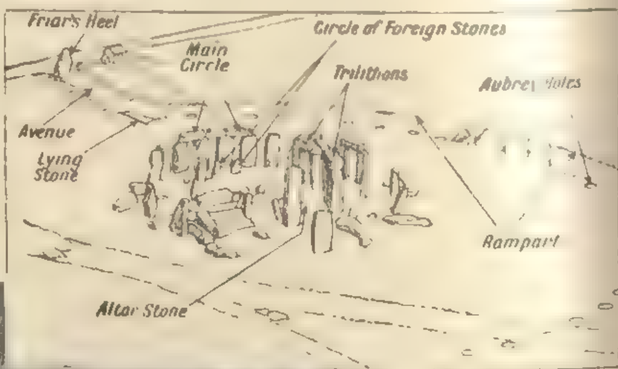
lintels touching each other all the way round. The inner circle is of the "foreign" stone brought all the way from Prescelly, near Milford Haven, in Pembrokeshire.

Within these two circles stood five great trilithons, forming not a circle but a horse-shoe; within these is another horseshoe of Prescelly stones, and within that again a stone lying flat and called the altar stone. All the stones have been squared, and the cross-bars of the outer ring and of the trilithons are fixed to their supports by mortise-and-tenon joints. This is a technique of wood-working, and suggests that the builders may have been carpenters rather than stonemasons. Only in one spot are there any toolmarks still visible; these marks were certainly made with stone tools.

In 1953 carvings on some of the sarsens were observed for the first time. Some of the designs are obscure, but among the clearly recognizable objects are numerous bronze axes and a bronze dagger of Early Bronze Age type. These carvings are likely to have been made shortly after the stones were erected.

Avebury and Stonehenge are the largest and most magnificent of these circular structures, but minor groups of stones are found in other parts of Great Britain.

In Brittany the most important non-funerary structures are the alignments. The three largest are at Carnac, where they are set up end to end and are over two miles in length. The stones are arranged regularly in rows, standing a little apart from one another, and forming wide avenues which lead up to a cromlech, or small stone circle. At the end of each row near the cromlech the stones are huge unhewn blocks; they diminish



STONEHENGE. The famous stone circle on Salisbury Plain, in south Wiltshire, dates from the Neolithic period and the Early Bronze Age. The outer ring is of the local sarsen stone. The inner circle is of "foreign" stone brought from Prescelly, near Milford Haven. The component parts are identified in the key diagram above. The significance of the Aubrey holes (named after the antiquary John Aubrey) is obscure.

in size the farther they are from the cromlech, till at the far end they are quite small. The cromlechs are probably burial-places; the alignments of monoliths, processional avenues.

Malta Temples

The chief megalithic monuments in Malta are the temples. They are all built on the same plan, which is peculiar to the island. There is a main aisle leading from the entrance to a semicircular sanctuary, and on each side of the aisle are two (in a few instances, three) semi-circular transepts The temple is always built of carefully squared slabs set upright, and the whole structure is surrounded by a wall of rough-hewn blocks; the transepts appear to have had corbelled stone roofs, but the main aisle was open to the sky.

ALTAR OF A MALTA TEMPLE. The stone-slab temples of Neolithic Malta are all built on the same plan, which is peculiar to the island. They were probably constructed by settlers from Sicily. The vast buildings at Hal-Tarxien (an altar is shown above) are the most impressive of these remains.

These temples usually occur in groups, often in pairs, one being always slightly larger than the other. The sculpture of spirals in relief with which the temples were decorated is so advanced in type that doubt has been expressed as to the early date of these buildings, but the excavated evidence proves conclusively that they belong to the Neolithic period. The pottery found in them shows that they were probably built by settlers from Sicily. The vast buildings at Hal-Tarxien are the most impressive of these remains.

Megalithic " Tables "

Peculiar to the Balearic Islands is the megalithic monument called the *taula* or table. It consists of two slabs. The lower is comparatively thin for its height and width ; the wide faces are the natural cleavage of the stone, but the narrow sides have been hammer-dressed so as to form surfaces at right angles to the wide sides. Each end of the slab was slightly bevelled, and the great mass was then raised upright and set into a shallow groove in the flat rock. It was probably held in place with rocks and earth until the upper stone was placed in position.

The upper stone, though not so large, is always considerably thicker than the supporting slab ; it also was hammer-dressed, but the working is far more careful. It is bevelled downwards, and on every side it overhangs the upright, to which it is fixed by a mortise-and-tenon joint like the cross-bars at Stonehenge.

Though the construction is the same as Stonehenge, the form of the structure is entirely different. At Stonehenge the trilithon has the form of a doorway ; in the Balearic Islands the taula has the form of a table. A taula stands in a more or less circular enclosure round which is a stone wall built without mortar, the wall being of later date. The earliest objects found in these enclosures are of the Bronze Age, though the working of the taula shows the Neolithic technique. As at Stonehenge, it would seem that the sacred stones were worked with the tools of a past civilization.

A TYPICAL TAULA. This photograph of a taula at Trapuco, Minorca, shows the characteristic megalithic monument of the Balearic Islands, found nowhere else.

Offerings of many kinds were often placed with the dead, and from these objects has been deduced a great part of the present knowledge of the civilization of the Neolithic and Early Bronze periods in the west of Europe. Pottery was common ; it was hand-made, usually badly baked, and the walls

of the vessels are thick and coarse. In England the rims are always carefully made, and the vessels are generally in the form of a round-bottomed bowl. The decoration is incised, and simple in style.

Brittany and Ireland were among the chief centres for the megalithic voyagers of the Neolithic era. The passage-graves in both those countries are often highly decorated with various sculptured designs. Of these the most important are at Newgrange in Ireland, and at Gavr'inis in Brittany ; in both places the whole of the inner faces of the stones is sculptured. The stream of trade, which carried the megalithic folk from Spain northwards, seems to have divided into two parts. One went up to Brittany ; the other went up the coasts of Ireland and Scotland.

LESSON 15

The Near East in the Daylight of History

History may be said to have begun on the banks of the rivers Nile and Euphrates. Here written records go back 5,000 years or more, and it is at the point where writing first conveys political information that history begins.

In Egypt the age coincides with that of the unification of Upper and Lower Egypt by the warrior-king whom later tradition called Menes. Thereafter the Pharaonic period of Egyptian history has been subdivided into dynasties (following the Egyptian priest-historian of Ptolemaic time, Manetho), and for the convenience of archaeologists into the wider classification of three main phases of greatest artistic activity : Old Kingdom, Middle Kingdom, and New Kingdom (*see* chart).

In Mesopotamia history may be said to begin, at approximately the same date, during that phase of the development of the city states at which the king lists cease to be myth and begin to contain names known to be historical.

There is a third area where civilization arose early, perhaps as early as in the Near East—the Indus valley of India (*see* Lesson 25). Here a developed script is found in association with a high level of material culture by at least 2400 B.C. But this script cannot yet be deciphered, and the Indus civilization cannot therefore be understood in the light of history. There is neither legend nor record to give any clue to the identity of the builders of magnificent cities like Mohenjo Daro and Harappa.

In the relatively short space of time, perhaps 200 years at most, which elapsed between the beginning of the dynastic period and the age of the great pyramid builders, the technical and artistic progress made by the Egyptians is astonishing. During this time the first tentative symbols scratched on slips of ivory had developed into the conventional inscriptions, beautifully designed and carved, of the Old Kingdom tombs. The rough stone floor of a royal tomb of the first dynasty led the way, two dynasties later, to the smooth, towering bulk of the pyramids.

Pyramids and Temples in Egypt

The unification of the country by the 1st dynasty kings (thought by some scholars to have been foreigners, or at any rate to have brought in ideas from outside) rapidly brought prosperity. Ample local resources in fine building stone were supplemented by trade. The early pharaohs sent expeditions to bring copper from Sinai, timber from Lebanon, incense from Arabia or Somaliland, and gold, ivory, and other luxury products from the Sudan.

Under the absolute despotism of the Old Kingdom pharaohs, large estates poured their produce into the royal granaries. Unlimited

TABLE OF MAIN HISTORICAL PERIODS IN THE NEAR EAST

EGYPT		Dates (approx.) B.C.	MESOPOTAMIA	Dates (approx.) B.C.
Archaic period	Dynasties 1 and 2	3000–2750		
Old Kingdom	„ 3–6	2750–2200	Early Dynastic period	2600–2350
1st Intermediate period	„ 7–10	2200–2100	Akkadian period	2350–2200
Middle Kingdom	„ 11 and 12	2100–1785	3rd dynasty of Ur	2100–2000
2nd Intermediate period	„ 13–17	1785–1580	1st dynasty of Babylon	1895–1595
			Early Assyrian Empire	1850–1780
New Kingdom	„ 18 and 19	1580–1200	Cassite period	1595–1154
Later Empire	„ 20–24	1200–715	3rd–10th dynasties of Babylon	1154–626
			Assyrian Empire	1189–612
Ethiopian and Saite period	„ 25 and 26	730–525	Neo-Babylonian dynasty	625–539
Persian period	„ 27–30	525–330	Persian period	539–330

man-power was at their disposal. With imagination and daring the royal architects designed temples for the gods with great monolithic granite pillars or delicate fluted columns. Massive piles of stone with no visible entrance —the pyramids—guarded the bodies of their royal masters, instead of the brick " mastabas " of the early dynasties which had proved vulnerable to the attacks of tomb-robbers.

Pyramids and temples alike were built by means of huge ramps of earth, gradually raised, up which the stone blocks were rolled on sledges and levered into position. The stone was dressed and finished, and as the earth was removed, the stone surface was carved or polished from the top downwards ; the lowest course of stone, first set in position, was the last to be finished. By means of earth ramps, too, colossal statues or obelisks could be raised on their pedestals.

Knowledge from the Tombs

Much present knowledge of life in ancient Egypt is derived from the excavation of tombs. The Egyptians had a passionate desire for survival, and they thought of the after-life as a continuation of life on earth. They sought to provide the dead with all the comforts and necessities of his earthly life, and buried tools, weapons, clothing, and furniture, and even food and drink, in the tomb to satisfy his needs.

They carved on the walls of his tomb, if he were wealthy enough to have a stone-built one, scenes of his pleasures and occupations, in the hopes that, by means of magical formulae, these scenes might come to life and be repeated for all eternity. Their concept of bodily survival led also to the craft of mummification and, as a precaution lest that craft would prove imperfect, to the school of realistic portrait sculpture in which Egyptian art excels.

The First Mesopotamian Civilization

The civilization of the Sumerian city-states was slower than that of Egypt to develop. The Sumerians enjoyed no meteoric career of political or cultural brilliance during the second millennium. But the gradual infiltration of vigorous peoples of Semitic speech resulted in the establishment first of an Akkadian dynasty, whose leaders opened trade routes into Cappadocia and campaigned in Northern Syria and Armenia, and three centuries later of the Babylonian dynasty of Hammurabi and his kinsfolk. The infusion of new energy into the old culture gradually transformed it and gave it new life.

The basic features of Sumerian civilization continued throughout Babylonian history. The pantheon of gods, though their names changed and they received Semitic colleagues into their ranks, retained their ancient shrines, their traditional insignia, and their main attributes, and were worshipped with Sumerian ritual and liturgy long after Semitic Akkadian had become the speech of everyday life.

HAMMURABI THE LAW-GIVER. Head of the stele of Hammurabi, showing the king receiving instruction from Shamash the sun-god. The stele was inscribed with a summary of laws in 182 paragraphs.

Ziggurat-temples, clay-tablets written in cuneiform, cylinder seals, mud-brick architecture with recessed wall-decoration, all continued to be typical features of Babylonian civilization.

On a number of sites in Northern Syria, South-eastern Turkey, and Iraq, cities of the 18th century B.C. have been found in which very similar objects occur, testifying to a similarity of civilization in this area. A fortunate find of diplomatic archives in the city of Mari, on the Euphrates, shows that these cities were in close commercial relationship and that their rulers maintained constant communication, by letter and oral message.

Hammurabi of Babylon

One of the correspondents was Hammurabi, king of Babylon. It is possible to trace his gradual policy of encroachment, his subtle and unscrupulous diplomacy, until he was able to conquer most of his neighbours and rule for a time considerable domains.

He is known now chiefly as an administrator ; his code of laws and administrative correspondence show him to have been an able organizer, and illustrate the political organization, social customs, and daily life of the period. Babylonian religious literature contains primitive elements, but the prayers reveal a high ethical tone, and a well-developed consciousness of sin and sense of personal responsibility.

LESSON 16

The Bronze Age in Europe

THE Bronze Age was an age of trade and the intercourse of nations peaceful on the whole, but with interludes of savage warfare. Both copper and tin are required for making bronze, and though copper is sufficiently common, tin is comparatively rare ; therefore those peoples who inhabited the countries where tin occurs were brought rapidly into contact with other civilizations.

Equally fortunate were the dwellers by the sea or by the side of large navigable rivers, such as the Nile, the Euphrates, and the Indus. But the very factors which brought to these countries riches and leisure to develop the arts of life roused the cupidity of other peoples, so that war between nations often had economic motives, though the pretext might be political.

Antiquity of Troy

Among the most important of the maritime peoples were those who lived in Anatolia (Asia Minor) on a site which was called by the Greeks in later times Troy. It was the key position for all trade between the Mediterranean and the Black Sea, and thus was in touch with the Danube and the Caucasus, as well as with Crete and the whole of the eastern Mediterranean. It is not surprising, then, to find that a settlement existed there from remote antiquity.

On this site one city after another rose on the ruins of its predecessor, till seven or eight cities of varied importance had risen and perished, leaving only a huge mound to cover their ruins. The lowest level is known as Troy I, Homer's Troy being Troy VIIa.

The first settlement was hardly more than a large village, but even at this early period bronze was coming into use. Troy II is of the utmost importance in any study of conditions in the Bronze Age in Europe. It was a big city ; its fortifications had stone foundations with brick battlements, and within the circle of the defences the houses were large and handsome. Troy II must have existed for many centuries, and perhaps had resisted more than one warlike attack, for the protecting wall had been rebuilt twice.

Then came disaster ; the city was stormed, and so ruthlessly destroyed that it remained desolate for many generations. The inhabitants of the captured town, however, had buried much of their wealth before the conquerors burst in and sacked the city, and from these hoards has been deduced what the civilization of Troy II was like. Small villages succeeded one another on the site till the rise of Troy VI, which about 1500 B.C. again dominated the trade of the Mediterranean and the Black Sea. This city had commercial relations with Mycenaean Greece.

FROM TROY TO YORKSHIRE. On pottery found in the Bronze Age settlement at Troy, the second city on the site, a characteristic owl-face design appears. Trade communications carried this and other designs all over Europe, and as far away as Yorkshire such designs carved on Bronze Age chalk drums (top) are found. Similar designs are found in Malta, Greece, and Spain.

From Schliemann "Atlas Trojanischer Alterthümer", and British Museum

The finds belonging to Troy II show cultural relations with other parts of the Near East and Europe. One remarkable example is the "owl-face" ornament on pottery and other objects, which is found in the Greek islands, in Malta, in Spain, and as far away from Troy as Yorkshire. In Saxony finger-rings of gold wire coiled in spirals, in Sicily bone ornaments, in Spain small figures of deities, all show that objects from Troy II were carried in trade right across Europe.

The Minoan Civilization

The whole of the Aegean area exercised a vast influence over Europe and the Near East, but as the chief excavations have been made in Crete and Troy it is natural to regard those two places as the principal centres from which civilization radiated.

Cretan objects of the Early Minoan I period are found in Egypt with

objects of the 1st dynasty, and Egyptian objects of the same date are found in Crete; while in the islands near Crete are copies in local stone of the Egyptian stone vases of this early period.

Trade did not end with the 1st dynasty, for stone bowls and perfume vases of the characteristic form of the Egyptian Old Kingdom are found in Crete, and also the Cretan copies of those forms. Again, in the 12th dynasty, vases of the fine polychrome Kamares pottery are found in Egypt, exactly dated to the reign of Senusret II, while scarabs of that king are found in Crete. Such synchronisms are of the utmost importance since they provide a chronological framework for the successive phases of Bronze Age civilization in Crete which are now named Early, Middle, and Late Minoan (after Minos, the legendary king of Crete).

Crete was in constant trade communication with the islands of the Aegean, where independent little civilizations arose, of which the most notable is the Cycladic. The Cyclades group was too small to support a large population, but their mineral and metallic resources, added to their geographical position, gave the islands unusual importance in the early Bronze Age. The group lies between Asia Minor and the mainland of Greece, and thus offered convenient ports of call between Crete, northern Greece, and the Black Sea.

EUROPEANS OF THE MIDDLE BRONZE AGE. Each detail of clothing, weapons, and ornaments of these figures of people of the second millenium B.C. is copied or inferred from archaeological finds. The woman's necklace, belt, and cloak-pins, and the jar she carries, are like those unearthed from a barrow-grave in Wurttemberg. The man's sword is of a type found in what is now Hungary. Fragments survive of cloaks, jerkins, and half-boots worn at the same period.

Westward Spread of Culture

The Bronze Age civilization of western Europe seems to have owed its origin to the Aegean culture. Carried by trade from the eastern Mediterranean, it first took root in Almeria on the east coast of Spain, spreading round the Iberian coast and also inland.

The aboriginal inhabitants of Spain and Portugal on the Atlantic sea-board were already hardy and daring mariners, and as traders they carried the new civilization north to France and the British Isles.

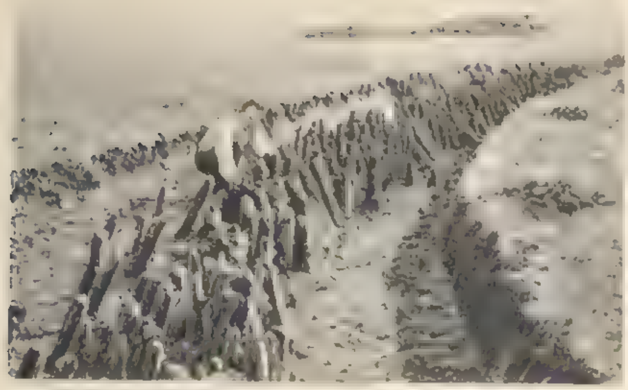
To some extent these traders may have been responsible for introducing in Ireland the more elaborate forms of passage-graves, and a very few objects, such as bone pins and flint spearheads, which are distinctively Portuguese in form. Their most important contribution was to introduce the working of gold and bronze, so that at a very early date Ireland was exporting the products of these industries to other areas still in the Stone Age—to Britain, north Germany, and Scandinavia.

After its promising beginning the Bronze Age culture of Spain and Portugal seems to have been cut off from contact with the centres of east Mediterranean civilization (though some customs, such as jar burial, may show relationships of some kind with Anatolia). A long period of stagnation set in which lasted until the Late Bronze Age. The reasons for this may lie partly in the interception of trade by inhabitants of islands lying nearer to the eastern centres, and partly in the diversion of traffic between Europe and the Near East to the inland Danube–Brenner routes.

Coming of the Beaker Folk

At the end of the Neolithic period there appeared in many parts of Europe, and in the British Isles a new people, now called the Beaker folk after the distinctive kind of pottery they used. Their place of origin is uncertain; they were nomadic, moving about in small groups, and probably depended more on flocks and herds than on cultivating the soil. Their favourite weapon seems to have been the bow, with arrows tipped with barbed-and-tanged heads; on their left wrists they wore archers' guards of stone or bone as protection from the snap of the bow-string.

Some of them may have been aggressive invaders, dominating the populations of the areas they entered, for nearly everywhere



VILLAGE STRONGHOLD IN THE LATER BRONZE AGE. At this period villages in various parts of central Europe were heavily fortified. In this remarkable example in Upper Swabia, the village stood on an island and was completely surrounded by a palisade of more than 50,000 pine trunks driven into the bed of the lake. The buildings had outer walls of heavy timbers arranged in regular courses like log cabins. The findings suggest an era of frequent warfare.

native cultures seem to have been modified by their advent. But it is unlikely that they exterminated native populations ; soon or late they were absorbed by the latter, whose numbers must have been much larger.

There is little evidence that the Beaker folk themselves manufactured objects of gold or copper. But they provided a market for these and, owing to their nomadic habits, doubtless were responsible for opening up new trade routes. Their appearance in central and western Europe and in Britain heralds the opening of the Bronze Age.

On the broad and fertile plain of the Danube the Neolithic farmers (Danubians) had vastly increased in numbers, for the loess (of which the plain mainly consists) is one of the finest soils for growing grain. Eventually the Danubians, following the loess deposits, spread as far west as Belgium.

The Aunjetitz Culture

Partly under the stimulus of contact with the Beaker folk there arose, on the middle and upper Danube and along the adjacent rivers, the Aunjetitz culture. Other stimuli were also coming in from Anatolia, for in the earliest deposits of this culture are found objects belonging to the period of Troy II.

Soon a brilliant local Bronze Age developed, and as the Aunjetitz territory lay on the amber route from the Baltic to the south its inhabitants amassed great wealth and were able to bury their chieftains in princely style accompanied by costly grave-gifts. They had trade relations with countries as far distant as Scandinavia and southern Britain.

There is little evidence that the peaceful Danubian farmers were ruled by princes or chieftains or that any individuals were wealthier than others. But with the Bronze Age came a rapid change : the many daggers and swords show that it was necessary, at least for the prosperous, to be armed ; and the special tombs and rich grave-gifts of a few burials show that some sort of aristocracy had arisen. Thus the new economy altered the structure of society.

In the part of the Danubian plain which is now Hungary, metallurgy was extensively practised, and many objects in bronze were manufactured and exported up and down the Danube, and into Germany, and as far as the Ukraine. The ancient Hungarians wore ornaments which were not made elsewhere ; one, which is very peculiar, was a bronze ribbon wound spirally round the arm or leg.

Other ornaments were bronze pendants for girdles, and for necklaces. Occasionally gold was used for this purpose ; though it is rare, the mere fact that it occurs indicates the wealth of these communities. The bronze pendants were made chiefly for export : the commonest type is in the form of an ivy leaf. Copper axes with a hole for hafting appear in south Russia and Hungary ; they are of Mesopotamian type, and show that intercourse with the Near East was constant. Another indication of the contact is the occurrence in central Europe of the Mesopotamian " adze-axe." More positive proof of the continual and close connexions with more easterly peoples is afforded by the houses with porches, and the concentric fortifications.

Finds in the Black-earth Lands

Stretching right across Galicia, Wallachia, and southern Russia, is a wide belt of park-like grassy country, dotted with woods and clumps of trees. These areas, called in Russia black-earth lands, are of great importance for the study of the early civilization of central Europe.

Just to the north of the Transylvanian Alps there once stood a small city, of which the modern name is Erösd. It was perched on a precipitous spur of the mountain, a site obviously chosen as being easily defended because only one side needed fortifying. There were three levels of occupation, and even in the lowest, metal was found.

The pottery on this site is very remarkable. The vases were painted with spirals and meanders in various colours, and the ware itself was fine and well fired. Part of an actual kiln and a pottery model of a kiln were discovered, so that the exact method of firing the pots can

be seen. The houses were rectangular, with posts to support the walls of mud and interlaced boughs.

In Rumania, at a site called Cucuteni, pottery painted with spiral designs has been found. Like Erösd, this settlement was fortified with a walled rampart of stones and a deep ditch.

The black-earth lands are inadequately explored as yet. A few sites on the Dnieper have been cleared ; in the largest of these, known as Tripolye, two periods can be distinguished, which are labelled A and B. The pottery on these sites had the same kind of painted spirals as that found farther west. Metal was known, and the ancient inhabitants of these villages possessed all the domestic animals, but it is not certain that they were cultivators of the soil.

The spread of the painted pottery gives the clue to the origin of much of the civilization of central Europe in the Bronze Age. Obviously it came from the east, and the evidence points to the Caucasus and Asia Minor.

LESSON 17

The Bronze Age in the British Isles

THE manufacture of bronze axes and of gold ornaments began earlier in Ireland than elsewhere in the British Isles, and the new techniques were introduced by metalworkers from Spain and Portugal. These Irish products were exported to Scotland and England (where it is thought that the Beaker folk helped to open up markets) and to western and northern Europe.

Denmark, which has no deposits of metal ores, and Sweden, whose ores were not worked in early times, received great numbers of Irish products, and it may even be that Irish smiths settled there and helped to develop the Scandinavian bronze industry, using imported ores. Thus at the beginning of the Bronze Age the British Isles played an important part in the culture and economy of Europe.

The Wessex Culture

The Early Bronze Age is the period of the Wessex culture, which was chiefly centred on Dorset, Wiltshire, Hampshire, and Berkshire. Here, in the mixed population resulting from the absorption of Beaker folk into the large and prosperous native Neolithic communities, there suddenly emerged in the 16th century B.C. a class of wealthy chieftains.

As in the Aunjetitz culture of central Europe, these chieftains were buried with their most costly possessions : decorated gold plaques, handsome bronze daggers, and stone maceheads (the symbol of authority). They were buried singly, each under his own round mound or barrow. This form of single-grave burial under round barrows was introduced to Britain by the Beaker folk and supplanted collective burial as practised in Neolithic times.

In the Wessex culture women, too, were buried with costly possessions—bronze knives and awls, beads of jet, amber, and a vitreous blue paste (faience). The grave-goods in the Wessex barrows show that these people were in trading relations with all the major centres of civilization. The amber is thought to come from the Baltic ; there are bronze pins from Aunjetitz ; bronze daggers from Aunjetitz and Brittany; the faience beads must have come from Egypt or the Near East. And connexions with the Aegean are seen in the gold cup from Rillaton, Cornwall, and in the gold-bound amber discs, two of which are known from Wessex graves and another from a Late Minoan II tomb in Crete.

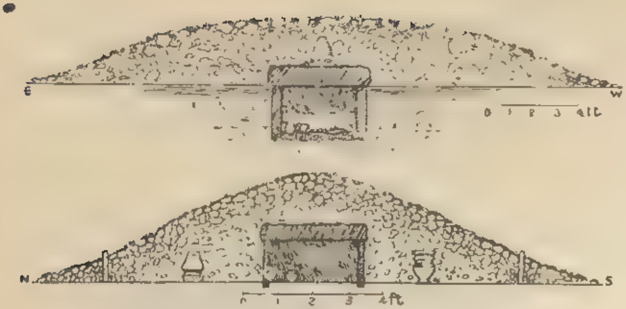
The wealth of the Wessex culture must have derived from trade. These chieftains perhaps were acting as middlemen in the distribution of Irish bronzes and also of Baltic amber ; a branch of the great north-to-south amber trade route may at this time have been diverted to pass through southern England. No settlements of the culture are known, but agriculture, stockbreeding, and hunting must have provided the necessities of life.

Contemporary with the Wessex culture, but outside its confined area, there flourished still some Beaker folk and, especially in Yorkshire, Scotland, and Ireland, the Food Vessel culture. This was a mixed culture of Beaker and native origins, characterised by a special kind of pottery known only in the British Isles. This culture was much less wealthy than Wessex, and the objects found most commonly in the graves, in addition to pots, are jet beads and flint knives.

Middle and Late Bronze Ages

The Wessex culture lasted about 200 years, and after its disappearance there are no more rich Bronze Age burials in Britain. Burial rites changed, and although round barrows were still being built it became customary to cremate the dead and place the burnt bones under the barrow, covered by an Overhanging-rim Urn. Sometimes the same barrow was used several times, the later deposits being placed in pits dug into the mound. Few objects other than urns are found with cremations. Hardly any settlements of this period are known.

The Middle Bronze Age is marked by progressive developments in bronze tools and weapons. Axes developed into palstaves, and



ROUND BARROWS IN SECTION. A widely prevalent type of grave in the Bronze Age was the round barrow composed of heaped earth and rubble. Many, like the Northumbrian examples seen above, contain stone receptacles or chambers in which the body was placed.
British Museum

daggers into long rapiers and spears. Ornaments now were mostly in the form of torques; twisted strips of bronze or gold worn round the neck or arm.

The Late Bronze Age saw a great increase in the amount of metal available and in techniques for its manufacture. In the earlier phases of the Bronze Age, metal was used chiefly for weapons and ornaments. With the important exception of axes and palstaves, essential for tree-felling and wood-working, it was little used for tools. But in the Late Bronze Age chisels, gouges, hammers, sickles, razors, and even large buckets and cauldrons, could be made of bronze.

To this late period belong the "founders' hoards," the stock-in-trade of bronze-smiths, who travelled the country buying up broken and worn-out implements, melting them down, and producing replacements for individual customers. Important new types introduced at this time were socketed axes, swords, and shields.

In a cave at Heathery Burn, Co. Durham, was found the equipment of a family of Late Bronze Age smiths, which included antler cheek-pieces (used instead of metal bits for controlling horses), and bronze nave-bands from wagon wheels. This is the earliest evidence in Britain of wheeled vehicles and the use of horses to draw them.

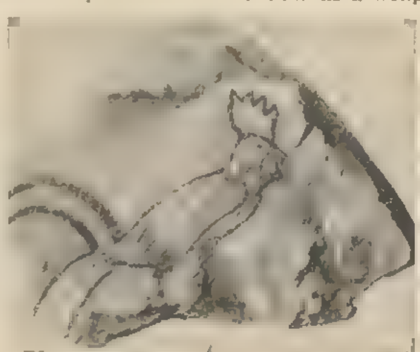
Burial customs were the same as in the Middle Bronze Age, though sometimes the cremated bones and urns were buried in flat cemeteries rather than in barrows. At the very end of this period a few objects from the European Iron Age Hallstatt culture were introduced.

LESSON 18

Ancient Egypt as a World Power

DURING the early part of the second millennium B.C. great movements of peoples took place in the Near East. The Hittites, a people of Indo-European speech, moved down into Anatolia and took possession of the rich metal-producing lands of Cappadocia. Cassite armies from North-western Persia swept down on Babyonia, and c. 1600 B.C. established their dynasty in the ancient capital. In Syria and Palestine, the Hurrian peoples from their mountain home came in increasing numbers to conquer and settle. All these peoples, with leaders speaking Aryan tongues, brought with them the use of the horse and light chariot.

The chariot arrived in Egypt with the Hyksos, Palestinian Semites who seized the throne and dominated the Delta. Their civilization was of a lower type than that of the people they conquered; the objects they have left are few, and as they were ignorant of writing, the records of their invasion and subsequent expulsion were made by their enemies. They appear to have been horsemen and archers. The Egyptians could never be induced to accept the bow as a weapon, but they took to horse breeding; at the beginning of the Iron Age they were exporting horses.



EARLIEST DRAWING OF A COCK that has survived is this one, found on a tomb of the XVIIIth Dynasty, during which Thothmes III introduced into Egypt many plants and animals, including domestic fowls.
British Museum of Natural History

The Hyksos were driven out, and Egypt became a united country under a dynasty of indigenous rulers. But as the Nile valley can never be secure from invasion unless the countries to the north and south are held in check, the warrior-king Thothmes III found himself obliged to subdue and hold Syria. This conquest is an important factor in the development of religious and social ideas in the Near East and throughout Europe.

Up to this time raids and invasions were well known—raids in which the victors massacred young and old and retired to their own country

FUNERAL TEMPLE OF HATSHEPSUT AT DEIR-EL-BAHRI. Partly built and partly rock-hewn, the great queen's temple rises in three terraces below the crags. Its famous reliefs tell the story of the expedition she sent to Punt (possibly Somaliland).

with all the booty on which they could lay hands; and invasions when the conquerors settled in the conquered country after killing or enslaving the population. Now, for the first time, an attempt was made to hold and organize conquered territory so that it might be exploited for the continued benefit of the conquerors.

In the capacity of general Thothmes was daring and cool, and he won the confidence of his army; as a conqueror he was merciful and generous; as a statesman he was far-seeing. When a town was taken by assault, there was no massacre or sack, as in the wars between the Israelites and the Midianites; terms were offered and the population was spared.

Thothmes III as Overlord

Egyptian governors were appointed over the conquered districts; or, if the local chiefs were prepared to swear fealty to the Egyptian pharaoh, they were left to rule the country, provided they ruled it in peace. Tribute, in the form of luxury goods, agricultural produce, and livestock, came in yearly to swell the coffers of the palace and the treasuries of the temples.

References to Syria in the Egyptian and Israelite records show the country split up into a number of little principalities, each governed by its own king. Wars between these little states were common, and political intrigues were rife. Syria suffered continually from civil war or from invasion. Thothmes III introduced the idea of peace to these warring peoples, who found that subjection to Egypt meant safety to themselves, as their suzerain was strong enough to protect them against all enemies.

CRETE AND EGYPT. The Late Minoan vase borne by this Aegean envoy to Egypt is evidence of commerce between the powers.

Another important step was now taken. In Egypt, as in all other countries, each district had its own tribal deity, who was regarded as the only god in his own territory. When the princes of Thebes united all Egypt under their rule in order to expel the Hyksos, the local god of Thebes, Amun, became the supreme god of Egypt; and when the pharaoh of Egypt became the overlord of other countries, Amun became thereby the overlord of the gods of those countries.

Power of the Priesthood

The material magnificence of the period is reflected in its architecture. The conquests and the trading expeditions of the early 18th dynasty, and the piety of the pharaohs, raised the priesthood to power. Temples were built whose very ruins still fill the beholder with wonder. Amun, and the other gods were honoured with endowments of gold, lands, and buildings such as Egypt had never before seen. Costly stones from distant quarries decked the temple shrines, and golden vessels were dedicated to the service of the god.

With the expansion of the Egyptian empire, new interests came into the country. The famous Queen Hatshepsut and her husband and immediate successor, Thothmes III, must be credited with the first recorded attempt to acclimatise foreign plants and animals in Egypt. Hatshepsut's expedition to the land of Punt, on the shores of the Red Sea, brought back

THIS BATTLE-AXE from Ras Shamra (Ugarit), here mounted on a dummy haft, was made by Mitannian craftsmen in the second millennium B.C. The head, ornamented with a boar's head and two lions' masks, was ingeniously shrunk on to the blade, obviating the use of rivets.

many incense-bearing trees, and these were planted in pits of earth to form a shady avenue to the gate of her temple at Deir-el-Bahri ; others were planted in the garden in the outer court. Subsequent excavation has laid bare these pits of earth, which still contain the roots of the trees.

Thothmes endowed his temple at Karnak with a garden of strange plants and animals, and his artists depicted on the walls of one of the chapels examples of these exotic creatures and flowers. One of the artists who drew the plants was also something of a botanist, for he has represented several consecutive stages of the germination of a seed. These are, as far as is known, the oldest scientific drawings in the world.

Egyptian Trade

The position of Egypt as the great world power in the 17th and 16th centuries B.C. brought her into contact with other lands and peoples; it is possible to gauge, from her records and from the objects found in her tombs and towns, the culture at which those peoples had arrived.

CRAFTSMANSHIP OF ANCIENT AFRICA. Imports into Egypt from the south included ebony stools inlaid with ivory. This one was part of the treasure in Tutankhamen's tomb.

Trade in resins and spices for perfumes and for religious and burial purposes was carried on with the land of Punt. The Egyptian expeditions bartered beads, trinkets, and weapons for raw materials, such as frankincense, gold in dust and in rings, and wood of different kinds, as well as elephant tusks and panther skins.

The sketches made by the artists accompanying Hatshepsut's expedition give a vivid picture of the landscape and houses of Punt, as well as the costume and ornaments of the inhabitants. Ebony stools, inlaid with ivory, and with leather seats, were the products of Africa. Elaborate but barbaric goldwork also came from Africa, and with it were brought unworked ebony and ivory, and stuffs woven in brilliant red and yellow patterns.

But it is from the north that the most interesting products came. Syria, Mesopotamia, and the Aegean all traded with Egypt and vied with one another in Egyptian markets. The North Syrian kingdom of Ugarit (Ras Shamra) was a centre of international commerce.

Excavations have brought to light immumerable examples of jewelry in gold and lapis lazuli, amber and cornelian beads, copper and ivory statuettes, the oldest "steel" weapon known—a battle-axe—and evidence of a remarkable system of drainage, and of well-equipped stables for race-horses. Later the Egyptian protectorate over Syria brought the two countries into close contact, Egyptian products being carried into Syria in return for Syrian manufactures.

Foreign Craftsmanship in Egypt

Syrian artisans entered Egypt, sometimes as prisoners of war, sometimes as free men, and brought their crafts with them. This is perhaps why glass-making became so great an industry in Egypt at this time. At first glass beads were made only in black and white: then, under Amenhotep III and Akhnaton, the whole range of colour was used for beads and small vases. Inlaid glazes, which had not been seen from the time of the 1st dynasty were re-introduced, and at Tell-el-Amarna were brought to perfection.

Syria was famous for its metal-working; gold vases and bowls, chased or repoussé and often jewelled, were brought as tribute to the pharaoh. Aegean islanders, among them perhaps Cretans, sent gifts ; the ambassadors

wore loin-cloths woven in brilliant colours. Some of the pottery found in Egypt at this period is probably from Syria.

In the reigns of the Tell-el-Amarna pharaohs (Amenhotep III, Akhnaton, and Tutankhamen) foreign imports are found and foreign connexions are strongly visible. It is possible that the impulse given to the art of that period came from abroad, though the actual sculpture and painting were carried out by Egyptian artists. Babylonia and Mitanni also sent their products to Egypt. Among the mass of objects placed with the dead pharaoh were three large couches, a throne, and a footstool, all of wood overlaid with gold, some being decorated with ivory. The beds are made in four pieces and are put together with bronze hooks and staples, obviously for convenience of carriage.

LESSON 19

Crete and Mycenae

DURING the Bronze Age a brilliant civilization arose in the Aegean quite different from, though not divorced from connexion with, the Near Eastern civilizations which surrounded the Eastern Mediterranean. It is noticeable that the great towns of Minoan Crete and of the Helladic mainland were built, almost without exception, in close proximity to harbours, without regard to the agricultural value of the land—an indication that maritime trade and perhaps piracy were of more importance to the islanders than farming.

Minoan Palaces

In the Middle Minoan period (corresponding to the Middle Bronze Age or the Middle Kingdom in Egypt) the two great towns were Knossos on the north and Phaistos on the south coast of the island, in both of which there were magnificent palaces. It is uncertain whether these were originally the capitals of two principalities, but at a later stage the two palaces were joined by a royal road across the island.

Knowledge of the civilization of Minoan Crete is now almost entirely dependent on the archaeological evidence of material finds since the records, imperfectly deciphered, are of a late period only and there is nothing that corresponds to the historical inscriptions of Egypt or the political correspondence of the Hittites, Syrians, and Babylonians.

Cretan palaces lay open to all comers. They had no ramparts, moats, or other warlike defences. In this they differ from palaces of the Mycenaean age on the mainland of Greece, with their Cyclopean defences. The inevitable conclusion must be drawn that an island which had command of the sea needed no fortifications.

The palace of Knossos—the Labyrinth of tradition—still shows ample evidence of the enormous wealth that was lavished on its decoration. The few frescoes which have escaped destruction give a faint idea of the beauty of naturalistic design which adorned its walls. But practical architects, as well as great artists, had taken part in the design, for great attention had been paid to the water supply, surface drainage, and sewerage. It has been said that Europe took more than three thousand years to regain the knowledge of sanitation lost when the Minoan empire collapsed.

Crete was at her zenith about 1600 B.C., when Egypt was still stagnant under the paralysing rule of the Hyksos. Tradition places Minos at this period, visualising him as a single king, though probably the legendary figure embodies a dynasty. Written documents begin to appear at this period, and the linear form of

STORE-ROOMS OF THE PALACE OF KNOSSOS. The "Corridor of the Magazines" in the palace is 200 ft. long, flanked on one side by 22 narrow store-rooms containing a large number of immense jars. These were probably receptacles for grain and oil brought from overseas.
From Daniel Baud-Bovey and Frédéric Boissonns, "Des Cyclades en Crète"

script, long undeciphered, is now recognized to be a syllabic system, devised for a language which was presumably the original tongue spoken in Crete, imperfectly adapted to write a primitive form of Greek.

That the lords of Knossos were early Greeks is therefore obvious. Similar tablets in primitive Greek are found at Mycenaean sites on the mainland, and this should argue a kinship between the Cretans of Knossos and the warrior-lords of Mycenae, Tiryns, and Pylos. But the tablets are of little historical value; they seem to be lists or dockets, parts of the economic registers of the palace, and it must be assumed either that the Minoans and Mycenaeans had no written literature and no history, or that they usually wrote on materials which have perished.

Minoan Pottery

The commerce of Crete carried her wares, particularly her pottery, to all parts; and this was not surprising, as the pottery was unsurpassed in beauty of decoration. Beginning, in the Middle Minoan II, with the polychrome Kamares ware—the colours laid on a dark ground—it passed to white line drawings on a pale ground, and then to dark designs on a light ground. The designs were spirals and conventionalised forms of natural objects, especially shells and marine creatures. One of the most characteristic motifs was the octopus.

OCTOPUS MOTIF. The Late Minoan potters excelled in the portrayal of marine life, as witness the octopus amid shells and seaweed on this jar.
British Museum

End of Knossos

The palace of Knossos was destroyed by fire at the end of Middle Minoan II, but it rose again soon afterwards in greater magnificence. It was not the exclusive personal abode of a monarch and his court, but a vast communal building. Among the remains are the "Corridor of Magazines," and the throne, a dignified chair of white stone. It lasted till the beginning of Late Minoan III; then invaders broke in, and sacked both town and palace with such violence that it has lain desolate and in ruins ever since. Fugitives from the invasion sought refuge in other parts and carried with them some of the Minoan culture, thus prolonging for a time the remembrance of the island civilization.

Cult of the Bull

An interesting part of the culture of the Cretans was their fondness for sports and public games. "The dancing-ground which Daedalus made in broad Knossos for fair-haired Ariadne" has been identified with a paved area, which has rows of steps at the sides on which the spectators could sit and watch the shows.

Bull-games were common, as is proved by frescoes where youths and girls are represented leaping over a charging bull; there is nothing, however, to show that the bull was killed, as in a modern bull-fight. There was probably some degree of sanctity attached to these displays, for one of the deities of Knossos seems to have been the Minoan bull, which was stabled within the palace itself, and the legend of the Minotaur hints at dark rites in connexion with this cult.

The Tell-el-Amarna letters—a collection of diplomatic correspondence called after its place of discovery—show the condition of trade in the Near East in the 15th century B.C. Communication was fairly regular, and envoys plied constantly between the courts of the great powers—Egypt, Babylonia, Assyria, Mitanni, Hittite-land, and Alashiya or Crete. On the land routes local princes were charged with the duty of protecting and providing for caravans which passed through their provinces.

Trade and Piracy

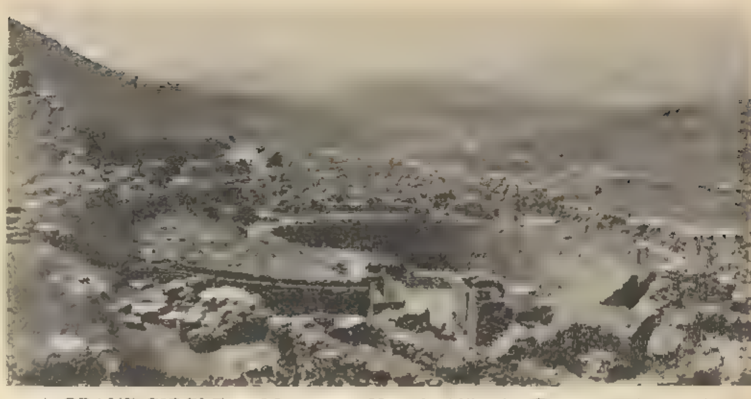
On the sea there was always the danger of piracy, and measures had to be taken to secure the safety of merchant vessels. Custom-houses were established both in Egypt and in Crete for the commodities which paid duty.

The state of trade and intercommunication in the eastern Mediterranean at this period must have been very much like the conditions in that part of the Near East after the fall of Roman power until organized policing of the seas in the Napoleonic wars put an end to piracy.

On the mainland of Greece another brilliant manifestation of Aegean culture began at about the same time, *c.* 1600 B.C. At Tiryns, Mycenae, Athens, Pylos, and elsewhere, strong cities arose which were in commercial contact with Crete but maintained their individual culture.

Tiryns was founded on a low rocky hill rising out of the swamps of the plain of Argos, about a mile from the sea. An earlier village, probably inhabited by aborigines who used obsidian implements, crowned the hill-top; the natives were ejected by the colonists, who made a settlement surrounded by a wall of megalithic blocks built without mortar. The wall was from 25 to 50 feet thick, with passages in the thickness. The citadel, perhaps also a

alace, contained a num-
: of rooms and court-
irds, with colonnades
nd flights of steps. The
escoes on the walls are
finitely Cretan in char-
cter, particularly the
ull-leaper, which must
ive been painted by a
etan artist. The ala-
aster frieze is of excep-
ional interest because
is carved and inlaid
vith blue glass which is of the same date as the glass of Thothmes III, or possibly a little earlier.



A GRAVE CIRCLE at Mycenae. Here in 1876 the German archaeologist Heinrich Schliemann found much treasure in the shaft-graves. Near by, in 1952, a second circle containing 16 royal graves was discovered by Professor Papadimitrov.

Golden Mycenae — Mycenae of the Wide Ways—began as a small but strong citadel on a rocky inland hill. In the rock, shafts were dug for the burial of the Mycenaean princes. Here many of the wonderful golden objects were of Cretan workmanship. Though the technique is often very fine, the artistic beauty cannot compare with similar objects of Egyptian work; the gold masks of Mycenae, if placed beside the gold mask of Tutankhamen, emphasise the crudeness of this early phase of Mycenaean art. The wealth of the city must have been immense as the rulers could afford to bury so great a mass of treasure. At a later date the royal cemetery was enclosed with a circular wall of upright slabs, possibly the original gravestones.

TOMB OF AGAMEMNON. This most magnificent and best-preserved of the Mycenaean beehive tombs was named by its discoverers the Treasury of Atreus.

The goldwork of Mycenae, though splendid in some ways, is often disappointing in technique. Among the 700 circular and leaf-shaped plates of gold found in the shaft-graves there are only 14 designs, repeated over and over again without variation. They seem to have been produced either by casting from moulds or by hammering the soft sheet-gold into flat moulds. The same method must have been used to produce the gold buttons and the strange ornaments in the shape of cuttlefish. The whole of the goldwork is therefore a study in mass production, by which the greatest display of wealth is effected with the least possible cost of time and labour.

After the shaft-grave dynasty came to an end, a new family of princes rose to power. These increased the area enclosed by fortifications, and to them belongs the famous Lion Gate, which gave admission to the fortress. The gate derives its modern name from the sculpture of a pillar with two lions as supporters, which is placed above the entrance; the Cretan origin of the design is seen in the form of the pillar, which narrows to the base like its prototypes in the Labyrinth. The royal burials were at this period in buildings which are called, from their shape, "Beehive" tombs.

These tombs were scattered all down the slopes of the hill, and in spite of their solid masonry and thick walls they were completely plundered long before any written record of

GOLD CUPS from a Mycenaean tomb at Vaphio, near Sparta. The cup on the left shows a man leading captive bulls. The other shows a bull leaping.
National Museum, Athens

Mycenae was made. Remains of the royal palace show that the walls were decorated with painted stucco.

After the fall of Knossos (*c.* 1400 B.C.) and the utter collapse of Minoan culture, caused perhaps by an invasion from the mainland Greeks, only Troy remained to dispute with the Mycenaean Greeks of the mainland the supremacy of the Aegean. The story of the Trojan war, possibly, tells us how a Greek confederacy, led by the ruler of Mycenae, broke the power of Troy and destroyed the city.

Mycenae in her turn sent out colonies, which settled in the island of Rhodes, along the coast as far as Macedonia, in Cyprus, Corfu, Italy, and Sicily. The products of Mycenae were carried in trade to Egypt, Palestine, and Syria. The most easily recognizable of these products are the small vases known as stirrup-vases from the shape of the handles; they probably contained perfume, or some precious liquid.

Mycenae and Tiryns fell when barbarian tribes from the north invaded peninsular Greece, and by the end of the 5th century B.C. they had lost all their traditional glory and had degenerated into small and insignificant villages.

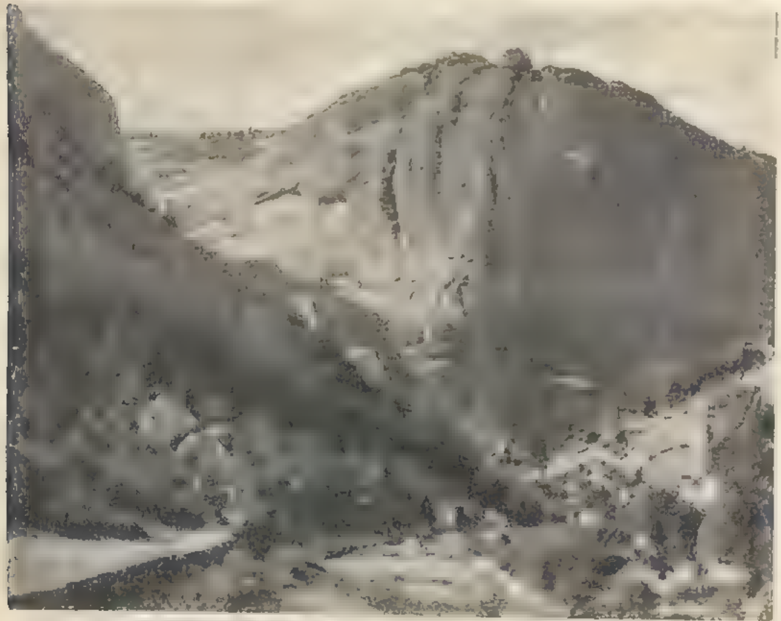
LESSON 20

The Heyday and Fall of the Hittite Empire

The Egyptians' greatest rivals at this period were the people known to them as Great Hatti, and to us, by the Biblical version of their name, as the Hittites. This people of Indo-European speech and high civilization established themselves on the Anatolian plateau not long after 2000 B.C., and by a gradual policy of conquest and annexation widened the bounds of their empire until by about 1400 B.C. most of Asia Minor fell under their sway.

One of the greatest of their kings, Shubbiluliuma, during the 14th century B.C. succeeded in winning from the Hurrians much territory in North Syria which had formerly belonged to Mitanni, and in gaining the allegiance of the North Syrian city-states which had hitherto been vassals of Egypt. The Amarna letters written to the pharaoh from these vassals show the success of his method of slow encroachment only too clearly; one by one they deserted their allegiance and went over to the Hittite.

From Ugarit have come a great number of tablets, found in the foreign office of the king of Ugarit, which are diplomatic documents from the time, 14th and 13th centuries B.C., when Ugarit was a dependant of the Hittites and obeyed the orders of "the Sun," as the Hittite king styled himself.



NATURAL DEFENCES OF THE HITTITE EMPIRE. On the east the Hittite country was defended from Armenia to Cilicia by mountainous country of the kind here shown. The river is the Jihan, the classical Pyramus, possibly the "Seha" of the Hittites, in one of the gorges of the Taurus Mountains through which it flows to the Mediterranean Sea near Tarsus.

The centre of the Hittite homeland was a group of cities in northern Anatolia, at no great distance from the Halys river, the modern Kizil Irmak. Their capital, Hattusas, was identified by its German excavators with the modern site Boghazkeui or Boghaz-kale. A succession of excavations in the early 20th century laid bare the impressive double wall with sculptured posterns which surrounded the citadel and made it impregnable. Palaces, temples, and other public buildings were found within and

without these walls, and a considerable part of the city streets was excavated. It is known what a Hittite house was like, what were the ordinary objects of domestic use ; and, thanks to the thousands of tablets found in the archives of the palace, much is known also about the social organization of the Hittite state, the laws that governed it, and the religious and mythological beliefs of the people. It is a notable triumph of archaeology that so much has been discovered about a vigorous, capable, and well-organized people whose record, before A.D. 1900 or thereabouts, had been but very vaguely traced.

ROCK CARVING SHOWING A HITTITE CEREMONY. Self-portraits of tall-hatted Hittites in a recess of their open-air sanctuary at Iasily Kaya near Boghaz-keui, site of their capital, Hattusas. The carving is believed to represent a harvest dance or procession, and the curved implements to be sickles.

The Hittites were a military nation, basing their might on a well-trained infantry, on horse and light chariot (whose use they seem to have acquired from the Hurrians), and on superior armour forged from metal ores of the Anatolian plateau. The king in person led his troops and received the submission of an enemy, often setting one of his sons to rule as vassal king in a conquered territory. Treaties drawn up with legal phraseology bound the vassal to aid the Hittite king in war and not to harbour fugitives or make separate alliances.

The King as High Priest

A strong sense of political responsibility seems to have been a characteristic of the Hittite king. Although supreme head of the administration in all matters, he was limited by tradition and religious obligations, and in the earlier period seems to have had a body of advisers whom he was bound to consult on certain important matters of state. There exists part of several codes or collections of laws showing how the law was constantly undergoing reform and revision, generally in the direction of greater humanity.

Capital offences were few, and the punishment for crimes usually took the form of compensation of the injured party rather than mutilation of the offender—a comparatively advanced concept in which the Hittites were ahead of their neighbours in the Near East. But the slave had few rights, and the father held patriarchal sway over his family.

The king was high priest of the cults of all the deities of the land, both those of the ruling class of Hittites themselves and those ancient local cults of the indigenous population whom they had conquered. Once a year he made a tour of all the chief temples in his realm and celebrated their festivals. Anatolia is a mountainous land, subject to sudden storms and torrential rains, and many of its deities were gods of storm and rain ; their worship was often performed in open-air rock sanctuaries or hill-top shrines.

The plan of a typical Hittite temple is very much like that of a Babylonian temple : a paved court, many small store rooms for the cult objects and offerings, and a shrine which held the divine statue, with an altar for offerings. As in Babylonia, omens and oracles played an important part in private and civic life.

The Hittite Scripts

The influence of Babylonian and Assyrian civilization is seen also in the literature, and in the fact that the Hittites, having no writing of their own when they entered Asia Minor, borrowed and adapted the cuneiform script for their own purposes. Another script, entirely different, developed in parts of the Hittite domain during the second millennium ; this is the Hittite hieroglyphic script, still only partly deciphered, which continued in use in the states of North Syria long after the disappearance of the Hittites as an imperial power.

Under the slackened administration of the preoccupied pharaoh Akhenaten, and the

continued pressure of the Hittites, Syria passed out of Egyptian control, and by the end of the 18th dynasty, shortly before 1300 B.C., little but Palestine remained of the former Egyptian empire. Seti I regained some territory, and Rameses II after much hard fighting succeeded in establishing a frontier with the Hittite king Murshilish in central Syria. After a war of 20 years, both sides were equally exhausted and were glad to make peace on equal terms; the treaty was cemented by the betrothal of the pharaoh to the daughter of the new Hittite great king, Hattusil.

End of the Hittite Power

Not long after this, perhaps about 1220 B.C., the records at Boghaz-keui cease, and the cities of the Hittite homeland (so archaeology reveals) were looted and burnt by invaders with a different and less advanced stage of civilization; the knowledge of writing was lost, the mighty cities were rebuilt only as small or insignificant townships, and the highly burnished, shapely pots characteristic of the empire were replaced by painted wares. It is thought that these newcomers were the people known later as the Phrygians, whose traditional home was Thrace, in eastern Europe, and who therefore may have crossed into Asia Minor over the Hellespont.

In the historical period there are many movements of peoples which must be regarded as parallel with those unrecorded migrations of early times that can be traced by archaeology only, with the aid of the study of material remains. One of these events was the movement of the sea-peoples through the eastern Mediterranean towards Egypt, which is recorded in the Egyptian annals of the 12th century B.C. and of which effects can dimly be discerned in the archaeological history of the countries through which they passed.

WARRIOR OF THE PELESET. Prominent among sea-peoples were the Peleset or Philistines, probably Cretan in origin.

Invasions by the Sea-peoples

After the fall of the Hittite empire and the collapse of Egyptian control in Syria and Palestine (under the pharaoh Merenptah) Egyptian troops were fully engaged in the west, and after his death the country was distracted by civil war; the eastern Mediterranean lands were in a turbulent condition. The story of the siege of Troy may echo movements of migration across the Aegean, and the Egyptian records reveal how the Peoples of the Sea combined for conquest and plunder. Peleset, Denen, and Sherden are among the nations named; from their islands and coast they moved down towards Egypt, which had never been plundered and was consequently the richest country of the ancient world.

Victories of Rameses III

Passing through Asia Minor, the horde of sea-peoples turned south through Syria. Ugarit and other cities fell before them, never to rise again. It was a migration rather than a war of aggression. Whole tribes were on the move, bringing their women and children and all their possessions in carts. It was like a swarm of locusts. The fighting men captured the land fortresses, and the warships seized and plundered the coast towns; no nation or citadel could stand against them.

Fortunately an able pharaoh, Rameses III, now sat on the throne. Battle was joined on land and sea, and Egypt was victorious.

The disappointed invaders united with the western tribes and attacked on the Libyan frontier, but without success. But though Rameses III raided Syria and brought back much plunder, Egypt's resources had been overstrained in the struggle for life; her Asiatic empire was never won back.

The records and sculptures in the temple of Rameses III at Medinet-habu reveal much about the culture of the peoples who attacked Egypt. Foremost among the enemy were the Peleset, whom we know as the Philistines, with their high feather-crowns which hid the helmet, fastened with a leather strap under the chin. They wore a protective corslet covering the whole body, but leaving the arms free. Their weapons were short two-edged swords, and they carried round shields.

Their galleys were equipped with rams at the prow, and at bow and stern there was a high protective bulwark. The Egyptian crews were armed with bows and arrows, spears, and maces; at the summit of Egyptian ships was a fighting top, from which a slinger cast his missiles. A weapon used with deadly effect was the grappling hook, of which four tied together are shown as having been thrown at, and entangled in, the sail of a Philistine craft which they are tearing to ribbons. Metal helmets seem to have been worn by the other invaders. Their weapons and armour were of bronze, and their short sword was the slashing sword of central Europe, probably introduced into the eastern Mediterranean by the tribes who destroyed Mycenae.

LESSON 21

The Iron Age

The Iron Age was ushered in with wars, general unrest, and redistribution of power, until conditions were adjusted and became stabilised.

It is difficult to understand why so complicated a process as bronze-making should have preceded the simpler method of working iron. Probably it was because of the comparative rarity of the ore. A little iron was always in use in Egypt from the Gerzean period onwards, but it was not a steady industry, and most of the iron seems to have been meteoric. In the same way, iron was used in Crete as early as Middle Minoan II, but it is very rare before the 12th century B.C. The use of iron among the Hittites seems to have begun rather earlier than in Crete. The Dorian invasion brought iron into Greece, and with it came geometric designs on pottery. Iron ousted bronze gradually. In the *Iliad* the Achaeans wore bronze armour and fought with bronze weapons, though iron was coming into use for agricultural and household purposes.

In Mycenae iron was still sufficiently rare to be considered fit for jewelry. To Egypt iron came from Syria as a costly gift to the pharaohs, and Tutankhamen possessed a dagger with an iron blade. In Palestine the earliest known iron was found in a Philistine grave excavated at Tell Fara, dating from the 15th century B.C. At that time the Israelites were just emerging from their Bronze Age, but 200 years later the Philistines still kept the traffic in metal in their own hands, and the Israelites were not permitted to sharpen, much less to make, their metal implements ; " there was no smith found throughout all the land of Israel . . . All the Israelites went down to the Philistines to sharpen every man his share, and his coulter, and his axe, and his mattock " (I Sam. xiii, 19, 20).

TUTANKHAMEN'S IRON-BLADED DAGGER. An early instance of iron for weapons in place of bronze is one of the daggers of Tutankhamen (reigned *c.* 1358–1353 B.C.). The haft is of jewel-encrusted gold with a crystal knob.

From Howard Carter, " Tomb of Tutankhamen "

Phoenician Trade

The immense importance of Phoenician trade during the Iron Age is one of the chief features of the period. Tyre and Sidon had trading posts on the African shore of the Mediterranean by 1100 B.C., and their ships plied the Eastern Mediterranean and the Red Sea. Cyprus had Phoenician Colonies early in the Iron Age.

All along the coast of north Africa there was a string of trading centres, of which the latest was Qarthhadasht, the New Town, known to us as Carthage. This city came into existence about the end of the 9th century B.C., when the Phoenicians had already made settlements in the islands of Sardinia, Malta, Sicily, and Iviza. It was in Sicily that the Greek colonial settlements and Phoenician trading posts first came into contact ; their relations appear to have been unfriendly from the first.

The Rise of Carthage

When Tyre collapsed, Carthage rose and established herself as the chief trading power in the Mediterranean. It was not an altogether peaceful rise, but she succeeded in driving, with the help of her Etruscan allies, the Greek colonies out of Sicily and the Greek shipping off the seas. Phoenician sailors circumnavigated Africa, the expedition being undertaken at the instigation and expense of Pharaoh Necho of Egypt. They started from the Red Sea and returned by way of Gibraltar.

Carthage never developed a civilization of her own, although she was the means of developing it in other lands. She was entirely a trading power, and held the whole of the carrying trade in the Mediterranean. All Carthaginian objects, whether found actually at Carthage or at her trading centres, show that her workmen were copyists without originality.

Her army was composed of foreign mercenaries officered by the citizens of the country. Her religion seems to have been copied equally from Tyre and from Egypt. To Tyre the Carthaginians yearly sent envoys to take part in the worship of Melkart, the great god of Tyre ; and the greater number of amulets and figures of gods show Egyptian influence.

Iron Age in Europe

In Europe the Iron Age is divided into two parts. The earlier is called the Hallstatt period, from the famous cemetery in Austrian Tirol where this civilization was first studied ; the second part, which is known as La Tène, from the Celtic settlement on Lake Neuchâtel, is subdivided into four sections. The beginning of the Iron Age saw the first spread of the Celtic peoples, and in the La Tène period the

GREEK AND CARTHAGINIAN ART. The Carthaginians were copyists rather than creators. Compare the crudely fashioned dancing girl (right), of Carthaginian workmanship, strongly suggestive of Grecian influence, with the beautifully modelled Greek flute-player (left).
From Musée Lavigerie E. Leroux, Paris

Celts were the chief power in continental Europe. They were hardly a nation, but consisted of tribes more or less closely allied, who acted sometimes in concert, sometimes alone ; the real bond among them was a common language.

They are first located in central Europe, especially in south-west Germany, whence they migrated in successive waves in every direction, reaching Britain in the west, Greece and Asia Minor in the east, and penetrating southward into Italy. The civilization of the Hallstatt II period (beginning about 700 B.C.) was carried in their early migrations, and the spread of the La Tène culture (beginning about 500 B.C.) was also due to them.

Westward Migration

European civilization was profoundly affected by the coming of iron. For unlike copper and tin, iron ore is to be found nearly everywhere ; therefore it is comparatively cheap and many more people could afford to own iron tools than ever owned bronze. This resulted in increased felling of forests, increased agricultural productivity, and a rapidly expanding population.

The demand for fresh acres, combined with a deterioration of the climate which made some areas less productive, led to unrest and competition for the more fertile soils. Thus whole tribes were dispossessed and there was a general westward migration from central Europe, the Rhineland, and northern France. Waves of such migrants were arriving in Britain from 500 B.C. onwards.

The first arrivals seem to have settled down to a peaceful life in small farmsteads, such as have been found at All Canning's Cross and Little Woodbury in Wiltshire, and on Plumpton Plain in Sussex. But before long so many more people had come from across the Channel that small-scale warfare and raids became common ; and many tribes found it necessary to construct for themselves hill-top forts, protected by deep ditches with high ramparts inside. Most of these were not permanently inhabited, but were used as refuges in times of danger.

The Three Ages

This first phase in Britain is known as Iron Age A, and the culture of the people is derived from the Hallstatt culture of central Europe. This connexion is shown most clearly by the shapes of their pots, for otherwise their possessions were few and poor. No Iron Age A cemeteries are known, although a few isolated burials have been found.

From the 3rd century B.C. new groups began to arrive in the south and in east Yorkshire, both coming originally from the Marne district of northern France. Their culture was that of La Tène on the Continent, and their period is referred to as Iron Age B in Britain. (This culture did not entirely supplant that of Iron Age A, which continued to flourish in inland districts.) The B people in Yorkshire are best known from their rich cemeteries, one at Arras and another near Driffield known as the Danes' Graves.

Chariots and Slings

Here are found the chariot-burials of chieftains, with the bronze trappings of their war-chariots. Their wives were buried with costly ornaments such as coral-ornamented pins. In the graves of retainers were put plain pots, each containing a leg of pork. In the south the Iron Age B people are known only from settlements. In some instances they took over and strengthened the defences of existing hill-forts. At Maiden Castle in Dorset the number of ramparts was increased, and several hoards of sling-stones belonging to this period were found. It seems, therefore, that the sling was then the main offensive weapon.

In about 100 B.C., Cornwall was entered by groups who came originally from north-western Spain. They built forts with stone

ramparts, and small villages of stone houses, each with a central courtyard. Some 50 years later began the history of the famous lake-villages of Glastonbury and Meare in Somerset. These villages were built over an area that was then a marshy lake and the round, one-roomed huts stood on platforms formed of brushwood and clay.

Contact with dry land (with its cultivated fields, and cattle) was maintained by boat. These villages were busy centres of industry and trade. Attractively decorated pots were exported as far afield as Dorset, and there may also have been trade in cloth, iron tools, and such things as small bowls made of sheet bronze.

The latest Iron Age B arrivals fled from France to escape the Roman occupation of Gaul. During this occupation began the last period of Iron Age Britain, that known as C; and it is marked by the coming of the Belgae—Gauls of mixed Celtic and Germanic ancestry. The Belgae, like the A and B peoples, came in successive waves of migration.

The first of these, the Catuvellauni, arrived about 75 B.C., settled first in Kent, and then extended their territory to Hertfordshire, where under Cassivellaunus (the first inhabitant of Britain whose name is known) they established a stronghold at Wheathampstead. This was stormed by Caesar in the second of his raids, 55–54 B.C., and Cassivellaunus was forced to come to terms with the Romans. After Caesar's withdrawal Britain remained free for still another 97 years. The capital of the Catuvellauni was subsequently established first at Prae Wood, St. Albans, and later at Camulodunum (Colchester), in the time of Cunobelin.

CELTIC MIRROR. **This beautifully engraved bronze mirror-back, with red enamel bead decorations in the handle, was found in Gloucestershire.**

In about 50 B.C. another Belgic tribe, the Atrebates, under their king Commius, fled from Roman Gaul, entered by the south coast, and spread westwards. The Belgae brought with them many of the elements of Roman civilization: coinage (from which comes much of the present knowledge of the names of tribes and their kings), the potter's wheel, and a taste for wine and other luxuries which had to be imported from the Continent. The wealth which enabled them to afford these things was due to improved farming methods; they had heavy ploughs suitable for tilling the clay soils, such as that of Hertfordshire, which all earlier peoples had avoided. The prehistoric period of Britain ended in 43 A.D., when under the Emperor Claudius it was taken over as a province of the Roman empire.

The highly developed artistic traditions of the Continental La Tène culture were brought to Britain by the Iron Age B people and the Belgae, and here a native style developed which made use of delicate curvilinear designs. Such designs appear on bronze mirrors, scabbards, and horse-trappings; the chariot fitments were often enamelled. The White Horse, cut into the chalk of the Berkshire downs above Uffington, is stylised in the same manner as the horses on some Belgic coins; for this reason the White Horse can be dated with some certainty to the last years of prehistoric Britain.

LESSON 22

The Military Might of Assyria

The king lists of the Assyrians preserved a memory of a time when they were ruled by "kings who lived in tents." Their original home is not known, but it is thought that they migrated from somewhere in the Syrian desert, or perhaps Arabia itself, at a comparatively early period in the history of the Land of the Two Rivers, and were akin to the Akkadians, whose language that of Assyria closely resembles.

Excavations on the site of their first capital, Asshur, the modern Qal'at Shergat on the Tigris, have shown that during the 3rd millennium B.C. its inhabitants enjoyed a civilization differing in no essential features from that of the Sumerian south. Asshur was within the orbit of the 3rd Dynasty of Ur, and it was not until the period of the confused rivalries preceding the Hammurabi age (*see* Lesson 15) that the first Assyrian warrior-king set out to extend the bounds of his kingdom to include the more northerly parts of Iraq, for a time also advancing towards the bend of the Euphrates to the south and west.

During the subsequent centuries able kings set up from time to time an ephemeral empire stretching across to the shores of the Mediterranean. In the 14th century B.C. a king of

Assyria wrote on equal terms to his brothers the kings of the great powers of the Near East. It was not till the 9th century that the Assyrian army began the final inexorable drive to north, south, east, and west which gradually swallowed up country after country until almost the whole of the civilized world was paying tribute to Nineveh and supplying slave labour to build her vast palaces.

Assyria's Conquering Army

From small beginnings as national militia, the Assyrian army became, over the centuries, the greatest fighting force in the ancient world, a professional body kept in training by annual campaigns and schooled to fight with equal efficiency in precipitous mountain terrain and in the parched desert. Reliefs found by excavators in the palaces of Nimrud, Khorsabad, and Nineveh have as their most frequent theme the battles and triumphs of the royal builders, and from these reliefs a great deal has been learnt about the tactics and armament of the Assyrian soldiery.

Field and Siege Tactics

The main attacking force comprised the bowmen of the infantry, protected by soldiers with shield and lance; coats of mail and helmets were worn. Among the auxiliaries, slingers, and others carrying a variety of weapons such as clubs, axes, daggers, and swords, are seen. The chariotry also fought in pairs, one man discharging his arrows while the other held a shield over him; later a third rider was added, to drive the chariot so that it could be used while in motion.

At first the chariots were clumsy affairs with six-spoked wheels; later they became higher and lighter. A third force, the cavalry, was introduced late in Assyrian history, perhaps when a sturdier breed of horse had evolved, capable of carrying an armed man. One of the great legacies of the Assyrians to the classical and medieval world was their development of siege warfare. Against strongly fortified walled cities such as those of the Aramaean Syrians and Elamite Persians, they used every kind of assault tactics.

Sappers are shown mining the walls, different kinds of battering rams, towers, and other siege engines are applied to them, and archers rain their arrows into the city under cover of a screen of shields. The investment of a city was followed by the erection of huge earth ramps against the defences, and few were the cities which could withstand this formidable attack.

DETAILS OF ASSYRIAN ARMY ORGANIZATION. These slabs portray in careful detail the activities of camp life in one of the Assyrian campaigns. On the left, army cooks are preparing food in a well-equipped field kitchen. On the right, horses are being groomed and watered.

Administration and Trade

Once conquered, each country was incorporated, to a greater or lesser degree, into the Assyrian empire and became a part of the greatest political unit the world had yet known. Governors in the provinces kept in touch with the central administration and sent and received frequent reports; some of their correspondence addressed to the Sargonid kings of Assyria has been found among the tablets of the royal libraries where it had been filed.

A system of roads and a rapid courier service linked the outermost parts of the empire with the capital, and these roads were of service alike to the armies which patrolled their territory and to the merchants, who travelled in ease and safety. Trade was further facilitated by the spread of Aramaic as the language of international intercourse.

Although of a warlike temper, the Assyrians were no ascetics. They enjoyed the luxury which wealth, in the form of the rich tribute which annually flowed into the coffers of the conqueror, brought to them. The arts of peace flourished, and relief sculpture, embroidery, metalwork, jewelry, ivory carving, and gem engraving, attained a high level.

ASSYRIAN INFANTRY. **Infantrymen of the army of Sennacherib (ruled from 705 to 681 B.C.), of the type which captured Sidon, had to retire before Jerusalem, and in 689 destroyed Babylon.**

The houses of even modest citizens of the Assyrian capital were equipped with an excellent drainage system, and were of several rooms. The palaces of the kings and of their chief officials were of considerable magnificence, being decorated with huge sculptured magical figures and fine series of reliefs depicting the king's triumphs and pleasures. Some of these palaces have been excavated, notably those at Nimrud, the ancient Calah or Kalkhu, at Nineveh, at Khorsabad, and at Asshur.

The king of Assyria was not only the leader in war and the religious leader of the community: he was expected to provide for the welfare of his subjects. Several of the Assyrian kings relate their care to promote agriculture, to improve the water supply, and to introduce new methods and new crops into the country. Huge irrigation works involving the digging of new canals, the building of aqueducts, and even the damming of rivers were undertaken by these energetic kings.

In the parks and market-gardens thus created, they planted foreign trees and flowering shrubs which they had encountered on their campaigns. Sennacherib describes how he introduced the cultivation of cotton, "the wool that grows on trees" into his capital. Much of Assyrian civilization was derived from Babylonia, but much also was due to their own genius; and Persia in her turn inherited from Assyria.

Outside the bounds of the Assyrian empire there were formidable rivals to check her advance and even threaten her borders. Her northerly neighbour, Urartu, the country around the mountains of Ararat and Lake Van, constituted a constant menace to the northern frontier of Assyria, and even at one time conquered much of North Syria, and cut her off from the metal trade with Anatolia.

Urartian metal-work is among the finest known from the ancient world. It was exported to the Aegean, and even to Etruria and farther west. Their rock-cut citadels, irrigation channels, and gigantic masonry bear witness to a high level of civilization. In Asia Minor, the Phrygians, whom the Assyrians knew as Mushki (Greek Moschoi), had taken over the hegemony once held by the Hittites and disputed the western frontier of Anatolia with the Assyrians. Their rock-tombs with elaborate carved façades, their large tumuli and rich grave-goods, bear witness to the wealth of king "Midas," whom the Assyrians knew as Mita, king of Mushki.

To the East of Assyria a third rival power was growing—the Medes, a people of Indo-European speech, migrating from Europe at a date unknown, were building up their kingdom on the Persian plateau around the Caspian Sea.

LESSON 23

Greeks and Persians: Clash of East and West

During the last century of Assyrian rule great movements of peoples had been taking place, while in south-eastern Europe simple Bronze Age village settlements were developing into the city-states of Greece and Etruria. Phoenician ships were vying with Greek in the colonisation of the Mediterranean seaboard, and the map of the civilized world was undergoing ethnic changes of momentous portent.

From the Russian steppes, advancing over the Caucasus, came wild nomadic horse-breeders with their wagons, the scalp-hunting Cimmerians, and their kinsmen, the Scythians. They put an end to the kingdoms of Urartu and Phrygia, and for a time penetrated far across

Asia Minor, threatening even the powerful and wealthy kingdom of Lydia on the western seaboard. To the east, the Medes were gaining ground, had found unity under a king, and had learnt military methods from their Urartian neighbours. Implacable enemies in the shape of the Chaldaean Babylonians, long oppressed but resilient in resistance, faced Assyria to the south.

Assyrian Empire Overthrown

At length the blow fell. Scythians, Medes, and Chaldaeans combined to bring about the overthrow of the Assyrian empire, and in 612 Nineveh succumbed to their combined attack. The empire was divided between the conquerors.

To the Babylonians, under their Chaldaean king Nabopolassar, fell Syria and Palestine, Arabia, and the Persian Gulf. A new and brilliant period of prosperity began, a period now called the Neo-Babylonian Age. The great city of Babylon, the ancient capital destroyed by the Assyrian conquerors, was now rebuilt on a far grander scale than ever before. The German archaeologist Koldewey and his assistants, excavating the site before the First World War, were able to reconstruct in imagination, from the towering remains of palaces and temples, the network of ramparts and canals, much of the ancient aspect of this noble city, later to be the marvel of Greek travellers.

THE SUPPOSED TOMB OF CYRUS. The limestone building at Pasargadae, which preceded Persepolis as the capital of ancient Persia, is believed to be the tomb of Cyrus the Great. The ruins of the city stand near the modern town of Murghab.

From Dieulafoy, " L'Art antique en Perse "

Babylonia has no stone, so the city was built almost entirely of brick. Glazed and moulded tiles decorated the façades of its temples and the great gateways that flanked the processional way down which images of the gods were borne at festivals. Little is now left of the huge ziggurat, or temple tower, which legend associated with the " Babel " of tongues. And the site of the Hanging Gardens which King Nebuchadnezzar built to solace his Median queen, homesick for her mountains, is disputed. But enough remains to show that at this time Babylon was a city without rival in the world.

No Median site of consequence has yet been explored by the excavator's spade ; and the capital, Ecbatana, lies under the modern Persian town of Hamadan. Therefore little is known of Median civilization. Scholars rely on Herodotus for most of the present knowledge of the country's history. The Median empire lasted for only three generations before their vassals and kinsfolk, the Persians, rebelled and took over the conduct of affairs.

Cyrus the Persian

Cyrus, the warrior king, descendant of Achaemenes, and one of the great figures of history, has left few monuments, and only scanty remains survive of his palace at Pasargadae. His greatest monument was the Persian empire itself, for it was almost certainly he who laid the foundations of that vast imperial organization, later revised and elaborated by Darius the Great, which found no rival in the ancient world until the days of the Roman empire.

Cyrus conquered territory from Lydia to Afghanistan, from the Euxine to the Arabian Sea. Cambyses, his son, added Egypt. At its greatest extent, under Darius, the Persian empire extended from Thrace in Europe across to the valley of the Indus, and from the Caucasus to Ethiopia. These vast territories were governed for the Great King by 20 governors, called satraps, often princes of the royal house, who maintained communication with the central administration along the magnificent roads which ran from end to end of the empire, and with a courier system based on, but much better organized than, the old Assyrian system.

The satrapal system worked well so long as the satraps were loyal and a strong hand held the reins. But the huge distances which separated outlying provinces from the capital at Susa or Babylon meant the inevitable rise, in times of weakened control, of quasi-independent princes, living like hereditary kings in their own provinces, and paying no more than lip-service to authority. Rebellion was constantly having to be crushed ; some provinces broke away ; and the disintegration of the empire had begun long before the Battle of Issus.

The greatest monument which Achaemenid Persian builders have left to the civilized world is the palace, or rather series of palaces, on the rock platform at Persepolis. The graceful pillars, crowned with carved foliage or bull's head capitals, have been the admiration of travellers in this desolate plain throughout the centuries. Excavations have uncovered the carved terraces with their wonderful processions of Medes and Persians who made up the royal bodyguard, of the officials of the Achaemenid court, and of embassies from conquered nations bringing their tribute to the Great King.

The palace of Darius the Great at Susa contained large panels in delicately coloured glazed tiles, each depicting a warrior, and these are now among the glories of the Louvre museum in Paris. The sculptors of Persepolis, Pasargadae, and Susa learnt much from Babylonia and Assyria and something from Egypt. It is known that the Achaemenid kings obtained craftsmen from Ionia and Lydia, Phoenician ivory workers and embroiderers, Babylonian tile-makers, and Armenian metal-workers, to fashion their palace furniture.

ATHENIAN SOLDIER. Memorial sculpture from a tomb. The heavily armed Athenian infantry were chiefly responsible for the great victory at Marathon.

Elements of Persian Art

Persian art contains elements from the civilizations of all the ancient countries of the Near East. But it is not merely derivative; it contains a freshness and beauty of its own. Persian art may have learnt from the Greeks, but Greece certainly also learnt from Persia, and Oriental influences are strong in Greek art of the 6th and 5th centuries B.C. The legacy of Achaemenid metalwork to the Sassanian age, and so to the Islamic world, has been considerable, and Persian and Byzantine influences mingled in much that the western world admired and copied.

Conflict of Two Worlds

It was on the coast of Ionia that the two worlds, European and Oriental, came into conflict. The cities of Ionia—Miletus, Ephesus, Halicarnassus, and the rest—were colonised early from the Greek mainland, and here, in contact with the rich Orientalised civilization of Lydia, many of the early experiments in Greek literature and science had taken place. Ionians had led the way in the colonisation of the west, and they had been responsible for the introduction of those two great promoters of commerce—coinage and an alphabetic script adapted from the old *alpha-beta* of Phoenicia whose history goes back to inscriptions of the 2nd millenium B.C.

Persian rule was in some ways welcomed by the conquered. Certainly there were economic advantages to be gained by co-operation with the satrap, and protection was assured against piracy, civil disturbance, and the aggression of neighbours. The Persians were tolerant, and encouraged the religion, customs, and local institutions of their subject peoples. But a fierce desire for freedom—that stubborn individualism which prevented the Greek city states from ever uniting into a great kingdom or building an empire—led the Ionians to revolt. Their six-years struggle ended in tragedy and led Darius to attempt the conquest of Greece itself; over the Hellespont bridge came the first great Oriental army to invade Europe.

Hoplites on the March

It is idle to speculate what changes in the history of our civilization might have resulted had the Persian army been successful, had the Battle of Salamis, at a critical moment, turned in favour of Xerxes. As it was, Persia continued to play a significant part in the political history of Greece and, in the end, invited nemesis in the shape of the avenging Alexander. Greek hoplites marched over the plains where Babylonian, Assyrian, and Persian armies had once, long ago, "swarmed like locusts." Greek and Persian, European and Oriental elements, became fused in that brilliant civilization which is called Hellenistic: the civilization to which the Roman genius added little but the ability to absorb it and the very efficient organization that enabled them to carry it with them into the farthest parts of their empire.

LESSON 24

Rome—the Distributor of Culture

The great claim which the Roman empire has on the respect of all archaeologists is that it took the best from the civilizations of the earlier periods of Europe, Asia, and Egypt, and carried the complex through the known world.

The Romans' method of doing this was entirely their own.

Underlying the whole of their policy towards the countries they ruled was the determination to be supreme. They allowed no political or other associations within their dominions ; the allegiance of subjects was due to Rome alone, and Roman rulers tolerated no breach in that allegiance. But the privilege of citizenship could be granted where Rome wished. Under the republic, to some of the Italians Rome gave full citizenship (so long at least as the citizen body needed recruits), to others partial citizenship, to others a smaller share of privilege, to all a certain measure of local self-government, always as the occasion and Rome's welfare directed.

Two Centuries of Single Rule

Under the empire this policy was continued among the provinces ; towns as well as individuals were admitted to the honour, till Rome had bound most of Europe into a single unit and reigned unchallenged over it.

This position was held in all its splendour for two centuries ; then the inevitable change set in. The frontiers, continually menaced by barbarian tribes, required larger and larger armies, and Rome, as the central authority, began to lose ground, till finally it was possible for the Praetorian guards to murder the emperor and put up the empire to auction. Then came an alteration in the government : the empire was divided into two parts, with an emperor and a capital city in each, Rome still retaining its former hold on the West while Constantinople ruled the East.

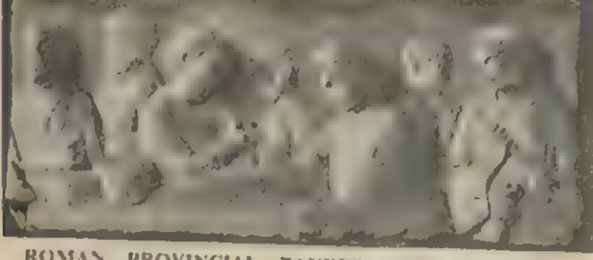
In the West the government was subdivided into " dioceses," probably following old tribal divisions. The force of tradition was such that the empire survived invasion after invasion from the Germanic barbarians and from the Scythic Huns. No emperor ruled in the West after the end of the fifth century, but Rome of the East was still a power to reckon with.

Romanisation of Provinces

The effect of Rome on the outlying provinces was almost as great as on those nearer the centre of government. Everywhere Rome followed the example first set by the Persian empire and made good roads and ensured the safety of travellers; for the Romans realized, as Persia had done, that easy and safe communication between countries is the basis of all advance ; the high road of commerce is the high road of civilization.

The Romans displayed genius for colonial government in all parts of the empire, educating subjects to look to Rome as the giver of all gifts, and romanising as far as possible the barbarian tribes within the imperial pale. Everywhere the barbarian nobles and chiefs modelled themselves on Rome. They built towns with a suite of public buildings—market-place, town hall, and baths, also an amphitheatre outside the walls.

They built villas in Roman style with tesselated pavements. They imported Roman products and tried to make their own manufactures resemble Roman goods. They incorporated Latin into their language, introduced Roman customs and Roman ways into their daily life, and adopted Roman styles of dress in preference to their own. Only in matters such as marriage rites and the methods of disposal of the dead did the provinces retain their ancient ways.



ROMAN PROVINCIAL BANKER. Wherever Roman colonisation extended, capitalistic development followed and the banker became a familiar figure. Here is one, with Roman clerks, receiving payment from Celtic peasants (Moselle region).
Trèves Museum

Intercommunication was safe, but that safety brought with it other dangers. The mineral wealth of the provinces was often exploited by speculators ; and foreigners, greedy of riches, often poured into a new province in hope of making their fortunes. Spain, as has always been the case from the beginning of the Bronze Age, produced gold and copper, wine and oil, and by reason of its natural wealth was one of the richest and most important of the provinces ; it

also did a thriving trade in steel blades, and linen and woollen textiles. Flour from the Balearic Islands came much into favour with the epicures of Rome.

The chief industries of Gaul were textiles, but there was quite a large output of pottery in imitation of the fine red Arretine ware. This imitation, often called Samian pottery, is characterised by a brilliant polished slip of a sealing-wax red colour. The bowls of this material were cast in a mould and were decorated with wreaths and cupids in the approved Roman style. The imitation ware was carried all over the provinces and was even taken to Italy; it is found on Roman sites in all countries of the old empire.

IMITATION ARRETINE POTTERY. The decorated terra cotta bowl (left) was cast in red clay in the mould (right). The bowl was then highly glazed and polished, giving a close imitation of metal work.
British Museum

Britain's minerals were not so important as those of Spain, for zinc instead of tin was used by the Roman metal-founders for alloying copper, the mixture producing brass, not bronze. Enamels still continued to be made, but Britain's chief exports were hides, corn, and textiles. Roman Germany, being on the border of the empire, did a large carrying trade of Roman products to the barbarians, and manufactured pottery, glass, and brass, all for export to the more distant provinces.

The trade of Rome on the east went as far as southern India and Ceylon, with which countries there was steady and regular communication, the imports being cotton, spices, and other Oriental luxuries. This trade passed across Egypt, with the result that Alexandria was as important under Rome as under the Ptolemys. Rome even sent an embassy to China, probably in connexion with the overland silk trade. One of the most lucrative of all the trades was the traffic in slaves, which was carried on with all countries throughout the known world.

A modern practice, first begun under the Romans, was the fashion of visiting interesting or beautiful places; this tourist traffic was in full swing in the 2nd century. Greece owed its prosperity to the tourists, Mediterranean cruises were very popular, and a journey up the Nile became fashionable after Hadrian's visit to Egypt.

Religion in the Empire

Rome was very tolerant of the religions of other countries; her provincials could worship as they chose. The persecution of the Christians was due not to hatred of their religion but to their refusal to acknowledge the supremacy of the emperor in the usual manner by burning incense before his statue. This was to the Romans a form of high treason, a political, not a religious, offence.

The eastern part of the empire became officially Christian under Constantine in the 4th century, and not long afterwards the West abandoned paganism. In the West the Roman legions had brought the worship of the soldiers' god, Mithras, to all parts of the empire; when Christianity was established as the official religion, it was carried far and wide and superseded, at least on the surface, the old pagan cults wherever Rome's power prevailed.

The Byzantine period begins after the accession of the emperor Constantine (306-337), when he laid the foundations of his new capital on the Bosporus in the year 324. Its history is largely the history of the Christian Church in the East, and the objects which remain are chiefly of religious significance or definitely Christian, because Byzantine art advanced along purely religious lines.

The Thousand Years of Byzantium

The wealth of Byzantium brought traders from all parts, and pilgrims passing to and from the holy places carried Byzantine influence to distant countries, so that bronze vessels of Coptic type from Egypt are found in Anglo-Saxon cemeteries. Ceramics, ivory carving, and woven and embroidered textiles all show a growing Orientalising influence.

From the accession of Constantine till the Muslim invasion the history of the Mediterranean is one of continual warfare. In the west the great empire of Rome collapsed under the continued attacks of barbarians; Goths, Huns, and Persians were pushing in, and many of the outlying provinces were lost. Towards the end of the 5th century the last emperor of Rome was deposed, and the centre of government in the West shifted to Ravenna under one of the conquerors.

In the East, Byzantium not only stood firm but carried on the old tradition of Rome's greatness. The advance of the Moslem hordes began in the first half of the 7th century; Persia, Egypt, Syria, North Africa, and Spain were reft away but Constantinople remained undefeated until the final capture by the Turks in 1453.

LESSON 25

Indian Archaeology

UNTIL the '20s of the 20th century the archaeologist's knowledge of ancient India extended only to the time of Alexander the Great's invasion, a period when the empires of Egypt, Babylon, Crete, Israel, the Hittites, and Assyria had passed almost into oblivion. But India must have had as old a civilization as Europe or the Near East.

To understand the problems which India presents, the geographical conditions must be taken into account. For archaeological purposes India can be divided into four regions: (1) the Indus basin, which has affinities with Persia and central Asia; (2) the Ganges valley, which also has connexions with central Asia and is in touch with the more eastern countries; (3) a *massif* of barren hills and deserts, which forms a barrier running across the whole country from the north-west corner of the Bay of Bengal on the east almost to the Indian Ocean on the west; (4) the peninsula of India.

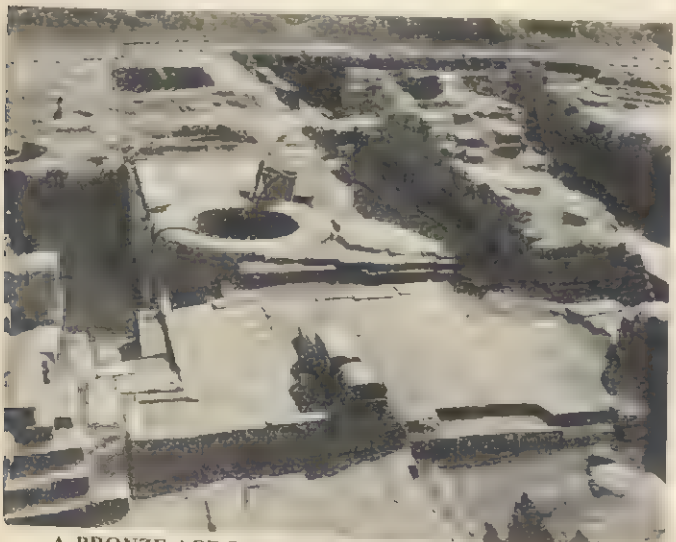
It is obvious that northern India's contacts with other countries, which were chiefly by land, must be entirely different from those of peninsular India, which were always by sea.

In the north, Afghanistan must be reckoned, geographically, as part of India, for the Kabul river is a tributary of the Indus, and the passes—of which the Khyber is the most celebrated—are practicable for traffic, either commercial or warlike. Afghanistan lies at the foot of the mountain range of the Hindu Kush, through which passes lead to the headwaters of the Oxus and so down into Bactria and thence into central Asia or Europe.

The land route from Persia lies through Kandahar, which controls the southern passes, and therefore controls the trade, through these passes, to Sind and Multan. The sea-borne traffic from Persia and the Persian Gulf entered India at the country of Makran, between Baluchistan and the sea, and went up the Indus valley.

The contacts and therefore the civilizations of the Ganges valley and of the Peninsula differ from each other and from those of the Indus valley. So little excavation of prehistoric sites has been done in the Ganges region that its archaeology is hardly known. In the Peninsula the contacts were by sea almost entirely. The belt of barren hills which divides India into a northern and a southern region reaches the sea on the east; and to the south of that belt lies the Chilka lake, which effectually blocks all access from the north to the Peninsula on the east except by sea.

On the west there is a strip of land between a belt of hills and the sea; through this, trade could filter to some extent. There are some good harbours down the west coast, so that sea-borne traffic could enter the Peninsula more easily on the west than on the east, where the harbours are poor.



A BRONZE AGE TRADE CENTRE. Mohenjo-Daro in the Indus valley was a city with substantial houses having private wells and bath-houses, and streets well paved and drained. The architecture was severely practical, with no sign of sculpture or pictorial art.

The Palaeolithic cultures are well represented in India, and it is a remarkable fact that the implements of the Lower Palaeolithic periods are found there as in all other parts of the Old World. The forms are completely stabilised, and the methods of chipping are the same wherever the implements are found; and on the sites where they occur they are in great quantities.

The wide distribution of these highly specialised implements is one of the many problems of archaeology. In the Neolithic period the people lived, as in Europe, on high, rolling country, where there were neither forests nor swamps. On such sites

Neolithic implements are found, and a microlithic industry was widespread in South-east Asia.

Until the excavations at Mohenjo-Daro and Harappa, the Bronze Age culture seemed hardly represented in India. Even now its evidences appear only in the Indus valley, where it may have been imported from the north or west. The Iron Age, however, begins so early and is so important that many authorities are of opinion that the smelting and working of iron began in India, and that the art filtered by slow degrees through the usual channels of communication to the Near East and Russia, and so to the West. The debt of pre-Roman Europe to India is as yet unknown, but it was probably very great.

The two principal excavations in the Indus valley are at Harappa and Mohenjo-Daro, which are about 400 miles apart. Mohenjo-Daro (the Mound of the Dead) lies in the Sind desert, about three-and-a-half miles from the Indus, but as the river is constantly changing its course the town was probably at one time actually on the river bank. Nine strata of buildings have been identified, all belonging to the Bronze Age.

FAMOUS FRESCOES OF AJANTA. The rock-cut monasteries and shrines of Ajanta, in Hyderabad, were parts of the Buddhist university which flourished there from about 200 B.C. to A.D. 600. The walls of these excavated dwellings and temples are covered with frescoes of mythological subjects.

For Trade Alone

Burnt brick was already the main material for all buildings at this early period ; it was known, though little used, in Mesopotamia in the contemporary period, but in Egypt it is not found till Roman times. The town was laid out with two arterial streets 35 feet wide, crossing each other at right angles ; other smaller streets and lanes branched off these, also at right angles. The houses were built, like modern Oriental houses, round a courtyard.

The alluvial plain in which Mohenjo-Daro stands has no metal and no stone ; both these materials had to be imported. The few large slabs, used for covering drains and for similar heavy work, were quarried 100 miles or more farther up the river and brought down by boat. Some semi-precious stones came with the metals from some other part of India, or even from Afghanistan or Tibet.

Some of the small stone seals found at Mohenjo-Daro and Harappa show connexions with seals discovered in Mesopotamia and Elam ; by these the date of the upper levels of Mohenjo-Daro can be considered with some certainty to be 2300 B.C. Other connexions with Mesopotamia and Elam are seen in the fragments of steatite vases carved with a mat pattern, found in all three places, and also the strangely etched cornelian beads which are known at Mohenjo-Daro and in Mesopotamia and Russia. Cotton was used for weaving, but there was no flax. The actual fibres of the cotton have been found preserved, corroded on a silver vase ; the threads which formed the fabric show its texture.

The whole civilization revealed at Mohenjo-Daro indicates a trading town. There is ample evidence of the existence of writing, in the shape of inscribed seals. The script cannot, however, be deciphered, and perishable materials must have been used for writing. Everything was severely practical—there was no attempt at art ; and though religion was manifest, the great temples with their beautiful architecture, which are a feature of Egypt and of later India, do not occur. Mohenjo-Daro existed for trade alone.

Asoka and Buddhist India

When India first comes into the light of history, about 600 B.C., the northern part of the country was subject to Hinduism. The expedition of Alexander the Great is the beginning of detailed and consecutive history. From that date until the beginning of the Middle Ages the chief periods are as follows : (1) Mauryan (in the Ganges valley), 323 B.C.–A.D. 100 ; this includes the Sunga, and also the Bactrian and Parthian intrusions into the north-west ; (2) Kushan (Indo-Scythian), A.D. 100–320 ; (3) Gupta (in the Ganges valley), A.D. 320–6th

TEMPLE AT ELLORA. This magnificent rock-hewn Hindu temple was begun about A.D. 760 as a thank-offering for victory, and was named the Kailasa, after Siva's mountain paradise in the Himalayas.

From Havell, "Ancient and Medieval Architecture of India," John Murray

century, conquered all western India; (4) Vakataka (in the Deccan); (5) Pallava (in peninsular India), A.D. 642–11th century; this includes the Chola.

The chief king of the Mauryan dynasty was Asoka (273–232 B.C.), who made Buddhism the state religion of his empire. This empire comprised practically the whole of the north of India, and stretched from the Hindu Kush mountains in the north to the barren belt of hills which divides northern India from the peninsula. Because it was the most powerful kingdom in India, ambassadors and traders flocked in, and Asoka's subjects traded with the lesser kingdoms of the south, including Ceylon, and with the whole of western Asia, eastern Europe, and Egypt.

With the full force of a remarkable character Asoka sought to introduce the tenets and practices of his religion into other countries. To this end he sent out missionaries, who established themselves in various lands, east and west.

Buddhist Influence on Christianity

In Ceylon and farther east their efforts were crowned with success. In the west the most important for Europe was the mission to Egypt, which preached and practised asceticism and the necessity of withdrawing from the wickedness of this world. The effect of this preaching and of the long contact of Egypt with Buddhist India was not seen till Christianity, which also preached renunciation of the world, swept over Egypt like a flood.

Egypt grafted Buddhist asceticism on Christian doctrine, monasteries were founded, hermits withdrew to the desert, until nearly a third of the inhabitants of Egypt were under religious vows. The missionary spirit, emanating first from India, was also introduced into Christianity, with far-reaching effect on the civilization of medieval and modern Europe.

Asoka appears to have been the first to use stonework for his monuments. His inscriptions were on stone, his commemorative pillars were of stone, and his buildings were of stone; the earlier temples and palaces were probably of sun-dried brick. The change was brought about by foreign influence, and the style of the beautiful monolithic pillars suggests that it was Persian.

After the fall of the Mauryan dynasty the Sunga kings continued as Buddhists; so also were their successors, the Kushan kings. The religious building of this period is the *stupa*, a solid mound of brick or stone surrounded by stone railings and carved stone gateways, with monasteries and chapels adjacent. The railings and gateways are obviously copies of constructions in wood, but the detailed and delicate carvings with which they are adorned suggest that the carver learnt his art on ivory or some equally fine material.

Wood was commonly used in the construction of private houses, a temple being usually only a more elaborate dwelling-place. In the Mauryan and Kushan periods are found the prototypes of those rock-cut halls and cells which later became so characteristic of Buddhist art in northern and central India.

Kings of the Gupta dynasty were patrons of all the arts. The earliest stone buildings which still survive belong to this period, and they are all Hindu, not Buddhist. They show a stage of development between the primitive caves and the fully developed temples and shrines of Ajanta, Ellora, and other caves of medieval India. Working in metal on a large scale is characteristic of this age. Statues are known

which weigh about a ton, and the celebrated Iron Pillar at Delhi dates from this dynasty.

Knowledge of the archaeology of this period derived chiefly from the sculptures in the temples; the general life of the people cannot be understood without excavation of ancient towns. ; in Egypt, working in stone, when once introduced, improved rapidly. The artificial caves of Ajanta and the rock-hewn temple of Ellora are among the greatest achievements in stone-working that the world has seen.

Ajanta and Ellora

The Ajanta caves show in their sculptures and frescoes the progress of Hinduism and the decadence of Buddhism; they are still Buddhist in type, but progressively Hindu in feeling. At Ellora the cave technique resulted in the carving of a temple out of a rocky hill. The hill has been completely cut away above and at the sides, leaving a solid four-square mass of rock standing. This mass has been hollowed out, and it forms the temple, which is enriched with carving. It stands upon a huge rock plinth, surmounted by a remarkable frieze of elephants which appear to carry the temple on their backs.

Aryan-Speaking Invaders

Names similar to those of Hindu gods occur on cuneiform tablets of the 15th century B.C. found at Boghaz Keui in Asia Minor, showing that some of the Vedic deities are connected with the Near East and the eastern Mediterranean. As the 15th century B.C. is the time of the Aryan invasion of northern India, it is very possible that the spread of this culture may have been due to that great movement of peoples.

Though the people who entered India spoke an Aryan language, Sanskrit, which is the foundation of a great number of the languages of India, it must not be supposed that the invaders were all of one race. They were probably a mixture of races speaking one language, like the Celts of the Iron Age and the Arabs of the Middle Ages; they were united by language but not necessarily by blood.

LESSON 26

The Dawn of Culture in China and Japan

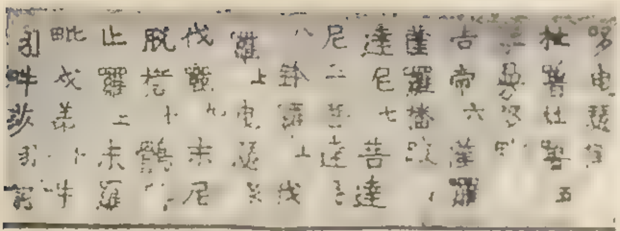
THOUGH the known history of China is late as compared with that of the countries of the Near East, the discovery of the remains of Palaeolithic man shows that China was inhabited as early as the more western parts of the Old World. Accurate dating does not begin till the 9th century B.C., but scientific excavation is carrying knowledge of ancient China back to a still earlier period.

The Bronze Age appears to have been established as early as in Europe, and there is some reason to suppose that the use of the mixed metal, bronze, may have originated in China. By the 12th century B.C. the Chinese, who have always been craftsmen, understood the full value of bronze, and were producing those bronze vessels which are the wonder and despair of all metal workers. Many of these vessels were used to contain the votive offerings dedicated to the worship of the ancestors, and on them was lavished all the skill of the artist.

ANCIENT CHINESE BRONZES. By the 12th century B.C. Chinese craftsmen were producing superb bronze vessels. These sacrificial bowls date from the Chou period which ended in 250 B.C. The lower bowl, 33 inches across, is inlaid with gold and silver.
Victoria and Albert Museum

Glazing of faience was known in Egypt before the beginning of the historic period, and elsewhere there was a certain amount of glazed pottery. But porcelain appears to have been a Chinese invention, and it was made by them so extensively that the common word in modern English for that material is the name of its country of origin, *china*. The invention consisted in subjecting a special kind of clay to sufficient heat to make it crystalline and translucent.

The Chinese were using glazed ware about the 2nd century B.C., having perhaps learned this art, together with the art of making glass, from the western peoples with whom they came in contact by trade. Porcelain does not occur till the 7th century A.D. The Chinese potters were not content with merely inventing a beautiful material; they used that material to form beautiful shapes, and they decorated the objects with exquisite and appropriate designs. No other country can show such a continuity of high achievement in these arts or such a rich variety in their application.



EARLIEST SPECIMEN OF CHINESE PRINTING. Found at Kichik-Hassar in Turkestan, these Chinese Buddhist charms, printed in black and red, date from the 8th century A.D.
British Museum

In Honan province excavations have been made on the sites of important centres of ancient Chinese life and culture. At one, on the left bank of the Hwan river, royal tombs of the Shang or Yin dynasty (1766–1122 B.C.) have yielded many masterpieces of Chinese art, including superb bronzes and beautifully carved limestone figures, stated to be the earliest examples of sculpture yet found in the Far East. Inscribed pictograms on "oracle-bones" of shell bear witness to a developed script, and more than 2,000 of these signs are known. The Chinese never evolved an alphabetic script like the peoples of the Near East, but they were the first to use movable wooden blocks for printing.

Tomb tiles of the 3rd century B.C., found at Old Loyang, in the valley of the Lo river, western Honan, depict hunting scenes and wild geese in flight of a remarkable naturalism, as well as stylised designs of birds, horses, and human figures.

The earliest known contacts between China and the Near East took place within two centuries after Alexander the Great's expedition to India, when Europe first became aware of vast territories far away to the east. Rome was spreading her empire across Europe and the countries of the Levant, and so came in touch with Persia and India. China was in close contact with India, and Buddhist missionaries carried their religion and their religious art—with its dogmatic symbolism, a bad element in art, and its devotion, a vitalising element—to China.

There is also a record that Chinese envoys visited the court of the Kushan kings at the very beginning of the Christian era, a fact which shows that relations between the two countries were not only amicable but probably frequent.

A regular route for trade from China to Europe ran north of the Hindu Kush, across the Pamirs, along the chain of oases bordering the Taklamakan desert. The silk trade, which followed this route, was very important for China. It began when the Han emperors were at war with the hereditary enemies of China, the Nomads. These fierce people rode small active horses, and the emperor desired large horses that he might overwhelm his foes in cavalry charge

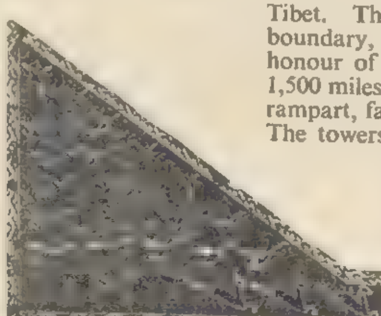
Monopoly of the Emperors

In order to obtain such horses he sent missions to other countries, which resulted in regular communication with the West: for if the emperor desired horses, the Europeans desired silk with equal fervour. The trade became so important that the production of silk was a jealously guarded monopoly of the emperors. No silkworms or eggs of the insect might be exported; only raw or woven silk was permitted to leave the country.

This rigid monopoly continued until about A.D. 550, when, at the instigation of the Byzantine emperor, two monks smuggled out of China a quantity of eggs hidden in a bamboo cane.

The first emperor of the Ch'in dynasty, Shi Hwang Ti, lives in the world's memory on account of his connexion with the Great Wall. This forms the northern boundary of China, and runs from the Yellow Sea to the mountains of Tibet. There had been other walls along the boundary, but to Shi Hwang Ti belongs the honour of combining them into one, which is 1,500 miles in length. It was originally an earth rampart, faced with either stone or burnt brick. The towers—there are said to be 25,000 great towers and 15,000 watch-towers—were of sun-dried or burnt brick and only occasionally faced with stone.

Confucianism and Taoism were probably the indigenous religions of China, though Buddhism, which was introduced from India, took root and flourished. Another foreign religion was Christianity, which came in from Persia in the form called Nestorianism. This



EARLY CHINESE MASTERPIECES. Left, an owl carved in limestone. It was discovered in a Shang or Yin dynasty tomb, and it is the oldest known Far Eastern sculpture. Right, a tomb tile of the 3rd century B.C. It displays dogs, deer, and flying geese.
Academia Sinica, Nanking

grew so popular that in the 8th century it appeared likely to become the state religion, but the opposition of Confucianism and Buddhism, and perhaps of Islam, overwhelmed it, and although it still survives it is far from being the national religion.

The civilization of the island of Japan is founded on the Chinese. Japan appears to have been inhabited in early times by a primitive race, perhaps originating in western Asia ; these are called Ainu. There are legends of a still earlier dwarf people who preceded the Ainu, which shows that the islands have a long, though almost forgotten, history. The people now called the Japanese appear to have come from the mainland by way of Korea during the Bronze Age, for they were already well acquainted with the use of metal.

PREHISTORIC POTTERY OF JAPAN. The art of pottery making was established in the southern islands of Japan in the Neolithic age. The features of these curious figures are indicated by bits of clay dabbed on and by incisions.

The history of Japan traditionally begins at 660 B.C., when the imperial dynasty was first founded. Continual wars with the Ainu and other savages, and perpetual conquests and reconquests of Korea, gave the Japanese little leisure to evolve a civilization of their own. They therefore turned to China, and in the 3rd century A.D. established close cultural contact with that highly civilized power. Chinese writing and all the Chinese arts and crafts were introduced into Japan, and with them priests from Korea also brought the religion of Buddha, which gradually overshadowed the earlier Shintoism and became the state religion.

LESSON 27

Temples and Shrines of The Far East

In the Far East the religious influence of India, whether as Buddhism or Hinduism, is visible throughout the Middle Ages ; it extends to Ceylon, Burma, Siam, Cambodia, Malaya, Java, China and Japan. The great Shwe Dagon of Lower Burma is one of the holiest shrines in the East, for here are preserved eight hairs which the Buddha plucked from his own head and gave to two pious Burmese traders who visited him.

This is the legend. But the extreme sanctity of the temple suggests that it was built on a site which was already regarded as sacred by the pre-Buddhist people. The Shwe Dagon is a solid mass of brickwork with shrines round it. It was begun before the Christian era, but was not finished in its present form until 1584, and during all those centuries it was one of the chief places of pilgrimage in the Far East. The dome is covered with sheets of gold, and the belfry-pinnacle on the top is encrusted with precious stones and hung with numbers of silver bells which are rung by the wind.

The Shwe Dagon is in Lower Burma. In Upper Burma between the 9th and 13th centuries there was a wealthy and powerful kingdom whose capital was Pagán. In the 11th century one of its kings married an Indian princess (showing that there was a considerable amount of intercourse between the two countries). This queen's son built at Pagán a magnificent temple which has many features in common with Indian architecture. The city of Pagán was full of splendid pagodas, for each king tried to outdo his predecessors in lavish piety. It was utterly destroyed by a Chinese army sent to avenge the murder of some Chinese envoys.

Even after its fall some of the pagodas retained their sanctity, and the holy relics of Buddha which they contained made them the objects of pilgrimage for all devout Buddhists throughout the Middle Ages. This custom of making pilgrimages to sacred places and visiting holy relics dates from the time when ancestor-worship was the vogue.

In Ceylon the dagobas, or " relic-receptacles," take the place of the stupas of India, and, like their prototypes, are solid masses of brickwork. The shape is not unlike an inverted basin or a bell, with the little belfry-pinnacle at the top like the handle of the bell. Each dagoba stands on a base, which may be circular, quadrangular, or polygonal ; these rise in three terraces, and around them the religious processions made their way. In some of the dagobas there is a secret chamber hidden in the mass of the building, where the sacred relic was kept. On the walls of the terraces were carved scenes from the life of Buddha.

The impulse underlying these carvings is the

THE SHWE DAGON PAGODA. This great Buddhist shrine at Rangoon, in Lower Burma, was founded, according to tradition, in 585 B.C. The main building is surrounded by about 1,500 smaller shrines.

same as that which produced the Poor Man's Bible of the Middle Ages in Europe ; those sacred pictures and sculptures by which the illiterate learned the dogmas of their faith are similarly exhibited in an easily understood form.

In Cambodia, Hindu influence is more manifest than in Burma and Ceylon—an interesting fact, showing that Hinduism had its missionaries as well as Buddhism. The magnificent temple of Angkor Vat was built to honour Hindu gods, but later it became a shrine of Buddha. This famous building was finished in the 7th century A.D.

Three miles away lay the city of Angkor Thom, where dwelt the Aryan invaders who brought Hinduism to the country. They built not only Angkor Vat but other temples in Cambodia in the same style and as vast in size. The Aryan invaders came from India, not in one overwhelming mass but in a series of waves, so that they were gradually absorbed into the general population. Buddhism smothered the old Hinduism

"The Way of the Gods"

In all these countries, though Buddhism became the official religion, the old primitive beliefs can often be traced underlying the temple cults. Many of the old gods, dispossessed by Buddha, are still revered and still receive sacrifices in village shrines.

The original religion of Japan was ancestor-worship, the modern name of which is Shinto, "the Way of the Gods." It was about the middle of the 6th century that Buddhism was first introduced into Japan, and with it came the arts of civilized China.

It was due to the Buddhist doctrine that all living creatures should be treated with consistent kindness that the emperor Temmu decreed in the 7th century that no traps or pitfalls should be used for catching game and that certain animals should not be used as food. As long as Buddhism kept within the bounds of religion all went well, but when in 1575 the priests tried to usurp the temporal power the emperor put them all to the sword.

In the following century the same danger to the government arose through the Jesuit and Franciscan missionaries, and the same means were used to suppress the danger. To prevent any further peril from "religious intrigues," Japan shut herself off from the rest of the world, and until the year 1868 foreigners were rarely permitted to enter the country.

LESSON 28

Civilizations of the Mayas and Aztecs

CENTRAL AMERICA is divided geographically, and to some extent archaeologically, into two parts by the depression across the isthmus of Tehuantepec. To the east the countries of Guatemala, Honduras, and Yucatan belong to the Maya civilization ; those to the north-west of the isthmus were Aztec.

The history of Central America resolves itself into a series of migrations coming in from the north, sometimes by way of southern California and Jalisco, sometimes by the eastern route. The invaders were probably all of the same race, but at different stages of civilization, for they appear to have arrived as nomadic hunters and adopted agriculture after settling in the country. The continued immigration of

new tribes brought in fresh ideas and so raised the level of civilization.

The Maya people were closely connected with the pre-Aztec (Toltec) inhabitants of Mexico. Their architecture, their calendar, their vigesimal (proceeding by twenties) system of counting were the same; and the Maya civilization, like the Toltec, ended before the Spanish invasion. The collapse of the Mayas appears to have been due to internal decay; they gradually relapsed into small, insignificant communities.

In Mexico the allied culture of the Toltecs also died under the invasions of the Chichimecs, whose kingdom served as the foundation on which the still later invaders, the Aztecs, raised their empire. The Toltec kingdom began in A.D. 752; the Aztecs founded their city and established their supremacy in 1325.

The Potter's Wheel Unknown

The architecture of the people of Central America is peculiar to the country. The most remarkable of all the buildings were those dedicated to the service of religion, i.e. the temples. These were raised on high platforms or foundation-mounds made of earth and stones faced with masonry, and as the Mexicans had a special reverence for the points of the compass, the mounds were built four-square. The steps, by which access was obtained to the temple on the top of the mound, were always on the west, the temple being built on the flat platform on the top.

The reason for such a structure seems to be the desire to build the earthly habitation of the god as close as possible to his heavenly abode in the sky. In many places the walls of these erections are decorated with geometric designs, obviously derived from the technique of weaving and basketry. The patterns are often worked in mosaic and the variations of design are amazing. Throughout Central America weaving and basket-work have exerted an immense influence on the art; the squareness so characteristic of the painting and relief-sculpture indicates its origin.

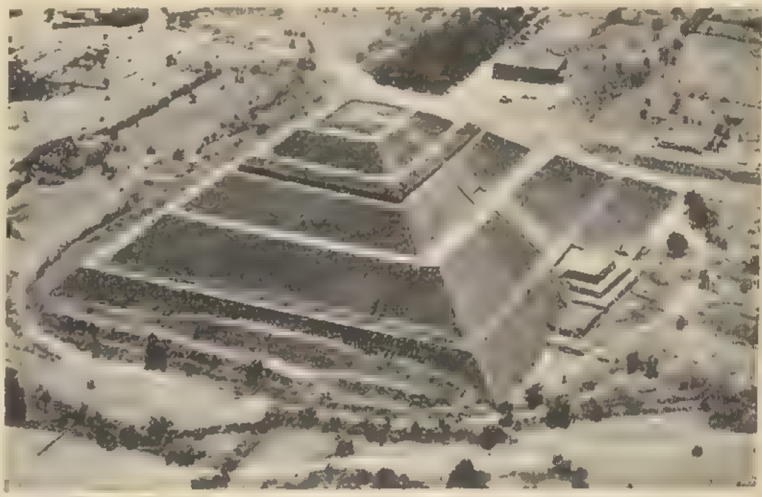
It is a strange fact that though the pottery is often very elaborate in design the potter's wheel was entirely unknown. The elaborate forms were made by hand, and were often fitted with pedestals or legs. The decoration was always geometrical in design.

The most important of the artistic products of the Mexicans was their stone mosaic; this was a product not found elsewhere in the ancient world, and may have been an invention of their own. One of the finest examples of this art is a human skull covered with horizontal bands of turquoise and jet set in resin; the eyes are iron pyrites surrounded with a ring of shell, and the nose is inlaid with pink shell. This object represented the god Tezcatlipoca, who was a fertility deity to whom human sacrifices were made. That the worship of this god began in very early times is shown by his emblem of a flint knife.

The Mexican calendar has always attracted a great deal of attention owing to its peculiarities. There were obviously two calendars, one superimposed on the other. The solar calendar, which was presumably the later of the two, gave 365 days to the year. The year was divided into 18 months of 20 days each, and there were five intercalary days at the end of the year; these were known as "useless" days and were regarded as unlucky.

Monthly Festivals

The monthly festivals, 18 to the year, followed this calendar, and it was then that human sacrifices were made, the victims being men, women, and children as occasion or custom demanded. The whole of the ceremonies were connected with agriculture; and, the length of the year being that of the agricultural year, it would seem that this calendar evolved when



AZTEC PYRAMID. The vast artificial mound of the Temple of the Sun at Teotihuacan, in Mexico, is the largest pyramid in Central America. It is 216 feet high, on a base line of nearly 1,000 feet. The actual temple was on the summit.

the tribes settled in the country. The ritual calendar, called the Tonalmatl, had a year of 20 weeks, each week being of 13 days, making a total of 260 days to the year ; this was the calendar for movable feasts and for divination, and it conforms to no other calendar in existence. There was also an elaborate system of naming the days, so that it was only the priests who could use the calendars with any facility.

The Mexicans had advanced far beyond the system of picture-writing common to the Indians of North America, and had evolved a form of script in which the signs were symbolical and not mere pictures. They were able to make maps and town-plans.

The religion of the Aztecs, which so horrified the Spaniards, throws a great deal of light on the beliefs and religious customs of early peoples. Death by sacrifice was the normal death of the fighting man, and ceremonial cannibalism was regarded as an act of communion with the god who was identified with the sacrificed victim. To primitive people death is neither abhorrent nor greatly to be feared, and this fact must always be borne in mind by the archaeologist.

The Mexican burial customs suggest that there was a strong belief among the Aztecs in the hereafter, for when the body was burnt the possessions of the dead man were burnt also, that they might go with him to the next world. Sometimes, as in other primitive countries, human beings were slaughtered at the grave to accompany their masters, and the Mexican was careful to take his dog also with him. The Aztec civilization is specially interesting as an instance of arrested development ; had the Spaniards not conquered the country, the tribes might have produced a civilization as high as, if not higher than, that of Peru.

LESSON 29

Developments of Culture in Peru

THOUGH Peru is linked in modern minds with the Inca civilization in power at the time of Pizarro's expedition, there was a great empire in that region before the Incas. The geographical features of that area of South America played a large part in influencing the development of culture. There are three main lines of country running roughly parallel from north to south : (1) the coastal desert, known to the Spaniards as Los Llanos, (2) the highlands and plateaux of the Andes, the Sierra, and (3) the forested eastern slopes of the Andes, the Montana.

Streams and small rivers flow from the Andes through the coastal desert, forming valleys, and it was by the sides of these streams that civlization arose and developed in that almost rainless land. The areas between the river valleys were, and still are, uninhabited, except by a few fisher-people living in tiny hamlets on the seashore, who cultivate maize.

Knowledge of the early periods depends chiefly on legends and oral tradition, and by this means, supplemented by some not very scientific excavation in Peru and Bolivia, it is possible to date approximately some part of the history :

	A.D.
Early Chimu (on the coast)	500
Tiahuanaco I (in the mountains)	500
Tiahuanaco II (coast and mountains)	600–900
Chaotic period	900–1100
Inca supremacy	1100–1530

About the beginning of the Christian era —a time of great movements of peoples throughout the world—tribes in small groups entered Peru from the north and settled in the mountains and in the river valleys of the coast. This period lasted till about A.D. 900, and was followed by a period of chaos, the causes of which are not known. Then the Inca tribe rose to power among the mountain people, conquered and held the coast, and ruled the country successfully until destroyed by the Spaniards.

Record-keeping by Knots

The accounts of the Chimu period are legendary but with foundations in fact, for these people had a system of record-keeping peculiar to themselves. This was a kind of tally formed by a series of knots tied on string —called quippu—the record-keepers being instructed as to the event associated with each knot, and passing on their knowledge orally.

The method continued through the Inca period, and the history thus chronicled was collected by some of the Spanish Jesuits. During the early Chimu period the country was divided into small independent communities, each ruled by its own chief. This is the invariable beginning of all civilized nations, and it is not until two or more small states coalesce to form one large community that any advance can be made.

The chief product of the Chimu period is the pottery, found buried in tombs. Though the potter's wheel was unknown, the art of pottery-making was highly developed. There were two contemporary "schools" of pottery-making—one at Truxillo in the north, where modelling was the principal consideration, the

other in the Nasca Valley in the south, where colour was of more importance than form. All the vessels were intended for holding water and are therefore of a peculiar type, as in that arid country water was exceedingly precious. The body of the vessel is almost globular, while the handle is a hollow loop with a vertical spout which could be easily stopped up in order to prevent evaporation.

The northern types are often in human form, surmounted by portrait heads naturalistically modelled. Sometimes the body of the vase is decorated with spirited sketches and scenes of daily life. The characteristic of Truxillo ware is the naturalistic and realistic quality of the artists' work. At Nasca the designs have the usual squareness of American art, but the colour in which those designs are worked out redeems their rather squat effect. These designs appear to have been influenced by a textile technique, the Chimu people, both north and south, being expert weavers. The Chimu used gold and copper for ornaments, but they had hardly reached the stage of using metal tools. They were builders in sun-dried brick, and had developed a system of irrigation which brought water in plenty into their chief city.

The masonry of the mountain people is surprising. They built with large blocks, having apparently no objection to using enormous boulders; these were fitted into other blocks of any size with re-entrant angles, so that each stone fits one place and one place only. These constructions were built without mortar, and they relied simply on mass and precision of fitting. At Cuzco walls 20 feet high are still standing, and the ruins of other walls extend for miles. At Tiahuanaco stood a great megalithic structure, probably a temple, of which there remains a huge doorway cut out of a single large block.

PERUVIAN POTTERY. This quaintly shaped water-bottle was found at Truxillo, in northern Peru. It dates from the Chimu period, c. A.D. 500.
British Museum

When the Inca civilization emerged from the storms which followed the Tiahuanaco II period, some of the ancient arts still remained. The Inca masonry had not the careless freedom of the earlier builders, but what was lost in size was gained in accuracy; the blocks used were smaller, but were more evenly squared and fitted. The monotony of long straight walls was broken by niches; these have been likened by superficial observers to Egyptian "false" doorways. But there is a difference: the Egyptian doorway always has vertical sides, whereas the Inca doorway broadens out from the top to the bottom.

The most amazing part of the Inca civilization was the organization of the country, which, owing to natural conditions, had never before been unified. Besides Peru, their empire, by 1500, included Ecuador, Bolivia, Chile, and part of Argentina. The Incas made roads, built causeways and bridges, and so linked together all outlying districts. They removed the inhabitants of any newly conquered country and settled them in the regions that were already Inca-ised, sending Inca colonists to fill the empty villages in the new territory.

Inca garrisons were stationed in outlying parts, and large trading centres were established. Agriculture was improved—no man was exempt from agricultural labour or military service except by special privilege—and irrigation works increased the wealth of the land. Mining, metallurgy, and stonework were in the hands of special artisans. Inca "propaganda" was carried out with great thoroughness. The Inca religion, language, and economics were imposed everywhere. But, with the wisdom of good rulers, the Incas refrained from any interference with the social organization of the people they conquered, and they seem to have treated the vanquished with justice and consideration.

LESSON 30

Archaeology and Human Progress

In looking back across the ages, it is possible to see how little has man changed in essentials since he first appeared on the earth. The same forces that surround him now surrounded him then, forces which he cannot control, yet which mould him and to which he must adapt himself if he would survive. It is possible to see how settlements arose in various parts, each with its own organization.

These became linked together by trade or by conquest, both these forces having a high value in the promotion of civilization. The clash of warring peoples and the subsequent compromise between victor and vanquished are often the prelude to a sudden outburst of culture; civilization is forced to take a quick step forward, and advances more rapidly than by the slow process of evolution. Political

changes have been continuous. Conquests, large or small, gave one nation or tribe the supremacy over others. That supremacy might be enforced by the extermination of the vanquished, as by the Spaniards in Central America, or it might be entirely nullified by the absorption of the victors into the vanquished nation. as in the Norman conquest of England.

Centres of civilization have shifted from country to country, the cause of the change being generally the decay of the paramount power. That decay was produced either by exhaustion following on a long-continued or violent war or by stagnation from a peace undisturbed by invasion or battle.

Geography and climate determine the conditions of life. A mountain people will not produce the same culture as the people of the plains or of the sea-shore. The tropics will create a different civilization from the temperate zone. The desert or forest people have other needs than those of the agriculturalists.

Climatic changes, such as the drying up of a region, will drive out the inhabitants to seek their living elsewhere ; overcrowding of a settlement will have the same effect. From the very earliest period of which there is any knowledge mankind has always been in movement, leaving traces of his handiwork wherever he settled. Palaeolithic man made his typical stone implements in all parts of the Old World, Metal-working spread with immense rapidity over large areas ; the only barriers to the advance of civilization were waterless deserts.

Man's Own Impulses

In every part of the world civilizations have been established, sometimes growing and spreading from very small beginnings until they become great centres for the distribution of culture, making their mark on the world as the chief contributors to the advance of mankind ; among these can be reckoned the ancient empires of the Near East. Others, like the Incas of Peru, were crushed by a less civilized power before they reached their full glory, and never had the strength to recover from their defeat ; thus they had no effect on the general culture of the world.

There are other factors which control civilization—these spring from man's own impulses. Of them one of the most important and powerful is the religious impulse. When man first used his brain to think, he found himself amid the marvels of nature ; and not knowing which were under his control and which were not, he attempted to propitiate all.

Though the primitive religious impulse was responsible for the savage rite of human sacrifice practised by the early Hebrews as by the more modern Aztecs, it was also responsible for the high ethical code of the ancient Egyptians, for the consciousness of sin among the Babylonians, for the realization of the sufferings of animals among the Buddhists.

The Desire for Beauty

As man rises in the scale of civilization, so does his conception of God also rise ; his beliefs and his civilization are so closely linked that one cannot rise or fall without affecting the other. The religious impulse is also responsible for some of the most remarkable and beautiful creations of human hands : the Palaeolithic paintings, the vast and magnificent temples in all lands, the sculptures of ancient Egypt and Greece.

Closely connected with religion in its earliest expressions is, therefore, the artistic impulse. This is manifested when man has attained to a little leisure, when the struggle for life is not too severe. Leisure implies wealth, and wealth implies civilization. The desire for beauty, however, is always present, even in the races that are regarded as of low mentality or culture.

The method of expressing that desire—whether in music, art, or words—necessarily varies according to the natural surroundings, the state of civilization, and the immediate conditions of the country in which that desire is shown. Poetry and music leave no trace until they are recorded, but there is no doubt that they were among the earliest of the arts. The finest epics were inspired by war. Love songs, lullabies, and hymns arose from the primitive impulses of sex, maternal love, and religion respectively.

It is when archaeology is studied from the standpoint of the rise of man that it assumes its true importance, for it brings into one connected whole the struggles, the defeats, and successes of mankind throughout the world.

BOOK LIST

Piecing together the Past, V. G. Childe (Routledge) : *Digging up the Past*, C. L. Woolley (Benn) ; *Foundations in the Dust*, Seton Lloyd (O.U.P.) ; *Man the Toolmaker*, K. P. Oakley (British Museum) ; *Later Prehistoric Antiquities of the British Isles* (British Museum) ; *Prehistoric Britain*, C. and J. Hawkes (Pelican Books) ; *Prehistoric Europe ; the Economic Basis*, J. G. D. Clarke (C.U.P.) ; *Wonders of the Past*, 2 vols. (Amalgamated Press) ; *New Light on the Most Ancient East*, V. G. Childe (Kegan Paul) ; *The Birth of Civilisation in the Near East*, H. Frankfort (Williams and Norgate) ; *A History of Technology, Vol. I*, C. Singer, E. J. Holmyard, and A. R. Hall (Clarendon Press) ; *Everyday Life in Babylonia and Assyria*, G Contenau (Arnold) ; *The Archaeology of Palestine*, W. F. Albright (Pelican Books).

MATHEMATICS

A SOLID *basis is provided in this Course for the study and comprehension of mathematical principles and methods, from numbers to the calculus. Necessarily compressed, it includes the major subjects listed below. The Course is the work of a group of expert instructors in this department of knowledge, a department upon which virtually every other form of science is dependent for its progress and practical development.*

28 LESSONS

LESSON 1

The Number System: Whole Numbers

MATHEMATICS is related to other branches of science in a simple way. Science is concerned with changes that occur in nature, the measurement of these changes, and predictions about future changes based on this. Fundamental in all this lies the study of numbers, how they are used and how new types of numbers have to be invented to represent new varieties of change.

Whole Numbers

A single mark, 1, is perhaps the simplest way to represent a single object. The simplest group of objects could then be represented by the simplest group of such marks, viz. 1, 1 or, "1 and 1." We find it better to give this group a special mark 2 and we call it *two*. So with a group 1, 1, 1 which we mark 3; and so on. Thus we quickly have the marks 1, 2, 3, 4, 5, 6, 7, 8, 9. The last, 9, stands for 1, 1, 1, 1, 1, 1, 1, 1, 1.

Each of these groups has been thought of as made up of single members, but it is easy to see that a group of 9 may be broken up into sets of smaller groups represented by the marks 3 and 6, or 4 and 5, or 2 and 7 for example. Thus:

9 is equivalent to 3 and 6, or $9 = 3 + 6$
9 „ „ „ 4 and 5, or $9 = 4 + 5$
The mark = is read *is equal to* or simply *equals*.
The mark + is read *add* or even simply *and* or *plus*.

Merely to use symbols to refer to groups does not help much if you have to invent a special symbol each time you form a new group. The difficulty is overcome in this way. Use 0 to stand for a "group" with no members—an empty group as it were. You then have groups represented by 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. When you form a new group by adding 1 to the group 9, you take the result as a new basic group, a new unit. To write it simply as 1 would confuse it with the more elementary unit, so you write it "one, nothing" meaning it is 1 large group and no individual members. The obvious mark for this is 10 which in ordinary language is called *ten*.

In this way 12 is one large "ten-group" and two individuals, 19 is one large "ten group" and nine individuals. 99 is nine large ten groups and nine individuals. To represent, say, 99 in strokes is a heavy task—hence the economy of this system of numbers. In the same way 100 is ten groups of ten units. If × stands for "times" then 9×10 stands for 9 times 10, i.e. nine groups of ten units, and 99 is $9 \times 10 + 9$ or 9 groups of ten units to which are added 9 units.

So any number of objects can be represented by means of the marks 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and as long as you are dealing only with whole objects these ten marks suffice. There are words in the language for the larger groups—ten groups of ten are called *one hundred*, ten groups of one hundred is *one thousand*, a thousand groups of one thousand is *one million*, and these are written as 100, 1,000, 1,000,000.

Thus the symbol 7,684 represents the number seven thousand six hundred and eighty-four. To facilitate reading, a comma is sometimes inserted between groups of three digits (starting from the right), thus 7,684 or 1,000,000. The number ten is said to be the base or scale of the notation (or the scale of counting); it is fairly certain that it was adopted because of the important role of the fingers in primitive and elementary counting.

Addition and Subtraction

The notation for whole numbers leads immediately to rules for adding together sets of numbers. For example, to add 135, 243, 274, you write them under each other like this:

$$\begin{array}{r} 135 \\ 243 \\ 274 \\ \hline 652 \end{array}$$

Add up the units $4 + 3 + 5 = 12$. This represents one group of ten and 2 units. Accordingly you write 2 in the answer to the extreme right and add 1 group of ten to $7 + 4 + 3$ which are groups of ten. The result is 15 groups of ten, i.e. 1 group of 100 and 5 tens. Thus you place 5 next to the 2 in the answer and add 1 to the hundreds giving $1 + 2 + 2 + 1 = 6$.

The final result is therefore 652. Then 652 is called the *sum* of 135, 243, and 274.

The opposite process to addition is *subtraction*, and for this is used the sign − (*minus*). It stands for the word "less." Thus $7 - 4 = 3$ reads "seven minus four equals three." We leave it to the student to justify the ordinary procedure for subtraction; notice that you cannot subtract a number from a smaller number.

Multiplication and Division

Multiplication and division of whole numbers are merely more elaborate forms of addition and subtraction. For the symbol 7×1428 is shorthand for the sum of seven sets or groups each containing 1428 objects. Rules for carrying out multiplications, based on the multiplication tables, avoid the necessity of doing

the equivalent additions. Thus, in the given example,

$$\begin{array}{r} 1428 \\ 7 \\ \hline 9996 \end{array}$$

7 groups of 8 are 56 units—that is to say, 5 groups of ten and 6 units. The 6 is written down and the 5 added to the next set of 10s, viz. 7 groups of 2 tens + 5 = 19 groups of 10. The 9 is written down and the 1 carried forward, and so on.

To calculate 27 × 1428, you first calculate 7 × 1428, as above, then calculate 20 × 1428 and add the two results. So 20 × 1428 = 28560, since 2 × 1428 = 2856, and you write the whole calculation in the following scheme :

$$\begin{array}{r} 1428 \\ 27 \\ \hline 9996 \\ 2856 \\ \hline 38556 \end{array}$$

(The 0 is usually left out in 28560 in such a multiplication).

It is seen that 1428 × 27 = 27 × 1428. This is true of the product of (i.e. the result of multiplying) any two numbers.

Division proceeds in the following way. If you have to divide 1428 into 7 equal groups you can see that 14 hundreds will divide into 7 groups of 200 each, and 28 units into 7 groups of 4. Thus :

1428 divided by 7 is 204 ; this may be written

$$1428 \div 7 = 204, \text{ or } 1428/7 = 204, \text{ or } \frac{1428}{7} = 204$$

If 1428 can be separated into 7 groups of 204 each, it can be separated into 204 groups of 7 each. Stated otherwise, this says that

$$1428 \div 7 = 204, \quad 1428 \div 204 = 7,$$
$$1428 = 7 \times 204$$

Thus every problem in division is equivalent to a problem in multiplication. Here is the systematic way in which a division would be performed. Divide 149527 by 413.

$$\begin{array}{l} 413)149527(362 \\ 1239 \\ \hline 2562 \\ 2478 \\ \hline 847 \\ 826 \\ \hline 21 \text{ units} \end{array}$$

You follow precisely the same method as in the simple case. You take the first four numbers 1495 ignoring the other two except to recognize that you are really dealing with 1495 hundreds. The largest whole number of times 413 divides into this is 3. Multiplying 413 by 3 gives 1239. Taking this from 1495 leaves 256, and since it was 1495 *hundreds*, you are left with 256 hundreds, or 2560 tens. Actually the original number was not 1495 hundreds but this plus 2 tens and 7 units. Accordingly you have 256 hundreds and 2 tens to consider, so you bring the 2 down alongside the 256, giving 2562 tens. Now 413 divides into this 6 times. You place the 6 in the *tens* position in the answer alongside the 3, and note that 6 times 413 is 2478. Taking this from 2562 leaves 84 *tens*. So far you have left out of account only the 7 units in the original number ; this with the 84 tens is equivalent to 847 units and 413 will divide into this 2 times, leaving 21 units over.

Thus finally you can say that 149527 can be split up into 413 equal groups each of 362 items, and 21 items are left over, an amount itself too small to be distributed among 413 equal groups. Thus :

$$149527 = 413 \times 362 + 21 \text{ over}$$

362 is the *quotient* and 21 the *remainder*. If one number divides another without remainder, it is said to divide it exactly. A number is even if 2 divides it exactly ; otherwise it is odd.

Factors and Prime Numbers

If one number divides exactly into a second number the former is said to be a *factor* of the latter. To *factorise* a number is to write it as the product of factors.

Thus 2, 7, 9, 20 are all factors of 2 × 7 × 9 × 20, that is of 2520.

A prime number is one which has no factors other than itself or 1. Thus 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, . . . are all prime numbers. It can be shown that there is no greatest prime number.

When a number has been factorised and all the factors are prime numbers, these factors are called prime factors. Thus :

$$2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$$

These are a few simple rules for finding factors.

1. Every even number has the factor 2.
2. If the sum of the *digits*, or individual figures, in a number is divisible by 3 or 9, then 3 or 9 is a factor of that number.
3. If the sum of the 1st, 3rd, 5th . . . digits of a number equals the sum of the 2nd, 4th, 6th . . . digits, the number is divisible by 11.
4. If a number ends in 5, it is divisible by 5.
5. If a number ends in 0, it is divisible by 10.

Powers of Numbers

Consider the numbers :

$$5,\ 5 \times 5,\ 5 \times 5 \times 5,\ 5 \times 5 \times 5 \times 5,$$
$$5 \times 5 \times 5 \times 5 \times 5, \ldots .$$

which for simplicity are written

$$5^1,\ 5^2,\ 5^3,\ 5^4,\ 5^5,\ \ldots .$$

The smaller numbers written above are called the *powers*.

Thus

10, 100, 1,000, 10,000, 100,000, 1,000,000, ... are

$$10^1,\ 10^2,\ 10^3,\ 10^4,\ 10^5,\ 10^6.$$

One million is 10 to the power 6. It is not customary to write the figure 1 for the first power, for there you are dealing only with the

number itself. Thus you could write:

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$
$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$
$$2400 = 2^3 \times 3 \times 10^2 \quad = 2^5 \times 3 \times 5^2$$

A number raised to the power 2 is said to be *squared*, because, as will be seen, the area of a square, the length of whose side is measured by the number, is itself measured by the same number raised to the power 2.

A number raised to the power 3 is said to be *cubed* because the volume of a cube, the length of whose side is measured by the number, is itself measured by the same number raised to the power 3.

NOTE. $2^3 \times 2^4 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^7 = 2^{3+4}$

$10^2 \times 10^3 = 10 \times 10 \times 10 \times 10 \times 10$
$= 10^5 = 10^{2+3}$

If two powers of the same number are multiplied, the result is the same number raised to the sum of the powers. The factorisation of 2520 into prime factors may be written $2520 = 2^3, 3^2, 5, 7$.

Least Common Multiple

What is the smallest number into which 24 and 36 will both divide exactly? They will evidently both divide into 24×36, but this is not the smallest number. Now

$$24 = 2^3, 3$$
$$36 = 2^2, 3^2$$

The least number into which they both divide exactly—the *least common multiple*, abbreviated to L.C.M.—must be divisible by 2^3 and 3^2, the former because 24 requires this, and the latter because 36 requires it. Thus the L.C.M. of these two numbers is $2^3, 3^2 = 72$.

Similarly the L.C.M. of any set of numbers may be found. The rule is that any prime number appears in the L.C.M. of a given set of numbers raised to the power equal to the *largest* power to which it occurs in the set.

(You will require this when adding or subtracting fractions.)

Highest Common Factor

This is the largest factor common to any given set of numbers. For example, if you look at the factors of 24 and 36 you can see that the highest factor in common is $2 \times 2 \times 3 = 12$. It is usual to write H.C.F. for "highest common factor."

Similarly the H.C.F. of any set of numbers may be found. The rule is that any prime number appears in the L.C.M. of a given set of numbers raised to the power equal to the *smallest* power to which it occurs in the set.

EXERCISES

To be worked by the student. Answers appear at the end of the following Lesson.

(1) Complete the statements: $7 = 1 +$ $24 + 3 =$, $4 + 7 = 1 +$, $9 =$.
(2) Add together 27, 42, 121.
(3) Add together all the odd numbers up to, and including, 17.
(4) Add one thousand and eight to two hundred and forty-seven. Then subtract six hundred and eighty from the total.
(5) Multiply 184 by 27.
(6) Verify that 342×16 is same as $16 \times$
(7) Divide 64 into 1088.
(8) Find what is left over when 2683 is divided by 51.
(9) Find the prime factors of 105, 62, 11
(10) Find the L.C.M. and H.C.F. of 25 and 155; 6, 9, and 21; 24 and 56.

LESSON 2

The Number System: Fractions

So far we have dealt only with whole numbers, and you know what we mean when we say: "Divide 27 by 3 or divide 19 by 6." The answer to the former is 9, and to the latter 3 with 1 over.

If you are dealing with a single group of 12 objects it is possible to divide it into 2, 3, 4, or 6 parts so that you could talk sensibly of

$$1 \div 2 \text{ or } \frac{1}{2},\ 1 \div 3 \text{ or } \frac{1}{3},\ 1 \div 4 \text{ or } \frac{1}{4},\ 1 \div 6 \text{ or } \frac{1}{6}$$

of the group. This would also be true if it were a length of distance or time, or the weight of a heap of sand. Sub-division is a normal physical process. A number that represents any element or group of elements into which a unit has been subdivided is called a *fraction*.

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots, \frac{2}{3}, \frac{2}{4}, \frac{3}{4}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \ldots$$

are all *fractions*.

To divide two units into three equal parts (each of which is therefore 2/3) is the same as dividing two separate units each into three parts and adding together two of the elementary parts. Thus

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3},\ \frac{2}{4} = \frac{1}{4} + \frac{1}{4},\ \frac{3}{5} = \frac{2}{5} + \frac{1}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$$

Two quarters are equal to one half; so also are three sixths or four eighths, etc. Thus:

$$\frac{2}{4} = \frac{1}{2};\ \frac{3}{6} = \frac{1}{2};\ \frac{4}{8}\ \frac{1}{2}$$

In each of the fractions 2/4, 3/6, 4/8 there is a factor common to the *numerator* and the *denominator*, i.e. the number at the top, and that at the bottom. Can you always cancel out a common factor from numerator and denominator?

This must be so, as can be seen in any special case. Suppose for example you have 10/13

w…ch is the result of taking a thirteenth part of something ten units long. If you take a length of 60 units instead of 10, you would require to divide this not into thirteen parts but into many more parts, viz. six times thirteen p…s in order to obtain the same final length. Thus:

$$\frac{10}{13} = \frac{6 \times 10}{6 \times 13}$$

Reversing this process you can write, say—

$$\frac{105}{117} = \frac{3 \times 5 \times 7}{3 \times 3 \times 13} = \frac{5 \times 7}{3 \times 13} = \frac{35}{39}$$

$$\frac{84}{108} = \frac{2 \times 2 \times 3 \times 7}{2 \times 2 \times 3 \times 3 \times 3} = \frac{7}{3 \times 3} = \frac{7}{9}$$

When all the factors that are common to numerator and denominator have been cancelled out as in these two examples the fraction …aid to be reduced to its *lowest terms.*

…ldition, Subtraction, Multiplication

…he foregoing considerations can be used …addition of fractions in this way:

$$\frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4}$$

$$\ldots \frac{3}{10} = \frac{7}{3 \times 5} + \frac{3}{2 \times 5} = \frac{7 \times 2}{2 \times 3 \times 5} + \frac{3 \times 3}{2 \times 3 \times 5}$$

$$= \frac{14}{30} + \frac{9}{30} = \frac{23}{30}$$

The denominator of each fraction is written in terms of its factors, and the least common multiple of the denominators is found. Each fraction is then brought to a form with this least common multiple as denominator and the addition follows at once. Thus consider:

$$\frac{5}{168} + \frac{3}{280} + \frac{1}{42}$$

Here

$$168 = 2^3, 3, 7$$
$$280 = 2^3, 5, 7$$
$$42 = 2, 3, 7$$
$$\text{L.C.M.} = 2^3, 3, 5, 7 = 840$$
$$\frac{5}{168} = \frac{25}{840}, \quad \frac{3}{280} = \frac{9}{840}, \quad \frac{1}{42} = \frac{20}{840}$$

Hence

$$\frac{5}{168} + \frac{3}{280} + \frac{1}{42}$$
$$= \frac{25}{840} + \frac{9}{840} + \frac{20}{840}$$
$$= \frac{54}{840} = \frac{2 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 3 \times 5 \times 7} = \frac{9}{140}$$

Subtraction can be carried through in the same way. Thus:

$$\frac{5}{168} - \frac{3}{280} + \frac{1}{42}$$
$$= \frac{25}{840} - \frac{9}{840} + \frac{20}{840}$$
$$= \frac{25 - 9 + 20}{840} = \frac{36}{840} = \frac{3}{70}$$

Fractions are multiplied simply by multiplying numerator and denominator. Thus:

$$\frac{15}{64} \times \frac{8}{27} = \frac{2^3, 3, 5}{2^6, 3^3} = \frac{5}{72}$$

Proper and Improper Fractions

We usually think of fractions as numbers less than 1, where the numerator is less than the denominator. Such a fraction is called a *proper fraction.* There is no reason however why the form of sub-division used to produce fractions should not be applied to cases where the numerator is greater than the denominator. For example, you could divide 6 into 3 parts and write it as:

$$\frac{6}{3} = \frac{2}{1} = 2$$

$$\text{Or, } \frac{4}{3} = \frac{3 + 1}{3} = \frac{3}{3} + \frac{1}{3} = 1 + \frac{1}{3}$$

$$\frac{22}{7} = \frac{21 + 1}{7} = \frac{21}{7} + \frac{1}{7} = 3 + \frac{1}{7}$$

The term *improper fraction* is used for one in which the numerator is greater than the denominator, and in such a case it can always be rewritten, as above, as a whole number plus a proper fraction. This you do by dividing the denominator into the numerator and forming the fraction by using the remainder for this purpose. For example you have found that 149527 when divided into 413 parts gives 362 and a remainder of 21 over (Lesson 1). Thus:

$$\frac{149527}{413} = 362 + \frac{21}{413} = 362 + \frac{3}{59}$$

NOTE.—It is usual to write combinations such as a whole number plus a proper fraction, like

$$3 + \frac{1}{7}, \text{ or } 362 + \frac{3}{59}, \text{ as } 3\frac{1}{7} \text{ and } 362\frac{3}{59}$$

Negative Numbers

We have indicated that mathematics is the servant of science. It has to discover the kind of numbers and the kind of numerical operations that represent those changes in nature that can be measured. So far we have dealt with whole numbers and fractions, both of which are needed in describing simple counting operations and in forming smaller groups from larger groups. We now propose to extend both these ideas of number.

If you will watch any kind of indicator or measuring instrument such as one that shows pressure or temperature, you will see that there is a certain normal position of the pointer usually referred to as the "zero position." When the temperature or pressure is greater than this, the pointer moves to one side or rises above the zero mark; when it is less than the normal, the pointer moves to the other side or falls below the zero mark.

Confusion would arise if you were merely to state, for example, that the temperature is ten degrees (or whatever the unit may be) away from the zero ; you must indicate whether it is above or below, to one side or the other, as the case may be.

How are you to indicate this ? We already have two symbols which stand for these, + and −. When you have written + 10 you have meant " add 10 " to what preceded it, and when you have written − 10, " take 10 away." In this case what precedes it is simply the zero position, viz. 0. Thus + 10 degrees on a thermometer would mean that it shows 10 degrees *above* zero, and − 10 degrees a reading 10 degrees *below* zero. − 10 degrees is as reasonable a temperature reading as + 10 degrees. It is therefore necessary for scientific reasons to consider that when you talk of 10 or 8 you mean + 10 or + 8 and when you talk of − 10 and − 8 you are equally indicating a real type of number.

What is true of whole numbers is equally true of fractions. In future, therefore, we shall accept negative whole numbers and negative fractions as valid forms of number and they will be indicated by the presence of a minus sign before them. When we have positive numbers, we may or may not insert the positive sign.

The rules of arithmetic for negative numbers are chosen to be consistent with their meaning, given above. Thus to subtract − 10, you add 10 ; to add − 10, you subtract 10 ; to multiply a number by − 10, you multiply by 10 and then change the sign of the answer. Notice that, by these rules, the product of two negative numbers is a positive number.

EXERCISES

Answers appear at the end of the following Lesson.

(1) Reduce the following fractions to their lowest terms :

$$\frac{16}{64}, \frac{81}{108}, \frac{19}{95}, \frac{168}{228}$$

(2) Complete the following statements

$$\frac{2}{7} + \frac{1}{5} = \frac{}{35}, \quad \frac{3}{11} + \frac{1}{4} = \frac{}{44}, \quad \frac{1}{12} + \frac{2}{9} \quad 36$$

(3) Add together $\frac{1}{11}, \frac{3}{8}, \frac{1}{6}$

(4) Find $\frac{1}{16} + \frac{3}{8} + \frac{5}{12}$

(5) Find $\frac{7}{9} + \frac{3}{4} - \frac{9}{10}$

(6) Show that $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = 1\frac{1}{12}$ and tha

$$\frac{7}{8} + \frac{1}{2} - \frac{3}{16} = 1\frac{3}{16}$$

(7) Show that $9 \times 81 = 3^6$; 7×21
$7^2 \times 3^3$; $16 \times 24 \times 72$ 6^7

ANSWERS TO EXERCISES IN LESSON

(1) 6, 27, 10, 3
(2) 190
(3) 81
(4) 57
(5) 4968
(7) 17
(8) 31
(9) 3, 5, 7 ; 2, 31 ; 3, 5, 7, 11
(10) 775, 5 ; 126, 3 ; 168, 8

LESSON 3

The Decimal System

CLOSELY interwoven with the method of writing numbers are *Powers of Ten*. For example, 24 stands for 2 groups of 10 plus 4 units :

$$24 = 2 \times 10 + 4$$

The 4 is added *after* the multiplication is performed. In the same way—

$$\begin{aligned} 396 &= 3 \times 100 + 9 \times 10 + 6 \\ &= 3 \times 10^2 + 9 \times 10 + 6 \\ 1428 &= 1 \times 10^3 + 4 \times 10^2 + 2 \times 10 + 8 \end{aligned}$$

so you see that the power of each 10 tells you the place occupied by the number multiplying it. Again in 11111 each 1 has a value equal to 1/10th of the 1 to its left. The least important place is 1 on the extreme right which represents just 1 unit. But if you use the principle that each 1 represents 1/10th of a 1 to its left, you ought to be able to represent fractions in this way by extending your numbers to the right beyond the unit place, provided you make clear where the unit place is. This you do by placing a dot after it. Thus 1·1 represents 1 unit (the left 1) and 1/10th of a unit (the right 1). In this way 1·2 would represent 1 unit and 2/10th ; 1·3, one unit and 3/10th and so on. For example—

$$743{\cdot}9 \text{ stands for } 7 \times 10^2 + 4 \times 10 + 3 + 9/10$$

The latter is a cumbersome way of writing, as a set of whole numbers and a fraction, something whose meaning is quite clear in the form 743·9. Notice that one of the positions might be occupied by a 0 and this would not affect the meaning to be attached to any subsequent number. For example 20·8 still means 20 and 8/10th.

Similarly, the symbol 37·62 stands for $37 + \frac{6}{10} + \frac{2}{100}$ or $37\frac{62}{100}$ and the symbol 124·757 stands for $124 + \frac{7}{10} + \frac{5}{100} + \frac{7}{1000}$ or $124\frac{757}{1000}$

Thus any fraction which has a denominator (when not necessarily in its lowest terms) which is a power of 10 can be represented by a

decimal. If you do not insist that the decimal terminates (i.e. has just a certain number of digits after the decimal point), then you may represent any number as a decimal ; and you may find a terminating decimal as near as you like to any given number.

Addition, Subtraction, Multiplication

The advantages of the system show themselves at once in the simple processes of addition, subtraction, multiplication, and division. The following will bring this out :

Addition	*Subtraction*	*Multiplication*
17·0350	17·0350	13·9995
4·1781	4·1781	4·1781
13·9995	12·8569	
35·2126		

```
   13·9995
    4·1781
   -------
    139995
   1119960
   979965
   139995
  559980
  ------------
  58·49131095
```

The multiplication has here been carried through without paying any attention to the decimal point. You know that the result must actually be approximately the same as 4 × 14, i.e. 56. Hence you insert the decimal point after the 8. Otherwise the rule would say that the number of digits after the decimal point is the sum of the number of digits after the decimal point in 13·9995 and 4·1781, that is, 8, and the decimal point must be inserted before the 8th figure from the right in the answer.

Division of Decimals

To divide 13·9995 by 4·1781

```
4·1781)13·9995(3·35068 . . .
       125343
       ------
        146520
        125343
        ------
         211770
         208905
         ------
          286500
          250686
          ------
           358140
           334248
           ------
            23892
```

The division proceeds in the usual way. You know that the result must be approximately, 3, hence the position of the decimal point is fixed. A series of 0s is brought down in succession, simply because 13·9995 is the same as 13·9995000 . . . The division is carried as far as is required.

Converting Factors to Decimals

You know already that—

$$\frac{1}{10} = 0{\cdot}1,\ \frac{2}{10} = \frac{1}{5} = 0{\cdot}2,\ \frac{3}{10} = 0{\cdot}3,\ \frac{4}{10} = \frac{2}{5} = 0{\cdot}4,$$

$$\frac{5}{10} = \frac{1}{2} = 0{\cdot}5,\ \frac{6}{10} = \frac{3}{5} = 0{\cdot}6,\ \frac{7}{10} = 0{\cdot}7,\ \frac{8}{10} = \frac{4}{5} = 0{\cdot}8,$$

$$\frac{9}{10} = 0{\cdot}9$$

Other fractions can be expressed as decimals by direct division. Thus :

$$\frac{1}{4} = 1 \div 4 = 1{\cdot}00 \div 4$$
$$= 0{\cdot}25$$
$$\frac{1}{8} = 1{\cdot}000 \div 8 = 0{\cdot}125$$
$$\frac{3}{8} = 3{\cdot}000 \div 8 = 0{\cdot}375$$
$$\frac{5}{8} = 5{\cdot}000 \div 8 = 0{\cdot}625$$
$$\frac{7}{8} = 7{\cdot}000 \div 8 = 0{\cdot}875$$

Recurring Decimals

If you try to convert fractions whose denominators contain prime factors different from 2 or 5, you obtain non-terminating decimals. For example :

$$\frac{1}{3} = 1{\cdot}0000\ldots \div 3 = 0{\cdot}3333\ldots,\quad \frac{1}{9} = 0{\cdot}1111\ldots,$$
$$\frac{2}{3} = 2{\cdot}00000\ldots \div 3 = 0{\cdot}6666\ldots,\quad \frac{2}{9} = 0{\cdot}2222\ldots$$

All of these consist of one recurring digit. To indicate this you write a dot above the digit that recurs thus :

$$1/9 = 0{\cdot}\dot{1},\ 1/3 = 0{\cdot}\dot{3},\ 2/3 = 0{\cdot}\dot{6}$$

A long chain of numbers such as those after a decimal point, all of which are single integers, either show no pattern at all—like

3·14159265 . . .

or recur after a certain interval, such as :

0·142857142857142857142857142 . . .

The former is the ratio of the circumference of a circle to its diameter and does not recur. The latter is the decimal for 1/7 as can easily be found by direct division, 1·0000 . . . ÷ 7. To show how they recur a dot is placed over the first and last digit of the section that repeats. Thus :

$1/7 = 0{\cdot}\dot{1}4285\dot{7}$	$2/7 = 0{\cdot}\dot{2}8571\dot{4}$
$3/7 = 0{\cdot}\dot{4}2857\dot{1}$	$4/7 = 0{\cdot}\dot{5}7142\dot{8}$
$5/7 = 0{\cdot}\dot{7}1428\dot{5}$	$6/7 = 0{\cdot}\dot{8}5714\dot{2}$

All non-terminating decimals obtained by converting fractions recur.

To convert a recurring decimal into a fraction :

(1) Write the decimal as the sum of a non-recurring and recurring part, thus

$$25{\cdot}67\dot{4}3\dot{8} = 25{\cdot}67 + 0{\cdot}00\dot{4}3\dot{8}\ ;$$

(2) Convert the recurring part to a fraction by putting into the denominator as many 9s as there are digits recurring, followed by as many 0s as there are 0s between the decimal point and the first recurring digit, thus

$$\cdot 00\dot{4}3\dot{8} = \frac{438}{99900}.$$

(3) Finally,

$$25{\cdot}67\dot{4}3\dot{8} = 25 + \frac{67}{100} + \frac{438}{99900}$$

Between 1·1 and 1·2 there is no limit to the numbers that can be inserted. Those below 1·15 are nearer to 1·1 than to 1·2; those above 1·15 are nearer to 1·2 than to 1·1.

Rounding off the Decimal

Hence if you wish to write such a decimal as 1·13 to one place of decimals, you will write it simply as 1·1. In the same way 1·18 to one place of decimals would be written as 1·2. You will have *rounded off* the second decimal place.

Example. 1·17832 is
1·1783 to *four* places of decimals,
1·178 to *three* places of decimals,
1·18 to *two* places of decimals and
1·2 to *one* place of decimals.

Finally, the nearest integer is 1.

EXERCISES

Answers appear at the end of the following Lesson.

(1) Write the following decimals as fractions: 21·7, 19·62, 214·03, 70·07, 142·618
(2) Add together 26·548, 3·006, and 42·71.
(3) Find 4·07295 + 11·20036 − 9
(4) Subtract 17·3214 from 27·2107
(5) Multiply 143·65 by 5.
(6) Multiply 16·3074 by 2·14 and confirm that the value is the same as multiplying 2·14 by 16·3074.
(7) Find $\frac{21{\cdot}4 \cdot 32{\cdot}7}{2{\cdot}6}$
(8) Express the following recurring decimals as fractions: $0{\cdot}\dot{2}$, $0{\cdot}\dot{6}$, $0{\cdot}8\dot{3}$, 2·16

ANSWERS TO EXERCISES IN LESSON 2

(1) $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{14}{19}$ (2) 23, 11
(3) $\frac{167}{264}$ (4) $\frac{41}{48}$ (5) $\frac{113}{180}$

LESSON 4

Mathematical Problems of Shape and Size

We have said that science is concerned with the changes that occur in nature, and mathematics with the measurement of these changes. A mathematical problem therefore arises as soon as one has disentangled from a complicated situation those parts of it that relate to shape, size or amount, area, time, etc.

This is not always easy to do; for instance, if you are listening to a Beethoven symphony it is not a simple matter to find a reasonable answer to the question—"*How* beautiful is it?" or "Does it stir you *more* than it stirs me?" That part of life that is linked up with feelings and emotions has not yet been, if it ever will be, reduced to mathematical form. But if you are watching a car speeding round a track, you can reasonably ask "*How long* will it take to make a complete circuit?" Or if a house has to be built of bricks, it is obvious that there could be ways and means of discovering the total *number* of bricks required, their total *cost*, their total *weight*, the greatest weight of roofing that can be placed on its walls. All these are questions of a mathematical kind because they involve quantities.

What is a Mathematical Problem?

The first step, therefore, in deciding whether a problem is of a mathematical nature is to examine how far it is concerned with shape and size and the changes of these with time. There are other aspects such as force, and temperature, and energy, which fall into the field of physical science, but they also are to that extent mathematical. It will be shown later how to analyse a mathematical problem in such a way as to bring mathematical knowledge to bear on it and solve it. For the moment all that is necessary to recognize is that a problem is of a mathematical nature when it is concerned with shapes, sizes, measurements, and the relations between these.

For the Lessons which follow you should provide yourself with the mathematical instruments that will enable you to handle such problems, and in doing so you will see how to disentangle the mathematical side of any complicated situation from those other aspects such as emotions and feelings with which we are not here concerned.

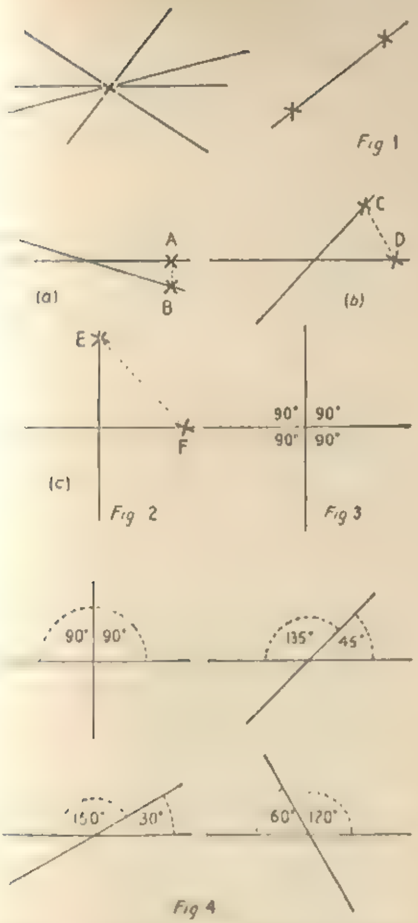
Shapes and Sizes

It is evident enough that the number of bricks required to build a house of a given shape and size depends on the size and shape of the brick and not on the feelings of the builder. He can be dismissed from the picture. Indeed, once the size and shape of the house is given it becomes merely a question of filling up so much space with bricks of certain sizes. Thus the composition of the brick itself also has nothing to do with the question. A mathematical problem is reduced to its proper form when you are left with a statement of what is required in terms of shape and size. We shall return to this later.

Our purpose now is to develop the tools, as we have already produced those of a simple arithmetical nature.

A mathematical problem is concerned with shape and size, and thus it becomes imperative to consider these terms more exactly. In seeking an answer to the question "What is a shape?" you will have to consider points and lines, and an examination of the meaning of size will lead to the discussion of lengths, areas, and volumes.

The branch of mathematics dealing with these problems is called *geometry* and the subject divides naturally into two parts, *plane geometry* and *solid geometry*. Plane geometry deals



Figs. 1 to 4. Fixing points, lines, and angles.

with points and lines on a *flat surface*, that is to say, the figures that you draw will have length and breadth (and therefore area) but no thickness. Solid geometry deals with objects having thickness as well (and therefore volume). This Lesson is concerned only with plane geometry.

Points and Lines

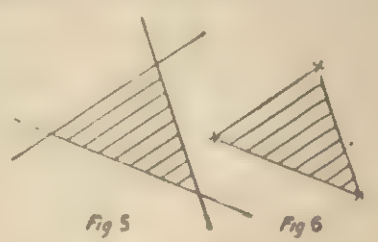
Suppose that you have a fixed point (as represented by a dot or a small cross on the paper (Fig. 1)) and wish to draw a straight line through this point. Since the direction of the line is in no way fixed you can draw any number of lines. If you wish to draw a line through two fixed points, then there is only one possibility. Stated otherwise, two points fix a straight line (Fig. 1). Again, if two straight lines cut one another, they fix a point. If two lines are drawn which do not cut one another, however far they are drawn in either direction, then those lines are said to be *parallel*.

If two lines cut one another, this can happen in various ways. Consider the three diagrams Figs. 2, 3, and 4. Each consists of two lines crossing one another (or *intersecting*, as it is called in geometry). What differs in the three diagrams is the "amount of spread." Consider the distance apart of pairs of points all at the same distance from the points of intersection. These points are lettered A, B ; C, D ; E, F. The distance CD is greater than the distance AB, and similarly EF is greater than CD. This "amount of spread" between pairs of intersecting straight lines is called the *angle* between them. In the third diagram (Fig. 2*c*) where the four angles made by the lines are all equal, the lines are said to be at *right angles* (Fig. 3), and if each right angle is divided into 90 parts, each small angle so formed is called one *degree* and written 1°. For closer measurement of angles each degree can be subdivided into 60 *minutes* (written 60′), and again each minute split up into 60 *seconds* (60″). It will be noticed that in the foregoing the sum of two *adjacent* angles (i.e. two angles next to one another) is 180°, and this is true however the intersecting lines cut, as seen in Fig. 4.

Triangles

So far we have considered only *two* lines intersecting. Suppose you have three lines intersecting, no two of which are parallel. Then each pair will have a point of intersection and you will have a shape formed by the part of each of the three lines between its intersections with the other two ; such a shape is called a *triangle* (Fig. 5). Thus a triangle is fixed by three lines or by three points (provided that the three points are not in the same straight line).

Every triangle has three sides and three angles (Fig. 6), but these are not all independent. We have not space here to give detailed geometrical proofs of all the relations between lines, angles and triangles ; these relations (or *theorems* as they are called) can be found in any text book on geometry. The purpose here is to consider some general properties, state some of the more important facts, and consider their bearing on mathematics in general.



Figs. 5 and 6. Triangles.

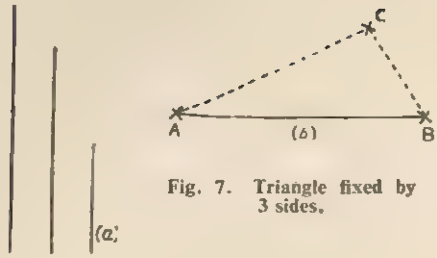


Fig. 7. Triangle fixed by 3 sides.

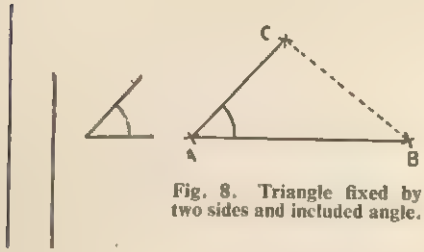


Fig. 8. Triangle fixed by two sides and included angle.

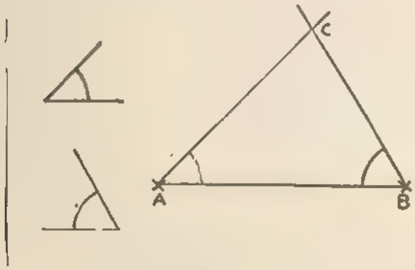


Fig. 9. Triangle fixed by one side and two angles.

The smallest number of quantities required to fix the shape and size of a triangle are three.

(1) A triangle is fixed if the three sides are given. Suppose the three sides are represented by three lines (Fig. 7*a*). Draw AB equal in length to the first line. Then only one point C can be found such that AC equals the second line in length and BC the third line (Fig. 7*b*).

(2) A triangle is fixed if two sides and the *included* angle are given (Fig. 8). (The included angle is the angle between the two given lines.) Draw AB equal to the first given line and at A make an angle equal to the given angle extending the arm to the point C and making AC equal the second given side. Then, B and C being now fixed, the third side BC is fixed so that only one triangle can be drawn with the given information.

(3) A triangle is fixed if one side and two of the angles are given (Fig. 9). Draw AB equal to the given side, at A draw one of the given angles and at B draw the other. By extending the arms of these angles they will meet in a fixed point C.

NOTE.—If only the three angles of a triangle are given, any number of triangles may be drawn.

These considerations of the least quantities necessary to fix the shape and size of a triangle have a very important bearing on the relation of *two* triangles to one another; for if two triangles have any one of the foregoing sets of necessary quantities *in common*, they must be equal in all respects. Triangles which are equal in all respects are said to be *congruent*.

Congruent Triangles

Two triangles will be congruent if they have

(1) Three sides of one equal to the three sides of the other.

(2) Two sides and the included angle of one of the triangles equal to two sides and the included angle of the other.

(3) Two angles and a corresponding side equal.

Similar Triangles

It has already been stated that if only the three angles of a triangle are given, any number of triangles may be drawn. There will, however, be a very important relation between the lengths of the sides of such triangles. To discover what this relation is, draw any triangle ABC taking AB equal to, say, two inches and making the angles at B and C a definite number of degrees (Fig. 10*a*). Then draw another triangle DEF making DE equal to, say, 5 inches and having the angles at E and D equal to those at B and A respectively (Fig. 10*b*).

Measure AC, BC, DF, EF. You have made DE two and a half times as long as AB and you will find that $DF = 2\frac{1}{2} \times AC$, and $EF = 2\frac{1}{2} \times BC$, except for some small practical error. In other words, you will find—

$$\frac{DE}{AB} = \frac{5}{2} = \frac{DF}{AC} = \frac{FE}{BC}$$

The only difference in the two triangles is in the *scale* to which you have drawn the sides; the angles are the same. Such triangles, where the angles are the same and the ratio of corresponding sides equal, are called *similar triangles*.

Facts True about ALL Triangles

(1) Two sides of any triangle are together greater than the third.

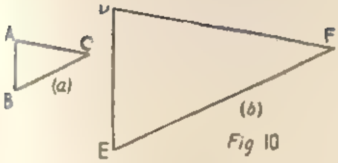
A straight line is often defined as the shortest distance between two points and on this definition, the foregoing theorem follows immediately. For suppose ABC is any triangle, then the path from A to B is shorter than the path A to C together with the path C to B (Fig. 11),

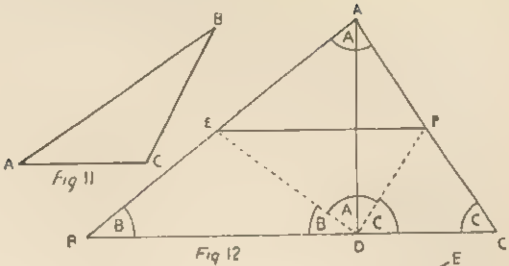
$$\text{i.e. } AB < AC + CB$$
$$\text{or } AC + CB > AB$$

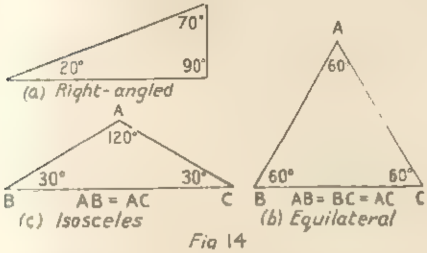
(The sign $<$ means "less than," and $>$ "greater than.")

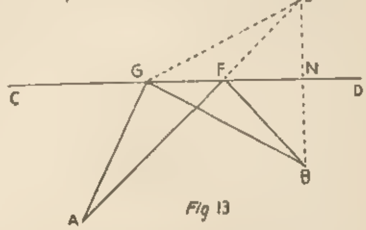
(2) The three angles of any triangle add up to 180°. This can be experimentally verified as follows:

Draw any triangle ABC and through the point A draw a line at right angles to BC









TRIANGLES. Fig. 10. Similar triangles. Fig. 11. Any two sides greater than third. Fig. 12. Three angles total 180°. Fig. 13. Problem. Fig. 14. Particular triangles.

(Fig. 12). Call this line AD. Divide AD in two by a line (EF) parallel to BC. Cut the triangle out and fold it about EF. A will coincide with D so that the angle between ED and DF (written $\angle$ EDF) is equal to A. Again draw lines through E and F parallel to AD, and fold the triangle about each of these lines in turn. Both B and C will coincide with D, so that $\angle$ EDB = B and $\angle$ FDC = C. Then since the three angles at D must add up to 180°, BDC being a straight line, you have $B + A + C = 180°$, i.e. the three angles of a triangle add up to 180°.

Problem. A pilot has to fly from a point A, bomb a straight railway line, and fly back to a point B. It is required to find the direction he must take in order that his total journey shall be as short as possible.

Consider the diagram Fig. 13. A, B represent the starting and finishing points respectively, CD the straight railway line. From B draw a line at right angles to CD cutting it at N and produce to E making NE = BN. Join AE, cutting CD in F. Then AF + FB will be the shortest distance the pilot need fly.

To prove that this is so, take any other point (G) in CD. Join AG, GB and GE. Then you require to show that AG + GB is always greater than AF + FB, wherever G may be.

Since BN = NE by construction, then the triangles FNB, FNE are congruent, i.e. equal in all respects because each of these triangles has two sides equal (FN is common, and BN = NE) and the included angle equal ($\angle$ FNB = 90° = $\angle$ FNE). Therefore FE = FB. Similarly GE = GB. Thus AF + FB = AF + FE = AE; and AG + GB = AG + GE. Using the property above, that two sides of any triangle are always greater than the third, you have

$$AG + GE > AE$$

i.e. $AG + GB > AF + FB$ for all positions of G. So that the point F you have found will be the point to bomb the railway line in order that the total distance flown will be a minimum.

Facts About Particular Triangles

(1) In any right-angled triangle, the sum of the two angles other than the right angle add up to 90°. This follows from the theorem above. Since the three angles of any triangle add up to 180°, then if one angle is 90°, that leaves 90° for the sum of the other two.

(2) In a triangle which has all its sides equal (an *equilateral* triangle) the angles are 60° each. The angles are all equal and therefore any one = 180°/3 = 60°.

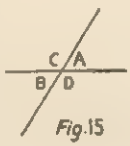
(3) An *isosceles* triangle is one which has two of its sides equal in length. The angles opposite these sides are also equal. Fig. 14 illustrates these three facts.

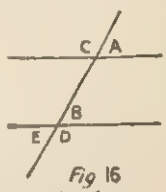
EXERCISES

The student will require a semi-circular scale (preferably transparent) for measuring angles (called a *protractor*).

(1) Draw any two intersecting lines as shown (Fig. 15). With the protractor measure each of the four angles A, B, C and D. Verify that A = B, C = D, A + C = 180°, A + D = 180°, A + B + C + D = 360°.

(2) Draw two parallel lines. (A convenient method is to place the ruler on the paper and draw lines along both edges.) Draw another line to cut both these parallel lines (Fig. 16). Verify that A = B, C = D, A + D = 180°, C + E = 180°, A = E, A + C + D + E = 360°.





Figs. 15 and 16. Exercises on triangles.

(3) Draw any triangle as shown (Fig. 17). Verify that A + B + C = 180°, D + E + F = 360°, E = A + C, F = A + B, D = B + C.

(4) Draw any triangle ABC. Measure the length of the sides and verify that AB + BC > AC, AB + AC > BC, AC + BC > AB. Draw another triangle DEF having the same angles, making the angles at D and E equal to those at A and B respectively. Measure the lengths of the sides and verify that DE/AB = EF/BC = FD/CA.

(5) Each of the following diagrams (Fig. 18), with the measurements as stated, is incorrect. Without measurement, give reasons [illegible] his being so.

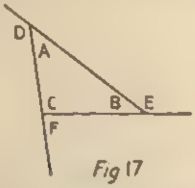
ANSWERS TO EXERCISES IN LESS[illegible]

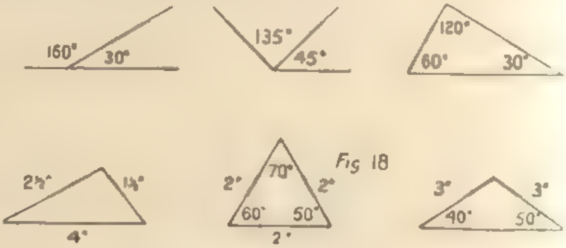
(1) $21\frac{7}{10}$, $19\frac{62}{100}$, $214\frac{3}{100}$, $70\frac{7}{100}$, $142\frac{61}{10}$[illegible]

(2) 72·264 (3) 5·57261 (4[illegible]

(5) 718·25 (6[illegible]7836

(7) 269·1461538 (8) [illegible] $2\frac{1}{6}$





Figs. 17 and 18. Exercises on triangles.

LESSON 5

Areas Enclosed by Simple Shapes, and Theorems on Triangles

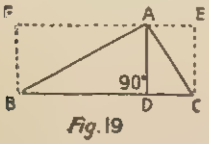
ONE of the most important facts concerning any shape is the area it encloses. We will now consider the areas of triangles and of some other simple plane figures.

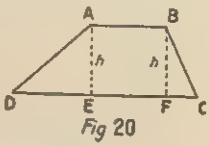
Rectangle

A rectangle is a four-sided figure, all the angles of which are right angles; its opposite sides are equal and thus its area is equal to the length of one side multiplied by the length of an adjoining side. If the sides are measured in inches, the area will be expressed in *square inches.*

Triangle Area

The area of any triangle is equal to one half the length of any side multiplied by the length of a line drawn from the opposite angular point (or *vertex*) at right angles to that side. (This line is called a *perpendicular.*) Thus in Fig. 19 the area of the triangle ABC = ½.BC.AD. This follows very simply from the expression for the area of the rectangle. Suppose a rectangle BCEF be drawn with BC as one side and the adjoining side made equal to AD. Then the area of the right-angled triangle ADB is half the area of the rectangle ADBF. Similarly area △ ADC = ½ area rectangle ADCE. By adding, therefore, the area of the △ ABC = ½ (rect. ADBF + rect. ADCE) = ½ rect. BCEF = ½.BC.AD.





Figs. 19 and 20. Areas enclosed by triangles and trapezia.

Trapezium

Any four-sided figure is called a *quadrilateral.* A *trapezium* is a particular quadrilateral with two sides parallel. Its area is found as follows. If ABCD represents the trapezium (Fig. 20), draw perpendiculars AE and BF from A and B, to CD. Since AB and CD are parallel, AE = BF. Suppose that each of these lines is h units in length, then the area of the trapezium—

= area rect. ABFE + area △ AED + area △ BFC

$= h.\,\mathrm{EF} + \tfrac{1}{2}.h.\mathrm{DE} + \tfrac{1}{2}.h.\mathrm{FC}$

$= h\left(\mathrm{EF} + \frac{\mathrm{DE}}{2} + \frac{\mathrm{FC}}{2}\right)$

$= \frac{h}{2}(2\mathrm{EF} + \mathrm{DE} + \mathrm{FC})$

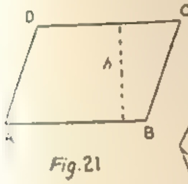
$= \frac{h}{2}(\mathrm{EF} + \mathrm{EF} + \mathrm{DE} + \mathrm{FC})$

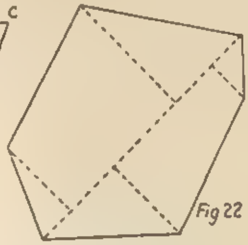
$= \frac{h}{2}(\mathrm{AB} + \mathrm{CD})$ since CD = DE + EF + FC and AB = EF

i.e. the area of a trapezium is equal to half the sum of the parallel sides multiplied by the distance between them.

A quadrilateral which has two pairs of parallel sides is called a parallelogram (Fig. 21). Its opposite sides are equal in length, i.e. AB = CD and CB = AD, and since it is also a trapezium its area is

$$\tfrac{1}{2} \times h \times (AB + CD) = \tfrac{1}{2} \times h \times 2 \times AB$$
$$= h \times AB$$





Figs. 21 and 22. Areas enclosed by parallelograms and polygons.

Convex Polygon

A polygon is convex if the line joining any two points inside it also lies inside it. Every convex polygon can be divided into a number of triangles and trapezia as indicated in Fig. 22, by drawing a line right across the figure from one vertex to another and drawing perpendiculars to this line from all other angular points.

This method of calculating the area enclosed by a convex polygon extends readily to any polygon.

Theorem of Pythagoras

The theorem of Pythagoras is one of the most important theorems in geometry and is supposed to have been discovered by the Greek philosopher Pythagoras (born about 530 B.C.). The theorem states that in any right-angled triangle (Fig. 23) the square drawn on the longest side (called the *hypotenuse*) is equal in area to the sum of the squares on the other two sides, i.e.:

sq. BCDE = sq. ACIH + sq. ABFG,
or, alternatively,
$(BC)^2 = (AC)^2 + (AB)^2$.

Where no confusion can arise this may, for convenience, be written

$$BC^2 = AC^2 + AB^2.$$

Thus, as an example, if AC = 3 inches, AB = 4 inches ; then—

$$BC^2 = 3^2 + 4^2 \text{ sq. inches}$$
$$\text{i.e. } BC^2 = 9 + 16 \text{ sq. inches}$$
$$= 25 \text{ sq. inches}$$
$$\text{i.e. } BC = 5 \text{ inches.}$$

The truth of this theorem can be seen in many ways, one of the simplest being by the use of similar triangles. Let ABC (Fig. 24) be any right-angled triangle having the right angle at A. From A draw a perpendicular AD on to the hypotenuse. Then, first of all, the △ DBA is similar to the △ ABC because they are both right-angled triangles and have the angle B in common. Therefore :

$$\frac{AB}{BC} = \frac{BD}{AB} \text{ or } AB^2 = BC \times BD.$$

Again, the triangles DAC, ABC are similar ; they are both right-angled and have the angle C in common. Thus :

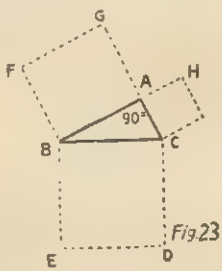
$$\frac{AC}{CD} = \frac{BC}{AC} \text{ or } AC^2 = BC \times CD$$

Adding these two relations together,

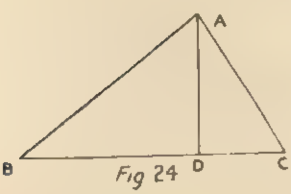
$$AB^2 + AC^2 = BC \times BD + BC \times CD$$
$$= BC\,(BD + CD)$$
$$= BC \times BC$$
$$= BC^2$$

Problem. A ladder 26 ft. long just reaches to the eaves of a house when the foot of the ladder is 10 ft. away from the house wall (Fig. 25). What is the height of the eaves above the ground? Suppose that h feet represent the height of the eaves above the ground. Then from the theorem above

$$h^2 + 10^2 = 26^2$$
$$\text{or } h^2 = 26^2 \quad 10^2$$
$$h^2 = 676 - 100 = 576$$
so that h = 24 feet.



Figs. 23 to 25. Theorem of Pythagoras with proof and application.



Theorems on Triangles

In the geometrical work we have done so far we have considered the fundamental facts concerning triangles. There still remain several theorems that are of importance in branches of applied mathematics. We will consider these now.

(1) The straight lines which bisect the angles of a triangle are concurrent.

(Three or more lines are said to be *concurrent* if they all pass through one point.)

Let ABC be any triangle (Fig. 26) and suppose that the angles at B and C are bisected by two lines meeting in I. From I draw perpendiculars to the sides, meeting them in D, E, F as shown, and join IA. If you can show that IA bisects the angle at A then you will have shown that the three bisectors of the angles of a triangle all pass through the same point.

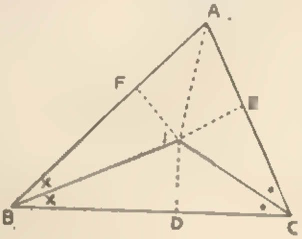


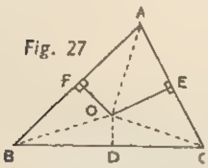
Fig. 26. Concurrent lines in a a triangle.

Fig 26

The proof is very simple : since BI bisects the angle at B, ∠ IBD equals ∠ IBF (indicated on the diagram by a small cross). In addition, the triangles BID and BIF are right-angled and have the hypotenuse (BI) in common. Thus they are congruent and ID = IF. By applying exactly similar reasoning to the triangles CID and CIE you have ID = IE, so that IF = IE, which proves that the right-angled triangles AIF, AIE are congruent. Thus ∠ FAI = ∠ EAI, or AI bisects the angle at A. (It is important to notice that since ID = IE = IF, a circle with centre I could be drawn to pass through D, E and F; it would touch the sides BC, CA, AB at these points.)

(2) The straight lines which bisect the sides of a triangle at right angles are concurrent.

Let ABC be any triangle (Fig. 27) and D, E, F the mid-points of the sides. Through F and E draw perpendiculars meeting in O. Join OD.



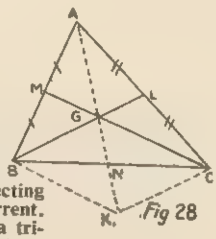


Fig. 27. Lines bisecting triangle sides are concurrent.
Fig. 28. Medians of a triangle are concurrent.

You require to prove that OD is at right angles to BC. The method of proof is similar to the previous ; the triangles AFO, BFO have two sides and the included angle equal, and are therefore congruent, so that OB = OA. Similarly OA = OC and consequently OB = OC. Now D is the mid-point of BC by construction, so that the triangles ODB, ODC have two sides equal and the third side common. Thus the triangles are congruent and therefore ∠ ODB = ∠ ODC, i.e. they are both right angles. O is thus the point of intersection of the three perpendicular bisectors of the sides of the triangle. (It should be noticed that, since OA = OB = OC, a circle with centre O will pass through the point A, B, C. This circle is called the *circumscribed* circle of the triangle ABC.)

(3) The *medians* of a triangle are concurrent. (A *median* of a triangle is a straight line drawn from a vertex to the mid-point of the opposite side.)

Let ABC be any triangle and BL, CM two medians meeting in G (Fig. 28). Join AG and continue this line until a point K is reached where GK = AG. Let this line cut the side BC in N.

You have to prove that AN is a median, i.e. that BN = CN. Join BK and CK. By your construction you have made G the mid-point of AK, and L the mid-point of AC. Therefore GL is parallel to KC, i.e. BL is parallel to KC. Similarly CM is parallel to BK so that you have proved that the quadrilateral BKCG is a parallelogram. One of the properties of a parallelogram is that its diagonals bisect each other, i.e. BN = NC and GN = NK.

The first of these relations proves our theorem because it shows that AN is a median and therefore the three medians are concurrent. The second of the relations fixes the position of G because GN = NK, i.e. GN is one half of GK, i.e. GN is one half of AG, i.e. GN is one third of AN so that the point of intersection of the medians is one third the way up either—or, stated otherwise, G is a point of *trisection* of a median. G is called the *centroid* of the triangle.

This geometrical theorem is of very great importance in connexion with problems on centres of gravity in mechanics.

(4) The *altitudes* of a triangle are concurrent. (An *altitude* of a triangle is a straight line drawn from a vertex perpendicular to the opposite side.) The proof is by a property of quadrilaterals inscribed in circles which is demonstrated in the next Lesson.

EXERCISES

(1) Draw a triangle having sides 4, 6 and 8 inches in length. Draw and measure the three perpendiculars from the angular points on to the opposite sides. Find the area of the triangle three ways by using each perpendicular in turn.

(2) Find the area of the figure, having the lengths (in feet) as shown (Fig. 29).

(3) Draw a straight line AE 4 inches in length, making AB = BC = CD = DE = 1 inch. Through the end A draw a perpendicular line AP, one inch in

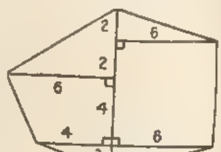


Fig. 29

length. Join PB, PC, PD, PE. Verify that these lengths will represent $\sqrt{2}$, $\sqrt{5}$, $\sqrt{10}$, $\sqrt{17}$ respectively and find them by measurement.

(4) Draw three triangles having sides of lengths 5, 12, 13 cms.; 8, 15, 17 cms.; 12, 35, 37 cms. By measuring the angles verify that they are right-angled triangles. Use the theorem of Pythagoras to confirm by calculation.

(5) A triangle ABC has sides AB = $2\frac{1}{2}$ inches, BC = $6\frac{1}{2}$ inches and AC = 6 inches. Verify by drawing that it is a right-angled triangle and confirm by calculation. Use a protractor to draw a perpendicular (AD) from A on to BC. From similar triangles calculate the lengths of AD, BD, and CD.

Answer to Exercise in Lesson 4

(5) (*a*) Angles do not add up to 180°; (*b*) angles add up to 180°, but should not; (*c*) angles do not add up to 180°; (*d*) two sides are not greater than the third; (*e*) angles of an equilateral triangle are equal; (*f*) angles at the base of an isosceles triangle are equal.

LESSON 6

Circles

EARLIER Lessons have restricted attention to shapes composed of straight lines. The smallest number of such lines that could enclose a space is three, giving a triangle. In this Lesson we give up the restriction that the elements of which the shape is composed are straight lines.

A space can be enclosed by a *closed* curved line, of oval or circular shape. Such shapes are of great importance in all forms of mechanism, the simplest of which is the wheel. Because the use of the tree-trunk, the rolling log, and the wheel as aids to transport dates from antiquity, it is not surprising that the properties of closed figures such as circles have been studied for many thousands of years. The extent of early knowledge of circular measurements may be judged from the specifications given in the Bible for the building of the Temple. The Hebrews, the Egyptians, and the Greeks all contributed to the development of the subject. This Lesson is concerned with shapes that are bounded by circles and straight lines.

A Circle Defined

A circle is defined as the figure enclosed by a curve traced out by a point which is always a fixed distance from a fixed point. The fixed point is called the *centre* of the circle, the fixed

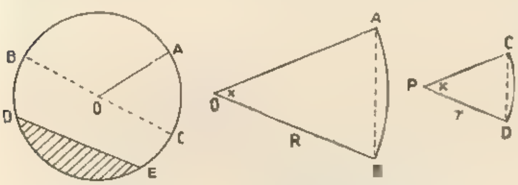


*Fig.*30 *Fig.*31

Figs. 30 and 31. Chords, segments, sectors, and arcs of circles. Calculating the circumference.

distance is the *radius*, and the curved path the *circumference*. Any line passing through the centre and terminated by the circumference is called a *diameter*, so that the length of a diameter is twice the radius. In Fig. 30 OA is a radius, BC a diameter. Any other line, such as DE, which divides the circle into two parts is a *chord*, and the two parts (one of which is shaded) are called *segments*. Also, any part of a circle bounded by two radii and part of the circumference (such as the shape OAC) is a *sector*, the curved part of the circumference, AC, being termed an *arc*.

Circumference of a Circle

One of the fundamental properties of the circle is that the ratio of the length of the circumference to that of the diameter is a constant, quite independent of the size of the circle, and this theorem may be expected from our knowledge of similar triangles. Consider two sectors of different circles, both sectors having the same angle at the centre (Fig. 31). Let the sectors be OAB and PCD and draw the chords AB and CD. Suppose the radius of one circle is R units and of the other, r; then because OAB and PCD are similar triangles AB/R = CD/r. Similarly we can show that

$$\frac{\text{arc AB}}{\text{R}} = \frac{\text{arc CD}}{r}$$

Again, it is reasonable to expect that the number which, multiplied by the arc AB, gives the circumference of the circle with centre O, will be the same as that which, when multiplied by the arc CD, gives the circumference of the circle with centre P, because the angles AOB and CPD are equal. Thus the circumference divided by the radius is a constant for all circles; or, since the diameter is twice the radius, you have

$$\frac{\text{circumference}}{\text{diameter}} = \text{constant.}$$

This constant has been denoted for centuries by the Greek letter π (*pi*). It is a non-terminating non-recurrent decimal; its value to 5 places of decimals is 3·14159. You can write:

Circumference of any circle
$= \pi \times$ its diameter
$= 2 \times \pi \times$ its radius.

The area of a circle of radius R is πR^2. This will appear reasonable by the following argument. Let a circle be drawn and divided up into an even number of sectors and suppose these sectors are placed side by side as shown in Fig. 32. Then by increasing the number of sectors the diagram above will resemble very closely a parallelogram, one of whose sides, AB, will be approximately equal to half the circumference in length since it is made up of the arcs of half the total number of sectors. Also AC is the radius of the circle (R). The area of the parallelogram is AB × AC which is approximately $R \times \pi R = \pi R^2$.

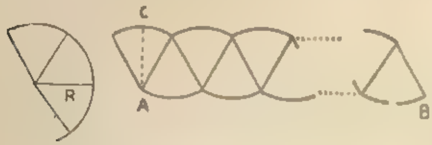
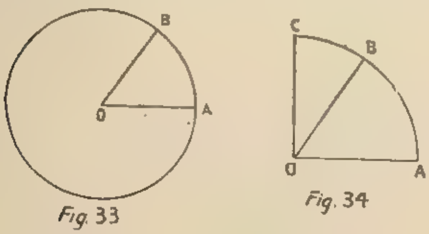


Fig. 32. Calculating the area of a circle.

Measurement of Angles

From the fact that the circumference of any circle divided by its radius is a constant, we have another method of measuring angles which is suitable for theoretical work in geometry. Draw any circle, and suppose that the length of an arc AB is made equal to the radius OA (or OB) (Fig. 33). Then the angle BOA will be the same whatever the size of the circle and is therefore conveniently taken as a unit angle. This angle is called a *radian*, and it may be defined as the angle subtended at the centre of a circle by an arc equal in length to the radius. Since we already have one method of measuring angles (i.e. degrees), there must be a relation between degrees and radians. From Fig. 33 one radian must be nearly equal to 60° since the *arc* AB is not very different in length from the *chord* AB.



Figs. 33 and 34. Relation between degrees and radians.

The exact relation can be found quite easily. Consider a quadrant of a circle (Fig. 34) and draw an angle of 1 radian and let the radius of the circle be R units. Then by definition of a radian, the arc AB = R units in length. Also the arc AC, being one quarter of the circumference of the circle, is $\frac{1}{4} \times 2$ [illegible] $\times$ R units in length, and the angle COA = 9[illegible]. Using the fact that the ratio of angles is e[illegible] to the ratios of corresponding arcs, we have

$$\frac{\angle BOA}{\angle COA} = \frac{\text{arc AB}}{\text{arc A[illegible]}}$$

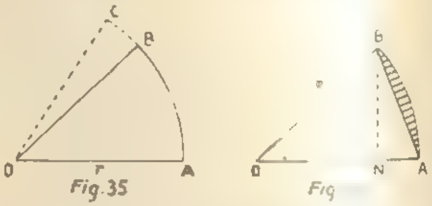
so that $$\frac{1 \text{ radian}}{90^\circ} = \frac{R}{\pi R/2}$$

Cross-multiplying,

π radians = 180 d[illegible],
i.e. 3·14159 radians = 180 de[illegible]es,
and therefore 1 radian = 57·29[illegible] [illegible]rees.

The following table gives the r[illegible]on between degrees and radians for the con[illegible]ner angles:

30° = $\pi/6$ (= 0·5236) ra[illegible]
45° = $\pi/4$ (= 0·7854) rad[illegible]
60° = $\pi/3$ (= 1·0472) r[illegible]
90° = $\pi/2$ (= 1·5708) r[illegible]
180° = π (= 3·1416) r[illegible]
360° = 2π (= 6·2832) rad[illegible]



Figs. 35 and 36. Areas of sectors and segments of a circle.

Length of a Circular Arc

One of the advantages of measuring angles in radians is that it enables us to obtain a very simple expression for the length of any circular arc. Consider Fig. 35: it consists of any arc (AB) of a circle of radius r units together with an arc AC which has been made equal to the radius, so that the ∠AOC = 1 radian by definition.

Now $$\frac{\text{arc AB}}{\text{arc AC}} = \frac{\angle AOB}{\angle AOC}$$

i.e. $$\frac{\text{arc AB}}{r} = \frac{\angle AOB \text{ (in radians)}}{\angle AOC \text{ (in radians)}} = \frac{\angle AOB \text{ (in radians)}}{1}$$

Thus $$\text{arc AB} = r \times \angle AOB \text{ (in radians)}.$$

We can now check our previous expression for the circumference of a circle. The angle at the centre of a circle is four right angles, i.e. 360° or 2π radians; therefore the length of the circumference $= r \times 2\pi$, which is the value already obtained.

Area of Sector of a Circle

Referring again to Fig. 35, you can find the area of the sector AOB by the same method as that for finding the area of the circle earlier in

this Lesson. There you divided the circle into an even number of sectors and placed them end to end (Fig. 32). You can divide the sector AOB into an even number of smaller sectors placed end to end, when you will obtain, by increasing the number of small sectors, a parallelogram having one side of length ($\frac{1}{2}$ arc AB), and the radius of the circle (r) as the perpendicular length between the parallel sides. Thus

area of sector AOB (Fig. 35)
$= r \times \frac{1}{2}$ arc AB
$= r \times \frac{1}{2} \times r \times \angle AOB$ (in radians)

from the theorem above giving the length of a circular arc,

$= \frac{1}{2} r^2 \times \angle AOB$ (in radians).

Area of Segment of a Circle

The area of the segment, shaded in the diagram (Fig. 36), is obviously the area of the triangle AOB subtracted from the area of the sector AOB, i.e.

area of segment = area of sector AOB
$- \text{area } \triangle AOB$
$= \frac{1}{2} r^2 \angle AOB - \frac{1}{2} r \times BN$, where BN is the perpendicular from B on to OA.

The student will not yet have sufficient knowledge to obtain an expression for the length of BN in terms of the $\angle AOB$. We shall return to this in Lesson 9.

Theorems on Circles

We give now a list of the more important facts concerning circles. We will indicate why the results are to be expected and how they are obtained, but for a rigorous proof the student is referred to a text-book on geometry.

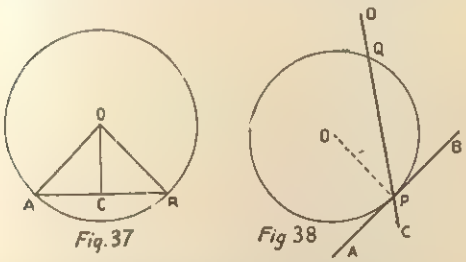


Fig. 37. Bisecting a chord. Fig. 38. A tangent to a circle is at right angles to the radius.

(1) *The line drawn from the centre of a circle to bisect a chord is perpendicular to it.*

If AB is the chord (Fig. 37), C its mid-point, and O the centre of the circle, then the triangles OAC, OBC are congruent because OA = OB = radius, AC = CB, and OC is common to both triangles.

Therefore $\angle OCA = \angle OCB = 90°$.

NOTE.—A *tangent* to a circle is a line which touches the circle and when produced does not cut it. Let CD represent a line (Fig. 38) cutting the circle in P and Q, and suppose that P is kept fixed, and CD is revolved about P so that Q approaches P. Then ultimately Q and P will become the same point and the line CD will lie along AB. This line (AB) is then a tangent to the circle at the point P, which is said to be the *point of contact*.

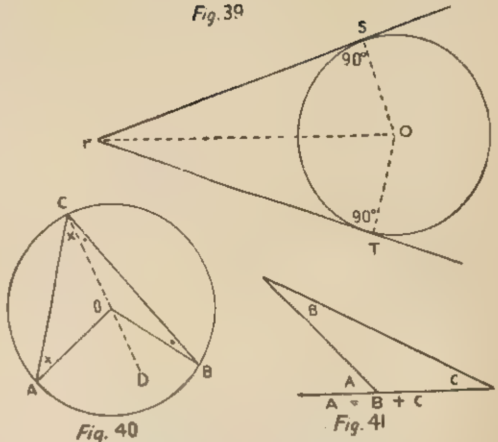


Fig. 39. Two tangents to a circle drawn from a point outside are of the same length. Figs. 40 and 41. Angles at the centre of a circle.

(2) *A tangent to a circle is at right angles to the radius drawn to its point of contact.*

In Fig. 38 $\angle OPA = \angle OPB = 90°$. Important results of this theorem are that

(a) only one tangent to a circle can be drawn through a given point on the circumference

(b) the straight line at right angles to a tangent, at its point of contact, passes through the centre of the circle.

(3) *The two tangents drawn to a circle from a point outside it are of the same length.*

Let PS and PT be the tangents drawn from a point P to the circle, centre O (Fig. 39). The triangles OPS, OPT are congruent because they are both right-angled, OS = OT, and OP is common. Thus PS = PT.

(4) *The angle at the centre of a circle is double the angle at the circumference standing on the same arc.*

Let AB be the arc (Fig. 40), O the centre of the circle so that $\angle AOB$ is the angle at the centre, and let C be any point on the circumference. Then $\angle ACB$ is the angle at the circumference and we have to show that $\angle AOB = 2 \times \angle ACB$. Join CO and produce to D. Then the triangle OAC is isosceles (OC equalling OA) and therefore $\angle OAC = \angle OCA$. Also, in Lesson 4, it was seen that the exterior angle of any triangle equals the sum of the two interior opposite angles (A = B + C, Fig. 41), so that finally (Fig. 40):

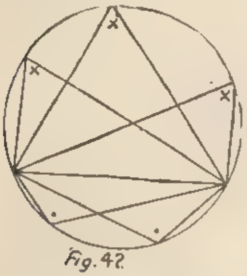
$$\angle DOA = \angle OAC + \angle OCA = 2 \times \angle OCA.$$

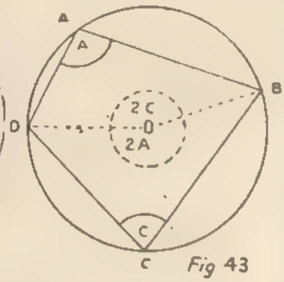
Similarly,

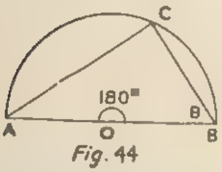
$$\angle DOB = \angle OBC + \angle OCB = 2 \times \angle OCB$$

Adding, $\angle AOB = 2 \times \angle ACB$.

(5) *Angles in the same segment of a circle are equal.* This follows immediately from the foregoing theorem ; since C was any point on the circumference, every angle at the circumference is half the angle at the centre and therefore all angles in the same segment are equal (Fig. 42).







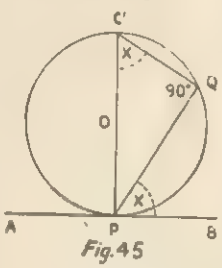


Fig. 42. Angles in same segment. Fig. 43. Cyclic quadrilateral. Fig. 44. Angle in a semicircle. Fig. 45. Angle between a tangent and a chord.

(6) *The opposite angles of a quadrilateral whose four corners lie on the circumference of a circle are together equal to two right angles.* (Such a quadrilateral is called *cyclic.*)

This theorem follows from (4). If ABCD represents the cyclic quadrilateral (Fig. 43) and if O denotes the centre of the circle, join DO, BO. Then the smaller angle DOB (i.e. less than 180°) equals twice the angle at C and the larger angle DOB (i.e. greater than 180°, sometimes called the *reflex* angle DOB) equals twice the angle at A, as indicated in the diagram. But these two angles DOB together add up to four right angles. Thus $2A + 2C = 360°$, or $A + C = 180°$, so that the opposite angles of a cyclic quadrilateral add up to two right angles.

NOTE.—In this theorem the quadrilateral was cyclic, and you had to prove that the opposite angles added up to 180°. If you had proved that if the opposite angles of a quadrilateral added up to 180°, then the quadrilateral was cyclic, you would have been proving the *converse* theorem. In this case the converse is also true, but often the converse of a true statement is untrue, for instance—"all dogs have two eyes" is a true statement, but the converse, that all animals which have two eyes are dogs, is obviously false.

(7) *The angle in a semicircle is a right angle.* This is just a special case (Fig. 44) of (4).

(8) *The angle between a tangent and the chord through the point of contact is equal to the angle in the opposite segment.*

Consider Fig. 45. AB is a tangent to the circle, centre O, P being the point of contact and PQ is a chord. PC is a diameter and you have to prove that ∠BPQ = ∠PCQ. The proof is as follows : ∠CQP = 90° because it is the angle in a semicircle and therefore ∠CPQ = 90° − ∠PCQ. But ∠CPQ also equals 90° − ∠BPQ, since OP is perpendicular to the tangent AB (see theorem (2) above). Therefore ∠BPQ = ∠PCQ.

EXERCISLS

(1) A square is drawn inside a circle of radius 2 in., so that each corner lies on the circumference of the circle. Thus four segments are formed. Find the areas of the circle, the square, and each segment.

(2) Which is the greater angle, 120° or [illegible] radians ?

(3) Draw a sector of a circle of radius 4 in., having an angle of 50° at the centre. Fix about 20 pins round the arc and find an approximation to the length of the arc with a piece of cotton. Convert 50° to radians and calculate the length of the arc accurately and also the area of the sector.

(4) Draw any circle together with two intersecting tangents. Measure the lengths of the two tangents from the point of intersection to the points of contact and verify that they are the same within the limits of practical error.

(5) Draw any circle, centre O, and any chord PQ. Join PQ to O and to any three points A, B, C in the same segment as the centre. Verify, by measurement, that ∠PAQ = ∠PBQ = ∠PCQ = ½ ∠POQ.

(6) Draw a circle and any cyclic quadrilateral. Verify that the sum of each pair of opposite angles is 180°.

(7) A tangent is drawn to a circle from a point P, T being the point of contact. Another line is drawn through P, cutting the circle in A and B. Prove that the triangles PAT, PTB are similar, and hence that $PA \times PB = PT^2$. Verify by drawing and measuring a particular case.

ANSWERS TO EXERCISES IN LESSON 5

(1) Area = 11·62 sq. in. (2) 84 sq. ft.
(3) 1·41, 2·26, 3·16, 4·12.
(5) AD = $2\frac{1}{8}$ in., BD = $\frac{11}{16}$ in., CD = $5\frac{1}{8}$ in.

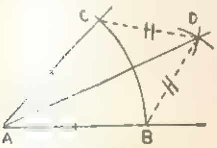
LESSON 7

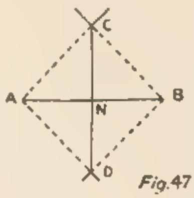
Practical Geometry

IN the two previous Lessons you have been studying plane geometry—the properties of points, lines, triangles, circles on a flat surface ; and frequently it was necessary, in order to prove some important fact concerning these figures, to imagine that some piece of construction had been carried through. For example, in one theorem it was supposed that the bisector of an angle had been drawn ; in another, the bisector of a line. It was also frequently assumed that a line perpendicular to the side of a triangle had been constructed. This was quite reasonable, but it is very important not to lose sight of the practical aspect.

One must be prepared, if one is to develop the subject logically, to supply answers to such questions as "*How* do you bisect an angle ?" or "*How* do you draw a perpendicular ?" and to supply those answers in a practical way with a pencil, a ruler, and a pair of compasses. It is the technique of this practical geometry that will now be considered.

In the constructions that follow, arcs of circles will be drawn with centres A, B, say, to intersect in C, say. It will always be assumed that the radii of the circles are sufficiently large for the arcs to intersect ; it is not difficult to see that this requires that the sum of the radii exceeds the distance from A to B.





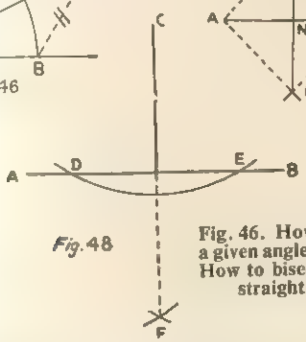


Fig. 46. How to bisect a given angle. Fig. 47. How to bisect a given straight line.

Fig. 48. How to draw a perpendicular to a straight line from a point outside it.

(1) *To bisect a given angle* (Fig. 46). Let A be the point of intersection of the lines forming the angle ; then with compass point on A draw an arc of a circle with any radius to cut the lines in B and C. With compass point first at B and then at C draw arcs with any, but the *same*, radius to cut at D. Join AD, which is the bisector of the given angle. The proof is simply that in the triangles ABD and ACD, AB = AC, BD = CD, and AD is common, so that these triangles are congruent, i.e. $\angle CAD = \angle BAD$.

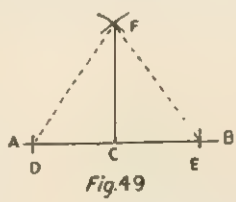
(2) *To bisect a given straight line.* Let AB be the given straight line. With centre first at A and then at B and any radius draw two arcs to intersect above the line in C and two arcs to intersect below the line in D. CD is the perpendicular bisector of AB at N (Fig. 47).

The proof is as follows. Join AC, BC, AD, BD ; these will all be equal since they are the radii of the same or equal circles. Thus the triangles CAD, CBD are isosceles with two sides of one equal to two sides of the other and the side CD common to both. Therefore these triangles are congruent and $\angle ACD = \angle BCD$. Therefore if CD cuts AB in N, the triangles ACN, BCN are congruent because AC = BC, CN is common, and the included angle ACN equals the included angle BCN. Thus AN = BN and $\angle CNA = \angle CNB = 90°$, i.e. CD is the perpendicular bisector of AB.

(3) *To draw a perpendicular to a given straight line from a given point outside it.* Let AB be the given straight line and C the point from which a perpendicular is to be drawn. With centre C draw any arc to cut AB in D and E and with these points as centres and the same radius draw arcs to cut in F (Fig. 48). CF is the perpendicular required. By joining CD, CE, FD and FE the proof follows exactly the same reasoning as in case (2) for the bisection of a given line, and we leave it as an exercise for the student.

(4) *To draw a perpendicular to a given straight line from a given point in it.* Let AB be the given line and C the given point (Fig. 49). You have to draw a perpendicular to AB at C. With centre C and any radius, draw arcs cutting AB in D and E. With these points as centres and radius greater than DC, draw arcs cutting in F. FC is the perpendicular required. The proof is this. Join FD and FE; then the triangles FCD, FCE are congruent because FD = FE, CD = CE, and CF is common. Therefore $\angle FCD = \angle FCE = 90°$.

(5) *To draw a straight line through a given point parallel to a given straight line.* Let AB be the given line, C the given point (Fig. 50). You require a line through C parallel to AB. With centre C and any radius sufficient to cut AB, draw an arc as shown in the diagram, cutting AB in D. With D as centre and the same radius, draw another arc to cut AB in E. With centre E and the same radius, draw an arc which must cut the first arc in D and another point F. CF is parallel to AB. The proof is that CD = CF = DE = EF and so CFED is a parallelogram, i.e. CF is parallel to AB.



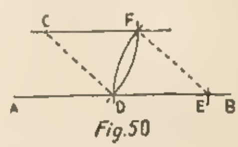
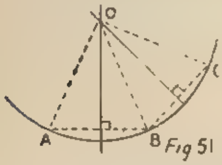


Fig. 49. Drawing a perpendicular. Fig. 50. Drawing a line parallel to a given line.

(6) *To draw a circle through three given points* (*not in the same straight line*). Let A, B and C be the three points. Join AB, BC. These will be chords of the required circle. Bisect AB and BC at right angles (*see* (2) left col.) and let these bisectors intersect in O. Then O is the centre of the required circle and OA (= OB = OC) is

the required radius (Fig. 51). The proof of the above construction we leave as an exercise for the student.



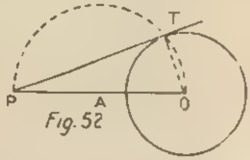


Fig. 51. Describing a circle through three points.
Fig. 52. Tangent to a circle from a point outside.

(7) *To draw a tangent to a given circle from a point outside it.* Let P be the given point, and O the centre of the given circle. Join OP. Bisect OP in A and, with this point as centre, draw a semicircle cutting the original circle at T. Join PT. It is a tangent to the given circle because $\angle PTO = 90°$ as it is the angle in a semicircle. Therefore OT is at right angles to PT, and OT is a radius of the circle. Therefore PT is a tangent (Fig. 52).

Projection

A very important idea in mathematical work is that of *projection.* Consider a line (AB) of fixed length and suppose that perpendiculars are drawn from A and B on to another line XY, the feet of the perpendiculars being C, D respectively. Then CD is said to be the projection of AB on the line XY. Similarly EF is the projection of AB on to another fixed line LM (Fig. 53).

Suppose you have any line (AB) straight or curved in space, and from every point in such a line you draw perpendiculars on to some plane (or flat) surface. You will obtain the projection of the line on the plane. Such projections are represented by the lines *ab* (Fig. 54). It is this idea of the projection of lines in space on to planes which forms the basis of the technical drawing and "blue print" of the engineer.

Technical Drawing

In engineering, the facts about a piece of machinery which has been made, or which it is proposed to make, have to be conveyed from one person to another, and while a picture is useful for showing the *shape* of the object it is generally useless as a means for giving the *size* of the object. Obviously the actual measurements of each part of the solid body are of extreme importance, so that diagrams have to be devised to give this information. A difficulty arises immediately. How are you to represent a solid body, having width, breadth, and thickness, with flat diagrams on paper?

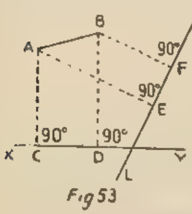
This problem is solved by means of projections. Since the size of a solid body will be determined by the lengths and positions of its edges, we will first consider the diagrams necessary to fix the measurement of a straight line in space. Refer again to Fig. 54; *ab* is the projection of AB on a horizontal plane, and this projection gives valuable information about the position in space of the line AB.

The projection of a line (or lines) on to a *horizontal* plane is called the *plan*, but this, in itself, is insufficient exactly to determine AB, for many lines can exist in space all having the same plan *ab*. To fix AB exactly you need its projection on to another plane, and generally the vertical plane is the most convenient. The projection on to a vertical plane is called an *elevation.* Consider Fig. 55: AB is a line in space and *ab* is its projection on a horizontal plane (H), *a′b′* its projection on to a vertical plane (V). *ab* is the plan and *a′b′* the elevation.

Once the plan and elevation are given, the line is fixed; and this fact gives a convenient method for practical work. XY, the line of intersection of the horizontal and vertical planes, is called the *ground line.* You take a sheet of paper and draw a horizontal line XY to denote the ground line. Above this line represents the vertical plane; below it, the horizontal plane, so that you have simply rotated the horizontal plane about XY and thus have the plan and elevation on the same sheet of paper. By comparing Figs. 55 and 56 the following facts are obvious: *a′*L, *b′*M are the heights of A and B respectively above the horizontal plane, and L*a*, M*b* are the distances of A, B respectively from the vertical plane.

Finding True Length

To find the true length of a line given its plan and elevation. By referring to Fig. 55 it will be seen that the true length of AB is the hypotenuse of a right-angled triangle whose base is equal to the length of the plan and whose perpendicular height is the difference in heights of A and B above the horizontal plane. Consider Fig. 57: *a′b′* is the elevation of a line in space and *ab* its plan. *b′*N is the difference in heights of A and B (Fig. 55), found from the elevation. Through the end *b* of the plan is shown a line at right angles to *ab*, its length being made equal to *b′*N. This gives the point C. *a*C is the true length of the line in space. $\angle$C*ab* is the angle that the line in space makes with its own plan.



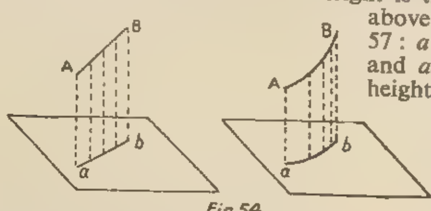


Fig. 53. Projecting line AB on to lines XY and LM. Fig. 54. Projecting line AB on to a plane; curved line on to plane.

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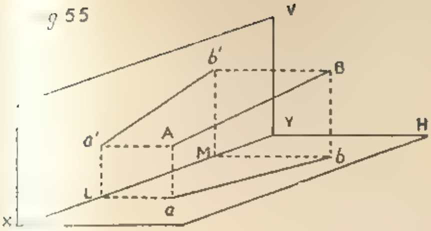


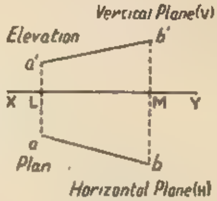


Fig 56

55. Projecting a ... in space on hori- ...tal and vertical ...nes. Fig. 56. Plan ... elevation. Fig. 57. ...gth of a line, given ...lan and elevation.

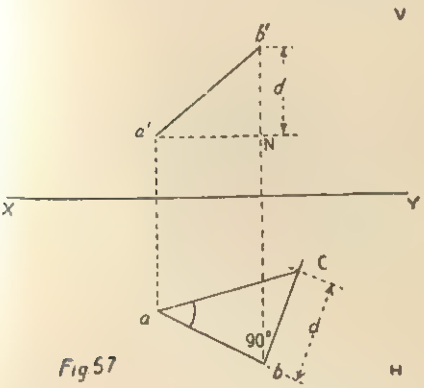


Fig 57

At this stage a few more definitions are needed to help formulate the subject exactly. The *angle between a line and a plane* is defined as the angle between the line and its own projection on the plane. The *angle between two planes* is defined as the angle between two lines, one in each plane, which are drawn at right angles to the line of intersection of the planes through a common point, i.e. in Fig. 58, if XY is the line of intersection of the two planes, and N is any point in XY, then the angle BNC is the angle between the two planes where BN and CN are drawn perpendicularly to XY. A *cube* is a solid, all of whose faces are squares. A *pyramid* is a solid, all of whose faces except one meet in a point, called the *vertex*. A *triangular pyramid* (i.e. a pyramid with all its faces triangular in shape) is called a *tetrahedron*. A *regular tetrahedron* is one of which every face is an equilateral triangle.

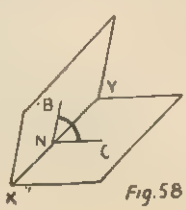


Fig.58

PROBLEMS

(1) Draw the plan and elevation of a cube of edge 2 in., resting on the horizontal plane and whose vertical faces are inclined at 30° and 60° to the vertical plane. Find the length of a diagonal and the angle between the diagonal and the horizontal plane. Draw the ground line XY (Fig. 59), and below it a square *abca* of side 2 in., one edge making 30° with XY. This square is the *plan* of the cube. From the four points *a, b, c, d,* project lines to the part above XY which represents the vertical plane. The height of the elevation must be 2 in. Notice which lines are dotted and which are thick; *bd* is the plan of a diagonal, so through *b* a perpendicular is drawn and *b*C is made equal to 2 in. (the height through which the diagonal rises in space). *d*C is the true length of the diagonal and $\angle bd$C the angle it makes with the horizontal plane. By calculation, using the theorem of Pythagoras, the diagonal is easily seen to be of length $\sqrt{12}$ in.

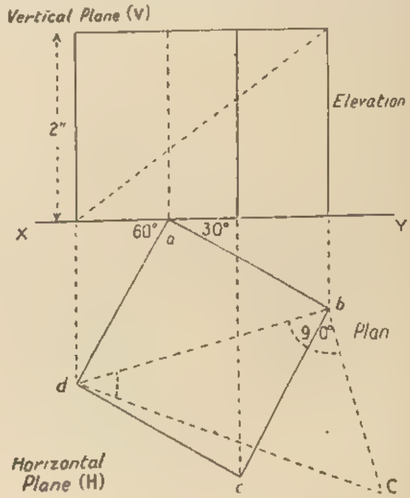


Fig 59

Fig. 59. Plan and elevation of a cube.

(2) The six edges of a triangular pyramid ABCD are of lengths as follows: AB = 2·6 in., BC = 3·6 in., CA = 2 in., AD = 2·8 in., BD = 3·7 in., CD = 3 in. Draw the plan of the pyramid when it is standing on the face ABC, and determine the height of D. Draw also the elevation of the pyramid on a vertical plane parallel to the edge AB.

The triangle ABC is first drawn, Fig. 60, then the point D_1 is found, making BD_1 = 3·7 in., CD_1 = 3 in. Then BCD_1 is the actual shape of the face BCD which has, in effect, been rotated about the edge BC until it is brought in to the horizontal plane. Draw a perpendicular from D_1 on to BC, cutting it in L and produce. Similarly draw ACD_2 which is the true shape of the face ACD. Draw a perpendicular from D_2 on to AC and continue to cut D_1L produced in *d*. *d* is the plan of the vertex. Join A*d*, B*d*, C*d* which are the plans of the edges AD, BD, CD respectively. Now L*d* is the plan of a line from L to the vertex D and this line is equal in length to LD_1, so that you can very easily find the height of the pyramid.

Draw a line through *d* perpendicular to D_1L produced. Cut this perpendicular (at D_3) by an arc of a circle, centre L and radius LD_1; then dD_3 is the height of the pyramid. A line XY is drawn parallel to AB and the elevation is constructed, the height being now known (dD_3). The student should cut out a model as indicated in Fig. 61, which will then close

Fig. 60. Plan and elevation of a triangular pyramid.

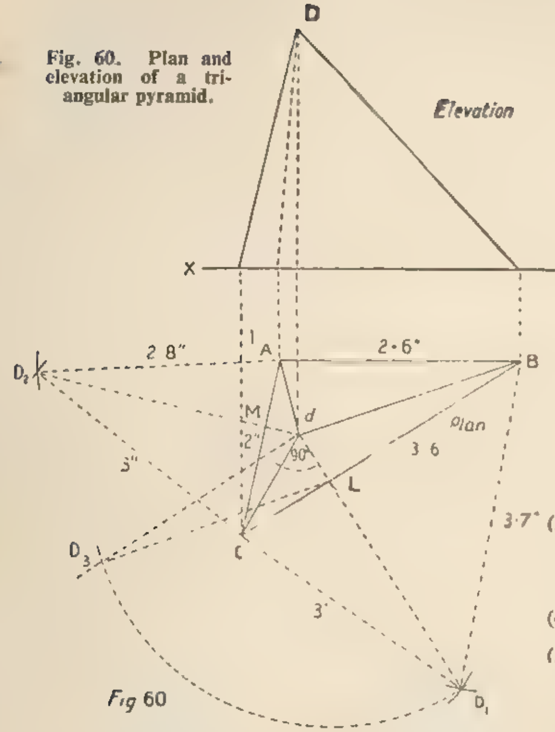
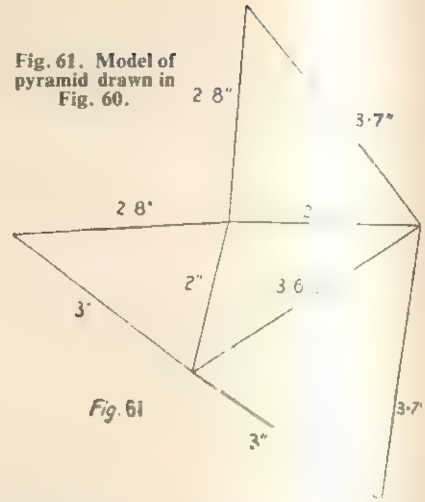


Fig. 61. Model of pyramid drawn in Fig. 60.



up to be the required pyramid, and by drawing lines on this model he will be able to convince himself of the correctness of the foregoing constructions.

EXERCISES

(1) Draw an angle of 66° and bisect it geometrically. Verify with a protractor.

(2) Draw two lines AB, BC each 3 in. long so that angle ABC = 120°. Find geometrically, the centre of the circle passing through A, B, and C, and measure its radius.

(3) Draw a triangle having sides 6 in., 8 in. and 10 in. Find a geometrical method for drawing a circle inside the triangle to touch all three sides and measure its radius (*see* Theorems of Triangles, Lesson 5).

(4) Draw a circumscribed circle of the triangle given in Question (3), and measure radius.

(5) Draw the plan and elevation of a rectangular block of edges 2 in., 3 in. and 4 in., resting on a 3-in. × 4-in. face such that the 3-in. edge makes an angle of 45° with ground line. Find, geometrically, length of a diagonal, and check by calculation.

(6) A regular tetrahedron has all its edges of length 3 in. Draw its plan when resting on any face, find the height of the vertex, and draw an elevation on a plane parallel to one of the edges of its base. Verify the height of the vertex by calculation, using the theorem on medians in Lesson 5

ANSWERS TO EXERCISES IN LESSON 6

(1) 4π, 8, $\pi - 2$ sq. in.
(2) 2·1 radians.
(3) arc = 3·49 in., area = 6·98 sq. in.

LESSON 8

First Steps in Trigonometry

ARISING directly out of the geometrical work of Lesson 4 on similar triangles, you can adopt a new technique to examine the relations between the lengths of the sides of a triangle and the magnitude of its angles. Consider the diagram, Fig. 62; it consists of a number of similar right-angled triangles AB_1C_1, AB_2C_2, . . . AB_nC_n, all having the same angle at A. From the geometry of similar triangles it follows that the ratio of any pair of corresponding sides is equal, i.e.

$$\frac{B_1C_1}{AB_1} = \frac{B_2C_2}{AB_2} = \frac{B_3C_3}{AB_3} = \dots = \frac{B_nC_n}{AB_n}$$

This ratio is constant, therefore, for the particular angle A. The ratio, the perpendicular divided by the hypotenuse, is useful to link up the sides of the triangle with the angle A because it will be constant for any particular angle at A and will not depend on the size of the triangle. This ratio is called the *sine* of the angle at A. Similarly from the same similar triangles:

$$\frac{AC_1}{AB_1} = \frac{AC_2}{AB_2} = \frac{AC_3}{AB_3} = \dots = \frac{AC_n}{AB_n},$$

a constant for the angle A. and this ratio is called the *cosine* of the angle at A.

Lastly,

$$\frac{B_1C_1}{AC_1} = \frac{B_2C_2}{AC_2} = \frac{B_3C_3}{AC_3} = \dots = \frac{B_nC_n}{AC_n}$$

and this constant ratio is called the *tangent* of the angle at A. These three ratios—the sine, cosine, and tangent—are fundamental in the

study of the relation between the sides and angles of any triangle and it is this study which forms the subject of *trigonometry*. The ratios are invariably abbreviated to sin (pronounced sine), cos (pronounced cŏss), and tan.

To summarise : in any right-angled triangle ABC (Fig. 63), if the angle at A be considered as being of magnitude A degrees, then the definitions we have stated are :

(i) $\sin A = \dfrac{BC}{AB}$ $\left\{\text{i.e. } \dfrac{\text{the side opposite to A}}{\text{hypotenuse}}\right\}$

(ii) $\cos A = \dfrac{AC}{AB}$ $\left\{\text{i.e. } \dfrac{\text{the side adjacent to A}}{\text{hypotenuse}}\right\}$

(iii) $\tan A = \dfrac{BC}{AC}$ $\left\{\text{i.e. } \dfrac{\text{side opposite to A}}{\text{side adjacent to A}}\right\}$

Before continuing with the study of trigonometry it is important for the student to become thoroughly familiar with the foregoing definitions. This can best be achieved by actual

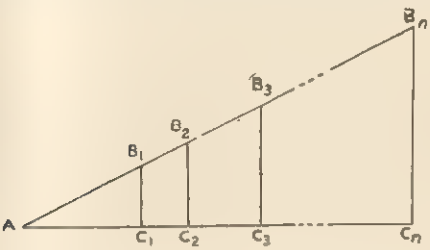


Fig. 62

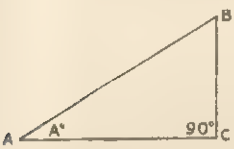


Fig 63

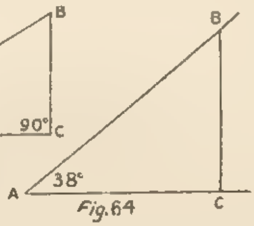


Fig. 64

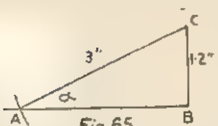


Fig 65

Figs. 62 to 65. Trigonometrical relations of sides and angles in right-angled triangles.

practical work, for which the student will require a protractor (a scale for measuring angles), a ruler for measuring lengths (preferably in inches and fiftieths), and two set squares, one giving angles of 30° and 60° and the other of 45°.

Example 1. Find the sine, cosine, and tangent of 38° (Fig. 64). Draw a horizontal line and from one end (A) set up an angle of 38° with the protractor. The line at this angle will be the hypotenuse, so that it will be convenient to make it some length which will be an easy divisor, say 2 in. Mark off the point B, such that AB = 2 in., and then by means of either of the set squares or the protractor draw a perpendicular from B on to the horizontal through A, meeting it in C. Measure BC and AC. Working through this problem, the lengths of BC and AC were respectively 1·24 in. and 1·58 in. The hypotenuse was made equal to 2 in. so that

$$\sin 38 = \frac{1{\cdot}24}{2} = 0{\cdot}62$$

$$\cos 38 = \frac{1{\cdot}58}{2} = 0{\cdot}79$$

$$\tan 38 = \frac{1{\cdot}24}{1{\cdot}58} = \frac{0{\cdot}62}{0{\cdot}79} = 0{\cdot}78$$

It should be noticed that $\tan 38° = \dfrac{\sin 38°}{\cos 38°}$. This is true in general ; the proof, following easily from the definitions, is given after Example 2.

Example 2. Find the angle whose sine is 0·4 (Fig. 65).

Let the angle whose sine is 0·4 be a, then $\sin a = 0{\cdot}4 = \dfrac{0{\cdot}4}{1}$; so that you require a right-angled triangle with one side equal to 0·4 and the hypotenuse equal to 1. This would give a small and inaccurate diagram, so you multiply numerator and denominator by some convenient number to give a larger diagram, e.g. $\sin a = \dfrac{0{\cdot}4}{1} = \dfrac{1{\cdot}2}{3}$ (multiplying by 3); thus you require a right-angled triangle, one side 1·2 in. and hypotenuse 3 in.

Draw a horizontal line about 4 in. long and from one end (B) set up a perpendicular of length 1·2 in., giving the point C on the diagram, and with this point as centre strike an arc of radius 3 in. to cut the horizontal through B in A. a is then the angle CAB which, measured by a protractor, is 23·2° or 23° 12′.

NOTE.—The angle whose sine is 0·4 is written $\sin^{-1}$ (0·4). Thus 23° 12′ = $\sin^{-1}$ (0·4) and sin (23° 12′) = 0·4 mean the same.

In these last two examples there is a margin of practical error. The actual values of the sines, cosines, and tangents of all angles between 0° and 90° have been calculated and set out in tables. The student will need such a set of tables and is advised to work through the last two examples himself, comparing his results both with those obtained above and with the values found from the tables, which are as follows :

Ex. (1) sin 38° = 0·6157, cos 38° = 0·7880, tan 38° = 0·7813 ;
Ex. (2) $\sin^{-1}$ (0·4) = 23° 35′.

In Example 1 it was found that $\tan 38° = \dfrac{\sin 38°}{\cos 38°}$ and this was only a particular case of a general relation that $\tan A = \dfrac{\sin A}{\cos A}$

For $\sin A = \dfrac{BC}{AB}$ and $\cos A = \dfrac{AC}{AB}$ by definition.

Therefore

$$\frac{\sin A}{\cos A} = \frac{\;\frac{BC}{AB}\;}{\frac{AC}{AB}} = \frac{BC}{AB} \cdot \frac{AB}{AC} = \frac{BC}{AC} = \tan A.$$

Another fundamentally important relation between sin A and cos A comes directly from Pythagoras' Theorem. This theorem applied to the triangle above gives (Fig. 63)

$$(BC)^2 + (AC)^2 = (AB)^2$$

or, dividing both sides by $(AB)^2$,

$$\left\{\frac{BC}{AB}\right\}^2 + \left\{\frac{AC}{AB}\right\}^2 = 1$$

But $\frac{BC}{AB}$ has been defined as sin A, so that

$$\left\{\frac{BC}{AB}\right\}^2 = (\sin A)^2 \text{ or } \sin^2 A \text{ (as it is written).}$$

Similarly $\left\{\frac{AC}{AB}\right\}^2 = (\cos A)^2$, which is written $\cos^2 A$

so that the relation above becomes

$$\sin^2 A + \cos^2 A = 1.$$

It will be noticed here that the two fundamental relations above have been proved for any values of A between 0° and 90°. Later it will be seen that when meanings have been given to sin A, cos A, and tan A for values of A greater than 90°, these relations are still true.

Since both the sine and cosine of A are one or other of the sides adjacent to the right angle divided by the hypotenuse, it follows that both sin A and cos A must be less than 1. Tan A, on the other hand, may have any value less than 1 or greater than 1 according as BC is less than AC or greater than AC. In mathematical work "less than" is denoted by the symbol $<$ and "greater than" by $>$. Thus the condition above can be expressed thus:

$$\tan A < 1 \text{ if } BC < AC, \text{ and}$$
$$\tan A > 1 \text{ if } BC > AC.$$

Example. Given cos A = 5/13, to find tan A without using tables.

From the identity

$$\sin^2 A + \cos^2 A = 1$$

we obtain, since cos A = 5/13,

$$\sin^2 A + (5/13)^2 = 1$$

i.e.

$$\sin^2 A = 1 - \frac{25}{169} = \frac{144}{169}$$

so that

$$\sin A = \sqrt{\frac{144}{169}} = \frac{12}{13}$$

Finally,

$$\tan A = \frac{\sin A}{\cos A} = \frac{12/13}{5/13} = \frac{12}{5} = 2{\cdot}4.$$

Cosec A, sec A, and cot A

Arising out of the three fundamental trigonometrical ratios sin A, cos A, and tan A, it has been found convenient to give special names to the reciprocals of these ratios.

Thus *cosecant* A is defined as 1/sin A, *secant* A is defined as 1/cos A, and *cotangent* A is 1/tan A. Cosecant A is shortened to cosec A, secant A to sec A, and cotangent A to cot A. Thus we have altogether six trigonometrical ratios, and there are, of course, a number of relations connecting them.

First, since

$$\tan A = \frac{\sin A}{\cos A} \text{ and } \cot A = \frac{1}{\tan A},$$

it follows that

$$\cot A = \frac{\cos A}{\sin A}.$$

Again, since

$$\tan A = \frac{\sin A}{\cos A}$$

$$\tan^2 A = \frac{\sin^2 A}{\cos^2 A}$$

$$1 + \tan^2 A = 1 + \frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A + \sin^2 A}{\cos^2 A}$$

$$= \frac{1}{\cos^2 A} \text{ because } \sin^2 A + \cos^2 A = 1$$

$$= \sec^2 A \text{ because } \sec A = \frac{1}{\cos A}.$$

Lastly,

$$1 + \cot^2 A = 1 + \frac{\cos^2 A}{\sin^2 A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin^2 A}$$

$$= \frac{1}{\sin^2 A}$$

$$= \operatorname{cosec}^2 A.$$

Since sin A and cos A were both less than 1, it follows necessarily that cosec A and sec A are greater than 1.

To sum up, the following relations have been stated:

$$\tan A = \frac{\sin A}{\cos A}, \cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A},$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \sec A = \frac{1}{\cos A},$$

$$\sin^2 A + \cos^2 A = 1, 1 + \tan^2 A = \sec^2 A,$$
$$1 + \cot^2 A = \operatorname{cosec}^2 A.$$

These relations are called *identities*, one side being the same as the other side but expressed in a different form. This is quite a different type of relation from an equation; something which is true for only a limited number of values of A. Identities are true for all values of A. It is very important in more advanced work to be able to manipulate identities from one form to another, and the technique of this manipulation will be considered at a later stage.

EXERCISES

(1) Draw an angle of 22°. Find its sine, cosine, and tangent.

(2) Repeat this question, but make the diagram about twice as large. Confirm that the values obtained are more accurate than those above by comparing with the values given in tables.

(3) Construct the angle whose cosine is 0·52. Measure it and find its sine and tangent graphically.

(4) A vertical post in the ground of height 2 ft. throws a shadow of length 4½ ft. How high is a flag-pole if its shadow measures 67½ ft.?

(5) ABC is a triangle right-angled at B. Find the lengths of the other two sides in terms of (i) AB and the angle A, (ii) AB and the angle C, (iii) AC and the angle A.

(6) If sin A = ½, find the value of $1 + \tan^2 A$.

(7) Given that sin A = 4/5, find the value of $(\sin A + \cos A)^2$ and $\sin^2 A + \cos^2 A$.

(8) Prove geometrically that the tangents of all angles between 45° and 90° are greater than 1.

Answers to Exercises in Lesson 7

(2) 3 in., (3) 2 in., (4) 5 in., (5) 5·385 in. = $\sqrt{29}$ in., (6) 2·45 in. = $\sqrt{6}$ in.

LESSON 9

Trigonometry of the Right-angled Triangle

We now proceed to the consideration of trigonometrical relations and ratios of the right-angled triangle, considering first a few cases of common occurrence.

Angles of 30°, 45°, 60° are of common occurrence and their trigonometrical ratios can be established very easily.

Consider a square ABCD of side 1 in., divided in two by the diagonal AC (Fig. 66). Then the angles of the triangle ABC are 45°, 45° and 90°. From Pythagoras' Theorem the length of the diagonal AC is $\sqrt{(1^2 + 1^2)} = \sqrt{2}$ in.

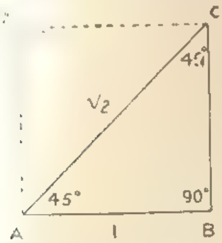


Fig 66

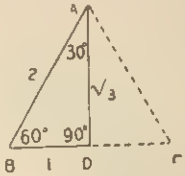


Fig 67

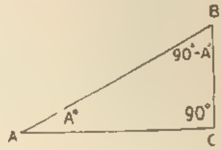


Fig 68

Figs. 66 to 68. Trigonometrical ratios of various angles.

Whatever the side of the original square had been, the ratio of the three sides of the triangle would always have been 1 : 1 : √2, since they are similar triangles. Thus by choosing 1 unit of length for the side of the square you are enabled at once to write down the trigonometrical ratios.

	$\sin 45° = BC/AC = 1/\sqrt{2}$,
also	$\cos 45° = AB/AC = 1\sqrt{2}$,
and	$\tan 45° = BC/AB = 1$;
thus	cosec 45° and sec 45° are both $\sqrt{2}$
and	cot 45° is 1

The ratios for the angles 30° and 60° are obtained by dividing an equilateral triangle in two by drawing a perpendicular from any vertex on to the opposite side as shown in the diagram (Fig. 67). For convenience take the side of the original equilateral triangle as 2 units, so that AB = 2 units, BD = 1 unit (since the perpendicular AD bisects the side BC) and consequently, from Pythagoras' Theorem, AD = $\sqrt{(AB)^2 - (BD)^2} = \sqrt{(4 - 1)} = \sqrt{3}$. Again, whatever length of side you had chosen for the original equilateral triangle the sides AB, BD, AD would have been in the ratio 2 : 1 : √3.

Thus	$\sin 60° = AD/AB = \sqrt{3}/2$,
	$\cos 60° = BD/AB = 1/2$,
and	$\tan 60° = AD/BD = \sqrt{3}$.

From the same triangle you can obtain the trigonometrical ratios of 30°,

for	$\sin 30° = BD/AB = 1/2$,
	$\cos 30° = AD/AB = \sqrt{3}/2$,
and	$\tan 30° = BD/AD = 1/\sqrt{3}$.

From these, by inverting the values, you find the cosec, sec, and cot of 30° and 60°.

It will be a great help to the student if he can memorise some of the above values. The most important are as follows :

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

It will be noticed in the table that sin 30° = cos 60° and cos 30° = sin 60°, which suggests that there may be a relation between the ratios of any angle A and its *complement*, that is, 90° − A. We will look into this question further. Consider any right-angled triangle ABC (Fig. 68) ; then, by definition,

$\sin A = \frac{BC}{AB}$. But $\frac{BC}{AB}$ is also the cosine of the angle at B which is 90° − A,

so that

sin A = cosine of 90° − A, which is written cos (90° − A).

Similarly,

$\cos A = \frac{AC}{AB}$ and $\frac{AC}{AB}$ is also sin B, i.e. sin (90° − A)

so that you have

$\cos A = \sin (90° - A)$.

Lastly,

$\tan A = \frac{BC}{AC} = \cot B = \cot (90° - A)$, and

$\cot A = \frac{AC}{BC} = \tan B = \tan (90° - A)$.

We have the following important results :

$$\sin A = \cos (90° - A),$$
$$\cos A = \sin (90° - A),$$
$$\tan A = \cot (90° - A),$$
$$\cot A = \tan (90° - A).$$

Example. Without using tables, show that

$$\cos 40°.\ \cos^2 50°.\ \cot 50° = \sin^3 40°.$$

Bearing in mind the foregoing relations, and noting that $40° + 50° = 90°$, you have $\cos 50° = \sin 40°$ and $\cot 50° = \tan 40°$, so that the left-hand side becomes

$$\cos 40° \sin^2 40° \frac{\sin 40°}{\cos 40°} = \sin^3 40°.$$

Simple Problems in Trigonometry

You can now use the trigonometry you have learned in solving simple problems. Suppose, for example, you are given the problem of finding the height of a flag-pole (Fig. 69). What measurements must be made on the ground in order that you can calculate the height? If you walk a distance away from the foot of the pole and notice the angle that the top of the pole makes with the horizontal (called the *angle of elevation* of the top of the pole) then you have sufficient information for your purpose. As an example, let FP represent the flag-pole and suppose that at a point 25 yds. away from the foot of the pole, the angle of elevation of P is 30° (Fig. 69). Then from trigonometry—

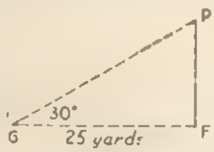


Fig 69

$$\frac{FP}{25} = \tan 30° = \frac{1}{\sqrt{3}}$$

so that $$FP = \frac{25}{\sqrt{3}} \text{ yds.} = 43{\cdot}3 \text{ ft.}$$

It is important to notice the valuable implications of the last example; that inaccessible distances may be calculated and inaccessible points located by measuring suitable distances and angles.

Let us consider some other cases and for the present restrict them to types where all the measurements of lines and angles are in the same vertical plane.

(1) To find the height and position of an object where the foot of the perpendicular from the object on to the ground is inaccessible.

Let LM (Fig. 70) represent the height to be found and suppose that you can find the angles of elevation of L at two points (A and B) in the horizontal line through M. Then from the definition of the tangent of an angle,

$$\frac{LM}{AM} = \tan A \text{ and } \frac{LM}{BM} = \tan B, \text{ or}$$

$AM = LM/\tan A = LM \times \cot A$ and
$BM = LM/\tan B = LM \times \cot B$.

Finally
$$\begin{aligned} AB &= AM - BM \\ &= LM \times \cot A - LM \times \cot B \\ &= LM (\cot A - \cot B) \end{aligned}$$

so that $$LM = \frac{AB}{\cot A - \cot B}$$

This formula gives the height LM in terms of the distance AB and the two angles of elevation. Knowing LM, the position of M is fixed, since $BM = LM \times \cot B$.

(2) To find the height and position of an object knowing the angles of elevation from two points a known distance apart, one on each side of the object.

As before let LM denote the height to be found, and AB the known distance (Fig. 71). The three points A, M, B are all in a straight line since you are considering only cases where all the measurements are in the same plane. Reasoning as in the last case.

$$\begin{aligned} & LM = AM \times \tan A \\ \text{and also} \quad & LM = BM \times \tan B \\ \text{i.e.} \quad & AM = LM \times \cot A \\ \text{and} \quad & BM = LM \times \cot B \end{aligned}$$

so that

$$\begin{aligned} AB &= AM + BM \\ &= LM \times \cot A + LM \times \cot B \\ &= LM (\cot A + \cot B) \end{aligned}$$

i.e. $$LM = \frac{AB}{\cot A + \cot B}$$

giving LM in terms of AB and the angles A and B. M is fixed since $AM = LM \times \cot A$.

The following examples illustrate the above, and similar types.

Example 1. From two points, 100 ft. apart on level ground, the angles of elevation of the top of a tower are 30° and 60°. Find the height of the tower and the distance of the foot from the nearest point.

Consider the diagram (Fig. 72):

$LM = BM \times \tan 60° = BM \times \sqrt{3}$,
also

$$LM = AM \times \tan 30° = AM \times \frac{1}{\sqrt{3}}$$

so that $$AM = LM \times \sqrt{3},$$

and $$BM = LM \times \frac{1}{\sqrt{3}}.$$

Thus $$AB = LM \left\{ \sqrt{3} - \frac{1}{\sqrt{3}} \right\}$$

and $$LM = \frac{AB}{\sqrt{3} - \frac{1}{\sqrt{3}}} = \frac{100}{\sqrt{3} - \frac{1}{\sqrt{3}}} = \frac{100\sqrt{3}}{2} = 86{\cdot}6 \text{ ft.}$$

Therefore $$BM = LM \times \frac{1}{\sqrt{3}} = 50 \text{ ft.}$$

Example 2. A flag-pole is placed vertically on a hill which has an even slope of 30°. From a point 60 ft. down the hill, the angle of elevation of the top of the pole is 45°. Find the height of the pole.

In the diagram (Fig. 73) LM represents the flag-pole, A the point from which the elevation is 45° so that

Fig. 70

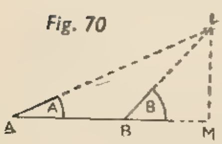


Fig. 71

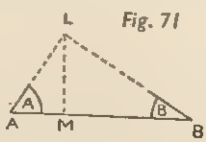


Fig. 72

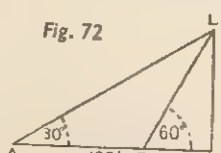
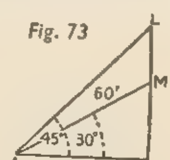


Fig. 73



PROBLEMS. Finding height from angles of elevation (Figs. 70 to 73). General and particular cases.

AM = 60 ft. AB is horizontal so that $\angle MAB = 30°$. $AB = 60 \times \cos 30° = 60 \times \frac{\sqrt{3}}{2} = 30\sqrt{3}$ ft., and since $\angle LAB = 45°$, AB = LB. Also $MB = 60 \times \sin 30° = 30$ ft.

Finally,

$$\begin{aligned} LM &= LB - MB \\ &= AB - MB \\ &= 30\sqrt{3} - 30 \text{ ft.} \\ &= 21{\cdot}96 \text{ ft.} \end{aligned}$$

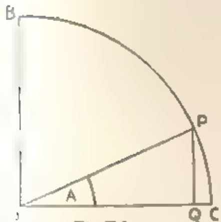


Fig.74

Fig. 74. Trigonometrical ratios of angles of 0 and 90 degrees.

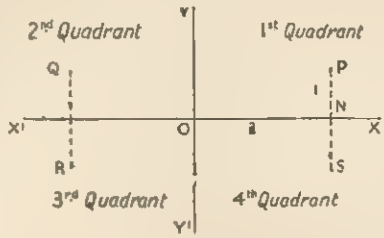


Fig.75

Fig. 75. Co-ordinate references.

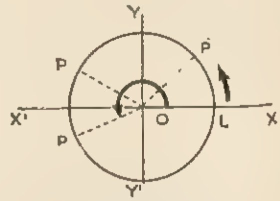


Fig.76

Fig. 76. Ratios of angles of any size.

We will now give meanings to the trigonometrical ratios of 0° and 90°. Draw a quadrant of a circle BOC (Fig. 74) and let P be any point on the circumference. Draw a perpendicular PQ. Then if $\angle POQ = A$, $\sin A = \frac{PQ}{OP}$. If P approaches C, then the angle A becomes smaller and smaller and consequently PQ becomes shorter. OP remains constant in length. As P becomes coincident with C, the angle A becomes zero and PQ also becomes zero so that $\sin 0° = \frac{0}{OP} = 0$.

Similarly $\cos A = \frac{OQ}{OP}$ and as P approaches C, OQ becomes equal to OC. Thus $\cos 0° = \frac{OC}{OC} = 1$, and therefore $\tan 0° = \frac{\sin 0°}{\cos 0°} = 0$. By making P approach B you obtain $\sin 90° = \frac{OB}{OB} = 1$ and $\cos 90° = 0$. Thus $\tan 90° = \frac{1}{0}$; there is, of course, no such number, but you observe that tan A gets larger and larger as A approaches 90°, so you write $\tan 90° = \infty$ (*infinity*).

Co-ordinates

In the trigonometry which we have dealt with so far, we have defined the ratios sine, cosine, and tangent only for angles between 0° and 90°. It is important to extend these definitions to include all angles whatever their size, and in order to do this we must, first of all, discuss the use of what are termed *co-ordinates*.

Consider two lines XOX′ and YOY′ drawn at right angles (Fig. 75). With these as reference lines you can fix the position of any point by giving it two numbers. For instance, the point P is reached from O by moving a certain distance (for example, say 3 units) along OX to N and then moving at right angles (parallel to OY) for a certain distance (say 1 unit). These distances, 3 units and 1 unit, are called the *x* co-ordinate and *y* co-ordinate respectively of the point P. They do in fact fix the position of P with respect to the reference lines OX and OY. The *x* co-ordinate is always written first so that P is designated as the point (3, 1).

If you consider the movement from O towards X as a positive direction, you must consider a movement from O to X′ as a negative direction. Thus the co-ordinates of the point Q, for instance, would be (− 3, 1) indicating that to reach Q from O you must proceed in the X′ direction (i.e. the negative X direction) for 3 units and then in the positive Y direction for 1 unit. Correspondingly, points within the quadrant X′OY′ have both the *x* and *y* co-ordinates negative, while points within XOY′ have the *x* co-ordinate positive but the *y* co-ordinate negative. Thus R is the point (− 3, − 1) and S the point (3, − 1). The four quadrants are referred to as the first quadrant, second quadrant, and so on, as indicated.

If you draw two reference lines (or *axes*, as they are called) XOX′ and YOY′ (Fig. 76) and choose any point L on the positive X axis, then you can imagine the line OL rotating about O in an anti-clockwise direction. If P be any point on the circumference in the first quadrant, then $\angle POL$ is less than 90°. If now OP is rotated until P lies somewhere in the second quadrant, then the $\angle POL$ will be between 90° and 180°. Similarly if P be in the third quadrant, the angle POL (always measured in the anti-clockwise direction) will have a size between 180° and 270°. Finally, if P lies in the fourth quadrant, $\angle POL$ lies between 270° and 360°. This provides a method of defining the sine, cosine, and tangent of any angle whatever its size, which is as follows :

$$\sin \angle POL = \frac{y \text{ co-ordinate of P}}{OP}$$

$$\cos \angle POL = \frac{x \text{ co-ordinate of P}}{OP}$$

$$\text{and } \tan \angle POL = \frac{y \text{ co-ordinate of P}}{x \text{ co-ordinate of P}}$$

Since in all the quadrants, except the first, one of the co-ordinates at least is negative, these extended definitions must mean that for angles greater than 90° one of the trigonometrical ratios at least is negative, OP always being taken as positive.

Example. Find the sine, cosine, and tangent of 120°: Draw an angle of 120° taking the positive X direction as the initial line. Then $\angle X'OP = 60°$ so that if ON is numerically 1 unit in length, $PN = \sqrt{3}$ units and OP = 2 units (Fig. 77). Then the co-ordinates of P are $(-1, \sqrt{3})$ and from our definitions

$$\sin 120° = \frac{y \text{ co-ordinate of P}}{OP} = \frac{\sqrt{3}}{2}.$$

$$\cos 120° = \frac{x \text{ co-ordinate of P}}{OP} = \frac{-1}{2}, \text{ and}$$

$$\tan 120 = \frac{\sqrt{3}}{1} = -\sqrt{3}.$$

For every position of P in the second quadrant, the x co-ordinate is negative and the y co-ordinate positive, so that for all angles between 90° and 180° the sine is positive, the cosine is negative and therefore the tangent is negative. For any point P in the third quadrant, the x co-ordinate is negative, the y co-ordinate is also negative, so that angles in the third quadrant have their sine and cosine negative, but their tangent positive. By similar reasoning, for the fourth quadrant you find that the sine and tangent are negative but the cosine is positive. This information is summarised in Fig. 78.

Example. Find by drawing and measurement the sine, cosine, and tangent of 326°. This angle falls in the fourth quadrant. Draw OP in position (Fig. 79) making it some convenient length, say 3 in. Draw a perpendicular PN. Measure ON and PN in inches.

Then $\sin 326° = \frac{-PN}{3}$ $\cos 326° = \frac{+ON}{3}$ and

$\tan 326° = \frac{-PN}{ON}$. Notice that the *numerical* values (that is, apart from the sign) of sin, cos and tan 326° will be the same as for 360° − 326°, i.e. 34°.

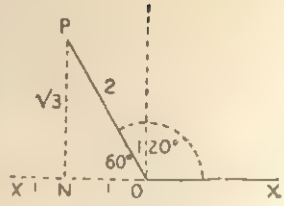
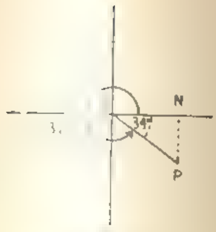


Fig 77



, 79

SIN +	SIN +
COS −	COS +
TAN −	TAN +
SIN −	SIN −
COS −	COS +
TAN +	TAN −

Fig 78

Fig. 77. Sine, cosine, and tangent of angle of 120°. **Fig. 78.** When sin, cos, and tan are positive or negative. **Fig. 79.** Finding sin, cos, and tan of angle exceeding three right angles.

EXERCISES

(1) A man stands on the straight bank of a river directly opposite to a tree on the other side. He walks along the bank for 25 yds. and finds that the tree makes an angle of 45° with the direction in which he has just moved. How wide is the river?

(2) From a certain point the elevation of a balloon is 60°. A person walks for 100 yds. in a straight line passing directly under the balloon. The angle of elevation is then 30°. What is the height of the balloon and how far did the person walk before he was directly under it?

(3) From a rowing boat the angle of elevation of the top of the cliffs is 5° 6′ and after rowing towards the shore for some time the angle is [illegible]. If the cliffs are known to be 100 ft. high, how far was the boat rowed between making the two observations and how far remains before land will be reached?

(4) Find by drawing and measurement the sine, cosine, and tangent of 156°, 200°, 236°, 315°.

(5) Deduce the sine, cosine, and tangent of 180°, 270°, 360°.

(6) Prove that the fundamental identities of Lesson 8 are true for angles of any size.

(7) Find by drawing the angle in the third quadrant whose tangent is 3·5.

ANSWERS TO EXERCISES IN LESSON 8

(1) and (2) From tables sin 22° = 0·3746, cos 22° = 0·9272, tan 22° = 0·4040.
(3) 58° 40′. From tables sin 58° 40′ = 0·8542, tan 58° 40′ = 1·6426.
(4) 30 ft.
(5) (i) AC = AB sec A, BC = AB tan A.
(ii) AC = AB cosec C, BC = AB cot C.
(iii) AB = AC cos A, BC = AC sin A.
(6) 4/3. (7) 49/25, 1.

LESSON 10

Elementary Ideas in Algebra

When you say that a group consists of three objects and use the symbol 3 to represent its *number* or its *size*, you use the same symbol 3 for any other group of the same size. The members of the group may differ from each other in a variety of respects, e.g. it may consist of a cup, a saucer, and a plate. The symbol 3 represents something about the group which is the same for a group consisting of a knife, a fork, and a spoon.

When you say that 3 + 2 = 5 you are describing the way in which groups combine as regards their size, and in doing so you ignore all other aspects of the situation. It is for this reason that arithmetic is of such universal value; it applies to widely diverse circumstances because all it demands is that *size* alone shall be the feature with which you are concerned. Were this not so a different kind of arithmetic would be required for each special occasion.

You can carry arithmetic a stage farther along these lines. As previously stated, an even number always has a factor 2 so that it can be written $2 \times n$ where n is any number what-

sort, even or odd. Thus, if you wish to discuss *even numbers in general* and not a particular even number like 2 or 4 or 6, you use the symbol $2 \times n$. In the same way, if you wish to discuss odd numbers in general and not a particular odd number, you use the symbol $2 \times n + 1$. It will be shown in a moment how such symbols are used, but it is important to realize at this stage that $2 \times n$ or $2 \times n + 1$ stand for sets of numbers, and that general statements about all the numbers of a given set may best be made by having a name or symbol for the whole set. For this enables us to discuss general problems and to arrive at general answers.

Remarks on Notation

When convenient we propose to drop the sign $\times$ for multiplication, provided no confusion arises. For instance, if we write $2n$ we mean $2 \times n$, or, what is of course the same thing, $n \times 2$. We do not usually write $n2$ since it is the custom to write the *particular* symbol first and the general symbol second.

Thus, $8n$ or $3x$ or $7y$ or $25z$ mean respectively $8 \times n$, $3 \times x$, $7 \times y$, $25 \times z$ where n, x, y, z are numbers not yet specified. In fact, they do not always need even to stand for numbers; we may use the notation $8n$ for 8 objects of the type n, for example 8 hairbrushes. In this sense, algebra is implicit in almost all day-to-day calculations.

Illustrations

Here we tabulate the values of $5x - 4$ for $x = 1, 2, 3, \ldots \ldots 9$.

$$
\begin{array}{ll}
x = 1, & 5x - 4 = 5 \times 1 - 4 = 1 \\
x = 2, & 5x - 4 = 5 \times 2 - 4 = 6 \\
x = 3, & 5x - 4 = 5 \times 3 - 4 = 11 \\
x = 4, & 5x - 4 = 5 \times 4 - 4 = 16.
\end{array}
$$

The student can easily complete this calculation. Note that the numbers 1, 6, 11, 16, ... increase each time by 5. Why is this?

Addition and Subtraction in Algebra

As we have said, in a sense everybody uses algebra. If you have to add 2 ft. 4 in. to 6 ft. 3 in., you would do it like this:

$$
\begin{array}{ll}
2 \text{ ft.} & 4 \text{ in.} \\
6 \text{ ft.} & 3 \text{ in.} \\
\hline
8 \text{ ft.} & 7 \text{ in.}
\end{array}
$$

But 2 ft. 4 in. is really 2 ft. plus 4 in., so that for economy in writing you might set it out like this:

$$
\begin{array}{l}
2f + 4i \\
6f + 3i \\
\hline
8f + 7i
\end{array}
\qquad \text{Here } f \text{ stands for feet and } i \text{ for inches.}
$$

This is algebra! Now the interesting point to notice about this is its generality. Suppose a company were holding a board meeting of financiers and industrialists. Suppose they entered the board-room in two groups, one of 2 financiers and 4 industrialists and the other of 6 financiers and 3 industrialists. You could add the two groups by exactly the same process as before, viz.:

$$
\begin{array}{l}
2f + 4i \\
6f + 3i \\
\hline
8f + 7i
\end{array}
$$

The same would hold if f stood for Frenchmen and i for Irishmen, and, indeed, if they stood for any two distinct types. For example, $f = 1$ group of 1 dozen eggs and i another group of 1 score eggs, the addition sum given above would read:

If you add 2 groups of 1 dozen eggs and 4 groups of 1 score eggs, to 6 groups of 1 dozen and 3 groups of a score, the result is 8 groups of a dozen eggs and 7 groups of a score of eggs.

In this case you check it up as follows:

$$
\begin{array}{l}
2f + 4i = 2 \times 12 + 4 \times 20 = 104 \text{ eggs} \\
6f + 3i = 6 \times 12 + 3 \times 20 = 132 \text{ eggs} \\
\hline
8f + 7i = 8 \times 12 + 7 \times 20 = 236 \text{ eggs.}
\end{array}
$$

What you have done with two items can equally be done with any number. Thus:

$$
\begin{array}{rrrr}
3a + & 4b + & 5c + & 6d \\
4a + & 5b + & 6c + & 3d \\
7a + & b + & c + & 2d \\
11a + & 3b + & 2c + & d \\
a + & 9b + & 70c + & 12d \\
\hline
26a + & 22b + & 84c + & 24d
\end{array}
$$

They need not all be positive signs, for after all it is not difficult to give a meaning to such an expression as $6f - 3i$ if f stands for dozens and i for scores.

Thus you could have addition sums like this:

$$
\begin{array}{l}
3a - 4b + 8c \\
5a + 7b - 6c \\
\hline
8a + 3b + 2c
\end{array}
$$

Subtraction follows the same way. Thus:

$$
\begin{array}{ll}
\text{from} & 7a + 8b \\
\text{take} & 3a + 5b \\
\hline
\text{Answer} & 4a + 3b
\end{array}
$$

Each item is taken from its appropriate column. The result in any column may itself be negative. Thus:

$$
\begin{array}{ll}
\text{from} & 8a + 3b \\
\text{take} & 6a + 8b \\
\hline
\text{Answer} & 2a - 5b
\end{array}
$$

This follows since $3 - 8 = -5$ or $3b - 8b = -5b$; and $8 - 6 = 2$, or $8a - 6a = 2a$.

These addition and subtraction sums might have been conducted without writing the items one under the other. For example, if you wish to add $7x + 5y + 4z$ to $3x - 4y + 11z$ you could write it like this:

$$
\begin{array}{rl}
& (7x + 5y + 4z) + (3x - 4y + 11z) \\
= & 7x + 5y + 4z + 3x - 4y + 11z \\
= & (7x + 3x) + (5y - 4y) + (4z + 11z) \\
= & 10x + y + 15z.
\end{array}
$$

If you wish to subtract them, you write :

$$(7x + 5y + 4z) - (3x - 4y + 11z)$$
$$= 7x + 5y + 4z - 3x + 4y - 11z$$
(see explanation below)
$$= (7x - 3x) + (5y + 4y) + (4z - 11z)$$
$$= 4x + 9y - 7z$$

Note that when we removed the bracket in the second line where a − sign stood in front of that bracket, we altered the sign of each term that had previously fallen within the bracket. Why was this ?

In the first place, to add − 6 to 10 is really to take 6 away. In symbols this means that $10 + (-6)$ is the same statement as $10 - (+6)$ and in both cases this becomes $10 - 6 = 4$. Thus the − sign becomes dominant.

Now consider what happens if you take − 6 *away* from a number, say 10. This would be equivalent to writing $10 - (-6)$. But to take − 6 *away* is really to *add* 6 ; thus this may also be written $10 + 6$; or $10 - (-6) = 10 + 6$.

A minus sign in front of a bracket changes a − sign inside to a +, and a + sign inside to a − sign. If you think of it in terms of direction :

+ 1 means take one step to the right
− 1 means take one step in the reverse direction
− (− 1) means take one step in the reverse direction to that of − 1, i.e. take one step to the right.
Thus $-(-1) = +1$.

This would be equally valid if you had $-5(-6) = 30$.

We are really dealing here with the multiplication of negative numbers.

Algebraically, therefore, we can write these rules as follows :

$$(+a)(+b) = +ab$$
$$(-a)(+b) = -ab$$
$$(+a)(-b) = -ab$$
$$(-a)(-b) = +ab$$

In this the two brackets standing alongside each other with no symbol between such as $(+a)(+b)$ imply that the contents of the brackets have to be multiplied together ; also a term ab signifies that a is multiplied by b. The *rule of signs* which summarises the foregoing conclusions in multiplication can be stated in the form : like signs give +, unlike signs give −.

Meaning of Brackets

If a number stands in front of a bracket, it means that it has to be multiplied by whatever stands inside the bracket, even if the number inside is composed of several added or subtracted or multiplied. Thus :

$$5 \times 6 = 5(4 + 2) = 5 \times 4 + 5 \times 2$$
$$5(x + y) = 5 \times x + 5 \times y = 5x + 5y$$
$$a(x + y) = ax + ay.$$

In the last example, we are regarding a as some unspecified number. Henceforth we will always suppose that the letters in our algebraic expressions represent numbers ; we preferred not to make this restriction from the outset in order the better to demonstrate the reasonableness of algebraic notation.

The meaning given to brackets implies that we must calculate $(a + b)(x + y)$ by the rule

$$(a + b)(x + y) = (a + b)x + (a + b)y.$$

To calculate further we broaden our rule so that a number standing *before* or *after* a bracket multiplies every term inside the bracket ; thus

$$(a + b)(x + y) = ax + bx + ay + by.$$

We could also have worked out this product as

$$(a + b)(x + y) = a(x + y) + b(x + y)$$
$$= ax + ay + bx + by$$

Similarly

$$(a + b)(x - y) = a(x - y) + b(x - y)$$
$$= ax - ay + bx - by$$
$$(x + y)(x - y) = x(x - y) + y(x - y)$$
$$= x^2 - xy + yx - y^2.$$

Here we have written $x \times x$ as x^2 to be read x-squared just as we did with ordinary numbers. In the same way we write y^2 for $y \times y$. Now xy is the same as yx. It follows that :

$$(x + y)(x - y) = x^2 - y^2.$$

You can verify this by writing definite numbers for x and y, say $x = 10$, $y = 3$. Then

$$x + y = 10 + 3 = 13 \qquad x - y = 10 \text{ [illegible] } = 7$$
$$x^2 = 10^2 = 100 \qquad y^2 = \text{ [illegible] } = 9$$

Thus the formula says that

$$13 \times 7 = 100 - 9 = 91,$$

which gives the factors of 91.

You can say therefore that the factors of $x^2 - y^2$ are $x + y$ and $x - y$.

Algebraic Powers

We have just introduced the notion of *powers* for algebraic symbols by writing :

$$x \times x = x^2 \text{ instead of } xx.$$

In the same way you write

$$x^3 \text{ for } x \times x \times x,\ x^4 \text{ for } x \times x \times x \times x,$$

and so on. A number so written above a symbol is called an *index* (plural, indices).

Rules of Indices

There are three rules of indices.

(i) You write $x^m = x \times x \times x \ldots . m$ times.

Here x appears as a multiplier m times in succession.
In the same way $x^n = x \times x \times x \ldots . n$ times

$$\text{Hence } x^m \times x^n = (x \times x \times x \ldots . m \text{ times}) \times (x \times x \times x \ldots . n \text{ times})$$
$$= x \times x \times x \ldots (m + n) \text{ times} = x^{m+n}$$

It is easy to verify in this way that

$$x^4 \times x^7 = x^{4+7} = x^{11}$$
$$x^5 \times x^8 = x^{5+8} = x^{13}$$

Thus, to multiply two powers of the same number you take that number raised to the sum of the two previous powers.

(ii) Suppose m is greater than n. You write this statement symbolically thus: $m > n$. Then

$$\frac{x^m}{x^n} = \frac{x \times x \times x \times x \ldots \ldots m \text{ times}}{x \times x \times x \times x \ldots \ldots n \text{ times}}$$

Whatever value x has, some of the x's above will cancel out with some below. How many will do so? Evidently all the x's underneath, n of them, since there are more x's in the numerator, m being greater than n. Thus there will be left $m - n$ such x's on top, while the denominator will be reduced to the product of a series of 1's. Finally, therefore,

$$\frac{x^m}{x^n} = x^{m-n}$$

Verification:

$$\frac{2^5}{2^3} = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} = 2 \times 2 = 2^2 = 2^{5-3}$$

(iii) What of a power of a power of x? Suppose x^n is itself raised to the power m, will the result still be a power of x?

$$(x^n)^m = x^n \times x^n \times x^n \times \ldots m \text{ times}$$

Thus:

$$(x^n)^m = x^{mn} = x^{nm} = (x^m)^n$$

Verification:

$$(x^3)^4 = x^3 \times x^3 \times x^3 \times x^3 = x^{3+3+3+3} = x^{12}$$

EXERCISES

(1) Find the value of $4p + 2s$ when $p = 20$, $s = 1$.

(2) Find the number of shillings in £4 2 0 and explain why the answer is the same as in question (1).

(3) Find the value of $2p + 17s + 6d$: (a) when $p = 240$, $s = 12$, $d = 1$ and (b) when $p = 20$, $s = 1$, $d = \frac{1}{2}$.

Compare the answers with the number of pence and the number of shillings respectively in £2 17 6.

(4) Add together
(a) $4a + 7b + 2c$, $a + 3b + c$, $2a + b + 6c$.
(b) $15x + 17y$, $13x - 4y$, $x + y$.
(c) $a + 3b$, $a - b + 2c$, $a - 2c$.

(5) Simplify $(4p + 6q - 7r) + (p + q + 2r) - (2p + q + r)$ and find its value if $p = 3$, $q = -1$, $r = 0$.

(6) How many ft. are there in 4 yds., 10 yds., x yds.?

(7) A rectangular table has sides of lengths p ft. and q ft. What is its area? What is the length of a side of a square table having the same area?

(8) A circle has a radius of r ft. What is the length of its circumference in inches and its area in square inches?

(9) Show that $(2a + b)(2a - b) = 4a^2 - b^2$ and verify this (a) when $a = 3$, $b = 1$ and (b) when $a = 3$, $b = 8$.

(10) Of a group of 15 men, x wear spectacles and 5 need spectacles but do not wear them. How many men of the group have good sight?

ANSWERS TO EXERCISES IN LESSON 9

(1) 25 yds.

(2) $75\sqrt{3}$ ft., (129·9 ft.).

(3) Distance rowed = 130 yds., distance remaining = $243\frac{1}{3}$ yds.

(4) Sin 156° = 0·41, cos 156° = − 0·91, tan 156° = −0·45. Sin 200° = −0·34, cos 200° = −0·94, tan 200° = 0·36. Sin 236° = −0·83, cos 236° = − 0·56, tan 236° = 1·48. Sin 315° = −0·71, cos 315° = 0·71, tan 315° = −1.

(5) Sin 180° = 0, cos 180° = −1, tan 180° = 0. Sin 270° = −1, cos 270° = 0, tan 270° = ∞. Sin 360° = 0, cos 360° = 1, tan 360° = 0.

LESSON 11

The Idea of a Function

THE student is now in a position to consider a set of terms involving powers of x combined by additions, subtractions or multiplications. Here, for example, are some such expressions:

$$x^2+5x+4\ ;\ x^3-7x^2+2x-1\ ;\ x^4-16\ ;\ x^7-x+5.$$

These are all called *functions* of x, of a special kind; mathematicians call them *polynomials*. In fact, any mathematical expression which involves x, and whose value therefore depends on the value assigned to x, is called a function of x. To put it in general terms, one quantity is said to be a function of another if a change in one correspondingly affects the other. It is usual to write $f(x)$ for the phrase "function of x." It means nothing more than what these words imply. If $y = f(x)$, then x, y are called *variables* and the variable y is said to *depend* on the variable x. Of course, there are other functions of x besides polynomials; in the next Lesson we shall consider trigonometrical functions.

Consider this function:

$$x - 7x^3 + 24 + 13x^4 - x^2.$$

The order in which the separate terms are added or subtracted does not affect its final value, but for some purposes it is better to use one order rather than another. For example, if you were calculating its value for very large values of x you would be more interested in the higher powers of x than in the lower powers. You would write these first—like this:

$$13x^4 - 7x^3 - x^2 + x + 24.$$

This is written, for obvious convenience in calculations, in descending powers of x.

You would write it in this order if x were a large number like 1,000, for the first term would be 13,000,000,000,000 as compared with the last term, which is only 24.

If you are interested in the lower powers rather than the higher powers you would write it in ascending powers of x—thus:

$$24 + x - x^2 - 7x^3 + 13x^4.$$

You would write it in this order if x were a small number like 0·001, for the first term is now 24 as compared with the last term, which is 0·000,000,000,013.

If you have to add the functions $24 + x - x^2 - 7x^3 + 13x^4$, $5x - x^3 - 26$ and $5x^3 - 10x^4 - 12 + x^2$ you would write them one under the other in descending powers of x:

$$\begin{array}{rrrrr} 13x^4 & -\ 7x^3 & -\ x^2 & +\ x & +\ 24 \\ & -\ x^3 & & +\ 5x & -\ 26 \\ -10x^4 & +\ 5x^3 & +\ x^2 & & -\ 12 \\ \hline \text{Answer: } 3x^4 & -\ 3x^3 & & +\ 6x & -\ 14 \end{array}$$

Terms with the same powers must be put in the same column.

The same principle applies to subtraction.

$$\begin{array}{lrrrrr} \text{From} & 13x^4 & -\ 7x^3 & -\ x^2 & +\ x & +\ 24 \\ \text{take} & -\ 10x^4 & +\ 5x^3 & +\ x^2 & & -\ 12 \\ \hline \text{Answer:} & 23x^4 & -\ 12x^3 & -\ 2x^2 & +\ x & +\ 36 \end{array}$$

Multiplication of Functions

When two expressions are multiplied together every item or term in the one is multiplied by every item or term in the other. This is best carried through by writing both functions in, say, descending powers of x, and ensuring that each term found for the product is written in its appropriate column corresponding to its power of x. This can best be explained by means of examples.

1. Multiply $x^2 - x + 1$ by $x^2 + x + 1$.

Write it in this way:

$$\begin{array}{rrrrrl} & & x^2 & -\ x & +\ 1 & \ldots\ldots\ldots\ldots \quad \text{(i)} \\ & & x^2 & +\ x & +\ 1 & \\ \hline & & x^2 & -\ x & +\ 1 & \ldots\ldots\ldots\ldots \quad \text{(ii)} \\ & x^3 & -\ x^2 & +\ x & & \ldots\ldots\ldots\ldots \quad \text{(iii)} \\ x^4 & -\ x^3 & +\ x^2 & & & \ldots\ldots\ldots\ldots \quad \text{(iv)} \\ \hline x^4 & & +\ x^2 & & +\ 1 & \ldots\ldots\ldots\ldots \quad \text{(v)} \end{array}$$

Line (ii) is obtained by multiplying each item in line (i) by $+ 1$, line (iii) by multiplying by x, and line (iv) by multiplying by x^2; since $1 + x + x^2$ is the multiplier. In the result each item is placed in the column corresponding to its appropriate power.

2. Multiply

$5x^4 + 10x^3 + 10x^2 + 5x + 1$ by $x^3 - 3x^2 + 3x + 1$

Again you set it out systematically as under:

$$\begin{array}{rrrrrrrr} & & & 5x^4 & +\ 10x^3 & +\ 10x^2 & +\ 5x & +\ 1 \\ & & & & x^3 & -\ 3x^2 & +\ 3x & +\ 1 \\ \hline & & & 5x^4 & +\ 10x^3 & +\ 10x^2 & +\ 5x & +\ 1 \\ & & 15x^5 & +\ 30x^4 & +\ 30x^3 & +\ 15x^2 & +\ 3x & \\ & -\ 15x^6 & -\ 30x^5 & -\ 30x^4 & -\ 15x^3 & -\ 3x^2 & & \\ 5x^7 & +\ 10x^6 & +\ 10x^5 & +\ 5x^4 & +\ x^3 & & & \\ \hline 5x^7 & -\ 5x^6 & -\ 5x^5 & +\ 10x^4 & +\ 26x^3 & +\ 22x^2 & +\ 8x & +\ 1 \end{array}$$

Division of Functions

To divide $x^3 + 3x^2 + 4x + 2$ by $x^2 + 2x + 1$, for example, again it is necessary to set both terms out in the same way. Here, for illustration, we carry the division through when the terms are arranged in descending powers of x, and then in ascending powers.

1. In descending powers of x.

$$\begin{array}{r|rrrrl} x^2 + 2x + 1 & x^3 & +\ 3x^2 & +\ 4x & +\ 2 & (x + 1 \\ & x^3 & +\ 2x^2 & +\ x & & \\ \hline & & x^2 & +\ 3x & +\ 2 & \\ & & x^2 & +\ 2x & +\ 1 & \\ \hline & & & x & +\ 1 & \end{array}$$

The first term x in the answer is found from the fact that x^2 is converted into x^3 by multiplying by x. The whole divisor $x^2 + 2x + 1$ is then multiplied by x and the result subtracted. The next term 2 is brought down and the divisor now requires only to be multiplied by 1. There is a remainder, $x + 1$.

2. The ascending powers of x.

$$\begin{array}{r|rrrrrl} 1 + 2x + x^2 & 2 & +\ 4x & +\ 3x^2 & +\ x^3 & & (2 + x^2 \\ & 2 & +\ 4x & +\ 2x^2 & & & \\ \hline & & & x^2 & +\ x^3 & & \\ & & & x^2 & +\ 2x^3 & +\ x^4 & \\ \hline & & & & -\ x^3 & -\ x^4 & \end{array}$$

The answer, if we stop at this stage, is $2 + x^2$, and the remainder is $- x^3 - x^4$.

Numerical Verification

(i) In the first case choose $x = 10$, say, then:

$$x^2+2x+1 = 10^2+20+1 = 121$$
$$x^3+3x^2+4x+2 = 10^3+3\times10^2+40+2 = 1342$$
$$x+1 = 11$$
$$\text{Now } \frac{1342}{121} = 11$$

and the remainder is 11 as expected.

(ii) In the second case, choose $x = -0{\cdot}1$.

$$1+2x+x^2 = 1-0{\cdot}2+0{\cdot}01 = 0{\cdot}81$$
$$2+4x+3x^2+x^3 = 2-0{\cdot}4+0{\cdot}03-0{\cdot}001 = 1{\cdot}629$$
$$2 \quad + x^2 \quad = 2+0{\cdot}01 \quad = 2{\cdot}01$$
$$\text{Now } \frac{1{\cdot}629}{0{\cdot}81} = 2{\cdot}01$$

and the remainder is $0{\cdot}0009$. But

$$- x^3 - x^4 = 0{\cdot}001 - 0{\cdot}0001 = 0{\cdot}0009,$$

which is the remainder.

Here you see how each method of writing out the function corresponds to a method of division suitable for large or for small values of x respectively, because in each case the remainder becomes small in comparison with the numbers originally used in the division. Of course, if there is no remainder the two methods give the same answer.

Algebraic Verification

You can easily show by direct multiplication that what you have found shows that

$$(x^2 + 2x + 1)(x + 1) + (x + 1)$$

and

$$(1 + 2x + x^2)(2 + x^2) - x^3 - x^4$$

are two ways of writing $x^3 + 3x^2 + 4x + 2$.

The first of these, moreover, indicates that since $(x + 1)$ appears in both terms, $x + 1$ must be a factor of $x^3 + 3x^2 + 4x + 2$.

Verify this by direct division, showing that the result is $x^2 + 2x + 2$.

Functions of x and y

An expression depending for its value on the values assigned to two quantities x and y is called a function of two variables x and y. For example,

$$x^2 + 2xy + y^2$$
$$x^2 + 2x + y - 5$$

are functions of this type. In such cases, instead of writing $f(x)$, you indicate the presence of the two variables by writing $f(x, y)$.

Degree of a Term

A term such as x^4 or y^4 is of degree 4. When x and y are both present in the same term, as for example in xy, or xy^3, or x^4y^2, the term is spoken of as of a degree equal to the sum of the two separate degrees. Thus, xy is of degree 2, xy^3 of degree 4, x^4y^2 of degree 6.

All the terms of a (polynomial) function of x and y are not necessarily of the same degree. When all the terms are of the same degree the function is said to be *homogeneous*.

For example, $x^3 + 3x^2y + 3xy^2 + y^3$ has all its terms of degree 3; it is therefore homogeneous.

Any function of x and y can be separated out into a series of groups each of which is a homogeneous function. For example:

$$x^6 + y^3 - 7x^2y + x^4y - x^3y^3 + y^5$$

can be rewritten in the form:

$$(x^6 - x^3y^3) + (x^4y + y^5) + (y^3 - 7x^2y)$$

in which the terms in brackets are respectively of degree 6, 5, 3.

Multiplication and Division

The processes of addition, subtraction, multiplication, and division can be carried through in this case in precisely the same way as for one variable. You fix on one of the variables, say x, and arrange the functions in descending or ascending powers of x. Thus, to divide

$$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 \text{ by } x + y$$

you write

$$\begin{array}{l}
x+y)x^4+4x^3y+6x^2y^2+4xy^3+y^4(x^3+3x^2y+3xy^2+y^3\\
\quad\;\; \underline{x^4+x^3y}\\
\qquad\quad 3x^3y+6x^2y^2\\
\qquad\quad \underline{3x^3y+3x^2y^2}\\
\qquad\qquad\qquad 3x^2y^2+4xy^3\\
\qquad\qquad\qquad \underline{3x^2y^2+3xy^3}\\
\qquad\qquad\qquad\qquad\quad xy^3+y^4\\
\qquad\qquad\qquad\qquad\quad \underline{xy^3+y^4}
\end{array}$$

Answer: $x^3 + 3x^2y + 3xy^2 + y^3$.

EXERCISES

(1) Verify that (a) $2^6 \times 2^3 = 2^9$, (b) $x^3 \times x^5 = x^8$, (c) $3^2 \times 3^4 \times 3 = 3^7$, (d) $8^2 = (2^3)^2 = 2^6$, (e) $5^6 \div (25)^2 = 25$, (f) $a^8 \times a^3 \div a^9 = a^2$, (g) $(2 \times 5)^3 = 2^3 \times 5^3$.

(2) A carpet has sides of length a ft., $a + 3$ ft. What is its area and the length of a diagonal?

(3) Multiply the following expressions together: (a) $7x^3 - x$ by $x^2 + 2x + 5$, (b) $x^2 - x + 1$ by $x^2 + 1$, (c) $4a + b - c + 2d$ by $a + 2c$, (d) $1 + a + a^2 + a^3$ by $1 - a$, (e) $x^2 + 4xy + 6y^2$ by $2xy$.

(4) Find n if 2^n is greater than 1,000 but less than 2,000.

(5) By taking numerical examples, verify that the terms $1, x, x^2, x^3, x^4, \ldots\ldots$ become smaller and smaller if $x<1$, but increase if $x>1$.

(6) Divide (a) $1 + x$ into $1 + x + x^2 + x^3$.
(b) $1 - x$ into $1 - x^5$.

(7) Show that

$$\frac{1+3x^2+4x^4}{1+x^2} = 1+2x^2+\frac{2x^4}{1+x^2} = 4x^2-1+\frac{2}{x^2+1}$$

(8) Divide $x^3 - 3x^2y + 3xy^2 - y^3$ by $x - y$.

(9) Show that $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

ANSWERS TO EXERCISES IN LESSON 10

(1) 82.. (2) 82.

(3) (a) 690, (b) $57\frac{1}{2}$.

(4) (a) $7a + 11b + 9c$, (b) $29x + 14y$, (c) $3a + 2b$.

(5) $3p + 6q - 6r$, 3. (6) 12, 30, $3x$.

(7) pq sq. ft., $\sqrt{(pq)}$ ft.

(8) $24\pi r$ in., $144\ \pi r^2$ sq. in.

(10) $10 - x$.

LESSON 12

Trigonometrical Functions of Compound Angles

In the last Lesson dealing with trigonometry we extended our definitions of sine, cosine, and tangent to include angles of any positive size, and we will now consider negative angles by adopting exactly the same reasoning as when considering negative numbers in arithmetic (Lesson 2). The distinction is between clockwise and anti-clockwise.

Negative Angles

The convention for measuring positive angles is to start from the positive x axis and rotate in the anti-clockwise direction. As is to be expected, *negative* angles are measured from the same initial line but rotating in the opposite direction, i.e. in the *clockwise* direction. As an example, consider an angle of 60° from which is subtracted 15° (Fig. 80). A rotation of 60° in the clockwise direction brings you to the position OA, then subtracting 15° you arrive at OB.

Notice that to arrive at OB you have rotated through 15° in the clockwise direction. We might have considered the angle 60° − 15°

as $60° + (-15°)$ which provides us immediately with the conception of a negative angle.

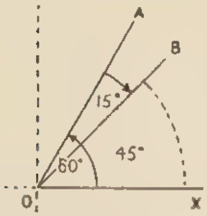


Fig 80

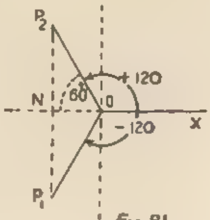


Fig. 81

Fig. 80. Measuring negative angles clockwise.
Fig. 81. Sin, cos and tan of −120 deg.

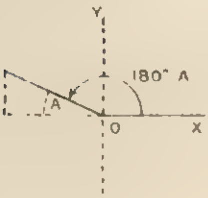


Fig. 82

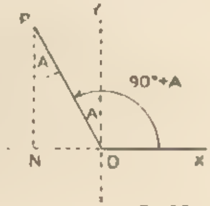


Fig. 83

Figs. 82 and 83. Simple compound angles.

Thus we may consider negative angles independently, and consequently a negative angle having a numerical size less than 90° lies in the fourth quadrant, a negative angle between $-90°$ and $-180°$ in the third quadrant and so on. From these positions it is a simple matter to determine the trigonometrical ratios of negative angles in terms of positive angles ; for instance, $\sin(-A) = -\sin A$, $\cos(-A) = \cos A$ and $\tan(-A) = -\tan A$.

Example. Find the sine, cosine, and tangent of $-120°$.

From Fig. 81 it is clear that

$$\sin(-120°) = \frac{NP_1}{OP_1} \text{ and } \sin(+120°) = \frac{NP_2}{OP_2}$$

so that $\sin(-120°) = -\sin(+120°)$.

But $\sin(120°) = \sin 60° = \sqrt{3}/2$

so that $\sin(-120°) = -\sqrt{3}/2$.

Both $\cos(-120°)$ and $\cos(+120°)$ are equal to

$$\frac{ON}{OP_3}$$

i.e. $\cos(-120°) = \cos(+120°) = -\cos 60° = -\frac{1}{2}$

and, finally,

$$\tan(-120°) = \frac{NP_1}{ON} = -\tan(120°)$$
$$= -(-\tan 60°) = +\sqrt{3}.$$

Simplified Expressions

In trigonometrical work angles such as $180° - A$, $90° + A$, $360° - A$ are of very common occurrence, and it is important to be able to write the sines, cosines, and tangents of these angles in as simple a form as possible, generally in terms of the angle A alone. If reference is made to Lesson 9, where angles greater than 90° were discussed, it will be seen how this can be done.

Example. Express the ratios of $180° - A$ in terms of those of A.

Fig. 82 represents the position of the angle $180° - A$ when A is less than 90°. From the figure it will be seen that $\sin(180° - A)$ is positive and numerically equal to sin A. $\cos(180° - A)$ is negative but numerically equal to cos A, so that $\sin(180° - A) = \sin A$, $\cos(180° - A) = -\cos A$ and consequently $\tan(180° - A) = -\tan A$. In exactly the same way these relations can be proved true for *any* value of A.

Example. Express the ratios of $90° + A$ in terms of those of A only.

Fig. 83 shows the position of the revolving line OP for an angle $90° + A$, when A is less than 90°. If a perpendicular is drawn from P on to the x axis it will be seen that $\angle POY = A = \angle OPN$. The sine of $90° + A$ is given by $\frac{PN}{OP}$ which is positive and equal numerically to $\cos \angle OPN$, i.e. cos A. Thus $\sin(90° + A) = \cos A$.

Similarly, $\cos(90° + A)$ is negative and numerically equal to $\sin \angle OPN$, i.e. sin A, so that

$\cos(90° + A) = -\sin A$. Finally, then,

$\tan(90° + A) = -\cot A$.

All these relations may be proved equally true if A is greater than 90°.

Relations Between Angles

An immediate use of the foregoing is provided in finding the relations between the trigonometrical ratios of the angles of any triangle. For then, if the three angles are A, B, C, we have $A + B + C = 180°$, so that $A + B = 180° - C$. Therefore, from the preceding considerations,

$\sin(A + B) = \sin(180° - C) = \sin C$ and
$\cos(A + B) = \cos(180° - C) = -\cos C$
and finally, since

$$\tfrac{1}{2}(A + B) = \tfrac{1}{2}(180 - C) = 90 - \tfrac{1}{2}C,$$
$$\sin \tfrac{1}{2}(A + B) = \sin(90 - \tfrac{1}{2}C) = \cos \tfrac{1}{2}C.$$

NOTE.—We have here used, for the first time, the expressions $\sin(A + B)$ and $\cos(A + B)$, and these mean exactly what they indicate : namely, the sine and cosine respectively of two angles added together. The student must be careful not to make the error of thinking that $\sin(A + B)$ is equal to $\sin A + \sin B$. This is *not* true. There is no reason why it should be. The word "sin" or "sine" is in no sense a quantity which multiplies $A + B$, it simply denotes a function of the combined angle $A + B$. Nevertheless, the sine of $(A + B)$ can be expanded in terms of sin A, cos A, sin B, and cos B, and it is this and similar expansions that we will now consider. First of all, in order to prove the various expansions we must consider the projections of lines in more detail.

Directed Lines

An important idea, useful in many branches of mathematics, is that of directed lines. We may choose one direction along a line as positive and then the opposite direction will be con-

sidered as negative; in fact we have already done this when defining co-ordinate axes. From left to right we chose as the positive x direction, so that from right to left was the negative x direction. Consider the four lines PQ in Fig. 84. The arrowhead indicates their direction; in every case perpendiculars have been drawn on to a horizontal line so that LM is the *projection* of each line on to this line. In each case the projection is PQ cos A, and, taking from left to right as a positive direction, you see from the diagram that in the first and last cases the projection is positive, but in the second and third it is negative. Finally, you may notice (Fig. 85) that if any triangle PQR is drawn, the projection of PQ = the projection of PR + the projection of RQ, i.e. LN = LM + MN. This is quite in order because MN is itself negative.

To Prove $\cos(A + B) = \cos A \cos B - \sin A \sin B$ and $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

Take any angle A and add to it an angle B (Fig. 86). On the boundary line of this combined angle choose any point P and draw a perpendicular PN on to the boundary line of the angle A, and produce to Q. (Two cases are drawn, but the following proof will be found to be true, whatever values are given to A and B). Thus you have a triangle ONP, and so, projecting this triangle on to OX you have:

projection OP = projection ON + projection NP . . . . (*i*)

The projection of OP on to OX is OP cos (A + B) and since $\triangle$ ONP is right-angled ON = OP cos B and NP = OP sin B.

Thus the relation (i) becomes

$$\begin{aligned} OP\cos(A+B) &= ON\cos A + NP\cos\angle XQP \\ &= OP\cos B\cos A + OP\sin B\cos\angle XQP \\ &= OP\cos B\cos A + OP\sin B\cos(90 + A) \end{aligned}$$

since $\angle XQP = 90° + A$; also $\cos(90 + A) = -\sin A$, so that

$$OP\cos(A + B) = OP\cos B\cos A - OP\sin B\sin A$$

or $\cos(A + B) = \cos A\cos B - \sin A\sin B$ . . . . (*ii*)

In exactly the same way, by projecting the triangle ONP on to a line at right angles to OX you obtain

$\sin(A + B) = \sin A\cos B + \cos A\sin B$ . . . . (*iii*)

The Expansion for sin (A − B) and cos (A − B). You can obtain these expansions most easily by replacing B by − B in (ii) and (iii). Thus, from (ii)

$$\cos(A + (-B)) = \cos A\cos(-B) - \sin A\sin(-B)$$

and, as seen earlier,

$$\cos(-B) = \cos B \text{ and } \sin(-B) = -\sin B,$$

so that

$\cos(A - B) = \cos A\cos B + \sin A\sin B$ . . . . (*iv*)

Again, from (iii)

$\sin(A + (-B)) = \sin A\cos(-B) + \cos A\sin(-B)$

i.e. $\sin(A - B) = \sin A\cos B - \cos A\sin B$ . . . . (*v*)

These four expansions, (ii), (iii), (iv) and (v), are of extreme importance and should be memorised. They provide the means of breaking up the sine or cosine of a combined angle into the sines and cosines of its constituent parts.

The Expansion for tan (A + B) and tan (A − B). These expansions can be obtained directly from what you now know, for

$$\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$$

and, dividing numerator and denominator of this last expression by cos A cos B, you obtain

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} \quad . . . . \quad (vi)$$

Again you may replace B by − B, you order to obtain tan (A − B); thus

$$\tan(A + (-B)) = \frac{\tan A + \tan(-B)}{1 - \tan A\tan(-B)}$$

$$\text{i.e. } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B} \quad . . . . \quad (vii)$$

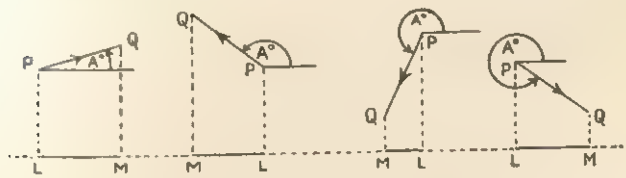


Fig 84

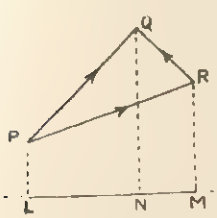


Fig. 85

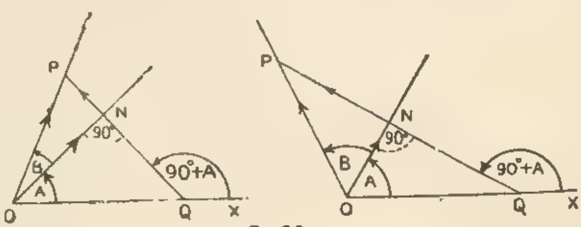


Fig 86

Fig. 84. Showing the directed line PQ with projections LM. Fig. 85. Showing the triangle PQR with projections LM, LN, and MN. Fig. 86. Proving relations in compound angles by projection.

The student should verify that this can be obtained equally well by dividing the expansion for sin (A − B) by that for cos (A − B).

Example. Find the value of cos 15°.

Since 15° = 60° − 45° you can use the expansion for cos (A − B) to determine cos 15° in terms of the sine and cosine of 60° and 45°; thus:

$$\begin{aligned}\cos 15° &= \cos (60° - 45°)\\ &= \cos 60° \cos 45° + \sin 60° \sin 45°\\ &= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}\\ &= \frac{1+\sqrt{3}}{2\sqrt{2}}.\end{aligned}$$

Example. Find the expansion of cot (A + B) in terms of cot A and cot B.

$$\begin{aligned}\cot (A+B) &= \frac{1}{\tan (A+B)} = \frac{1 - \tan A \tan B}{\tan A + \tan B}\\ &= \left\{1 - \frac{1}{\cot A}\cdot\frac{1}{\cot B}\right\} \Big/ \left\{\frac{1}{\cot A} + \frac{1}{\cot B}\right\}\\ &= \frac{\cot A \cot B - 1}{\cot A + \cot B}\end{aligned}$$

by multiplying numerator and denominator by cot A cot B.

Example. Show that, in any triangle ABC,

sin A = sin B cos C + cos B sin C.

As seen in an earlier paragraph, since A + B + C = 180°, B + C = 180° − A, so that sin (B + C) = sin (180° − A) = sin A,

i.e.
$$\begin{aligned}\sin A &= \sin (B + C)\\ &= \sin B \cos C + \cos B \sin C.\end{aligned}$$

Sin 2A, Cos 2A, tan 2A

A very important application of the foregoing expansions occurs when B = A. Then, for instance, sin (A + B) = sin (A + A), i.e. sin (2A), and since no ambiguity arises, this is written sin 2A and it simply means the sine of an angle 2A in size. By replacing B by A in each of the expansions sin (A + B), cos (A + B) and tan (A + B) you obtain the following:

$$\begin{aligned}(1)\ \sin 2A &= \sin A \cos A + \cos A \sin A\\ &= 2 \sin A \cos A \quad \ldots\ldots\ldots \qquad (viii)\\ (2)\ \cos 2A &= \cos A \cos A - \sin A \sin A\\ &= \cos^2 A - \sin^2 A \quad \ldots\ldots\ldots \qquad (ix)\\ (3)\ \tan 2A &= \frac{\tan A + \tan A}{1 - \tan A.\ \tan A}\\ &= \frac{2 \tan A}{1 - \tan^2 A} \quad \ldots\ldots\ldots \qquad (x)\end{aligned}$$

These three relations should also be memorised as they are of frequent occurence in further mathematical work. It should be noticed that since they are independent of the size of the angle A, you can, by writing 2A = P, obtain the sine, cosine, and tangent of any angle in terms of its half-angle. Thus, if

$$2A = P, \text{ then } A = \tfrac{1}{2}P$$

and you obtain

$$\sin P = 2 \sin \tfrac{1}{2}P \cos \tfrac{1}{2}P,\quad \cos P = \cos^2 \tfrac{1}{2}P - \sin^2 \tfrac{1}{2}P$$

and

$$\tan P = \frac{2 \tan \frac{1}{2}P}{1 - \tan^2 \frac{1}{2}P}.$$

Relation (ix) should be especially noticed as it is capable of alternative forms since $\sin^2 A + \cos^2 A = 1$. Thus:

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= \cos^2 A - (1 - \cos^2 A)\\ &= 2 \cos^2 A - 1 \quad \ldots\ldots \qquad (xi)\end{aligned}$$

or, alternatively,

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= (1 - \sin^2 A) - \sin^2 A\\ &= 1 - 2 \sin^2 A \quad \ldots\ldots \qquad (xii)\end{aligned}$$

From these relations you can obtain $\sin^2 A$ and $\cos^2 A$ in terms of cos 2A. For example, (xi) can be rewritten

$$2 \cos^2 A = 1 + \cos 2A,$$

i.e.
$$\cos^2 A = \tfrac{1}{2}(1 + \cos 2A)$$

and from (xii)

$$\sin^2 A = \tfrac{1}{2}(1 - \cos 2A)$$

In more advanced work it will be necessary to change expressions involving the powers of sin A and cos A into forms containing only multiples of A, and it is then that the usefulness of these relations will become evident.

The Expansions for Three Angles

You can obtain the expansion of expressions such as sin (A + B + C), tan (A + B + C), where the three angles may all be independent (i.e. not necessarily the angles of a triangle), by using expansions (ii) to (vii) and keeping two angles together in the first instance. We illustrate with an actual example:

$$\begin{aligned}\sin (A + B + C) &= \sin (A + (B + C))\\ &= \sin A \cos (B + C) + \cos A \sin (B + C)\\ &= \sin A (\cos B \cos C - \sin B \sin C)\\ &\quad + \cos A (\sin B \cos C + \cos B \sin C)\\ &= \sin A \cos B \cos C + \cos A \sin B \cos C\\ &\quad + \cos A \cos B \sin C - \sin A \sin B \sin C \qquad (xiii)\end{aligned}$$

by re-arrangement. Notice particularly the symmetry of this expression; the first three terms each contain the product of two cosines and a sine, the last term is the product of the three sines. The expansion for an expression such as sin (A − B + C) is obtained by replacing B by − B in (xiii), thus:

$$\begin{aligned}\sin (A - B + C) &= \sin A \cos (-B) \cos C + \cos A \sin (-B) \cos C + \cos A \cos (-B) \sin C - \sin A \sin (-B) \sin C\\ &= \sin A \cos B \cos C - \cos A \sin B \cos C + \cos A \cos B \sin C + \sin A \sin B \sin C.\end{aligned}$$

By a similar method you can obtain the expansions for cos (A+B+C), tan (A+B+C), and so on.

From relation (xiii) you can obtain the expansion for sin 3A by putting B = C = A as follows:

$$\begin{aligned}\sin 3A &= \sin A \cos^2 A + \cos A \sin A \cos A + \cos A \cos A \sin A - \sin A \sin A \sin A\\ &= 3 \sin A \cos^2 A - \sin^3 A\\ &= 3 \sin A (1 - \sin^2 A) - \sin^3 A\\ &= 3 \sin A - 4 \sin^3 A.\end{aligned}$$

Of course, if sin 3A is required in the first instance, there is no need to work through the

expansion of $\sin(A + B + C)$. It can be found directly by writing $\sin 3A = \sin(A + 2A)$ and then expanding.

EXERCISES

(1) Find the values of $\sin(-225°)$, $\tan(-45°)$, $\cos(-330°)$.

(2) By drawing quadrant diagrams, simplify $\sin(180° + A)$, $\cot(180° + A)$, $\cos(360° - A)$, $\operatorname{cosec}(360° + A)$, and check the results obtained by expansion.

(3) From a table of values of sines and cosines, verify that $\sin 48° = 2 \sin 24° \cos 24°$.

(4) By writing $\sin 75°$ as $\sin(45° + 30°)$ and expanding prove that $\sin 75° = (\sqrt{6} + \sqrt{2})/4$.

(5) Prove that $\sin(A + B) + \cos(A - B) = (\sin A + \cos A)(\sin B + \cos B)$. By replacing B by $-B$, deduce the factors of $\sin(A - B) + \cos(A + B)$.

(6) Obtain the expansion of $\cos(A + B + C)$ and from it deduce (1) the expansion of $\cos(B + C - A)$ and (2) that $\cos 3A = 4\cos^3 A - 3\cos A$.

(7) Express $\tan(A + B + C)$ in terms of $\tan A$, $\tan B$, and $\tan C$. Deduce that $\tan 3A = (3\tan A - \tan^3 A) / (1 - 3\tan^2 A)$ and verify by expanding $\tan(A + 2A)$ directly.

(8) If A, B, C are the angles of a triangle, show that
(1) $\cos A = \sin B \sin C - \cos B \cos C$,
(2) $\sin \frac{1}{2}A = \cos \frac{1}{2}B \cos \frac{1}{2}C - \sin \frac{1}{2}B \sin \frac{1}{2}C$,
(3) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$.

ANSWERS TO EXERCISES IN LESSON 11

(2) $a(a + 3)$ sq. ft., $\sqrt{(2a^2 + 6a + 9)}$ ft.
(3) (a) $7x^5 + 14x^4 + 34x^3 - 2x^2 - 5x$.
(b) $x^4 - x^3 + 2x^2 - x + 1$.
(c) $4a^2 + ab + 7ac + 2ad + 2bc + 4cd - 2c^2$.
(d) $1 - a^4$.
(e) $2x^3y + 8x^2y^2 + 12xy^3$.
(4) 10. (6) (a) $1 + x^2$; (b) $1 + x + x^2 + x^3 + x^4$.
(8) $x^2 - 2xy + y^2$.

LESSON 13

Trigonometrical Identities

It has already been indicated that the transformation of trigonometrical expressions into equivalent expressions of a different form plays an important part in the application of trigonometry to more advanced mathematics, and some more fundamental transformations can now be considered.

Transformation of Products and Sums

These can be obtained directly from the expansions of the last Lesson. Consider those for $\sin(A + B)$ and $\sin(A - B)$; namely

$$\sin(A + B) = \sin A \cos B + \cos A \sin B \quad \ldots \quad (i)$$

and

$$\sin(A - B) = \sin A \cos B - \cos A \sin B \quad \ldots \quad (ii)$$

By adding these two we obtain

$$\sin(A + B) + \sin(A - B) = 2 \sin A \cos B \quad \ldots \quad (iii)$$

and by subtracting (ii) from (i)

$$\sin(A + B) - \sin(A - B) = 2 \cos A \sin B \quad \ldots \quad (iv)$$

Notice what these last two relations imply: that the product of the sine of one angle and the cosine of another can be expressed as the sum or difference of two sines. We proceed similarly with the expansions for the cosines; thus:

$$\cos(A + B) = \cos A \cos B - \sin A \sin B \quad \ldots \quad (v)$$

and

$$\cos(A - B) = \cos A \cos B + \sin A \sin B \quad \ldots \quad (vi)$$

Adding (v) to (vi)

$$\cos(A + B) + \cos(A - B) = 2 \cos A \cos B \quad \ldots \quad (vii)$$

and subtracting (v) from (vi)

$$\cos(A - B) - \cos(A + B) = 2 \sin A \sin B \quad \ldots \quad (viii)$$

In this case it is desirable to subtract $\cos(A + B)$ from $\cos(A - B)$ in order to obtain a form on the right-hand side with a positive expression. This is because the cosine of any acute angle is greater than the cosine of a larger acute angle. The relations (vii) and (viii) give expressions for the product of two cosines and of two sines as the sum and difference of two cosines respectively.

If you wish to use relations (iii), (iv), (vii), and (viii) to express the sum or difference of two sines or cosines as products, they are more conveniently put in a slightly modified form: if you represent the angle $A + B$ by a single term C and the angle $A - B$ by a term D, then you have $A + B = C$, $A - B = D$, so that adding and subtracting, $2A = C + D$ and $2B = C - D$, i.e. $A = \frac{1}{2}(C + D)$ and $B = \frac{1}{2}(C - D)$. Then (iii) may be written:

$$\sin C + \sin D = 2 \sin \tfrac{1}{2}(C + D) \cos \tfrac{1}{2}(C - D) \quad \ldots \quad (ix)$$

From (iv):

$$\sin C - \sin D = 2 \cos \tfrac{1}{2}(C + D) \sin \tfrac{1}{2}(C - D) \quad \ldots \quad (x)$$

from (vii):

$$\cos C + \cos D = 2 \cos \tfrac{1}{2}(C + D) \cos \tfrac{1}{2}(C - D) \quad \ldots \quad (xi)$$

and from (viii):

$$\cos D - \cos C = 2 \sin \tfrac{1}{2}(C + D) \sin \tfrac{1}{2}(C - D) \quad \ldots \quad (xii)$$

Some examples will serve to show how these transformations are applied.

Example. Express as products: (1) $\sin 3A + \sin A$; (2) $\cos 5A - \cos A$; (3) $\sin 34° - \sin 14°$; (4) $\sin 20° + \cos 60°$.

(1) From (ix)

$$\sin 3A + \sin A = 2 \sin \tfrac{1}{2}(3A + A) \cos \tfrac{1}{2}(3A - A) = 2 \sin 2A \cos A$$

(2) Relation (xii) might equally well have been written

$$\cos C - \cos D = -2 \sin \tfrac{1}{2}(C + D) \sin \tfrac{1}{2}(C - D)$$

and you can use this to transform $\cos 5A - \cos A$, for then

$$\cos 5A - \cos A = -2 \sin \tfrac{1}{2}(5A + A) \sin \tfrac{1}{2}(5A - A) = -2 \sin 3A \sin 2A$$

(3) Using relation (x)

$$\sin 34° - \sin 14° = 2 \cos \tfrac{1}{2}(34° + 14°) \sin \tfrac{1}{2}(34° - 14°) = 2 \cos 24° \sin 10°.$$

The student is advised to take this example and verify it by working out both sides from a set of tables of sine and cosine; there will be a very slight discrepancy due to the fact that the tables give only approximate values, correct to four places of decimals.

(4) No relation which we have obtained gives the sum of a sine and a cosine as a product; for that we must first express either sin 20° as a cosine or alternatively cos 60° as a sine. Remembering that $\cos A = \sin (90° - A)$ we obtain

$$\begin{aligned} \sin 20° + \cos 60° &= \sin 20° + \sin (90° - 60°) \\ &= \sin 20° + \sin 30° \\ &= 2 \sin \tfrac{1}{2}(20° + 30°) \cos \tfrac{1}{2}(30° - 20°) \\ &= 2 \sin 25° \cos 5°. \end{aligned}$$

Example. Express as a sum or difference of two trigonometrical ratios (1) $2 \sin 4\theta \cos \theta$, (2) $\sin 3\theta \sin \theta$, (3) $2 \sin \frac{1}{2}A \cos \frac{1}{2}A$.

(1) Relation (iii) is

$$2 \sin A \cos B = \sin (A + B) + \sin (A - B)$$

so that

$$\begin{aligned} 2 \sin 4\theta \cos \theta &= \sin (4\theta + \theta) + \sin (4\theta - \theta) \\ &= \sin 5\theta + \sin 3\theta \end{aligned}$$

(2) The product of two sines was expressed as a difference of two cosines; namely:

$$2 \sin A \sin B = \cos (A - B) - \cos (A + B)$$

Thus

$$\begin{aligned} \sin 3\theta \sin \theta &= \tfrac{1}{2} (\cos (3\theta - \theta) - \cos (3\theta + \theta)) \\ &= \tfrac{1}{2} (\cos 2\theta - \cos 4\theta) \end{aligned}$$

(3) From relation (iii)

$$\begin{aligned} 2 \sin \tfrac{1}{2}A \cos \tfrac{1}{2}A &= \sin (\tfrac{1}{2}A + \tfrac{1}{2}A) + \sin (\tfrac{1}{2}A - \tfrac{1}{2}A) \\ &= \sin A + \sin 0° \\ &= \sin A \end{aligned}$$

It will be remembered that this was obtained when sin A was expanded in terms of $\frac{1}{2}$A.

Example. Show that

$$\cos 3A \sin 2A - \cos 2A \sin A = \cos 4A \sin A.$$

From (iv) $\cos 3A \sin 2A = \frac{1}{2}(\sin 5A - \sin A)$

and $\quad \cos 2A \sin A = \frac{1}{2}(\sin 3A - \sin A)$

so that

$$\begin{aligned} &\cos 3A \sin 2A - \cos 2A \sin A \\ &\quad = \tfrac{1}{2}(\sin 5A - \sin A - \sin 3A + \sin A) \\ &\quad = \tfrac{1}{2}(\sin 5A - \sin 3A) \\ &\quad = \tfrac{1}{2}.2 \cos 4A \sin A \ . \ . \ . \text{ from (x)} \\ &\quad = \cos 4A \sin A. \end{aligned}$$

Identities

Consider the two relations $\sin^2 A + \cos^2 A = 1$ and $\sin A = 1$. They are of very different types. The first of these is *true for all values of A* and is said to be an *identity*; but the second relation, i.e. $\sin A = 1$, is true only for special values of A, namely, 90°, 360° + 90°, 720° + 90°, . . . and so on, and is said to be a *trigonometrical equation*. As already explained, the study of identities is very important in more advanced work; it is essential to be able to manipulate a trigonometrical expression into different equivalent forms. The study of trigonometrical equations is left until later.

It will now be realized that all the expansions and transformations we have obtained are examples of identities; the expression to one side of the "equals" sign being equivalent to that on the other side of the "equals," but expressed in a different form.

It is unfortunate that the same sign (=) is generally used for both, although an attempt is sometimes made to overcome this difficulty by using the symbol ≡ between the two parts of an identity. An identity is generally most conveniently proved by taking one side and showing by successive transformations that it is equivalent to the other side, although it may sometimes be necessary to work with both sides and show that they are both equivalent to the same expression. We shall denote the left-hand side by L.H.S. and the right-hand side by R.H.S.

We will now work out some examples of simple identities, using the fundamental relations obtained in Lesson 8.

Example. Prove that $3 - 4 \sin^2 A = 4 \cos^2 A - 1$

$$\begin{aligned} \text{L.H.S.} &= 3 - 4 \sin^2 A \\ &= 3 - 4 (1 - \cos^2 A) \text{ since } \sin^2 A + \cos^2 A = 1 \\ &= 3 - 4 + 4 \cos^2 A \\ &= \text{R.H.S.} \end{aligned}$$

Example. Prove that $\cos^2 A(1 + 3 \tan^2 A) = 1 + 2 \sin^2 A$

$$\begin{aligned} \text{L.H.S.} &= \cos^2 A(1 + 3 \tan^2 A) \\ &= \cos^2 A + 3 \cos^2 A \,.\, \frac{\sin^2 A}{\cos^2 A} \quad \text{since } \tan A = \frac{\sin A}{\cos A} \\ &= \cos^2 A + 3 \sin^2 A \\ &= (1 - \sin^2 A) + 3 \sin^2 A \\ &= 1 + 2 \sin^2 A \\ &= \text{R.H.S.} \end{aligned}$$

Example. Prove that $\sin^4 A - \cos^4 A = \sin^2 A - \cos^2 A.$

It will be remembered from the algebra that $x^2 - y^2 = (x - y)(x + y)$ so if you put $x = \sin^2 A$ and $y = \cos^2 A$ this will mean that $\sin^4 A - \cos^4 A = (\sin^2 A - \cos^2 A)(\sin^2 A + \cos^2 A)$ and $\sin^2 A + \cos^2 A = 1$, so that $\sin^4 A - \cos^4 A = \sin^2 A - \cos^2 A$.

Example. Prove the identity

$$\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = \frac{2}{\cos A}.$$

By taking a common denominator of $\cos A (1 + \sin A)$ you can write

$$\begin{aligned} \text{L.H.S.} &= \frac{\cos^2 A + (1 + \sin A)^2}{\cos A (1 + \sin A)} \\ &= \frac{\cos^2 A + 1 + 2 \sin A + \sin^2 A}{\cos A (1 + \sin A)} \\ &= \frac{2 + 2 \sin A}{\cos A (1 + \sin A)} \text{ since } \cos^2 A + \sin^2 A = 1 \\ &= \frac{2 (1 + \sin A)}{\cos A (1 + \sin A)} \\ &= \text{R.H.S.} \end{aligned}$$

Identities involving the manipulation of expressions such as sin 2A, sin 4A, cos 2A will be valuable in later work. Here are two examples.

Example. If $t = \tan \frac{1}{2}A$, show that (1) $\sin A = 2t/(1 + t^2)$, (2) $\cos A = (1 - t^2)/(1 + t^2)$

(1) From Lesson 12 you can write

$$\begin{aligned} \sin A &= 2 \sin \tfrac{1}{2}A . \cos \tfrac{1}{2}A \\ &= 2\frac{\sin \frac{1}{2}A}{\cos \frac{1}{2}A} . \cos^2 \tfrac{1}{2}A \end{aligned}$$

multiplying numerator and denominator by $\cos \frac{1}{2}A$

$$\begin{aligned} &= \frac{2t}{\sec^2 \frac{1}{2}A} \text{ since } \cos^2 \tfrac{1}{2}A = \frac{1}{\sec^2 \frac{1}{2}A}. \\ &= \frac{2t}{1 + t^2} \text{ since } \sec^2 \tfrac{1}{2}A = 1 + \tan^2 \tfrac{1}{2}A \end{aligned}$$

(2) Similarly

$$\cos A = \cos^2 \tfrac{1}{2}A - \sin^2 \tfrac{1}{2}A$$
$$= \cos^2 \tfrac{1}{2}A.\left(1 - \frac{\sin^2 \tfrac{1}{2}A}{\cos^2 \tfrac{1}{2}A}\right)$$
$$= \cos^2 \tfrac{1}{2}A.\ (1 - t^2)$$
$$= \frac{1 - t^2}{1 + t^2} \text{ as in example (1).}$$

Example. Show that $\cos^2 2A - \sin^2 A = \frac{1}{2}(\cos 4A + \cos 2A)$.

You have already obtained (in Lesson 12) the expansion of cos 2A as

$$\cos 2A = 2\cos^2 A - 1$$

hence, on replacing A by 2A,

$$\cos 4A = 2\cos^2 2A - 1$$

i.e. $2\cos^2 2A = 1 + \cos 4A$.

Since also $\cos 2A = 1 - 2\sin^2 A$ as an alternative form, then

$$2\sin^2 A = 1 - \cos 2A$$

i.e.

$$\cos^2 2A - \sin^2 A = \tfrac{1}{2}(1 + \cos 4A) - \tfrac{1}{2}(1 - \cos 2A)$$
$$= \tfrac{1}{2}(\cos 4A + \cos 2A)$$

Again, you can use the transformations of products and sums obtained earlier in this Lesson to prove the equivalence of identical forms. In fact, we have already proved such an identity as an example of the use of the transformations, and we will now include a few more examples to illustrate them further.

Example. Prove that
$2\sin A(\cos A + \cos 3A + \cos 5A + \cos 7A) = \sin 8A$.

To prove this identity you need the expression

$$2\sin B\cos A = \sin(A + B) - \sin(A - B)$$

Thus:

	$2\sin A\cos A = \sin 2A - \sin 0,$
also	$2\sin A\cos 3A = \sin 4A - \sin 2A$
and	$2\sin A\cos 5A = \sin 6A - \sin 4A.$
Again	$2\sin A\cos 7A = \sin 8A - \sin 6A.$

If you add these four expressions together, the left-hand side is $2\sin A\cos A + 2\sin A\cos 3A + 2\sin A\cos 5A + 2\sin A\cos 7A$, which is the L.H.S. of the given identity. On adding the R.H.S. it will be seen that all the terms cancel except sin 8A in the last line. Thus you have proved the identity.

Notice that the process could have been continued indefinitely; it is a very simple matter to generalise it. You would then have $2\sin A(\cos A + \cos 3A + \cos 5A + \ldots + \cos(2n - 1)A) = \sin(2nA)$, where we have taken the last term of the L.H.S. as $\cos(2n - 1)A$, since $2n - 1$ is always odd and can be made equal to any odd number by suitably choosing n. This extension of the original identity provides a means of finding the sum of a set of cosine terms where the angles are odd multiples of A; for the above relation may be rearranged as

$$\cos A + \cos 3A + \ldots + \cos(2n - 1)A = \frac{\sin(2nA)}{2\sin A}$$

This Lesson ends with two examples of identities of a slightly different type, both of which are concerned with three angles A, B, and C which add up to 180°, i.e. they may be considered as being the angles of a triangle. This in no way conflicts with what we have stated concerning identities; A, B, and C are not tied down to particular values; they may be of any size, but subject to the overriding condition that $A + B + C = 180°$.

Example. If $A + B + C = 180°$, prove that $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

L.H.S. $= \sin 2A + \sin 2B + \sin 2C = 2\sin(A + B)\cos(A - B) + \sin 2C$

by transforming the sum of sin 2A and sin 2B into a product

$$= 2\sin(A + B)\cos(A - B) + 2\sin C\cos C$$

by expanding sin 2C. Now, as seen in Lesson 12,

if $A + B + C = 180°$
then $A + B = 180° - C$, so that
$\sin(A + B) = \sin C$ and $\cos(A + B) = -\cos C$

i.e. L.H.S. $= 2\sin C\cos(A - B) - 2\sin C\cos(A + B)$
$= 2\sin C(\cos(A - B) - \cos(A + B))$
$= 2\sin C.\ 2\sin\tfrac{1}{2}(A - B + A + B)\sin\tfrac{1}{2}(A + B - A + B)$
$= 4\sin C\sin A\sin B$
$=$ R.H.S.

Example. If $A + B + C = 180°$, prove that $\sin A + \sin B + \sin C = 4\cos\tfrac{1}{2}A\cos\tfrac{1}{2}B\cos\tfrac{1}{2}C$.

You will be dealing here with the half angles of A, B, and C and so you have the following relations:

since $A + B + C = 180°$, $A + B = 180° - C$, i.e. $\tfrac{1}{2}(A + B) = 90° - \tfrac{1}{2}C$.

Thus $\sin\tfrac{1}{2}(A + B) = \sin(90 - \tfrac{1}{2}C) = \cos\tfrac{1}{2}C$
and $\cos\tfrac{1}{2}(A + B) = \cos(90 - \tfrac{1}{2}C) = \sin\tfrac{1}{2}C$.

Returning now to the identity to be proved, you obtain

L.H.S. $= \sin A + \sin B + \sin C$
$= 2\sin\tfrac{1}{2}(A + B)\cos\tfrac{1}{2}(A - B) + 2\sin\tfrac{1}{2}C\cos\tfrac{1}{2}C$

by transforming sin A + sin B into a product and expanding sin C exactly as in the last example

$= 2\cos\tfrac{1}{2}C\cos\tfrac{1}{2}(A - B) + 2\cos\tfrac{1}{2}(A + B)\cos\tfrac{1}{2}C$

in virtue of the relations above

$= 2\cos\tfrac{1}{2}C(\cos\tfrac{1}{2}(A - B) + \cos\tfrac{1}{2}(A + B))$
$= 2\cos\tfrac{1}{2}C\ .\ 2\cos\tfrac{1}{2}(A - B + A + B)\cos\tfrac{1}{2}(A + B - A + B)$
$= 4\cos\tfrac{1}{2}C\ .\ \cos\tfrac{1}{2}A\cos\tfrac{1}{2}B$
$=$ R.H.S.

EXERCISES

(1) Express as products $\sin 3A - \sin A$, $\cos 3A + \cos A$, $\sin 2A + \sin 2B$, $\cos 15° - \cos 59°$, $\sin 72° - \cos 30°$.

(2) Express as the sum or difference of two trigonometrical ratios $2\sin 5A\cos A$, $2\sin A\sin 3A$, $\sin 2A\cos 2B$, $2\sin(A - B)\cos(A + B)$, $\cos 70°\cos 30°$.

(3) Prove the following identities:
(i) $\sin A\tan A\cos A = 1 - \cos^2 A$
(ii) $\cos^2 A(\tan^2 A - \sin^2 A) = \sin^4 A$
(iii) $\sin^2 A + 4 = 5 - \cos^2 A$
(iv) $(1 - \sin A)(1 + \sin A) = \cos^2 A$
(v) $1 + \cot A = \cot A(1 + \tan A)$

(4) If $t = \tan\tfrac{1}{2}A$, using the expressions obtained in an example for sin A and cos A in terms of t, show that

(i) $\dfrac{\sin A + \cos A + 1}{\sin A - \cos A + 1} = \dfrac{1}{t}$
(ii) $\sec A + \tan A = (1 + t)/(1 - t)$

(5) Prove that $\tan A = \dfrac{\sin A + \sin 2A}{1 + \cos A + \cos 2A}$

(6) Prove the following identities:
(i) $\dfrac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$
(ii) $\cos A + \cos B + \cos C + \cos(A + B + C) = 4\cos\tfrac{1}{2}(B + C)\cos\tfrac{1}{2}(C + A)\cos\tfrac{1}{2}(A + B)$.

(7) By writing sin 5A as sin (2A + 3A) and expanding, find sin 5A in terms of powers of sin A only.

(8) From the identity $2\cos^2 A = \cos 2A + 1$, multiplying both sides by 2 cos A, show that

$$4\cos^3 A = \cos 3A + 3\cos A.$$

By repeating the process, find $8\cos^4 A$ and $16\cos^5 A$ in terms of the cosines of multiples of A only.

(9) If A + B + C = 180°, show that

(i) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A\cos B\cos C$

(ii) $\sin^2 A + \sin^2 B + \sin^2 C = 2(1 + \cos A\cos B\cos C)$

(iii) $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A\cos B\cos C$

(iv) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{1}{2}A\sin\frac{1}{2}B\sin\frac{1}{2}C$

ANSWERS TO EXERCISE IN LESSON 12

(1) $1/\sqrt{2}$, -1, $\sqrt{3}/2$

(2) − sin A, cot A, cos A, cosec A

(5) (cos A + sin A) (cos B − sin B)

(6) $$\begin{aligned}\cos(A+B+C) &= \cos A\cos B\cos C\\ &\quad - \cos A\sin B\sin C\\ &\quad - \cos B\sin C\sin A\\ &\quad - \cos C\sin A\sin B\\ \cos(B+C-A) &= \cos A\cos B\cos C\\ &\quad - \cos A\sin B\sin C\\ &\quad + \cos B\sin C\sin A\\ &\quad + \cos C\sin A\sin B\end{aligned}$$

(7) $\tan(A+B+C) =$

$$\frac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A}$$

LESSON 14

Indices and Logarithms

We have already considered the fundamental law of algebra called the Index Law. In Lesson 10 it was shown that if m and n were two positive integers, then

$$a^m \times a^n = a^{m+n}$$

and this can be immediately extended to any number of terms: for

$$a^m \times a^n \times a^p \times \ldots = a^{m+n} \times a^p \times \ldots = a^{m+n+p+\cdots}$$

The property of algebra which distinguishes it from arithmetic is that in algebra numbers are represented by letters. This means that if a result is obtained in terms of letters, it will be equally true if the letters are replaced by any numbers, providing that during the course of the algebraic work no restrictions were placed on the numbers which the letters could represent.

Thus the above laws concerning indices are very much hampered in their application by the fact that m and n were of necessity positive integers, since a^m is only defined if m is a positive integer. The question thus arises of whether it is possible to give a meaning to a^m when m is any fraction in such a way as to preserve the Index Law (you would, of course, also require that $(a^m)^n = a^{mn}$). It should be noted that the use of algebra has suggested the need of important new definitions. Obviously no meaning can be given to the notion of multiplying a by itself m times unless m is a positive integer, so we must define a^m directly from the Index Law; we may remark that the Index Law gives the unique and correct meaning to a^m, where m is a positive integer, if we start from $a^1 = a$. We will first take a particular example and attempt to find a meaning for, say, $2^{\frac{1}{3}}$.

If the index law is to hold,

$$2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} = 2^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 2^1$$
$$\therefore (2^{\frac{1}{3}})^3 = 2$$

and taking cube roots of both sides you have that $2^{\frac{1}{3}}$ means $\sqrt[3]{2}$. This can be very easily generalised; if n is a positive integer, then $\frac{1}{n}$ is a positive fraction and

$$\begin{aligned}a^{\frac{1}{n}} \times a^{\frac{1}{n}} \times a^{\frac{1}{n}} \times \ldots &\text{ to } n \text{ factors}\\ &= a^{\frac{1}{n}+\frac{1}{n}+\frac{1}{n}} \ldots \text{ to } n \text{ terms}\\ &= a^{n(\frac{1}{n})}\\ &= a\end{aligned}$$

i.e. $$(a^{\frac{1}{n}})^n = a$$

and taking the n^{th} root of both sides $a^{\frac{1}{n}}$ means $\sqrt[n]{a}$.

At this stage it is convenient to derive a meaning for a^0, still subject to the condition that the index law shall hold; thus $a^3 \times a^0$ is to equal a^{3+0}, i.e. a^3, so that a^0 must equal $\frac{a^3}{a^3}$ i.e. 1, which says that any number raised to the power zero is to be given the meaning 1. We can use this to derive a meaning for a negative index, for example:

$$2^3 \times 2^{-3} = 2^{3-3} = 2^0 = 1$$

Therefore, dividing both sides by 2^3, 2^{-3} must mean $\frac{1}{2^3}$ and this can be generalised so that a^{-m} means $\frac{1}{a^m}$.

Finally, consider $4^{3/2}$

$$\begin{aligned}4^{3/2} &= 4^{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}}\\ &= 4^{\frac{1}{2}} \times 4^{\frac{1}{2}} \times 4^{\frac{1}{2}} \text{ (if the Index Law holds)}\\ &= (4^{\frac{1}{2}})^3\end{aligned}$$

and therefore

$$4^{3/2} \text{ means } (\sqrt{4})^3 \text{ i.e. } 2^3, \text{ or } 8,$$

but alternatively since

$$4^{3/2} \times 4^{3/2} = 4^{3/2+3/2} = 4^3$$

then $$(4^{3/2})^2 = 4^3$$

so that $4^{3/2}$ can be considered as the square root of 4^3, i.e.

$$4^{3/2} \text{ means } \sqrt{4^3} \text{ i.e. } \sqrt{64} \text{ or } 8$$

Thus

$$4^{3/2} = (4^3)^{\frac{1}{2}} = (4^{\frac{1}{2}})^3$$

The following examples illustrate these extensions of the Index Law.

Example. Simplify $3 . 2^{-2} + 8 . 4^{-1} + 16^{-\frac{1}{2}}$

$$\text{This expression} = 3 \cdot \frac{1}{2^2} + 8 \cdot \frac{1}{4} + \frac{1}{16^{\frac{1}{2}}}$$
$$= \frac{3}{4} + \frac{8}{4} + \frac{1}{\sqrt{16}}$$
$$= \tfrac{3}{4} + 2 + \tfrac{1}{4}$$
$$= 3$$

Example. Multiply $2^{\frac{1}{2}} . 3^{\frac{1}{3}}$ by 18

$$18 \times 2^{\frac{1}{2}} 3^{\frac{1}{3}} = 2 \times 3^2 \times 2^{\frac{1}{2}} \times 3^{\frac{1}{3}}$$
$$= 2^{1+\frac{1}{2}} \times 3^{2+\frac{1}{3}}$$
$$= 2^{\frac{3}{2}} 3^{\frac{7}{3}}$$

Example. Simplify $(x^{-4} y^{\frac{2}{5}})^{-5}$

$$(x^{-4} y^{\frac{2}{5}})^{-5} = x^{(-4)(-5)} y^{\frac{2}{5}(-5)}$$
$$= x^{20} y^{-2}$$
$$= 1$$

Example. Simplify $\dfrac{ab(b^{-1} - a^{-1})}{a - b} + \dfrac{a - b}{b^{-1} - a^{-1}}$

$$\text{This expression} = \frac{ab}{a - b}\left(\frac{1}{b} - \frac{1}{a}\right) + \frac{a - b}{\frac{1}{b} - \frac{1}{a}}$$
$$= \frac{ab}{a - b}\left(\frac{a - b}{ab}\right) + \frac{(a - b)ab}{a - b}$$
$$= 1 + ab$$

The student has now seen that a^m is capable of an intelligible meaning when m is a fraction. The cases considered so far have been very simple, but they show the proper meaning in the general case of fractional values of m. It is often convenient to consider these fractional values in their decimal forms. Thus, $5^{\frac{273}{1000}}$ means the thousandth root of 5^{273}, or alternatively $(\sqrt[1000]{5})^{273}$ and may be written $5^{0.273}$.

If a is a fixed positive number, not every number can be expressed as a^m for some fraction m. However, we may get as close as we like to any number by choosing a suitable value for m. It is this fact which leads to a very important application of the Index Law, namely logarithms.

By means of the theory which we shall establish, the amount of labour involved in the working out of numerical results can be very much reduced—multiplication being replaced by addition, division by subtraction, and the extraction of roots by division by integers. It must be stressed, however, that logarithms are used only for approximate, not for exact, calculations.

Definition of Logarithms

Suppose that a^m, when evaluated for a particular value of a and m, has a numerical value represented by N ; then m is said to be the *logarithm* of N to the base a (written $\log_a N$). This simply means that if m is the power to which a (termed the *base*) is raised in order that $a^m = N$, then m is the logarithm of N i.e. if $a^m = N$, then $m = \log_a N$.

For example, since $2^3 = 8$, then $3 = \log_2 8$.
Again, since $16^{0.25} = 2$, then $0{\cdot}25 = \log_{16} 2$, and since $10^{-2} = 0{\cdot}01$, then $-2 = \log_{10} 0{\cdot}01$.

The definition of a logarithm is conveniently expressed by the formula :

$$\text{number} = (\text{base})^{\text{logarithm}}$$

Example. Find $\log_9 (27\sqrt{3})$

Let $x = \log_9 (27\sqrt{3})$

Then from the definition of a logarithm,

$$9^x = 27\sqrt{3}$$
i.e. $(3^2)^x = 27\sqrt{3} = 3^3 \cdot 3^{\frac{1}{2}} = 3^{\frac{7}{2}}$

i.e. $3^{2x} = 3^{\frac{7}{2}}$

$\therefore\ 2x = \frac{7}{2}$, equating the indices of 3.

Thus $x = \frac{7}{4}$ so that $\log_9 (27\sqrt{3}) = 1{\cdot}75$

The student can now consider the properties of logarithms which make them especially useful in computational work.

Properties of Logarithms

I. The logarithm of 1 to any base is 0.

Since, from the theory of indices, a^0 had to be given the meaning 1, whatever the value of a, i.e. $a^0 = 1$, it follows from the definition of a logarithm that 0 is $\log_a 1$, i.e. the logarithm of 1 to any base is 0.

II. The logarithm of the base itself is 1.

$$\text{Since } a^1 = a$$
$$\text{Then } \log_a a = 1$$

III. The logarithm of a product to any base is the *sum* of the logarithms of the factors to the same base. Let M × N represent a product whose logarithm to any base (a) is required ; and suppose that M is equal to "a" raised to a power m, and N is "a" raised to a power n, then

$$M = a^m \text{ and } N = a^n$$
$$\therefore m = \log_a M \text{ and } n = \log_a N$$

Also $M \times N = a^m \times a^n = a^{m+n}$

i.e. M × N is equal to "a" raised to the power $(m + n)$

This means that $(m + n)$ is the logarithm of (M × N) to base a, i.e.

$$\log_a(M \times N) = m + n$$
$$= \log_a M + \log_a N$$

This is extendable to any number of factors, for

$$\log_a (M \times N \times P) = \log_a(MN \times P)$$
$$= \log_a(MN) + \log_a P$$
$$= \log_a M + \log_a N + \log_a P,$$
and so on

Thus, for example,

$$\log_{10} 42 = \log_{10} (2 \times 3 \times 7)$$
$$= \log_{10} 2 + \log_{10} 3 + \log_{10} 7$$

IV. The logarithm of a quotient is the logarithm of the denominator subtracted from the logarithm of the numerator, where the logarithms are taken to any base, providing it is the same throughout, i.e.

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

The proof is similar to that of property III. If $M = a^m$ and $N = a^n$, then $m = \log_a M$ and $n = \log_a N$.

Also $\dfrac{M}{N} = \dfrac{a^m}{a^n} = a^{m-n}$

so that $\log_a\left(\dfrac{M}{N}\right) = m - n = \log_a M - \log_a N$

As an example,

$$\begin{aligned}\log_3 (2\tfrac{1}{7}) &= \log_3 (\tfrac{15}{7})\\ &= \log_3 15 - \log_3 7\\ &= \log_3 (3 \times 5) - \log_3 7\\ &= \log_3 3 + \log_3 5 - \log_3 7\\ &= 1 + \log_3 5 - \log_3 7.\end{aligned}$$

V. The logarithm of a number (M) raised to a power (p) is p multiplied by the logarithm of the number,

i.e. $\log_a(M^p) = p \log_a M$

To prove this, let $M = a^m$ and therefore as before, $m = \log_a M$. Then

$$M^p = (a^m)^p = a^{pm}$$

i.e. M^p is equal to the base raised to a power (pm) so that $pm = \log_a(M^p)$

$$\begin{aligned}\text{i.e. } \log_a(M^p) &= p \times m\\ &= p \times \log_a M\end{aligned}$$

Thus, for instance:

$$\begin{aligned}\log_{10}81 &= \log_{10}(3^4)\\ &= 4 \log_{10}3\end{aligned}$$

This could also have been seen from property III, because—

$$\begin{aligned}\log_{10}(3^4) &= \log_{10} (3 \times 3 \times 3 \times 3)\\ &= \log_{10}3 + \log_{10}3 + \log_{10}3 + \log_{10}3\\ &= 4 \log_{10}3\end{aligned}$$

But this argument works here only because in this case p is a positive integer.

Example. Express $\log\left(\dfrac{r^5 s^{-\frac{1}{2}}}{\sqrt[3]{r}}\right)$ in terms of $\log r$ and $\log s$ (the logarithms to be taken to any base).

From the extended laws of indices

$$\begin{aligned}\frac{r^5 s^{-\frac{1}{2}}}{\sqrt[3]{r}} &= \frac{r^5 s^{-\frac{1}{2}}}{r^{\frac{1}{3}}}\\ &= r^{5-\frac{1}{3}}\, s^{-\frac{1}{2}}\\ &= r^{\frac{14}{3}}\, s^{-\frac{1}{2}}\\ \therefore\quad \log_a \frac{r^5\, s^{-\frac{1}{2}}}{\sqrt[3]{r}} &= \log_a (r^{\frac{14}{3}} s^{-\frac{1}{2}})\\ &= \log_a (r^{\frac{14}{3}}) + \log_a(s^{-\frac{1}{2}})\\ &= \tfrac{14}{3}\log_a r - \tfrac{1}{2} \log_a s\end{aligned}$$

where you have taken logarithms to any base (a).

Example. Show that $\log_a b \times \log_b a = 1$.

Let $\log_a b = x$, then from the definition of a logarithm, $a^x = b$. Taking the xth root of both sides

$$a = b^{\frac{1}{x}}$$

i.e. $\dfrac{1}{x} = \log_b a$

$\therefore \log_a b \times \log_b a = x \cdot \dfrac{1}{x} = 1$

Example. Given $\log_{10}2 = 0{\cdot}30103$ (this is approximate), to find $\log_4 25$.

Let $\log_4 25 = x$

Then $4^x = 25$

or multiplying both sides by 4

$$\begin{aligned}4.4^x &= 100\\ \text{i.e.}\quad 4^{x+1} &= 100\\ \text{or}\quad (2^2)^{x+1} &= 10^2\\ \text{i.e.}\quad 2^{2x+2} &= 10^2\end{aligned}$$

Taking logarithms of both sides to base 10,

$$\begin{aligned}\log_{10}(2^{2x+2}) &= \log_{10}(10^2)\\ \text{or}\quad (2x + 2) \log_{10}2 &= 2 \log_{10}10\\ &= 2\\ \text{so that}\quad 2x + 2 &= \frac{2}{\log_{10}2}\\ &= \frac{2}{0{\cdot}30103}\\ \text{i.e.}\quad x + 1 &= \frac{1}{0{\cdot}30103} \quad \text{on dividing both sides by 2}\\ &= 3{\cdot}32193\\ \text{i.e.}\quad x &= 2{\cdot}32193\end{aligned}$$

EXERCISES

(1) Multiply

(i) $x^2 + x^{-2}$ by $x^2 - x^{-2}$

(ii) $x^{\frac{1}{2}} + x^{-\frac{1}{2}}$ by $x^{\frac{1}{2}} + x^{\frac{1}{4}}$

(iii) $a^{\frac{2}{3}} + 2a^{\frac{1}{3}}b^{\frac{1}{3}} + 4b^{\frac{2}{3}}$ by $a^{\frac{1}{3}} - 2b^{\frac{1}{3}}$

(2) Evaluate

(i) $(x^{-\frac{1}{2}})^{-2} \times (x^{-\frac{1}{3}})^3$ if $x = 1{\cdot}5$

(ii) $(2y)^{-4} \times x^{-2} \times (xy)^5$ if $x = 2$, $y = 3$

(iii) $2^{x^2} - (2^x)^2$ if $x = 3$

(3) Show that, if the index law is to hold for fractional indices, then $a^{h/k}$ means either $\sqrt[k]{a^h}$ or $(\sqrt[k]{a})^h$ where h, k are positive integers. Evaluate $9^{\frac{5}{2}}$ in both forms and show that the result is 243.

(4) Find the least integral value of n such that $(\frac{1}{2})^n < 0{\cdot}01$.

(5) Given $x^{\frac{1}{2}} = 3{\cdot}1623$, $x^{\frac{1}{4}} = 1{\cdot}7784$, $x^{\frac{1}{8}} = 1{\cdot}3335$, $x^{\frac{1}{16}} = 1{\cdot}1549$, $x^{\frac{1}{32}} = 1{\cdot}076$, calculate to four figures $x^{\frac{3}{4}}$, $x^{\frac{7}{16}}$, $x^{0{\cdot}625}$, $x^{-0{\cdot}25}$.

(6) Find the values of $\log_8 128$, $\log_6 \dfrac{1}{216}$, $\log_a(a^6)$.

(7) Show that

(i) $\log_{10}(13\frac{1}{3}) = 1 + \log_{10}4 - \log_{10}3$

(ii) $\log_3(9 \times 27) = 5$

(iii) $\log_2(8\sqrt{8}) = 4{\cdot}5$

(iv) $\log_{10}(100\sqrt{10}) = 2{\cdot}5$

(8) Find a, b, c given that $\log_8 a = 2$, $\log_{10}b = -3$, $\log_4 c = \frac{1}{2}$

(9) Given $\log_{10}2 = 0{\cdot}30103$, find $\log_{10}64$

(10) Show that, to any base,

(i) $\log[(27^{\frac{1}{2}})(14^{-4}) . \sqrt[3]{56}] =$
$\dfrac{3}{2}\log 3 - \dfrac{11}{3}\log 7 - 3 \log 2$

(ii) $\log [(2x)^3\, y^{\frac{3}{2}}\, \sqrt{8xy}] =$
$\dfrac{9}{2}\log 2 + \dfrac{7}{2}\log x + 2 \log y$

Answers to Exercises in Lesson 13

(1) 2 cos 2A sin A, 2 cos 2A cos A, 2 sin (A + B) cos (A − B), 2 sin 37° sin 22°, 2 cos 66° sin 6°

(2) sin 6A + sin 4A, cos 2A — cos 4A, $\frac{1}{2}$ sin (2A + 2B) + $\frac{1}{2}$ sin (2A − 2B), sin 2A − sin 2B, $\frac{1}{2}$ cos 100° + $\frac{1}{2}$ cos 40°

(7) 16 $\sin^5$A − 20 $\sin^3$A + 5 sin A

(8) 8 $\cos^4$A = cos 4A + 4 cos 2A + 3,
16 $\cos^5$A = cos 5A + 5 cos 3A + 10 cos A

LESSON 15

Application of Logarithms: The Slide Rule

The properties arising out of the definition of a logarithm are quite independent of the number chosen as base. However, in the applications of logarithms which result from these properties the base plays an important part. The most useful base is 10, which is also the base of our numerical notation (*see* Lesson 1). But in purely theoretical work, involving the calculus, logarithms turn up naturally to a different base. This base is a number, usually written e, which is approximately 2·71828. Logarithms calculated to this base are called *natural* or *Naperian logarithms*. For the present we shall confine our attention to common logarithms, i.e. to base 10.

Logarithms to Base 10

Consider 10^3 as an example of 10 raised to a simple power. This equals 1,000 and therefore, from the definition of a logarithm, $\log_{10}1000 = 3$. In a similar manner a table of a few powers of 10 and their corresponding logarithms can be drawn up as follows:

$$10^0 = 1 \quad, \therefore \log_{10}1 = 0\,;$$
$$10^1 = 10 \quad, \therefore \log_{10}10 = 1\,;$$
$$10^2 = 100 \quad, \therefore \log_{10}100 = 2\,;$$
$$10^3 = 1000, \therefore \log_{10}1000 = 3.$$

In this table we have considered only integral powers of 10, and thus the logarithms obtained were whole numbers. Now consider $10^{1\cdot5}$. From the theory of indices

$$10^{1\cdot5} = 10^{1+\frac{1}{2}} = 10.10^{\frac{1}{2}} = 10\sqrt{10}$$
$$= 10 \times 3\cdot1623 \,.\,. = 31\cdot623 \,.\,.$$

Thus

$$\log_{10}31\cdot623 \,.\,. = 1\cdot5,$$

and we have as a result found in this case that the logarithm of a number between 10 and 100 has a value between 1 and 2. Had we considered $10^{2\cdot5}$, we should have found that $10^{2\cdot5} = 100\sqrt{10} = 316\cdot23 \ldots$ so that here we have a number between 100 and 1,000 whose logarithm is between 2 and 3. It is obvious that we shall obtain a general rule that numbers between 1 and 10 have logarithms between 0 and 1, numbers between 10 and 100 have logarithms between 1 and 2, between 100 and 1,000 logarithms between 2 and 3, and so on. This property holds only for logarithms to base 10.

Thus any logarithm is conveniently split up into two parts; the whole number, which is called the *characteristic*; and the decimal part, which is called the *mantissa*. Thus for every number between 10 and 100 the characteristic is 1, but the mantissa will vary, and it is these mantissae which have been tabulated and form the logarithm tables so useful in computational work.

Use of Logarithm Tables

Turn to such a set of tables and consider how you can use them to find the logarithm of any number. In the left-hand column the numbers range from 10 up to 99. Along the top of the page are the numbers 0, 1, 2, . . . 9, each one at the top of a column of four figures, and then farther across the page is a second set of numbers 1 to 9 (sometimes headed "Fourth Figure" or "Mean Differences"). Remember that numbers in this table give mantissae only.

Consider some actual examples. To find the logarithms of 42, 42·6, 42·67 from the tables. In each of these cases, since the numbers are between 10 and 100, the characteristic is 1. From the tables look down the extreme left-hand column until the number 42 is reached (*see* page 1231). In this row, but in the next column (i.e. that headed 0), is found the mantissa 6232. Thus

$$\log_{10}42\cdot0 = +\,1 + 0\cdot6232$$
$$= 1\cdot6232$$

The row across the top of the page gives the third figure, so that continuing along the row opposite 42 until you come to the column headed 6 you find the mantissa 6294. This, then, gives the mantissa for 42·6, i.e. $\log_{10}42\cdot6 = 1\cdot6294$. If the logarithm of a number containing four figures is required, the additional part of the mantissa for the "fourth figure" is found from the section 1 to 9 in the end columns of the page (i.e. those which may be headed "Mean Differences"). Thus to find $\log_{10}42\cdot67$, first find $\log_{10}42\cdot6$ as above (i.e. 1·6294) and then add on to the last place of decimals the amount in the same row but under the column 7, i.e. 7. Thus

$$\log_{10}42\cdot67 = 1\cdot6294 + 0\cdot0007$$
$$= 1\cdot6301$$

Suppose you require the logarithm of 426. This number equals $10 \times 42\cdot6$, so that

$$\log_{10}426 = \log_{10}10 + \log_{10}42\cdot6$$
$$= 1 + 1\cdot6294 \text{ (from the above)}$$
$$= 2\cdot6294$$

Notice that the only difference between $\log_{10}42\cdot6$ and $\log_{10}426$ is in the characteristic; the *mantissa is the same*. Again,

$$\text{since } 4260 = 1000 \times 4\cdot26$$
$$\log_{10}4260 = \log_{10}1000 + \log_{10}4\cdot26$$
$$= +\,3 + 0\cdot6294$$
$$= 3\cdot6294$$

As an example of the further use of this method, consider the following table and the logarithms which it provides:

$$\begin{aligned}
4260 &= 1000 \times 4{\cdot}26\\
426 &= 100 \times 4{\cdot}26\\
42{\cdot}6 &= 10 \times 4{\cdot}26\\
4{\cdot}26 &= 1 \times 4{\cdot}26\\
0{\cdot}426 &= 10^{-1} \times 4{\cdot}26\\
0{\cdot}0426 &= 10^{-2} \times 4{\cdot}26
\end{aligned}$$

Therefore:

$$\begin{aligned}
\log_{10} 4260 &= +3 + 0{\cdot}6294\\
\log_{10} 426 &= +2 + 0{\cdot}6294\\
\log_{10} 42{\cdot}6 &= +1 + 0{\cdot}6294\\
\log_{10} 4{\cdot}26 &= +0 + 0{\cdot}6294\\
\log_{10} 0{\cdot}426 &= -1 + 0{\cdot}6294\\
\log_{10} 0{\cdot}0426 &= -2 + 0{\cdot}6294
\end{aligned}$$

and so on. Thus we have passed from numbers whose logarithms involve positive characteristics to those whose logarithms have negative characteristics. Note that logarithms of all numbers less than 1 have negative characteristics. Consider the last one. $\text{Log}_{10}0{\cdot}0426 = -2 + 0{\cdot}6294$, i.e. the mantissa is still positive, but the characteristic is -2. The fact that only this integer is negative is indicated by writing $\log_{10}0{\cdot}0426 = \bar{2}{\cdot}6294$. The minus sign is placed over the 2 and is read as " bar two." Remember that this is *not* the same as $-2{\cdot}6294$.

How to Construct a Logarithmic Table

Before we show the value of a logarithmic table for calculations it is desirable to give some idea of how such a table can be constructed. It is now clear that we require only the logarithms of numbers lying between 1 and 10, for the logarithms of all other numbers can be derived from these. You know that $10^0 = 1$, $10^{0\cdot5} = \sqrt{10} = 3{\cdot}1623\ .\ .\ .$ and $10^1 = 10$. Hence $10^{\cdot25} = \sqrt{3{\cdot}1623} = 1{\cdot}778\ldots$, and $10^{\cdot125} = \sqrt{1{\cdot}778\ldots} = 1{\cdot}33\ldots$

Again

$$\begin{aligned}
10^{\cdot375} &= 10^{\cdot25} \times 10^{\cdot125} = 1{\cdot}778\ldots \times 1{\cdot}33 = 2{\cdot}371\ldots\\
10^{\cdot625} &= 10^{\cdot5} \times 10^{\cdot125} = 3{\cdot}1623 \times 1{\cdot}33\ldots = 4{\cdot}216\\
10^{\cdot75} &= 10^{\cdot5} \times 10^{\cdot25} = 3{\cdot}1623 \times 1{\cdot}778\ldots = 5{\cdot}623\\
10^{\cdot875} &= 10^{\cdot75} \times 10^{\cdot125} = 5{\cdot}623 \times 1{\cdot}33\ldots = 7{\cdot}497
\end{aligned}$$

Already, therefore, you have the following items for a logarithm table:

Logarithm of	1	= 0
" "	1·33 . .	= 0·125
" "	1·778 . .	= 0·250
" "	2·371 . .	= 0·375
" "	3·162 . .	= 0·500
" "	4·216 . .	= 0·625
" "	5·623 . .	= 0·750
" "	7·497 . .	= 0·875
" "	10	= 1·000

A simple way of extending this table is to draw a graph of the above logarithms against the numbers of which they are the logarithms, and read off intermediate values from this graph. The ordinary logarithm table is calculated in a much more elaborate way and with much greater accuracy than we have taken. For our purpose, however, it suffices to show that such a table can be constructed.

Consider the way in which the logarithm tables can be used in working out an actual computation.

Example. In this case we shall use only the table we have constructed above. To evaluate $x = 1{\cdot}778 \times 4{\cdot}216$.

Now the logarithm of a product is the sum of the logarithms of the factors. Thus

$$\begin{aligned}
\log x &= \log 1{\cdot}778 + \log 4{\cdot}216\\
&\quad 0{\cdot}250 + \quad 0{\cdot}625 \text{ from th[illegible] table}\\
&= 0{\cdot}875
\end{aligned}$$

Again from this table log 7·497 = 0[illegible]

Hence 1·778 × 4·216 [illegible] . .

as accurately as our tables will allow.

For subsequent examples reference should be made to a 4-figure logarithmic table

Example. Evaluate $\dfrac{138{\cdot}7 \times 0{\cdot}037}{36{\cdot}9}$

From the properties of logarithms already established

$$\begin{aligned}
\log_{10}\frac{138{\cdot}7 \times 0{\cdot}037}{36{\cdot}9} &= \log_{10}138{\cdot}7 + \log_{10}0{\cdot}037\\
&\quad - \log_{10}36{\cdot}9\\
&= 2{\cdot}1422 + 2{\cdot}5682 - 1{\cdot}5670
\end{aligned}$$

from tables; since 138·7 lies between 100 and 1,000, its logarithm has the characteristic 2, and since 0·037 lies between 0·01 and 0·12 its characteristic is 2 It will be found that the characteristic of the logarithm of a number less than one is always numerically one greater than the number of zeros after the decimal point. Proceeding with our evaluation:

$$\begin{aligned}
\log_{10}\frac{138{\cdot}7 \times 0{\cdot}037}{36{\cdot}9} &= +2 + 0{\cdot}1422 + (-2 + 0{\cdot}5682)\\
&\quad - 1{\cdot}567\\
&= 0{\cdot}7104 - 1{\cdot}567
\end{aligned}$$

We must always arrange that the mantissa is positive, so we write $0{\cdot}7104 - 1{\cdot}567$ as $-1 + 1{\cdot}7104 - 1{\cdot}567$. Thus we obtain $-1 + 0{\cdot}1434$, i.e. $\bar{1}{\cdot}1434$. This is the logarithm of our original expression, so that we have only to find from our tables the number which has this logarithm to know the evaluation of our expression. The $\bar{1}$ simply tells that the logarithm is that of a number between 0·1 and 1. To find its actual value we look up the mantissa 1434 in the body of the logarithm table. As accurately as we are able, from our tables, we find that 1433 is the mantissa of 1391, so that our evaluation is 0·1391; this, then, is the value of our original expression.

It should be noticed that the *antilogarithms* have also been tabulated; we say that a is the antilogarithm of b if b is the logarithm of a. In these tables we look up ·1433 in exactly the same way as for logarithms. Wherever possible a rough check should be made to ensure that the decimal point has been correctly placed; for instance, in our example,

$\dfrac{138{\cdot}7 \times 0{\cdot}037}{36{\cdot}9}$ is approximately $\dfrac{138{\cdot}7}{1000}$ i.e. 0·1387

Example. Find the value of $(0{\cdot}00596)^{1/5}$

$$\begin{aligned}
\log_{10}(0{\cdot}00596)^{1/5} &= \frac{1}{5}\log_{10}(0{\cdot}00596)\\
&= \frac{1}{5}\,(\bar{3}{\cdot}7752)
\end{aligned}$$

which means $\frac{1}{5}(-3 + 0{\cdot}7752)$. Since you must have

Number	0	1	2	3	4	5	6	7	8	9	Fourth figure 1	2	3	4	5	6	7	8	9
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	7	11	13	16	18	21	24
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

Portions of a Table of Logarithms including calculations on page 1229.

the mantissa positive, you write this as $\frac{1}{5}(-5+2{\cdot}7752)$, and then on dividing you obtain $(-1 + 0{\cdot}5550)$, i.e. $\bar{1}{\cdot}555$.

Thus $\log_{10}(0{\cdot}00596)^{\frac{1}{5}} = \bar{1}{\cdot}555$

so that $(0{\cdot}00596)^{\frac{1}{5}} = \text{antilog } \bar{1}{\cdot}555 = 0{\cdot}3589$

Notice especially that in the antilogarithm tables, only the 0·555 is looked up, the $\bar{1}$ tells that the number lies between 0·1 and 1.

Theory of the Slide Rule

The slide rule is an instrument which consists of two scales, one of which slides within the other, and its principal purpose is the rapid numerical evaluation of expressions involving multiplications and divisions. A modern slide rule at first sight appears to be an intricate system of graduated scales and numbers which is apt to be confusing, but the theory of the slide rule is very simple—it is only a mechanical device for adding and subtracting the logarithms of numbers.

Suppose you take the numbers from 1 to 10 and find their logarithms to base 10 from a set of tables. You obtain, to three decimal places:

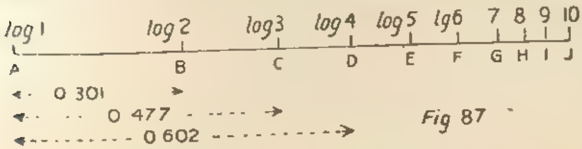
Number	1	2	3	4	5
Logarithm	0	·301	·477	·602	·699
Number	6	7	8	9	10
Logarithm	·778	·845	·903	·954	1

It is very important to notice that although the numbers increase by 1, the logarithms do not increase by constant amounts, but by amounts which are continuously *decreasing*. Thus

$$\log 2 - \log 1 = 0{\cdot}301,$$
$$\log 3 - \log 2 = 0{\cdot}176,$$
$$\log 4 - \log 3 = 0{\cdot}125,$$
$$\log 5 - \log 4 = 0{\cdot}097, \text{ and so on.}$$

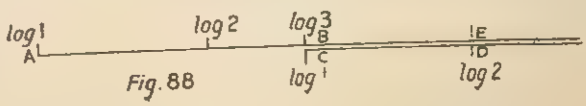
Since we are now dealing with common logarithms there is no ambiguity if we drop the suffix 10 to indicate the base. Thus $\log_{10}N$ will be written simply as log N.

From the foregoing set of figures it is an easy matter to construct a scale showing the relative positions of log 1, log 2, log 3, . . . to each other (Fig. 87). AJ is taken to represent one unit of length and B, C, D, . . . are placed so that AB = 0·301, AC = 0·477, AD = 0·602, . . .

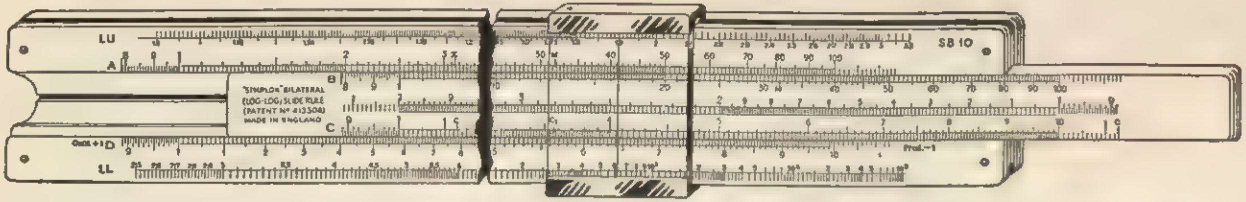


If you have two identical scales of this type which will slide one against the other, you have, in effect, a slide rule.

Consider how a pair of such scales can be used (Fig. 88) to multiply two integers together. In this



SLIDE RULE. Fig. 87. Scale of relative positions of logs 1, 2 onwards. Fig. 88. Pair of adjustable scales to multiply integers by adding logs.



This instrument enables arithmetical computations to be performed quickly by mechanical means. There are four principal scales : A and D on the rule itself ; B and C on the slide. The illustration of a typical slide rule (above) also shows Log-log scales (LU and LL) on top and lower edges respectively. Scales A, B, C, D are, in effect, tables of logarithms plotted out to scale on the rule. A and B are alike.

MULTIPLICATION. This and division may be carried out on either upper scales (A and B) or lower scales (C and D). The lower scales permit more accurate working, but the upper ones are generally used where possible. An example is given in Fig. 1. Two numbers are multiplied by adding together the distances corresponding to those numbers on rule and slide. Thus $2{\cdot}5 \times 3 = 7{\cdot}5$: Set 1 on B under 2·5 on A ; move cursor line over 3 on B ; read result (7·5) under cursor line on A. The position of slide, etc., is shown in Fig. 1.

DIVISION. Performed by subtracting the distance representing the divisor on one of the scales from the distance defining the dividend on the other adjacent scale. Fig. 2 illustrates an example : $7{\cdot}65 : 4{\cdot}5 = 1{\cdot}7$. Set cursor line over 7·65 on D ; slide 4·5 on C under the cursor line ; read the result 1·7 on D under 1 on C.

SQUARES AND SQUARE ROOTS. Readings on A are squares of exactly opposite readings on D, so that squares and roots may be read off directly.

CUBES AND CUBE ROOTS. A number can be raised to the third power by continued multiplication. See Fig. 3 : $1{\cdot}5^3 = 3{\cdot}375$. Set 1 on C to 1·5 on D ; move cursor line over 1·5 on B ; read result 3·375 under cursor line on A. In finding the cube root of 27 you would move the cursor line over 27 on A ; draw out slide (to the right in this instance) until the same number (3) comes under the cursor line on B which registers simultaneously on D under 1 on C.

Illustrations by courtesy of Dargue Bros. Ltd., Halifax

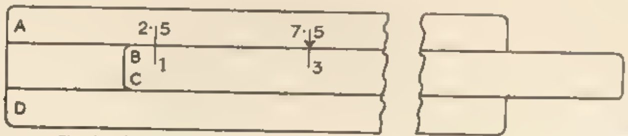


Fig. 1. Multiplication

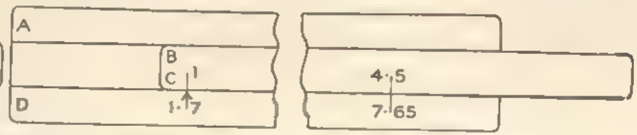


Fig. 2. Division

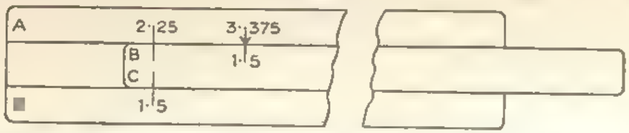


Fig. 3. Cubing

PRINCIPLE OF THE SLIDE RULE. These illustrations, with the accompanying practical description of the use of the slide rule in rapid calculations, are complementary to the text in pages 1231 and 1233, where the theory of the rule is discussed. The operations of multiplication, division, and cubing are demonstrated. The scales are, in effect, tables of logarithms plotted to scale. See also Lessons 14 and 15 on Logarithms.

diagram is shown the end of one of the scales (C) placed opposite to the mark representing log 3 on the other scale (B). Suppose that E is the mark on this scale opposite to log 2 on the first scale. To the logarithm of which number will E correspond ? This is a simple matter using a property of logarithms, for

$$\begin{aligned} AE &= AB + BE \\ &= AB + CD \\ &= \log 3 + \log 2 \\ &= \log (3 \times 2) \\ &= \log 6 \end{aligned}$$

Thus the reading E represents the point log 6. Alternatively, if the log 6 is read off from the upper scale, then you have automatically multiplied 3 by 2. Equally well you could have divided 6 by 2 by subtracting log 2 from log 6. Thus if you had placed log 2 on the lower scale against log 6 on the upper scale, you would have been able to read off log 3 on the upper scale as the mark opposite to log 1 on the lower.

Now that you know that the scales are logarithmic, it is unnecessary to clutter them up with the words "log" in front of each number. If in addition you fill up the spaces between log 1, log 2, log 3 . . . with graduations representing log 1·1, log 1·2, log 1·3, . . . and even log 1·11, log 1·12, log 1·13 . . . you will have scales from which you can read off multiplications and divisions.

Numbers with Product Greater than 10

There is one difficulty which the student may have noticed. If we had, in our previous example, multiplied two numbers together whose product was greater than 10, we should not have been able to obtain the reading at E, as we should have run off the end of the scale. This difficulty is overcome by working from the right-hand end of the lower scale instead of the left-hand end. Suppose you wish to use the slide rule to multiply 6 × 8 (Fig. 89). (The method will be the same even if the numbers involve decimals.) The end B of the lower scale is placed opposite 6 on the upper scale. The reading E opposite 8 on the lower scale is taken ; it will be found to be 4·8. The proof of this is as follows :

$$\begin{aligned} AE &= AD + DE \\ &= AD + BC \\ &= \log 10 - \log 6 + \log 10 - \log 8 \\ &= 2 \log 10 - \log (6 \times 8) \end{aligned}$$

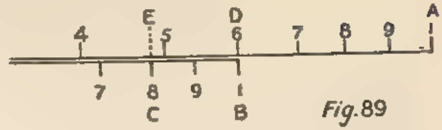


Fig. 89

SLIDE RULE. Reading logs for a multiplication.

But AE is log 10 — log (reading at E)

$$\begin{aligned} \therefore\ \log 10 - \log (\text{reading at E}) &= 2 \log 10 - \log (6 \times 8) \\ \therefore\ \log (\text{reading at E}) &= \log (6 \times 8) - \log 10 \\ &= \log (\tfrac{48}{10}) \\ &= \log 4\cdot8 \end{aligned}$$

so that the reading at E is 4·8.

It will be noticed that the onus is on the operator to fix the decimal point—the slide rule gives the numerical value.

As stated earlier, a modern slide rule gives a vast amount of information additional to the two scales above : further scales providing sines, tangents, squares, and so on. For the use of these and the *cursor* (a sliding piece which enables several multiplications or divisions to be done before the final answer is read off) the student is referred to the explanatory handbook sold with any slide rule.

EXERCISES

Use logarithms to solve the following problems :

(1) Find the product of 432·7 and $\frac{13\cdot9}{167\cdot5}$

(2) Evaluate $\frac{94 \times 174 \times 5}{176}$

(3) Divide 0·0009197 by 475·2.

(4) Find (i) the cube root of 0·0003,
(ii) $(0\cdot03)^{\frac{1}{4}}$,
(iii) $\sqrt[10]{20}$.

(5) Find x if $10^x = 3\cdot284$.

(6) If $e = 2\cdot718$, find $\log_e 12$.
Find x if $e^x = 11\cdot02$.

(7) Find $\log_5 71$.

(8) Construct a slide rule by drawing Fig. 87 on as large a scale as possible, extending the short vertical lines to cut AJ, inserting as many intermediate values as possible, duplicating the numbers above and below the line AJ and cutting along AJ with a knife. Use this slide rule to solve the first three questions of this exercise.

Answers to Exercises in Lesson 14

(1) (i) $x^4 - x^{-4}$; (ii) $x^3 + x^2 + x + 1$; (iii) $a^2 - 8b^4$.
(2) (i) 2·25 ; (ii) 3/2 ; (iii) 448.
(4) 7.
(5) 1·4330, 2·9389, 2·3715, 0·5623.
(6) $2\frac{1}{3}$, -3, 6.
(8) $a = 9$, $b = 0\cdot001$, $c = 2$.
(9) 1·80618.

LESSON 16

Analysing a Problem: Algebraic Identities

ALREADY a considerable amount of ground has been covered that has concerned itself with the technique of arithmetic, algebra, geometry, and trigonometry. These are tools, like hammers and chisels, that must serve in the handling of problems.

Factors in the Problem

The first step is to picture the problem—the shapes and sizes of the things involved. If they are simple geometrical shapes such as circles, lines, triangles, and squares, then so much the better. This attempt to picture the problem in this way is, in effect, our method of asking whether the problem can be adequately described in such simple terms.

For example, if you seek where to place four electric lamps in a room to give the most uniform illumination over a table in the centre of the room, consistent with giving sufficient light everywhere for reading, can you represent the lamps as points—points of light? Can you represent the table as a square? Can you draw the room simply as a box consisting of plane faces? What of the reflected light from the walls and the ceiling? Will it matter what size of book is to be read? Will it matter if the reader is ill-tempered? Does he suffer from eye-strain? Will it matter if the book is very broad, or if the print is large or small, or if there are pictures—coloured, or black and white?

A few moments' thought and it will become evident that there is no limit to the possible factors that may enter into the question, for that depends on how *deeply* one proposes to study it. The answer to this dilemma is simple. The tools you have at your disposal settle the depth to which you can penetrate, and if it turns out that this does not suffice for your purpose, you will simply have to discover deeper mathematical methods, and mathematical tools better for the purpose.

Geometrical Analysis

Accordingly the first step in analysis is to ask: What is the geometry of the problem? What shapes are involved and how do they lie with reference to each other? The next question is: What are the quantities or sizes involved?

There may be no obvious picture. It may be simply a question of adding sums of money or distances together. So much the easier. Once you have settled the picture and the sizes of the parts you know, you must decide whether you know enough. If you do not know enough, you cannot yet solve your problem.

How can you tell whether you have enough data? It is frequently not possible to decide this at once, but where the problem can be represented initially in geometrical form you can decide whether you have enough data by a simple test—can the figure or diagram be drawn from the given data?

For example, suppose you know that of three towns A, B, and C, the town A is 40 miles from B and 60 miles from C, while B is 50 miles from C, then by actual drawing on paper you can make a small-scale picture of the relative positions of the three towns, with fair accuracy. This first step shows that already there is some justification for thinking of the subject in mere geometrical form and for fixing the picture in this definite way. Suppose it is proposed to erect a factory as nearly as possible at a site at equal distances from the three towns—is this requirement sufficient to settle the location of the factory?

On the picture (Fig. 90) the problem would have to be expressed in this way:

Is there a point P such that
$PA = PB = PC$?

This requires only to be stated, for you to see that this would mean that you could fix the point of your compass at P, step off PA, and, swinging the compass around P, a circle would be traced out that passed through B and C. Is there such a point P? (*See* Lesson 5.)

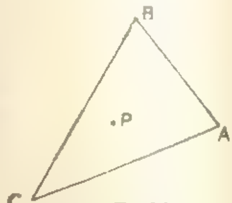


Fig 90

The question asks, in fact: Is there a circle that passes through 3 given points? The answer is yes. The point P is therefore its centre. To determine the point P, you require the centre of a circle that is to pass through 3 given points. By the time the problem of the location of the factory has been reduced to a statement of this nature you have extracted the mathematical essence from it and can turn your mathematical tools towards its solution.

Artificial Problems and Reality

The analysis of a problem proceeds by putting to oneself a series of questions so selected as to bring out the geometrical and numerical features and then so directed as to enable one to decide whether the problem as so stated is a *definite* mathematical problem. By the time this has been achieved it is usually

reduced to a form dependent on one of our abstract geometrical propositions. At this stage the ordinary technique comes into operation, and all that is thereafter necessary is to reinterpret the mathematical results in terms of the flesh and blood of reality.

What we have been discussing is how to extract the mathematical features from a situation in order to see what information this provides concerning that situation. Most books on mathematics in that sense provide artificial problems. They are usually stated in a form already devoid of all but the mathematical features essential for the solution of the problem. Even in the foregoing illustration we have done this ; we have given the distances apart of the towns A, B, and C—*along the straight lines joining them* instead of, say, along the roads leading from the one to the other ; we have represented the three towns as *points*, whereas they are a complicated grouping of streets, houses, shops, schools, churches, and human beings ; we have assumed that the result we sought did not depend on the weather, the day of the week, the language of the people, and a multitude of other factors.

These have little or nothing to do with the question, with the possible exception of the position of the roads. In assuming that the roads are straight we imply that deviations of existing roads from straightness are of little importance. If those deviations are great, then the problem would require reconsideration. What the mathematician does is to bring his special additional knowledge and experience to bear in order to set out positively which features in all this medley are of importance for his purpose. In this sense he is only carrying forward a tradition already followed by us all in the circumstances of everyday life.

Simplification of Algebraic Expressions

In an earlier Lesson (10) on algebra it was seen that the expression $a-(b-c)$ means $a-b+c$ and we will now consider the simplification of more complicated algebraic expressions. Suppose that the whole of $a-(b-c)$ is to be subtracted from x ; then this would be written

$$x-\{a-(b-c)\}$$

Here "curly" brackets have been used to indicate the whole of the expression $a-(b-c)$. When dealing with more than one bracket it is advisable, in order to minimise the possibility of making an error, to deal with the *innermost* bracket first, thus :

$$\begin{aligned} & x-\{a-(b-c)\} \\ = \; & x-\{a-b+c\} \\ = \; & x-a+b-c \end{aligned}$$

since it will be remembered that a minus ($-$) sign outside a bracket changes the sign of every term inside. Occasionally it is necessary to have yet a third distinctive type of bracket ; thus, if three times the above expression is to be subtracted from 10, say, we write this as

$$10-3\,[x-\{a-(b-c)\}]$$

Here we have used "square" brackets.

Example. Simplify $1-[10a^2-5a\{(a+b)-(b-c)\}]$.

$$\begin{aligned} \text{This expression } &= 1-[10a^2-5a\{(a+b-b+c)\}] \\ &= 1-[10a^2-5a\{a+c\}] \\ &= 1-[10a^2-5a^2-5ac] \\ &= 1-[5a^2-5ac] \\ &= 1-5a^2+5ac \end{aligned}$$

Exactly the same procedure is adopted if powers of several expressions are required. For instance $\{1-(a+b)^2\}^3$ means that a is to be added to b, this sum is to be squared, and subtracted from 1, and the whole resulting expression is to be cubed. We will work through an example which will involve the laws of indices.

Example.

Simplify $x^{-7}[y\{x^{-3}(x^2y^3)^{\frac{1}{6}}\}^{-\frac{3}{2}}]^2$

The method of simplification is to begin with the innermost bracket and work outwards. Thus the expression

$$\begin{aligned} &= x^{-7}[y\{x^{-3}\,x^{\frac{2}{6}}\,y^{\frac{3}{6}}\}^{-\frac{3}{2}}]^2 \\ \text{since } (ab)^n &= a^n\,b^n, \\ &= x^{-7}[y\{x^{-3}\,x^{\frac{1}{3}}\,y^{\frac{1}{2}}\}^{-\frac{3}{2}}]^2 \\ &= x^{-7}[y\{x^{-3+\frac{1}{3}}\,y^{\frac{1}{2}}\}^{-\frac{3}{2}}]^2 \\ &= x^{-7}[y\{x^{-\frac{8}{3}}\,y^{\frac{1}{2}}\}^{-\frac{3}{2}}]^2 \\ &= x^{-7}[y\,x^{(-\frac{8}{3})(-\frac{3}{2})}\,y^{(-\frac{3}{4})}]^2 \end{aligned}$$

Notice particularly at this stage that the y is not raised to the power $-\frac{3}{2}$, only the quantity inside the "curly" bracket. Continuing,

$$\begin{aligned} \text{the expression } &= x^{-7}[yx^{4}\,y^{-\frac{3}{4}}]^2 \\ &= x^{-7}[y^{1-\frac{3}{4}}x^4]^2 \\ &= x^{-7}[y^{\frac{1}{4}}\,x^4]^2 \\ &= x^{-7}\,y^{2(\frac{1}{4})}\;x^8 \\ &= x^{8-7}\,y \\ &= xy \end{aligned}$$

Algebraic Factors

When we were considering the multiplication of algebraic expressions we found that

$$(x-y)(x+y) = x^2-y^2.$$

If you begin with the expression x^2-y^2 and split it up into $x-y$ multiplied by $x+y$ you are factorising x^2-y^2 in exactly the same way as we factorised numbers in arithmetic. It should be noticed that when we write

$$x^2-y^2 = (x-y)(x+y) \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;(i)$$

we are stating an algebraic identity, i.e. this relation is true for all values of x and y. Thus :

$$\begin{aligned} 12^2-5^2 &= (12-5)(12+5), \\ 6^2-5^2 &= (6-5)(6+5), \\ 256^2-255^2 &= (256-255)(256+255). \end{aligned}$$

The student should verify these arithmetical examples. This process can be extended ; thus

$$\begin{aligned} (4z)^2-7^2 &= (4z-7)(4z+7), \\ x^2-(y+z)^2 &= \{x-(y+z)\}\{x+(y+z)\} \end{aligned}$$

Obviously relation (i) is very important ; it

is known as the "*difference of two squares.*" By its aid you can factorise other expressions involving higher powers of x and y. Consider $x^4 - y^4$. This is

$(x^2)^2 - (y^2)^2$, and so we write

$$\begin{aligned} x^4 - y^4 &= (x^2)^2 - (y^2)^2 \\ &= (x^2 - y^2)(x^2 + y^2) \\ &= (x - y)(x + y)(x^2 + y^2), \end{aligned}$$

and here you have been enabled to find the three factors of $x^4 - y^4$. Notice that $x^2 + y^2$ will not factorise in this way.

The difference and the sum of two cubes are two other fundamental forms which factorise; the results are:

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2) \quad \ldots \quad (ii)$$

and

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2) \quad \ldots \quad (iii)$$

These are important, and should be memorised; they may be verified in each case by multiplying out the right-hand side (R.H.S.). It is also a good plan to check by arithmetical examples. For instance:

$$7^3 - 3^3 = (7 - 3)(7^2 + 7 \times 3 + 3^2)$$

i.e.

$$\begin{aligned} 343 - 27 &= 4\,(49 + 21 + 9) \\ &= 4\,(79) \\ &= 316 \qquad \text{which is true.} \end{aligned}$$

The student should verify that

$$5^3 + 4^3 = (5 + 4)(5^2 - 5 \times 4 + 4^2)$$

and

$$17^3 - 1^3 = (17 - 1)(17^2 + 17 \times 1 + 1^2).$$

Many algebraic expressions can be factorised by comparing them with one of the fundamental forms (i), (ii), or (iii). Some examples are:

$$\begin{aligned} (1)\ a^3 + 8b^3 &= a^3 + (2b)^3 \\ &= (a + 2b)\{a^2 - a(2b) + (2b)^2\} \\ &= (a + 2b)(a^2 - 2ab + 4b^2), \\ (2)\ x^6 - y^6 &= (x^3)^2 - (y^3)^2 \\ &= (x^3 - y^3)(x^3 + y^3) \\ &= (x - y)(x^2 + xy + y^2) \\ &\ (x + y)(x^2 - xy + y^2). \end{aligned}$$

Another method of factorising algebraic expressions is by the grouping of terms together. Consider an example. Suppose you wish to find the factors of $3 + 7x - 3x^2 - 7x^3$. You may rewrite the expression

$$\begin{aligned} &= 3 - 3x^2 + 7x - 7x^3 \\ &= 3\,(1 + x^2) + 7x\,(1 - x^2) \end{aligned}$$

Now $1 - x^2$ divides into each term, i.e. the above expression must have been obtained by multiplying out $(3 + 7x)(1 - x^2)$ so that these are the factors. Since $1 - x^2$ is itself the difference of two squares, the expression factorises one stage further into $(3 + 7x)$ $(1 - x)(1 + x)$.

Quadratic Expressions

Dealing now with a very important form, suppose you multiply $x + 2$ by $x + 3$ you find—

$$\begin{aligned} (x + 2)(x + 3) &= x(x + 3) + 2(x + 3) \\ &= x^2 + 3x + 2x + 6 \\ &= x^2 + 5x + 6. \end{aligned}$$

The expression on the R.H.S. is called a *quadratic* expression in the variable x because it involves a term in x^2 and no terms of higher powers. An algebraic problem of frequent practical occurrence is to split up quadratic expressions into their factors. You obtain help in this direction by multiplying $x + a$ by $x + b$. You obtain

$$\begin{aligned} (x + a)(x + b) &= x(x + b) + a(x + b) \\ &= x^2 + bx + ax + ab \\ &= x^2 + (a + b)x + ab \end{aligned}$$

and you can now see the connexion between the numbers a and b in the factors, and in the terms on the R.H.S. It will be noticed that the coefficient of x on the R.H.S. is the sum of a and b, while the third term is their product. This means, for example, that if you wish to factorise $x^2 + 9x + 20$ you must find two numbers which when added together give 9 and when multiplied together give 20. By trial and error it will quickly be found that the numbers are 4 and 5, thus

$$\begin{aligned} x^2 + 9x + 20 &= x^2 + (4 + 5)x + 4 \times 5 \\ &= (x + 4)(x + 5) \end{aligned}$$

Other important forms are

$$(x + a)(x - b) = x^2 + (a - b)x - ab$$

and

$$(x - a)(x - b) = x^2 - (a + b)x + ab$$

Example. Factorise $x^2 - 2x - 3$.

Since this is of the form $(x + a)(x - b)$ (which it must be because the last term is $- 3$), you require $ab = - 3$ so that either $a = 1$, $b = - 3$ or $a = - 1$, $b = 3$, i.e. the factors are either $(x + 1)(x - 3)$ or $(x - 1)(x + 3)$. The first of these on multiplication gives $x^2 - 2x - 3$ whereas the second gives $x^2 + 2x - 3$.

Thus $x^2 - 2x - 3 = (x + 1)(x - 3)$.

We shall return to a study of quadratic expressions in the Lesson on algebraic equations.

Algebraic Identities

It has already been stated that when you write $x^2 - y^2 = (x - y)(x + y)$ you are stating an algebraic identity, but it must be realized that an identity need not be in the form of factors. Whenever you have two algebraic expressions which are equal for all values of x, but are in different forms, you have an identity. As an example consider

$$3x = 2(x - 1) + x + 2.$$

The R.H.S. when simplified reduces exactly to $3x$, quite independently of any value of x. That this statement is true can be shown by substituting *any* value for x and verifying that the numerical value of both sides is equal. We may consider here a general problem in identities which is of practical importance. Suppose you had been asked to find the values of the constants A and B in order that

$$3x = A(x - 1) + B(x + 2)$$

shall be an identity. You can see by comparing this relation with the one above that $A = 2$, $B = 1$, but the problem is to find these values without any previous knowledge. You proceed in the following way. Since

$$3x = A(x - 1) + B(x + 2)$$

is to be an identity, it is to be true for *all* values of x; this means that you may give whatever

values you please to x and the relation will still be true. This fact provides you with the necessary information to determine A and B. Notice that if you choose $x = 1$ as one value, the first term of the R.H.S. equals $A \times 0$, i.e. zero, so that you will find B immediately. Thus when $x = 1$

$$3 = 0 + B(1 + 2)$$
$$\text{i.e.} \quad B3 = 3$$
$$\text{so that} \quad B = 1.$$

Similarly, by putting $x = -2$, the second term on the R.H.S. becomes zero and you find A directly. If $x = -2$

$$3(-2) = A(-2 - 1) + 0$$
$$\text{i.e.} \quad -6 = -3A$$
$$\text{or} \quad A = 2.$$

Of course, you have not *proved* that $3x$ is expressible as $A(x - 1) + B(x + 2)$, but you have found the values of A and B as quickly as possible on the assumption that it is.

Example. If $x^2 + 9 = A(x + 1)(x + 2) + B(x + 2)(x + 3) + C(x + 3)(x + 1)$ find A, B, and C.

As in the last example, we choose values of x which make some of the brackets on the R.H.S. become zero, namely $x = -1$, $x = -2$, $x = -3$.

If $x = -1$

$$1 + 9 = A.0.1 + B.1.2 + C.0.2$$
$$\text{i.e.} \quad 10 = 2B$$
$$\text{so that} \quad B = 5$$

If $x = -2$

$$4 + 9 = A. -1.0 + B.0.1 + C. -1.1$$
$$\text{i.e.} \quad 13 = -C$$
$$\text{or} \quad C = -13$$

Lastly, if $x = -3$

$$9 + 9 = A. -2. -1 + B. -1.0 + C. -2.0$$
$$\text{i.e.} \quad 18 = 2A$$
$$\text{or} \quad A = 9$$

Thus

$$x^2 + 9 = 9(x + 1)(x + 2) + 5(x + 2)(x + 3) - 13(x + 1)(x + 3).$$

The student should multiply out the R.H.S. and verify that it simplifies to $x^2 + 9$.

Functions

It was stated in Lesson 11 that an expression which depends for its value on x is called a *function* of x, usually denoted by $f(x)$ or $F(x)$. We may conveniently use this notation to indicate the value of the function for a particular value of x. Consider an example—

If $\quad f(x) = x^3 - 2x + 1$

then $f(3)$ means the value of this function (in this case a *cubic* function) when $x = 3$,

$$\text{i.e.} \quad f(3) = 3^3 - 2.3 + 1 = 27 - 6 + 1 = 22$$
$$\text{Again} \quad f(1) = 1 - 2.1 + 1 = 0$$
$$\text{and} \quad f(0) = 0 - 0 + 1 = 1$$
$$\text{and} \quad f(x + a) = (x + a)^3 - 2(x + a) + 1.$$

You simply replace x by a particular value or quantity throughout the whole expression. You can write any polynomial function quite generally as

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots a_{n-1}x + a_n$$

where $a_0, a_1, \ldots a_n$ are constant *coefficients* (some may be zero) and n denotes the *degree* of the function. Thus a function such as $x^2 - 3x + 7$ is a *quadratic* function because in this case $n = 2$. It is said to be of the *second* degree. Again $x^7 + x - 3$ is of the seventh degree because 7 is the highest power in this function. A function of the first degree, for example $3x - 4$, is said to be *linear*.

The Remainder Theorem

The remainder theorem states that if a polynomial function of x, $f(x)$, is divided by $x - a$, then the remainder is $f(a)$. Before we prove this theorem we will consider what it means in terms of an actual example. Consider a function of x given by $f(x) = x^3 + 5x^2 - 7x + 15$, and suppose that this is divided by $x - 2$.

The theorem states that the remainder is $f(2)$. Now

$$f(2) = 2^3 + 5.2^2 - 7.2 + 15 = 29$$

so that this is the remainder on dividing $f(x)$ by $x - 2$. This may easily be verified by long division; it will be found that

$$\frac{x^3 + 5x^2 - 7x + 15}{x - 2} = x^2 + 7x + 7 + \frac{29}{x - 2}$$

i.e. the remainder is 29.

Notice that the R.H.S. of this expression could be written

$$\frac{(x^2 + 7x + 7)(x - 2) + 29}{x - 2}$$

You can now proceed with the proof of the remainder theorem: if $f(x)$ denotes any polynomial function of x, when this function is divided by $x - a$, the quotient will be another function of x, and the division may be continued until only a numerical remainder is left. Thus you can write

$$\frac{f(x)}{x - a} = F(x) + \frac{R}{x - a} \quad \ldots\ldots\ldots\ldots \quad (i)$$

where $F(x)$ denotes the quotient and R denotes the numerical remainder. For instance, in our example $F(x) = x^2 + 7x + 7$, $R = 29$.

Multiplying through by $x - a$, you obtain

$$f(x) = (x - a)F(x) + R \quad \ldots\ldots\ldots\ldots \quad (ii)$$

This is an identity and therefore holds even when $x = a$. Thus

$$f(a) = 0 \times F(a) + R = R$$

so that the numerical remainder equals $f(a)$.

This theorem is of great help in finding the factors of an expression, for if it should happen that $f(a) = 0$ then $R = 0$, i.e. that there is no remainder when $f(x)$ is divided by $x - a$. If there is no remainder when $f(x)$ is divided by $x - a$ the latter is a factor of $f(x)$.

Example. Show that $x - 1$, $x - 2$, $x - 3$ are all factors of $x^3 - 6x^2 + 11x - 6$.

If $f(x) = x^3 - 6x^2 + 11x - 6$, then you have only to show that $f(1)$, $f(2)$, $f(3)$ are all zero to prove that $x - 1$, $x - 2$, $x - 3$ are factors.

$$\begin{aligned} f(1) &= 1^3 - 6.1^2 + 11.1 - 6 \\ &= 1 - 6 + 11 - 6 \\ &= 0 \\ f(2) &= 2^3 - 6.2^2 + 11.2 - 6 \\ &= 8 - 24 + 22 - 6 \\ &= 0 \\ \text{and } f(3) &= 3^3 - 6.3^2 + 11.3 - 6 \\ &= 27 - 54 + 33 - 6 \\ &= 0 \end{aligned}$$

i.e. $x - 1$, $x - 2$, $x - 3$ are all factors of $x^3 - 6x^2 + 11x - 6$.

Example. Find a so that $x^3 + ax^2 + (a + 10)x + 15$ may be divisible by $x + 3$.

If $f(x) = x^3 + ax^2 + (a + 10)x + 15$, then there is to be no remainder when $f(x)$ is divided by $x + 3$, i.e. $f(-3)$ must equal zero.

$$\begin{aligned} f(-3) &= (-3)^3 + a(-3)^2 + (a + 10)(-3) + 15 \\ &= -27 + 9a - 3(a + 10) + 15 \\ &= 6a - 42\,; \end{aligned}$$

i.e. if $f(-3) = 0$

then $\quad 6a - 42 = 0$

i.e. $\quad 6a = 42$

$\quad a = 7$

so that if $a = 7$, $f(x)$ is divisible by $x + 3$, i.e. $x^3 + 7x^2 + 17x + 15$ is divisible by $x + 3$.

EXERCISES

(1) Simplify

(i) $(x + x^{-1})^2 - (x^2 + x^{-2})$

(ii) $\dfrac{(2^{x+2})^y}{2^{x(y+2)}} \times \dfrac{2^{3x}}{2^{6y}} \times 2^y$

(iii) $\left\{\left(2 + \dfrac{1}{x}\right)^5 \left(2 - \dfrac{1}{x}\right)^2\right\} \div \left\{\left(x + \dfrac{1}{2}\right)^5 \left(x - \dfrac{1}{2}\right)^2\right\}$

(2) Show that

$$\left(\frac{x}{y} - \frac{y}{x}\right) \left(\frac{x + y}{x - y} - \frac{x - y}{x + y}\right)$$ [illegible]

(3) Factorise

(i) $a^3 + 8b^3$

(ii) $x^3 + 27y^6$

(iii) $(a + b)^2 - (c + d)^2$

(iv) $x^2 + 2zx - y^2 - 2zy$

(v) $\sin^3 A + \cos^3 A$

(vi) $x^2 + 8x + 15$, (vii) $x^2 -$ [illegible]

(viii) $x^2 - 7x + 12$

(4) Use the remainder theorem to fi[illegible] remainder when $x^3 - 8x^2 + 7$ is divided by x [illegible] check by long division.

(5) Show that $x + 1$, $x - 3$, x [illegible] factors of $x^3 - 6x^2 + 5x + 12$.

(6) If $f(x)$ is divided by $mx - a$ show that the remainder is $f\left(\dfrac{a}{m}\right)$. Hence show th[illegible] 3 is a factor of $6x^3 - 9x^2 - 8x + 12$.

(7) If $f(x) = x^3 - 4x^2 + 8$, find $f($[illegible] $f(2)$, and hence find a factor of $f(x)$.

(8) If $3x + 7 = A(x + 1) + B(x + 2)$, find A and B.

(9) Find A, B and C if $13 + 4x - x^2 = A(x^2 - 1) + B(x + 1)(x - 3) + C(x - 3)(x - 1)$

(10) Factorise (i) $x^3 + 3x^2 - x - 3$

(ii) $x^3 - 1$

(iii) $x^3 + 7x^2 - 36$

ANSWERS TO EXERCISES IN LESS[illegible]5

(1) 35·90 (2) 464·6.
(3) 0·000, 001, 935.
(4) (i) 0·06694. (ii) 0·3107. (iii) 1·54[illegible]
(5) 0·5164. (6) 2·4849, 2·3996. (7) 2·618.

LESSON 17

Simple Algebraic Equations

ALGEBRAIC equations are statements true only for particular values of the letter or letters involved. For instance

$x^2 = 9$; $3 + x = 6$; $x + y = 8$; $x^3 + 1 = 0$

are all algebraic equations and we will attempt to *solve* these equations. *Solving* an equation means finding the value or values of the unknown quantity for which the equation is true. Thus, $x^2 = 9$ has two *solutions*, because if x is either $+3$ or -3 the equation is true. But $3 + x = 6$ has only one solution ; by subtracting 3 from both sides, you see that $x = 3$.

Now examine the equation $x + y = 8$; it is obvious that any number of different solutions may be found ; for instance

$$x = 1,\ y = 7$$

satisfies the equation, so also does

$$x = 16,\ y = -8$$

in fact, for every value of y, there is a value of x satisfying the equation.

The last example given, namely $x^3 + 1 = 0$, can be solved by subtracting 1 from both sides so that $x^3 = -1$, and by taking the cube root of both sides you obtain $x = -1$ apparently as the only solution. It is clear that if equations are to be studied, they must be studied systematically, and we begin with the simplest type.

Linear Equations

Linear equations is the name given to equations in which there are no letters raised to a higher power than the first, and the name is derived from the fact (*see* Lesson 20) that this type of equation is associated with a straight line in the study of algebraic geometry. The solution of a linear equation with one *unknown* (as the variable appearing in an equation is called) can always be found by simplifying the algebra until the value of the unknown letter is obtained.

Example. Solve the equation

$$\tfrac{1}{2}(x - 2) - 4(x - 3) = 4$$

Simplifying the L.H.S., we obtain

$$\tfrac{1}{2}x - 1 - 4x + 12 = 4$$

i.e. $\quad -3\tfrac{1}{2}x = -7$

or, by multiplying both sides by -2, $\quad 7x = 14$

i.e. $\quad x = 2.$

It was found in an earlier example that an equation with two unknowns could have an

infinite number of solutions, and consequently if you have two unknowns, you require more than one equation to fix both unknowns. It turns out that two equations in general suffice; this fact corresponds, for linear equations, to the geometric fact that two non-parallel lines have a single point in common. If two equations are both to be satisfied they are called *simultaneous*.

Consider the following problem: John and Harold have together the sum of £5. How much has each? Obviously that cannot be answered without further information: if x is the amount of John's money, y the amount of Harold's, then you only have the single equation

$$x + y = 5 \quad (i)$$

But if you are told at the same time that John has four times as much as Harold, then you are given a second equation, namely

$$x = 4y \quad (ii)$$

This additional information now fixes the amount of money each has because you can replace the x in (i) by $4y$ since equation (ii) states that $x = 4y$. Thus (i) becomes

$$4y + y = 5$$

i.e. $5y = 5$

or $y = 1$

so that Harold has £1 and John has £4.

Notice particularly the method of solution: between the two equations you have *eliminated* x, i.e. you have found a linear equation involving y only and this you solved to find the value of y.

We can generalise this method of solving simultaneous linear equations. Suppose you have

$$ax + by = m \quad (iii)$$

and $cx + dy = n \quad (iv)$

where a, b, c, d, m, n are constants, then to solve these you require to eliminate either x or y. This may most easily be done by multiplying (iii) by the coefficient of y in (iv), and (iv) by the coefficient of y in (iii) and subtracting, thus:

(iii) multiplied by d gives

$$adx + bdy = md \quad (v)$$

and (iv) multiplied by b is

$$bcx + bdy = nb \quad (vi)$$

Notice that now you have exactly the same term in y in both equations, therefore subtracting (vi) from (v) you obtain an equation in x only, thus:

$$adx - bcx = md - nb$$

i.e. $x(ad - bc) = md - nb$

or $$x = \frac{md - nb}{ad - bc}$$

Here you have found the value of x in terms of our constants and by substituting this value of x in either (iii) or (iv) you obtain the value of y.

The method appears to break down if $ad - bc = 0$, but the truth is that the equations are then inconsistent unless $md - nb = 0$. That is, unless $md - nb = 0$, when there is really only one equation, there are no values of x and y simultaneously satisfying $ax + by = m$, $cx + dy = n$, with $ad - bc = 0$. This corresponds to the geometrical fact that parallel lines do not intersect (*see* Lesson 20).

Quadratic Equations

If you put a quadratic expression equal to zero, you have what is called a quadratic equation. These equations are of very common occurrence in practical work and consequently are important. The example considered earlier in this Lesson, namely $x^2 = 9$, is an example of the simplest type of quadratic equation. Notice that there are *two* solutions, and they can be written down on sight by taking the square root of both sides. You see that if $x = +3$ or -3 the equation is satisfied. If a quadratic equation factorises, its solution is simply obtained; indeed you can solve the foregoing example by this method, for if

$$x^2 = 9$$

then $x^2 - 9 = 0.$

The L.H.S. is the difference of two squares, so that you may write

$$(x - 3)(x + 3) = 0.$$

If you have a product, such as this, equal to zero, then either $x - 3 = 0$ or $x + 3 = 0$. If $x - 3 = 0$, then $x = +3$, and alternatively if $x + 3 = 0$, then $x = -3$ so that you have the same values for x as before.

Consider another example of a quadratic equation soluble by factorising.

Example. Solve the quadratic equation

$$x^2 + 14x - 32 = 0.$$

To factorise the L.H.S. you require two numbers such that their product is -32 and their sum 14. By trial and error you find these numbers to be 16 and -2, so that you may write the equation

$$(x + 16)(x - 2) = 0.$$

Thus either $x + 16 = 0$ or $x - 2 = 0$

i.e. $x = -16$ or $x = 2$.

You require at this stage to notice a method of solving quadratic equations which does not depend on factorisation. It is generally known as "solving by completing the square" and is carried through as follows.

Consider the example solved above: you may write it

$$x^2 + 14x = 32.$$

Now $(x + a)^2 = x^2 + 2ax + a^2$. Thus any expression of the form $x^2 + 2ax$ may be made into a "perfect" square by adding a^2, or the square of half the coefficient of x. In this case the coefficient of x is 14, so that half this

coefficient is 7. By adding 7^2, i.e. 49, to the L.H.S. you make the L.H.S. exactly $(x + 7)^2$. In order not to alter the truth of the equation you must add this same quantity to the R.H.S. Thus the equation becomes

$$x^2 + 14x + 49 = 32 + 49$$

i.e. $$(x + 7)^2 = 81.$$

Taking the square root of both sides you obtain

$$x + 7 = \pm 9$$

i.e. $$x + 7 = +9 \text{ or } x + 7 = -9$$

i.e. $$x = +2 \text{ or } \quad x = -16$$

which agree with the values obtained by factorisation. The value of this method is that it can be applied to cases which cannot be factorised.

Example. Solve $x^2 + 6x + 7 = 0$.

This may be written

$$x^2 + 6x = -7.$$

The L.H.S. will become $(x + 3)^2$ if you add the square of half the coefficient of x namely, 3^2, i.e. 9, so that

$$x^2 + 6x + 9 = 9 - 7$$

or $$(x + 3)^2 = 2.$$

On taking the square root of both sides you obtain

$$x + 3 = +\sqrt{2} \text{ or } x + 3 = -\sqrt{2}$$

i.e. $$x = -3 + \sqrt{2} \text{ or } x = -3 - \sqrt{2}.$$

Taking $\sqrt{2}$ as 1·414 these values become

$$x = -1{\cdot}586 \text{ or } x = -4{\cdot}414.$$

At this stage a difficulty may arise if before you take the square root, the number on the R.H.S. is negative. Consider the equation $x^2 + 9 = 0$. To solve this you would write

$$x^2 = -9$$

but you cannot take the square root because no number can be found such that when it is squared the result is -9. Every number squared is positive, so that when the square of the unknown is negative, no solutions of the equation exist. For further mathematical work it is useful to invent new numbers, called *complex* numbers, so that all quadratic equations have solutions. This is done by introducing a symbol i corresponding to $\sqrt{-1}$ and then putting $\sqrt{-a} = i\sqrt{a}$ for any positive a. Thus $x^2 = -9$ yields $x = \pm\sqrt{-9} = \pm i\sqrt{a} = \pm 3i$. Notice that this process of inventing new numbers has occurred before. Fractions were invented for such equations as $2x = 1$ to have a solution and negative numbers were invented for such equations as $x + 4 = 3$ to have a solution.

The method of solution of a quadratic equation by "completing the square" is so important that we will consider the general case.

Every quadratic equation may be written in the form

$$ax^2 + bx + c = 0 \qquad (vii)$$

where a, b, c are constants. (Either b or c may in some cases be zero, but this will not affect the argument. If a were zero, the equation would no longer be quadratic.)

If you divide (vii) all the way through by a, you obtain

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

and keeping only the terms involving x on the L.H.S. this becomes

$$x^2 + \frac{b}{a}x = -\frac{c}{a}.$$

To make the L.H.S. a perfect square you require to add to it $\left(\frac{b}{2a}\right)^2$ and in order not to destroy the relationship of equality you must add this same amount to the R.H.S. as well. Thus:

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}.$$

The L.H.S. is now

$$\left(x + \frac{b}{2a}\right)^2 \text{ so that}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}.$$

Taking the square root of both sides

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$= \pm\frac{1}{2a}\sqrt{b^2 - 4ac}$$

so that

$$x = -\frac{b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \ldots\ldots (viii)$$

This formula allows you to write down the solutions of any quadratic equation by substituting for a, b, and c.

A more convenient form for substitution is available if a, b, c are integers and b is even. Then, dividing top and bottom of (viii) by 2, you have

$$x = \frac{-\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - ac}}{a} \quad \ldots\ldots (ix)$$

To revert to the previous example $x^2 + 14x - 32 = 0$, you see that $a = 1$, $b = 14$, $c = -32$, so that, using (viii), the two solutions are given by

$$x = \frac{-14 \pm \sqrt{(14^2 + 4.1.32)}}{2}$$

$$= \frac{-14 \pm \sqrt{(196 + 128)}}{2}$$

$$= \frac{-14 \pm 18}{2}$$

$$= -7 \pm 9$$

$$= +2 \text{ or } -16.$$

The student should try applying formula (ix) to this equation.

The formula (viii) (or (ix)) should be remembered. If the solutions of any quadratic are to be real, this requires that the quantity under the square root sign is positive, i.e. $b^2 - 4ac$ must be positive, i.e. $b^2 > 4ac$. If $b^2 < 4ac$ the quantity $b^2 - 4ac$ will be negative and you will have complex solutions. If $b^2 = 4ac$, the two solutions are real and equal.

The values of x which satisfy an equation are called the *roots* of the equation and are often denoted by the Greek letters α (alpha) and β (beta). Thus you can take

$$\alpha = \frac{-b + \sqrt{(b^2 - 4ac)}}{2a} \quad \text{and}$$

$$\beta = \frac{-b - \sqrt{(b^2 - 4ac)}}{2a}.$$

Relations between Roots and Coefficients of a Quadratic Equation

Suppose the equation $ax^2 + bx + c = 0$ has the roots $x = \alpha$, $x = \beta$. Now the equation

$$(x - \alpha)(x - \beta) = 0$$

also has the roots α, β, so that $ax^2 + bx + c$ must be identically equal to $a(x - \alpha)(x - \beta)$ —the factor a being included to make the coefficients of x^2 the same; multiplying up, you have

$$ax^2 + bx + c = ax^2 - a(\alpha + \beta)x + a\alpha\beta.$$

Comparing coefficients, you get

$$a(\alpha + \beta) = -b$$
$$a\alpha\beta = c,$$

so that

$$\alpha + \beta = -\frac{b}{a} \quad \ldots\ldots\ldots \quad (x)$$

and

$$\alpha\beta = \frac{c}{a} \quad \ldots\ldots\ldots \quad (xi)$$

These results should be remembered: (1) that the sum of the roots of a quadratic equation is equal to the coefficient of x divided by the coefficient of x^2, with the sign changed; and (2) the product of the roots is the independent term divided by the coefficient of x^2.

Example. Solve the equation

$$(x - 3)(x + 6) = (a - 3)(a + 6)$$

where a is some constant.

From the form of this quadratic equation you see *by inspection* that $x = a$ is one solution because when x is put equal to a, the equation is obviously satisfied. Thus a is one root. By considering the L.H.S. alone you see that on multiplication it becomes $x^2 + 3x - 18$, i.e. the coefficient of x^2 is 1 and the coefficient of x is 3. Thus the sum of the roots is -3, but one root is a; therefore the other root is $-3 - a$. The two solutions are therefore $x = a$ and $x = -3 - a$.

Equations of Any Degree

There are no simple general methods for solving equations of a higher degree than a quadratic and where such solutions are required it is usually necessary to obtain first approximate solutions and then to refine these approximations. This problem is considered in Lesson 19. There are, however, some particular methods; for instance an equation such as $ax^4 + bx^2 + c = 0$ can be solved as a quadratic in x^2. Again, the remainder theorem (p. 1237) may be useful. Any equation of the nth degree in x may be written $f(x) = 0$, and so if a value of x can be found, say α, such that $f(\alpha) = 0$, then α must be a root of the equation because if $f(\alpha) = 0$, this means that $f(x)$ is divisible by $x - \alpha$, i.e. $x - \alpha$ is a factor of $f(x)$, i.e. $x = \alpha$ is one of the solutions of $f(x) = 0$.

Example. Solve the equation

$$x^3 + 9x^2 + 6x - 56 = 0 \quad \ldots\ldots \quad (xii)$$

Put $f(x) = x^3 + 9x^2 + 6x - 56$ and find $f(1)$, $f(2)$, $f(3)$, . . . $f(-1)$, $f(-2)$ . . . and so on. Thus:

$$f(1) = 1 + 9 + 6 - 56 = -40$$
$$f(2) = 2^3 + 9.2^2 + 6.2 - 56$$
$$= 8 + 36 + 12 - 56$$
$$= 0$$

so that $f(x)$ is divisible by $x - 2$. Therefore one of the roots of $f(x) = 0$ is $x = 2$ and by long division of $\dfrac{f(x)}{x - 2}$ you will obtain a quadratic expression which will provide two more roots.

By long division $x^3 + 9x^2 + 6x - 56$ divided by $x - 2$ equals $x^2 + 11x + 28$, so that you may write (xii) as

$$(x - 2)(x^2 + 11x + 28) = 0$$

The quadratic term factorises so that this becomes

$$(x - 2)(x + 7)(x + 4) = 0$$

Thus there are three roots and their values are 2, -7, and -4.

EXERCISES

(1) Solve the equations:
 (i) $3(x - 3) = 7(x - 7)$
 (ii) $3(x + 1)^2 + (x - 1)^2 = 4x^2$
 (iii) $(x + 1)(x + 2) - (x - 1)(x - 2) = 9$

(2) Solve the simultaneous equations:
 (i) $x - y = 8$, $x + y = 4$
 (ii) $2x + 3y = 13$, $5x - 7y = 18$
 (iii) $\frac{1}{2}(x + 1) + \frac{1}{3}(y + 1) = 6$, $x + y = 13$

(3) Solve the following equations by factorisation:
 (i) $x^2 - 3x - 28 = 0$
 (ii) $x^2 + x - 72 = 0$
 (iii) $x^2 + 12x = 0$
 (iv) $x^2 + 9 = 6x$
 (v) $x^3 + 6x^2 + 5x = 0$
 (vi) $3x^2 - 8x - 3 = 0$

(4) By "completing the square," solve the equations:
 (i) $x^2 - 9x + 17 = 0$
 (ii) $\frac{1}{3}x^2 - \frac{1}{2}x = 9$
 (iii) $x^2 + 6x + 7 = 0$

(5) By means of the rule for writing down the roots of a quadratic equation, check the solutions obtained for the equations in questions (3) and (4).

(6) Find the sum and product of the roots of the following equations without solving them:
 (i) $3x^2 + 7x + 11 = 0$
 (ii) $x^2 + x - 10 = 0$
 (iii) $x(x + 1) + (x + 1)(x + 2) = 2$

In which of these equations are the roots real?

(7) Solve the equation $x^4 - 13x^2 + 36 = 0$.
(8) Use the remainder theorem to show that the equation $x^3 - 2x^2 - x - 6 = 0$ has only one real root, and find it.
(9) Solve the equation $x^3 - 5x^2 + 2x + 8 = 0$.

ANSWERS TO EXERCISES IN LESSON 16

(1) (i) 2, (ii) $2x$, (iii) $128x^{-7}$
(3) (i) $(a + 2b)(a^2 - 2ab + 4b^2)$
(ii) $(x + 3y^2)(x^2 - 3xy^2 + 9y^4)$
(iii) $(a + b + c + d)(a + b - c - d)$
(iv) $(x - y)(x + y + 2z)$
(v) $(\sin A + \cos A)(1 - \sin A \cos A)$
(vi) $(x + 3)(x + 5)$
(vii) $(x - 3)(x + 2)$
(viii) $(x - 3)(x - 4)$
(4) -17.
(7) $f(0) = 8$, $f(1) = 5$, $f(2) = 0$, $x = 2$
(8) A = -1, B = 4
(9) A = 2, B = 4, C = 1
(10) (i) $(x - 1)(x + 1)(x + 3)$
(ii) $(x + 1)(x^2 - x + 1)$
(iii) $(x - 2)(x + 3)(x - 6)$

LESSON 18

Trigonometrical Equations

HAVING studied algebraic equations in the preceding Lesson, we will devote this one to the study of trigonometrical equations since there are many points of similarity between the two.

Consider a very simple example such as $\sin A = \frac{1}{2}$, and examine what will be meant by the *solution* of this equation. Since $\sin 30^\circ$ is $\frac{1}{2}$ it is clear that $A = 30^\circ$ is one value of A which satisfies our equation. This value is a *root* in exactly the same sense as we had a root of an algebraic equation. Also there will be a large number of additional roots, for there is an angle in the second quadrant, i.e. between 90° and 180°, for which the sine is $\frac{1}{2}$, and again any multiple of 360° can be added to either of the two solutions already indicated to give further roots.

To find all these values exactly, the procedure is as follows. Draw a diagram as in Fig. 91, and find the solution in the first quadrant (in this case 30°). Since you are dealing with a sine in this example, and since any sine is positive also in the second quadrant, you have the solution 180° − 30°, i.e. 150°. In the third and fourth quadrants the sine is negative, so that there can be no solutions of the equation in these quadrants; but the angle 360° + 30° gives an angle in the first quadrant again, still with the sine equal to $\frac{1}{2}$, so that the general formula $n.360^\circ + 30^\circ$ will account for all solutions in the first quadrant by putting n equal to 0, 1, 2, ... successively.

The general formula $n.360^\circ + 150^\circ$ will similarly give all solutions lying in the third quadrant. It should be noticed that these two sets of solutions may be combined into the one general formula :

$$A = m.180^\circ + (-1)^m\, 30 \quad \ldots\ldots\ldots \quad (i)$$

by successively putting $m = 0, 1, 2, 3, \ldots$

Thus, if $m = 0$, A = 30°,
$m = 1$, A = 180° − 30°,
$m = 2$, A = 2.180° + 30°,
$m = 3$, A = 3.180° − 30°, and so on.

Remembering that negative angles are measured in a clockwise direction, it will be seen that there will be negative angles which also have their sine equal to $\frac{1}{2}$. For instance, $-180^\circ - 30^\circ$ is such an angle in the second quadrant and $-360^\circ + 30^\circ$ is a negative angle in the first quadrant having this same sine. It

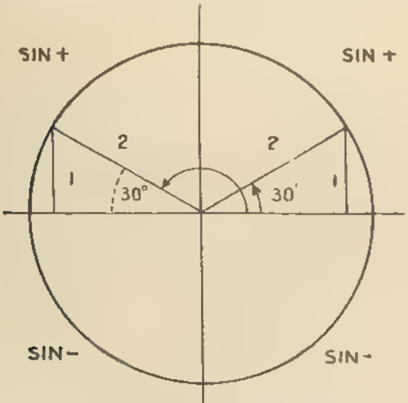


Fig 91

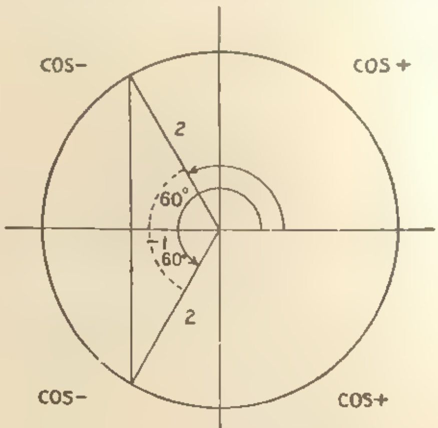


Fig. 92

EQUATIONS IN TRIGONOMETRY. Fig. 91. Finding root values in the first and second quadrants for $\sin A = \frac{1}{2} = 30^\circ$ (positive angles). Fig. 92. Finding angles that satisfy the equation $\sec A = 2$.

will be seen that the general formula (i) for the solutions of $\sin A = \frac{1}{2}$ will also include these negative values for when

$$m = -1,\ A = -1.180^\circ - 30^\circ,$$
$$m = -2,\ A = -2.180^\circ + 30^\circ,$$

and so on.

We will now consider some other examples, and in the first we shall find solutions only for a r[illegible]icted range of values of the angle.

Ex[illegible]ple. Find all the angles between 0° and 360° whi[illegible] satisfy the equation

$$\sec A = -2.$$

I[illegible] A [illegible] 2, then this means that $\cos A$ [illegible] $\frac{1}{2}$ and [illegible] a cosine is negative only in the second and thi[illegible]drants, there can be only two solutions to the eq[illegible], one in each of these quadrants. From the dia[illegible] (Fig. 92) it is obvious that the solution in the seco[illegible]uadrant is

$$A = 180^\circ - 60^\circ, \text{ i.e. } 120^\circ$$

and [illegible] solution in the third quadrant is $A = 180^\circ + 60^\circ$ [illegible] 240°. These are the only solutions in the given ran[illegible]

Y[illegible]u can use the knowledge of quadratic eq[illegible]tions gained in the preceding Lesson to help in the solution of trigonometrical equations if they can be put in the form of a quadratic equation in terms of any trigonometrical ratio.

Example. Solve the equation $\cot A = \tan A$ for values of A between 0° and 360°. (Note that this range may be written $0^\circ < A < 360^\circ$, meaning A greater than 0° but less than 360°.)

If $\qquad \cot A = \tan A$

then $\qquad \dfrac{1}{\tan A} = \tan A$

so that $\qquad \tan^2 A = 1.$

On taking the square root of both sides you have

$$\tan A = \pm 1$$

Thus there will be four solutions in the given range, for if $\tan A = +1$, you obtain a solution in the first quadrant (45°) and also in the third quadrant ($180^\circ + 45^\circ$). If you take the solutions given by $\tan A = -1$, you will have one in the second quadrant ($180^\circ - 45^\circ$) and one in the fourth quadrant ($360^\circ - 45^\circ$). Finally, then,

$$\text{if } \tan A = +1, \text{ then } A = 45^\circ \text{ or } 225^\circ,$$
$$\text{if } \tan A = -1, \text{ then } A = 135^\circ \text{ or } 315^\circ.$$

All these four values are solutions of the original equation.

Example. Solve the equation

$$2 \sin^2 A = 3 \cos A$$

for the range $0^\circ < A\ 3 < 60^\circ$.

You can obtain this equation wholly in terms of cosines by using the identity $\sin^2 A = 1 - \cos^2 A$.

Thus the equation becomes

$$2(1 - \cos^2 A) = 3 \cos A$$

i.e. $\qquad 2 \cos^2 A + 3 \cos A - 2 = 0.$

This is now a quadratic equation in $\cos A$ exactly similar to $2x^2 + 3x - 2 = 0$. Therefore you can factorise the expression into

$$(2 \cos A - 1)(\cos A + 2) = 0,$$

so that either

$$2 \cos A - 1 = 0 \text{ or } \cos A + 2 = 0$$

i.e. $\qquad \cos A = +\frac{1}{2}$ or $\cos A = -2.$

Since no angle can have a cosine numerically greater than 1, there can be no solutions of $\cos A = -2$, therefore you discard this possibility. The only solutions of the equation will be given by $\cos A = \frac{1}{2}$

If $\cos A = \frac{1}{2}$ then the solutions will lie in the first and fourth quadrants, and since $\cos 60^\circ = \frac{1}{2}$, our solutions are $A = 60^\circ$, $A = 300^\circ$.

Equations Involving Multiple Angles

If you are dealing with equations involving multiple angles you may be able to solve directly for the multiple angle or you may be able to use the transformations of products and sums to provide an equation which can be factorised.

Example. Solve the equation

$$\tan 2A = 2 \text{ for the range } 0^\circ < A < 180^\circ.$$

Since you require solutions for A in the range 0° to 180°, you must consider values of 2A in the range 0° to 360°.

If $\tan 2A = 2$, there will be solutions of 2A in the first and third quadrants. The value in the first quadrant is obtained from a table of tangents: you find that $\tan 63^\circ 26' = 2$, so that

$$2A = 63^\circ 26' \text{ (in the first quadrant)}$$

or $\qquad 2A = 180^\circ + 63^\circ 26'$ (in the third quadrant)

Thus $\qquad A = 31^\circ 43'$ or $90^\circ + 31^\circ 43'$,

i.e. the solutions are $A = 31^\circ 43'$, $A = 121^\circ 43'$.

Example. Solve the equation

$$\sin 2A + \sin 4A = \sin 3A$$

for the range $0^\circ \leqslant A \leqslant 180^\circ$. (Notice that this means A is greater than, or equal to, 0° and less than, or equal to, 180°).

You can combine the L.H.S. of the equation into a product by the transformation

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B).$$

Thus you obtain

$$2 \sin 3A \cos A = \sin 3A \quad \ldots\ldots\ldots\ (ii)$$

The student is particularly warned not to fall into the common error of supposing that at this stage the $\sin 3A$ occurring on both sides of the equation can be cancelled. Obviously $\sin 3A = 0$ will satisfy this equation and must not be disregarded. To avoid making this error, write (ii) as

$$2 \sin 3A \cos A - \sin 3A = 0,$$

so that you obtain

$$\sin 3A \,.\, (2 \cos A - 1) = 0.$$

Thus either $\sin 3A = 0$ or $2 \cos A - 1 = 0$. This provides two sets of solutions: if $\sin 3A = 0$ then $3A = 0$, 180°, 360°, 540°. You will have to consider all these values because if the range for A is $0 \leqslant A \leqslant 180^\circ$, then obviously the range for 3A is $0 \leqslant 3A \leqslant 540^\circ$. The solutions arising from these values of 3A are found by dividing by 3, i.e. $A = 0$, 60°, 120°, 180°. In addition there are the solutions obtained from $2 \cos A - 1 = 0$, i.e. $\cos A = +\frac{1}{2}$. The only solution of this in the given range is $A = 60^\circ$, so that there are only the four distinct solutions. Notice that if the range had been given as $0^\circ < A < 180^\circ$, there would only have been two distinct solutions, $A = 60^\circ$ and $A = 120^\circ$.

Example. Solve the equation

$$4 \cos^2 A = 2 + \sin A \cos 2A,$$

giving all the solutions.

You can rewrite the equation as

$$\begin{aligned} \sin A \cos 2A &= 4 \cos^2 A - 2 \\ &= 2(2 \cos^2 A - 1) \\ &= 2 \cos 2A \qquad \text{i.e.} \end{aligned}$$

$$\cos 2A (\sin A - 2) = 0,$$

so that either $\cos 2A = 0$, or $\sin A = 2$.

There are no solutions to $\sin A = 2$, since there is no angle which has its sine greater than 1 and therefore

the only solutions to the equation are those given by $\cos 2A = 0$. These are $2A = 90°, 180° + 90°, 360° + 90°, \ldots$ and can be included in the general formula

$$2A = 90° + n.180° \quad \ldots\ldots \quad (iii)$$

where n has the values 0, 1, 2 . . . . . . successively

i.e. $$A = 45° + n.90° \ldots\ldots\ldots\ldots \quad (iv)$$

These give the positive values; the negative solutions of $\cos 2A = 0$ are $2A = -90°, -180° - 90°$, so that these are also included in (iii) by giving n the values $-1, -2, \ldots$ and so on. Thus (iv) gives *all* the solutions of the original equation; these solutions are in degrees. Expressed in radians, they are given by the relation

$$A = \frac{\pi}{4} + n\frac{\pi}{2}$$
$$= (2n + 1)\frac{\pi}{4}$$

where n is zero or any integer.

Equations of the Form $a \cos A + b \sin A = c$

There are two alternative methods for solving equations of this type.

(1) By transforming sin A and cos A into $\tan \frac{1}{2}A$ and solving the resulting quadratic in $\tan \frac{1}{2}A$

(2) by the use of a *subsidiary* angle. We will consider both methods and apply them to the same example.

Method 1. It will be remembered that identities were obtained for sin A and cos A in terms of $\tan \frac{1}{2}A$ as follows:

$$\sin A = \frac{2 \tan \frac{1}{2}A}{1 + \tan^2 \frac{1}{2}A} \text{ and } \cos A = \frac{1 - \tan^2 \frac{1}{2}A}{1 + \tan^2 \frac{1}{2}A}$$

The procedure will become clear by taking an actual example.

Example. Solve the equation

$$2 \cos A + \sin A = 1$$

for the range $0° < A < 360°$.

If you write t for $\tan \frac{1}{2}A$ and transform, this becomes

$$\frac{2(1 - t^2)}{1 + t^2} + \frac{2t}{1 + t^2} = 1$$

i.e. $$2 - 2t^2 + 2t = 1 + t^2,$$

or $$3t^2 - 2t - 1 = 0.$$

This factorises into

$$(3t + 1)(t - 1) = 0,$$

so that either $3t + 1 = 0$ or $t - 1 = 0$,

i.e. $$\tan \tfrac{1}{2}A = -\tfrac{1}{3} \text{ or } \tan \tfrac{1}{2}A = 1.$$

Since you require a range for A of 0° to 360°, you require a range of $\frac{1}{2}A$ from 0° to 180°. From tables you find that $\tan 18° 26' = +\frac{1}{3}$, so that $\tan (180° - 18° 26') = -\frac{1}{3}$,

i.e. $\frac{1}{2}A = 161° 34'$ is the only value in the range 0° to 180°,

i.e. $A = 323° 8'$ is the only solution from the first factor. From the second factor $\tan\frac{1}{2}A = 1$, i.e. $\frac{1}{2}A = 45°$ so that $A = 90°$. Thus the only two solutions of the original equation are $A = 90°$, $A = 323° 8'$.

Method 2. You can write the general equation

$$a \cos A + b \sin A = c$$

in the form

$$\frac{a}{\sqrt{a^2 + b^2}} \cos A + \frac{b}{\sqrt{a^2 + b^2}} \sin A = \frac{c}{\sqrt{a^2 + b^2}} \quad (v)$$

If you choose a *subsidiary* angle B, such that

$$\cos B = \frac{a}{\sqrt{a^2 + b^2}}, \text{ and } \sin B = \frac{b}{\sqrt{a^2 + b^2}}, \text{ then}$$

you will have relation (v) in the form

$$\cos B \cos A + \sin B \sin A = \frac{c}{\sqrt{a^2 + b^2}}$$

i.e. $$\cos (A - B) = \frac{c}{\sqrt{a^2 + b^2}}$$

Also provided

$$c < \sqrt{a^2 + b^2}$$

you can find from the tables the angle whose cosine is

$$\frac{c}{\sqrt{a^2 + b^2}}$$

Knowing this angle and the angle B, you can find A. We illustrate this method with the same example:

$$2 \cos A + \sin A = 1$$

may be written

$$\frac{2}{\sqrt{5}} \cos A + \frac{1}{\sqrt{5}} \sin A = \frac{1}{\sqrt{5}}$$

You require to find the angle B such that

$$\cos B = \frac{2}{\sqrt{5}} \text{ and } \sin B = \frac{1}{\sqrt{5}} \text{ i.e. } \tan B = \tfrac{1}{2}.$$

From tables $B = 26° 34'$, so that you have

$$\cos (A - 26° 34') = \frac{1}{\sqrt{5}}$$
$$= 0{\cdot}4472$$

and 0·4472 is the cosine of 63° 26′ or 360° − 63° 26′. To consider any further values would obviously give A outside the range you require, thus:

$$A - 26° 34' = 63° 26' \text{ or } 360° - 63° 26'$$

so that

$$A = 90° \text{ or } 323° 8'$$

The student may wonder why we did not adopt what at first sight appears to be the obvious method of solving an equation of this type. We could have written the equation in the example as

$$2 \cos A = 1 - \sin A$$

and by squaring we should have obtained

$$4 \cos^2 A = (1 - \sin A)^2$$

or $$4(1 - \sin^2 A) = 1 - 2 \sin A + \sin^2 A$$

and this can be solved by rearranging as a quadratic in sin A. The disadvantage of such a method is that when both sides are squared the solutions of a different equation are bought in, namely

$$-2 \cos A = 1 - \sin A.$$

Consequently, if solutions were found this way, they would have to be put into the original equation and checked to exclude the solutions of this spurious equation.

EXERCISES

(1) Solve the following equations for values of A between 0° and 360°:

(i) $\cos A = -\frac{1}{2}$
(ii) $\operatorname{cosec} A = 2$
(iii) $2 \tan^2 A = 1$
(iv) $8 \cos^2 A = 9 - 6 \sin A$

(2) Give the general solutions of the following equations both in degrees and in radians:

(i) $\sin A = 0$
(ii) $\sin A = 1$
(iii) $\cos^2 A = 1$
(iv) $\tan^3 A = -1$

(3) Solve the following equations for the range $0^\circ \leqslant A \leqslant 360^\circ$:

(i) $\sin A + \sin 2A = 0$
(ii) $\cos 4A + \cos 2A = 2\cos^2 A$
(iii) $\sin A + \cos A = \sin 3A + \cos 3A$
(iv) $\cos 5A + \cos 3A = \cos A$.

(4) Show that the following equations have *no* real roots:

(i) $3\sin A = 4\operatorname{cosec} A$
(ii) $\tan A + 5\cot A + 2 = 0$
(iii) $\sin 2A + 6 = 3\sin A + 4\cos A$.

(5) The following relations consist partly of identities and partly of equations. Prove the identities and solve the equations, taking the range as $0^\circ < A < 360^\circ$.

(i) $2\sin A = \sin 2A$
(ii) $\sin 2A = 1 + \cos 2A$
(iii) $4\sin\frac{1}{2}A\cos\frac{1}{2}A = 1$
(iv) $\sin^2\frac{1}{2}A = (1 - \cos\frac{1}{2}A)(1 + \cos\frac{1}{2}A)$
(v) $\sin A + \sin 2A = \sin 3A$
(vi) $\sin 3A = \sin A(1 + 2\cos 2A)$.

(6) Find the general solutions of the equation:

$$\cos A - \sin A = \cos 2A.$$

ANSWERS TO EXERCISES IN LESSON 17

(1) (i) $x = 10$; (ii) $x = -1$; (iii) $x = 1\frac{1}{5}$
(2) (i) $x = 6, y = -2$; (ii) $x = 5, y = 1$
(iii) $x = 5, y = 8$
(3) (i) $x = -4, x = 7$ (ii) $x = -9, x = 8$
(iii) $x = 0, x = -12$ (iv) $x = 3$
(v) $x = 0, x = 1, x = -5$ (vi) $x = 3$, $x = -1$
(4) (i) $x = \frac{1}{2}(9 \pm \sqrt{13})$ (ii) $x = +6$, $x = -4\frac{1}{2}$ (iii) $x = -3 \pm \sqrt{2}$
(6) (i) sum $= -{}^7/_3$, product $= {}^{11}/_3$
(ii) sum $= -1$, product $= -10$
(iii) sum $= -2$, product $= 0$.

Roots in (i) are imaginary, in (ii) and (iii) are real.

(7) Four roots, $x = \pm 3, \pm 2$
(8) Real root is $x = 3$
(9) $x = 2, x = 4, x = -1$

LESSON 19

The Binomial Theorem

THIS important mathematical method, first formulated by Newton, the greatest of English mathematicians, provides a method of raising the sum of two terms to any power (binomial means "two names"). It is widely used for transforming and simplifying algebraic expressions.

It can easily be verified by direct multiplication that

$$(1 + x)^2 = 1 + 2x + x^2$$
$$(1 + x)^3 = 1 + 3x + 3x^2 + x^3$$
$$(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$$
$$(1 + x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5.$$

If you care to take sufficient trouble it is possible to obtain the coefficients of each power of x in the expression for $1 + x$ raised to any given power.

Can a general expression be found to represent this process? Can one find the expansion of $(1 + x)^n$ where n is any positive whole number? If we write:

$$(1 + x)^n = 1 + ax + bx^2 + cx^3 + \ \ldots\ldots + x^n$$

can we find the numbers $a, b, c, \ldots$ in terms of n, so that these coefficients could be written down at once no matter how large n is?

A glance at the formulae gives several clues to the general form.

Consider the following triangle of numbers (invented by the French mathematician, Pascal).

$$\begin{array}{lllllll}
1 \\
1 & 1 \\
1 & 2 & 1 \\
1 & 3 & 3 & 1 \\
1 & 4 & 6 & 4 & 1 \\
1 & 5 & 10 & 10 & 5 & 1 \\
1 & 6 & 15 & 20 & 15 & 6 & 1
\end{array}$$

in which each term equals the sum of the term immediately above and the term above and to the left. It may be noticed that the numbers in any row are just the coefficients of successive powers of x in the expansion of $(1 + x)^n$ for the appropriate value of n. These coefficients are called binomial coefficients, and you write $\binom{n}{r}$ for the coefficient of x^r in the expansion of $(1 + x)^n$. Pascal's triangle gives the clue how to express $\binom{n}{r}$ and to prove the binomial theorem. We introduce the symbol $n!$ (called *n factorial* or *factorial n*) for $1 \times 2 \times \ldots \times n$. Thus $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, and so on. What we will prove is that

$$\binom{n}{r} = \frac{n!}{r!\,(n - r)!} \qquad \ldots\ldots\ldots\ (i)$$

This formula also works for $r = 0$ and $r = n$ if you put $0! = 1$. Notice that

$$\binom{n}{r} = \binom{n}{n - r} \text{ and that,}$$

$$\text{since } \frac{n!}{(n - r)!} = n(n - 1)\ \ldots\ (n - r + 1),$$

$$\binom{n}{r} = \frac{n(n - 1)\ \ldots\ (n - r + 1)}{r!} \qquad \ldots\ (ii)$$

We will prove (i) by an argument of great importance in mathematics, namely, proof by induction.

Proof by Induction

We demonstrate this method of proof by the following example.

Example. $1 + 2 + 3 + \ \ldots + n = \dfrac{n(n + 1)}{2}$

This formula is certainly true if $n = 1$, since $1 = \dfrac{1.2}{2}$

We now suppose that k is a particular value of n for which the formula is true. Thus

$$1 + 2 + 3 + \ldots + k = \frac{k(k + 1)}{2}.$$

It follows that $1 + 2 + 3 + \ldots + k + (k + 1) = \dfrac{k(k + 1)}{2} + k + 1 = \dfrac{k + 1}{2}(k + 2) = \dfrac{(k + 1)(k + 2)}{2}$'

which is the formula for $n = k + 1$. The principle of induction states the obvious fact that if a formula holds for $n = 1$ and holds for $n = k + 1$ whenever it holds for $n = k$, it then holds for all (positive integer) values of n. Thus the formula is completely proved. We carry out one further example.

Example. $1 + r + r^2 + \ldots + r^n = \dfrac{1 - r^{n+1}}{1 - r}$.

This holds if $n = 1$, since $1 + r = \dfrac{1 - r^2}{1 - r}$.

Now suppose it holds for $n = k$,

so that $1 + r + r^2 + \ldots + r^k = \dfrac{1 - r^{k+1}}{1 - r}$.

Then $1 + r + r^2 + \ldots + r^{k+1} = \dfrac{1 - r^{k+1}}{1 - r} + r^{k+1}$

$$= \frac{1 - r^{k+1} + r^{k+1} - r^{k+2}}{1 - r} = \frac{1 - r^{k+2}}{1 - r}.$$

This shows that the formula holds for $n = k + 1$ and completes the proof by induction.

Proof of the Binomial Theorem

We now return to the binomial theorem. Recall that this says that

$$(1 + x)^n = 1 + \binom{n}{1}x + \ldots + \binom{n}{r}x^r + \ldots + x^n,$$

where $\dbinom{n}{r} = \dfrac{n!}{r!\,(n - r)!} = \dfrac{n(n - 1)\ldots(n - r - + 1)}{r!}$.

The theorem is true if $n = 1$, since $(1 + x)^1 = 1 + x$. We suppose it holds for $n = k$. Then

$$(1 + x)^k = 1 + \ldots + \binom{k}{r}x^r + \ldots + x^k, \text{ and}$$

$$(1 + x)^{k+1} = (1 + \ldots + \binom{k}{r}x^r + \ldots + x^k)(1 + x).$$

Multiplying out the R.H.S. we find that the coefficient of x^r is $\dbinom{k}{r} + \dbinom{k}{r - 1}$. Thus all that remains is to prove that $\dbinom{k + 1}{r} = \dbinom{k}{r} + \dbinom{k}{r - 1}$ . . . . . . *(iii)*

Before proving this, we remark that (iii) is exactly the rule given earlier for calculating the entries in Pascal's triangle, where, of course, k refers to the row and r to the column of the triangle.

Finally, we prove (iii). Now $\dbinom{k}{r} + \dbinom{k}{r - 1} =$

$$\frac{k(k - 1)\ldots(k - r + 1)}{r!} + \frac{k(k - 1)\ldots(k - r + 2)}{(r - 1)!}$$

$$= \frac{k(k - 1)\ldots(k - r + 2)}{r!}\left\{k - r + 1 + r\right\}$$

$$= \frac{(k + 1)\,k(k - 1)\ldots(k - r + 2)}{r!} = \binom{k + 1}{r},$$

and the binomial theorem is completely proved.

Example. Let $n = 6$, then

$$(1 + x)^6 = 1 + 6x + \frac{6 \times 5}{2!}x^2 + \frac{6 \times 5 \times 4}{3!}x^3$$
$$+ \frac{6 \times 5 \times 4 \times 3}{4!}x^4 + \frac{6 \times 5 \times 4 \times 3 \times 2}{5!}x^5 + \frac{6!\,x^6}{6!}$$
$$= 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$$

If in the expression for the Binomial Theorem you write x/a in place of x you find

$$\left(1 + \frac{x}{a}\right)^n = 1 + n\frac{x}{a} + \frac{n(n - 1)}{2!}\frac{x^2}{a^2} + \frac{n(n - 1)(n - 2)}{3!}\frac{x^3}{a^3} + \ldots$$

But $\left(1 + \dfrac{x}{a}\right)^n = (a + x)^n/a^n$.

Hence multiplying both sides by a^n you have

$$(a + x)^n = a^n + na^{n-1}x + \frac{n(n - 1)}{2!}a^{n-2}x^2 + \frac{n(n - 1)(n - 2)}{3!}a^{n-3}x^3 + \ldots$$

In this a and x may have any values you please, but n must be a positive whole number.

Example. If $a = x = 1$ then

$$1 + n + \frac{n(n - 1)}{2!} + \frac{n(n - 1)(n - 2)}{3!} + \ldots + n + 1 = 2^n,$$

that is, the sum of the coefficients in the binomial expansion is 2^n.

Example. If $a = 1$, x [illegible] 1, then

$$1 - n + \frac{n(n - 1)}{2!} - \frac{n(n - 1)(n - 2)}{3!} + \ldots\ldots = 0.$$

Example. To estimate the value of $1{\cdot}01^7$ to five decimal places

$$(1{\cdot}01)^7 = (1 + 0{\cdot}01)^7$$
$$= 1 + 7 \times 0{\cdot}01 + \frac{7 \times 6}{2!}({\cdot}01)^2 + \frac{7 \times 6 \times 5}{3!}({\cdot}01)^3 + \ldots$$

= 1·00000
·07000
·00210
·000035
1·072135

We have brought the expansion of $(1{\cdot}01)^7$ to an end at the term in $({\cdot}01)^3$ since subsequent terms cannot affect the fifth or even the sixth decimal place.

Application of the Binomial Theorem

Suppose you have to solve the equation

$$x^3 - 6x^2 + 11x - 6 = 0 \quad \ldots\ldots\ldots\ldots \quad (i)$$

To do this you try first to factorise the L.H.S. by means of the Remainder Theorem. It is then easily found that the equation is equivalent to

$$(x - 1)(x - 2)(x - 3) = 0,$$

with roots $x = 1$, $x = 2$, $x = 3$.

Suppose, however, the equation were not exactly (i) but, say,

$$x^3 - 6x^2 + 11x - 6{\cdot}1 = 0 \quad \ldots\ldots\ldots \quad (ii)$$

the difference being simply that the last term is 6·1 in place of 6. It is then obvious that the solutions of (ii) must be only slightly different from 1, 2, 3.

Suppose then that $x = 1 + y$ where y is expected to be very small. This value of x must satisfy (ii) ; hence

$$(1 + y)^3 - 6(1 + y)^2 + 11(1 + y) - 6{\cdot}1 = 0.$$

By means of the Binomial Theorem you can write this :

$$(1 + 3y + 3y^2 + y^3) - (6 + 12y + 6y^2) + 11 + 11y - 6{\cdot}1 = 0$$

or $\quad -0{\cdot}1 + 2y - 3y^2 + y^3 = 0 \quad \ldots\ldots\ldots \quad (iii)$

Now y is small in comparison with 1. It follows that y^2 and y^3 must be small compared with y. Hence in equation (iii), tentatively assume that

the terms $-3y^2$ and y^3 can be neglected in comparison with the others. Thus equation (iii) takes the approximate form

$$-0{\cdot}1 + 2y = 0, \text{ i.e. } y = 0{\cdot}05$$

You can now test whether you can neglect $-3y^2 + y^3$; for with this value of $y = 0{\cdot}05$, $-3y^2 + y^3$ is 0·007625, which is less than 8 per cent. of each of the other terms retained. Accordingly as a first approximation to a solution of equation (ii) you have

$$x = 1{\cdot}05$$

For an even closer approximation you could repeat this process by replacing x in equation (ii) by $x = 1{\cdot}05 + z$, and neglecting terms of higher order than the first in the resulting equation.

Again, the remaining two roots of equation (ii) can be found approximately by inserting $x = 2 + y$ and $x = 3 + y$ respectively in the equation since the solutions cannot be far removed from 2 and 3.

Thus inserting $x = 2 + y$ into equation (ii),

$$(2 + y)^3 - 6\,(2 + y)^2 + 11\,(2 + y) - 6{\cdot}1 = 0$$

Expanding by the Binomial Theorem and neglecting all terms in y beyond the first, you have

$$8 + 12y - 6\;(4 + 4y) + 11\;(2 + y) - 6{\cdot}1 = 0$$

i.e. $$-y - 0{\cdot}1 = 0$$

Thus

$$y = -0{\cdot}1$$

and the approximate root is

$$x = 2 + y = 1{\cdot}9$$

Again, for the solution which is close to $x = 3$ you write $x = 3 + y$ and insert in equation (ii). Thus:

$$(3 + y)^3 - 6\,(3 + y)^2 + 11\,(3 + y) - 6{\cdot}1 = 0$$
$$27 + 27y - 54 - 36y + 33 + 11y - 6{\cdot}1 = 0$$
$$2y - 0{\cdot}1 = 0$$

i.e. $$y = 0{\cdot}05$$

Hence $$x = 3 + y = 3{\cdot}05$$

is the approximate solution.

EXERCISES

(1) Write down the expansions:

(i) $(1 + x)^4$
(ii) $(a - b)^5$
(iii) $(y + y^{-1})^6$

(2) By expanding $(1 + x)^7$, check that the sum of the coefficients is 2^7.

(3) By using the Binomial Theorem evaluate $(1{\cdot}11)^8$ correct to 3 places after the decimal point.

(4) Show by expanding $(1 - x)^4$ and $(1 + x)^4$ by the Binomial Theorem and multiplying the results together that the final result is the same as expanding $(1 - x^2)^4$ by the Binomial Theorem.

(5) Write down the first five terms in the expansion of $(x + y)^{12}$.

(6) Show that the approximate value of $(1 - x)^7 (1 + x)^4 (1 - 2x)$ is $1 - 5x + 5x^2$ if powers of x above the second are negligible.

(7) Use the Remainder Theorem to factorise $x^3 + x^2 - 14x - 24$.

Hence state an approximation to the positive root of

$$x^3 + 0{\cdot}9x^2 - 14{\cdot}1x - 24 = 0$$

and find a closer approximation.

(8) Prove by induction that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n + 1)\;(2n + 1)}{6}$.

(9) Find all the roots of the equation

$$x^3 - 6x^2 + 11{\cdot}2x - 6 = 0$$

to one figure after the decimal place.

(10) Find a so that $x + 3$ is a factor of

$$x^3 + ax^2 + 4x + 3,$$

and find the other factor. Calculate accurately to the first decimal place, the real root of

$$x^3 + 3{\cdot}9x^2 + 4x + 3 = 0.$$

Answers To Exercises In Lesson 18

(1) (i) 120°, 240° (ii) 30°, 150°
(iii) 35° 16′, 144° 44′, 215° 16′, 324° 16′
(iv) 14° 29′, 30°, 150°, 165° 31′

(2) (i) m. 180° or m π radians
(ii) m. 360° + 90° or (4m + 1) $\frac{\pi}{2}$ radians
(iii) m. 180° or m π radians
(iv) m. 180° − 45° or (4m − 1) $\frac{\pi}{4}$ radians

(In each case m is any integer)

(3) (i) 0°, 120°, 180°, 240°, 360°
(ii) 0°, 90°, 180°, 270°, 360°
(iii) 0°, 22½°, 112½°, 180°, 202½°, 292½°, 360°
(iv) 15°, 75°, 90°, 105°, 165°, 195°, 255°, 270°, 285°, 345°

(5) (i) Equation: 180°
(ii) Equation: 45°, 90°, 225°, 270°
(iii) Equation: 30°, 150°
(iv) Identity
(v) Equation: 120°, 180°, 240°
(vi) Identity

(6) 2m π, (4m + 1) $\frac{\pi}{2}$, (4m + 1) $\frac{\pi}{4}$ radians

LESSON 20

Algebraic Geometry: First Steps

Let us review for a few moments the course that these mathematical studies has taken so far. We began with arithmetic and geometry, the ideas of number and shape, and from these beginnings we developed new branches; the arithmetic became generalised as we pursued the study of algebra, the geometry became more definite with the ideas of trigonometry. Now we are going to link the subjects of algebra and geometry, because they have a very close affinity.

Consider the following simple example. Suppose you have two lines at right angles and a whole series of points which are equidistant from both these lines (Fig. 93), then if you choose the lines at right angles to be co-ordinate axes as in Lesson 9, for every one of the points the x co-ordinate is equal to the y co-ordinate.

In addition a straight line will pass through all the points ; also for every point on this line you will have the relation

$$y \text{ co-ordinate} = x \text{ co-ordinate} \quad \dots\dots \quad (i)$$

Again, it is evident that this relation will not be true for any point which does not lie on the straight line ; so that relation (i) fixes a particular straight line. You can rewrite this relation more simply as

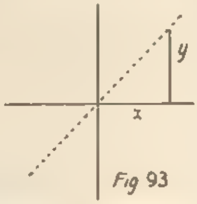


Fig 93

$$y = x,$$

an algebraic equation which defines a geometrical line.

The student now has an inkling of the connexion between the two subjects, and thus is on the threshold of a new branch of study, the subject of *algebraic geometry*, or *co-ordinate geometry* as it is frequently called because of the use that is made of the idea of co-ordinates. This idea was introduced in Lesson 9, and this use of co-ordinates must be considered in more detail.

Distances Between Points

If the co-ordinates of two points are given, you can easily find the length of the straight line joining them. Consider an actual example. To find the length of the straight line joining (3, 1) to (5, 5). In problems on co-ordinate geometry it is always of great help to draw a rough diagram as in Fig. 94. If A, B represent the two points and AN is drawn parallel to the X axis, the length of AN is the x co-ordinate of B—the x co-ordinate of A, i.e. 2 units. Similarly BN is the y co-ordinate of B—the y co-ordinate of A, i.e. 4 units. Thus, from the Theorem of Pythagoras,

$$AB^2 = 4^2 + 2^2 = 20, \text{ i.e. } AB = \sqrt{20}.$$

Consider one more example. To find the distance of (3, 1) from (−5, −5). Draw a diagram (Fig. 95) and construct the right-angled triangle ABN as in the previous example. BN is numerically equal to 3 + 5, i.e. 8, and AN is numerically equal to 1 + 5, i.e. 6 units. Thus

$$AB = \sqrt{(8^2 + 6^2)} = \sqrt{100} = 10 \text{ units.}$$

Division of a Line in Any Ratio

If you know the co-ordinates of the ends of a straight line, you can use similar triangles to find the co-ordinate of a point dividing this line in any ratio. Suppose A is the point (3, 6) and B the point (−7, 1) and you require the co-ordinates of a point P such that $AP/PB = 2/3$ (Fig. 96). Draw AN and BN parallel to the Y and X axes respectively and do similarly with PM and PL. Now BN = 3 + 7, i.e. 10 units, and AN = 6 − 1, i.e. 5 units. From similar triangles PM/AN = BP/BA

i.e. $$\frac{PM}{5} = \frac{3}{5}$$

i.e. $$PM = 3 \text{ units.}$$

The y co-ordinate of P = PM + 1 because M is at a distance of 1 unit from the axis of X, i.e. y co-ordinate of P = 4 units.

Similarly,

$$PL/BN = AP/AB$$

i.e. $$\frac{PL}{10} = \frac{2}{5}$$

i.e. $$PL = 4 \text{ units.}$$

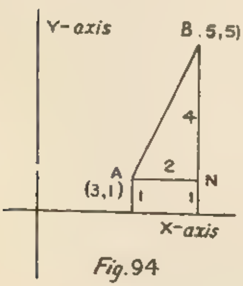
But L is 3 units from the Y axis, therefore the x co-ordinate of P is −1, i.e. P is the point (−1, 4).

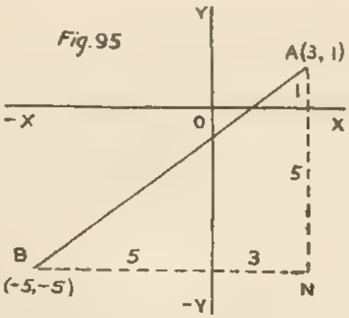
Areas

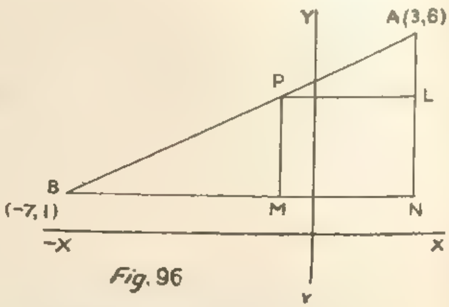
If you know the co-ordinates of the vertices of a triangle or any other figure, its area can be easily calculated. Again an actual example will suffice to explain the method.

Example. To find the area of the triangle formed by joining the three points (1, 1), (3, 5) and (7, 3). ABC (Fig. 97) represents the triangle, and through its vertices are drawn lines parallel to the axes, thus making a rectangle round the sides of which have been placed numbers representing the lengths of the various portions found from the co-ordinates of the vertices.

The three figures in circles denote the areas of the three right-angled triangles formed, in square units.







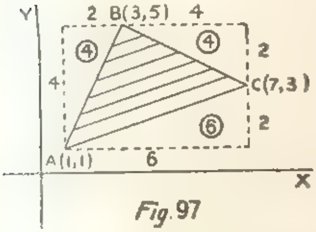
CO-ORDINATE GEOMETRY. Figs. 94 and 95. Given co-ordinates, how to find the length of lines joining them. Fig. 96. Given co-ordinates, how to divide a line in any ratio.

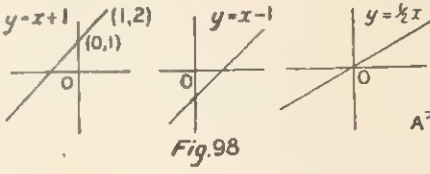
The area of the whole rectangle is 6×4, i.e. 24 square units, so that the area of ABC is $24 - 4 - 4 - 6 = 10$ square units.

Straight Lines

In the early part of this Lesson we found the equation which represented a particular line ; the equation was $y = x$, the line bisected the angle between the axes and passed through the *origin* (the point of intersection of the axes). We may equally well consider the equation $y = x + 1$ or $y = x - 1$ or $y = \frac{1}{2}x$.

It will not be difficult to convince yourself that these equations also represent lines, for you have only to plot a few points whose co-ordinates satisfy the equation to see the position of the line. Thus (Fig. 98) for the line $y = x + 1$, when $x = 0$, $y = 1$, when $x = 1$, $y = 2$, and when $x = 2$, $y = 3$, and so on, so that you find that $y = x + 1$ represents another straight line parallel to that given by $y = x$ but above it.





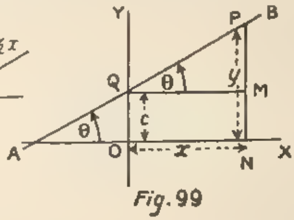


Fig. 97. Finding the area of a triangle. Figs. 98 and 99. Equations for straight lines.

The lines represented by $y = x - 1$ and $y = \frac{1}{2}x$ are also shown ; notice that the line defined by $y = \frac{1}{2}x$ is closer to the horizontal than the others. The student should try drawing some other lines, for example $y = x + 2$, $y = 3x$, $y = \frac{1}{3}x$, $y = \frac{1}{4}x + 1$, and two important facts emerge.

First, that $y = Ax$ (where A is any number, positive, negative or fractional) always represents a line passing through the origin ; and, second, that the value of A has some connexion with the angle that the line makes with the X axis.

For instance, you know that when $A = 1$ the line is at 45° to the X axis, and it will be found that when $A > 1$, the line is steeper and when $A < 1$ the line is less steep than $y = x$. Again, when A is negative the line will be in the second and fourth quadrants. More light will be thrown on this if you attempt to find the equation of *any* straight line.

Equation of a Straight Line

Take any straight line AB (Fig. 99) and choose any point P on it. Draw PN perpendicular to the X axis. If O is the origin, then ON, PN represent the x and y co-ordinates respectively of the point P. We shall denote them simply by x and y.

The problem is thus to determine the relation between x and y which holds for the line AB and for no other line. Suppose that the angle which AB makes with the X axis is represented by the Greek letter θ, then the line AB is still not fixed because any number of parallel lines may all be drawn making the same angle θ with the X axis. If Q is the point of intersection of AB with the Y axis, let $OQ = c$. Draw QM parallel to the X axis, then

$$PN = PM + MN,$$
$$= PM + OQ \text{ since OQMN is a rectangle,}$$

i.e. $$y = PM + c \quad \ldots\ldots\ldots\ldots \quad (i)$$

Also $\frac{PM}{QM} = \tan\theta$, and $QM = ON = x$;

thus $PM = x \tan\theta$,

and (i) becomes

$$y = x\tan\theta + c \quad \ldots\ldots\ldots \quad (ii)$$

Conversely, any point whose co-ordinates satisfy (ii) certainly lies on the line. Thus (ii) is the equation of AB, and if $\tan\theta$ (termed the *slope* or *gradient* of AB) is written as m (ii) becomes

$$y = mx + c \quad \ldots\ldots\ldots\ldots \quad (iii)$$

This is the fundamental equation of a straight line ; it represents any particular straight line in terms of its slope (m) and its intercept on the Y axis (c), provided that you always measure θ as the angle made with the positive X axis.

Example. Find the equation of the straight line passing through $(-1, 0)$ and $(0, -2)$.

If a rough sketch is made it will be seen that this line cuts both the negative X and negative Y axes, and consequently c (the intercept on the Y axis) is -2. Also $\theta > 90°$ so that $\tan\theta$ is negative, its numerical value being $\frac{2}{1}$ i.e. $m = -2$. Therefore the equation of this line is $y = -2x - 2$.

Equation of a Line of Given Slope through a Fixed Point

If the slope of a straight line is given and the co-ordinates of a point in it are known, then the straight line is completely fixed, so that you are able to find its equation in terms of these fixed quantities. It is convenient to denote the

co-ordinates of a fixed point in general terms by (x_1, y_1), then if the given slope is m, the equation to our line is

$$y = mx + c \quad \ldots\ldots\ldots\ldots \quad (iv)$$

Since this line is to pass through the fixed point (x_1, y_1), these co-ordinates must satisfy relation (iv); hence

$$y_1 = mx_1 + c \quad \ldots\ldots\ldots\ldots \quad (v)$$

Subtracting (v) from (iv), we obtain

$$\begin{aligned} y - y_1 &= mx - mx_1 \\ &= m(x - x_1) \quad \ldots\ldots\ldots\ldots \quad (vi) \end{aligned}$$

Example. To find the equation of the straight line passing through $(-3, 7)$ such that the angle (θ) which it makes with the X axis is given by $\tan\theta = 2$.

Here $m = 2$, so that the general equation of any straight line (iii) becomes in this case

$$y = 2x + c \quad \ldots\ldots\ldots\ldots \quad (vii)$$

This has to pass through the point $(-3, 7)$ so that (vii) must be satisfied by $x = -3$, $y = 7$

i.e. $\quad 7 = 2(-3) + c.$

This gives $c = 13$ directly so that (vii) becomes

$y = 2x + 13.$

You can obtain the same result by using the general equation (vi), for

$$y - 7 = 2(x + 3)$$

i.e.

$$\begin{aligned} y &= 7 + 2x + 6 \\ &= 2x + 13. \end{aligned}$$

Equation of a Line Joining Two Known Points

Suppose the co-ordinates (x_1, y_1), (x_2, y_2) of two points are known. Then from the diagram (Fig. 100) it is clear that

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

so that equation (vi) may be written:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \quad \ldots\ldots \quad (viii)$$

You may remember (viii) most easily in the form

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

The student is advised to consider the geometrical significance of this equation. Of course, (viii) may also be found directly by the repeated use of the fundamental equation $y = mx + c$. Consider an example.

Example. Find the equation of the straight line joining the points (3, 4) and $(-3, -1)$.

Since both these points must, in turn, satisfy the equation $y = mx + c$, you obtain

$$4 = 3m + c \quad \ldots\ldots\ldots\ldots \quad (ix)$$

and

$$-1 = -3m + c \quad \ldots\ldots\ldots\ldots \quad (x)$$

Adding these two equations,

$$3 = 2c, \text{ i.e. } c = \frac{3}{2}$$

and subtracting (x) from (ix)

$$4 - (-1) = 3m - (-3m), \text{ i.e. } 6m = 5 \text{ or } m = \frac{5}{6}$$

Thus $\quad y = mx + c$ becomes $y = \frac{5}{6}x + \frac{3}{2}$ or

$6y = 5x + 9.$

This is the equation to the required line and it can be written down immediately by using the general equation (viii). Taking (x_1, y_1) as the point (3, 4) and (x_2, y_2) as $(-3, -1)$, equation (viii) becomes

$$y - 4 = \frac{-1 - 4}{-3 - 3}(x - 3)$$

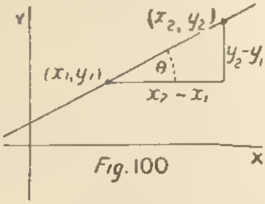
i.e. $\quad y - 4 = \frac{5}{6}(x - 3)$

or $\quad 6y = 24 + 5x - 15$, i.e. $6y = 5x + 9$.

Point of Intersection of Two Lines

If you know the equations of two straight lines you can find the co-ordinates of their point of intersection (provided the lines are not parallel) simply by solving the two equations together, i.e. by treating the two equations as simultaneous (*see* Lesson 17). The reason for this is that the point of intersection is the only point on both lines, i.e. its co-ordinates must satisfy both equations at the same time.

Example. Find the point of intersection of the lines represented by $y = x + 1$ and $y = 2x + 3$.



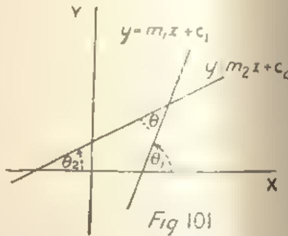


Fig. 100. Equation for a line joining two given points. **Fig. 101.** Finding the angle between two lines.

Treat these as simultaneous equations, thus:

$$y = x + 1 \quad \ldots\ldots\ldots\ldots \quad (xi)$$
$$y = 2x + 3 \quad \ldots\ldots\ldots\ldots \quad (xii)$$

Subtract (xi) from (xii)

$$0 = x + 2, \text{ i.e. } x = -2.$$

Therefore from (xi) $y = x + 1 = -2 + 1 = -1$. Thus the point of intersection is $(-2, -1)$. The student should draw these lines accurately and verify this result.

Angle Between Two Straight Lines

Consider the diagram (Fig. 101); it represents two straight lines which have the equations $y = m_1x + c_1$ and $y = m_2x + c_2$. These lines make angles θ_1 and θ_2 respectively with the X axis, so that $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$. You require to find the angle θ between these two lines. From geometry you know that the exterior angle θ_1 is equal to the sum of the two interior opposite angles (θ_2 and θ), thus

$$\theta_1 = \theta_2 + \theta$$

so that

$$\theta = \theta_1 - \theta_2$$

therefore

$$\tan\theta = \tan(\theta_1 - \theta_2)$$

and you can use your knowledge of trigonometry to expand the R.H.S. Thus

$$\begin{aligned} \tan\theta &= \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1\tan\theta_2} \\ &= \frac{m_1 - m_2}{1 + m_1 m_2} \quad \ldots\ldots\ldots \quad (xiii) \end{aligned}$$

This gives the tangent of the angle between the two lines in terms of their slopes. You can find the angle between the two lines used in the last example, namely $y = 2x + 3$ and $y = x + 1$. Here

$$m_1 = 2, m_2 = 1$$

so that

$$\tan \theta = \frac{2 - 1}{1 + 1.2} = \frac{1}{3}$$

and from tables $\theta = 18° 26'$. Notice that if you had taken $m_1 = 1$ and $m_2 = 2$ you would have found $\tan \theta = -\frac{1}{3}$ and then you would have found the obtuse angle between the two lines i.e. $180° - 18° 26'$.

Relation (xiii) is very important since it enables you to find the relation which exists between the slopes of two lines if they are either parallel or at right angles. If two lines are parallel, $\theta = 0°$ and so you must have

$$\frac{m_1 - m_2}{1 + m_1 m_2} = 0, \text{ i.e. } m_1 - m_2 = 0$$

or $\quad m_1 = m_2.$

This states what has already been discovered, namely, that if two lines are parallel, their slopes are equal. If two lines are at right angles, then $\theta = 90°$ and $\tan 90°$ is infinitely large. Therefore, in this case, you must have

$$1 + m_1 m_2 = 0$$

or $\quad m_1 = -\frac{1}{m_2}$

i.e. the slope of one line must be the negative reciprocal of the slope of the other. For example, $y = \frac{1}{3}x + 7$ is at right angles to $y = -3x + 4$ because $-\frac{1}{(-3)} = +\frac{1}{3}$.

EXERCISES

(1) Plot the points $(-4, 3)$, $(5, -6)$. Calculate the distance between them and check by measurement.

(2) Find by calculation the mid-point of the line joining $(-3, -4)$ to $(8, 7)$ and check by drawing.

(3) A triangle ABC has its vertices A, B, C at the points $(3, 0)$, $(-2, -1)$, $(0, 2)$. Find the lengths of AB, AC, BC and show that ABC is an isosceles right-angled triangle. Find its area.

(4) In the previous question the square ABCD is completed: what are the co-ordinates of D? Find the equations to the diagonals, prove that they are at right angles and find their point of intersection.

(5) Find the equations of the straight lines :
(i) of slope 5 through the point $(1, 7)$;
(ii) of slope $\frac{1}{2}$ through the point $(1, -3)$;
(iii) of slope -3 through the origin.

(6) Show that the line joining $(1, 1)$ to $(-3, -1)$ is parallel to that joining $(4, 0)$ to $(0, -2)$.

(7) Show that the line joining $(-1, 3)$ to $(5, 1)$ is perpendicular to the line joining $(2, 2)$ to $(0, -4)$.

(8) Calculate the angle between the lines $2y = x + 4$ and $x + 4y = 20$ and find their point of intersection. Check these results by drawing.

(9) Find the equation to the line which is perpendicular to $y = 4x - 3$ and passes through the point $(7, 3)$.

(10) Show that if the lines $y = m_1 x + c_1$ and $y = m_2 x + c_2$ are inclined at 45°, then $m_2 = \frac{m_1 - 1}{m_1 + 1}$.

ANSWERS TO EXERCISES IN LESSON 19

(1) (i) $1 + 4x + 6x^2 + 4x^3 + x^4$
(ii) $a^5 - 5a^4 b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$
(iii) $y^6 + 6y^4 + 15y^2 + 20 + 15y^{-2} + 6y^{-4} + y^{-6}$

(3) 2·304

(5) $x^{12} + 12x^{11}y + 66x^{10} y^2 + 220x^9y^3 + 495x^8y^4$

(7) $(x + 2)(x + 3)(x - 4)$: first approximation is $x = 4$, closer approximation is $x = 4{\cdot}0487$.

(9) $x = 0{\cdot}9$, $x = 2{\cdot}5$, $x = 2{\cdot}7$

(10) $a = 4$, $x^2 + x + 1$, $x = -2{\cdot}9$

LESSON 21

Problem Solutions : Lines and Circles

It has been seen (Lesson 16) how the scientist or technician extracts the measurable features of a complex situation and how the mathematician subjects them to calculation.

What is Meant by Solution of a Problem ?

There is a danger at this stage of which the student must beware. Contrast the picture presented by the mathematician with the real situation. He examines a building. Its walls are, say, 30 ft. long by 10 ft. high. A door has to be inserted in one wall 3 ft. 6 in. wide and 6 ft. high, and two windows each 3 ft. by 4 ft. in another. The picture he presents to himself is one of rectangles bounded by idealised lines. In this way he can calculate the wall space—if it has to be whitewashed—the amount of wood required for door and windows, and the amount of glass. He is concerned only with shape and size, and the result he finds is an absolutely correct answer *for his mathematical picture*.

The builder, the painter, the carpenter, and the glazier are made of different stuff, however. They have to deal with real walls, doors, and windows, not figments of the mathematician's imagination. They know the walls are not plane surfaces, that the edges are not mathematician's straight lines but boundaries formed by bricks and mortar, and that the door, window-frames, and glass must have a certain amount of "play" if they are to function. The mathematician's calculations are therefore wrong from their standpoint. It is not that they are not exact enough or that they are too

exact ; it is simply that the mathematician has solved a *different problem.*

Practical and Mathematical Problems

It is not very different, however ; but it is different. Two conclusions follow. First, the practical men are glad to have the mathematician's calculation, but they have to readjust his results in the light of the " give and take " with which they have to work. Second, the mathematician need not work out approximations to his solution of the problem. If he finds on calculation that the length of a certain beam is $\sqrt{5}$ ft., this is exact for his picture, but as it stands it is not the solution for the practical man. Now

$$\sqrt{5} = 2{\cdot}23606 \; . \; . \; . \; .$$

If the practical man wants to use his mathematics, he would regard the length of the beam not as $\sqrt{5}$ ft., but as

2 ft., or 2·2 ft. or 2·24 ft. or 2·236 ft. or 2·2361 ft. according to the degree of detail with which the practical man works.

To say it is 2·2 ft. is to neglect 0·04 ft. approximately, or about $\frac{1}{2}$ in.

To say it is 2·24 ft. is to neglect 0·004 ft. approximately, or $\frac{1}{20}$ in., which is a reasonable amount for a carpenter to neglect in his measurements.

To say it is 2·236 ft. is to neglect 0·0001 ft. approximately, or about 1/1000 in.

A sensible answer for the practical man, therefore, is not that the beam is $\sqrt{5}$ ft. long but that it is 2·24 ft., or 2 ft. $2\frac{7}{8}$ in. if the carpenter is working to sixteenths of an inch.

We can proceed a step further in this discussion. The mathematician will often present, as the solution to a problem, a rather complicated formula asserting that this is the answer. As an answer to a mathematician it may be exemplary in the sense that from it he can derive a series of rather general propositions about the class of problem in question. It may provide the basis for a close study of the properties of the abstract problem he has posed to himself ; but it may in that form be of little use to the practical man. In the end the latter requires an answer in terms of definite numbers that he can set out as measurements or as a guide in practice. Therefore all formulae must be tabulated to some stated degree of accuracy in order that they may be reduced to a usable form ; and until the formula has been presented in this way it cannot be said adequately to represent the solution.

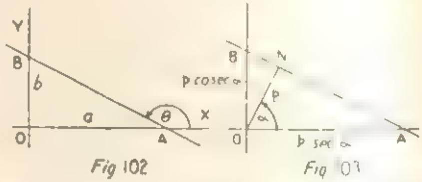
Since the mathematician's picture is necessarily a simplification of the hard reality from which it is drawn, the conclusions at which he arrives cannot be said to be proved *true* of the situation. They are attempts—perhaps groping attempts—to visualise the complexities of the real problem. The results of the mathematics must be tested against the real situation (by experiment, for example) in order to find out whether the simplifying assumptions introduced to render the problem susceptible to mathematical analysis, have led to a practically useful conclusion.

LINES AND CIRCLES

The study of the straight line is of fundamental importance in co-ordinate geometry and we will consider some further properties. In Lesson 20 we derived the equation to any straight line in the form $y = mx + c$, i.e. in terms of its slope and its intercept on the Y axis, and we used this equation to obtain various results. In this Lesson we shall obtain the general equation to any straight line in various forms, these being more suitable for certain purposes.

Equation to a Straight Line in "Intercept" Form

Suppose that a and b are the intercepts on the X and Y axes respectively, made by any straight line (Fig. 102). Then it is a simple



Figs. 102 and 103. Equations to straight lines in " intercept " and in " perpendicular " form.

matter to obtain our general equation $y = mx + c$ in terms of the intercepts a and b. For the line we have drawn, $180° > \theta > 90°$, and so tan θ is negative and numerically equal to b/a. Thus $m = -\frac{b}{a}$ and, in addition, the intercept on the Y axis is b in this case. Thus the general equation $y = mx + c$ becomes :

$$y = \frac{-b}{a}x + b$$

or, dividing all through by b,

$$\frac{y}{b} = \frac{-x}{a} + 1$$

$$\text{i.e. } \frac{x}{a} + \frac{y}{b} = 1 \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (i)$$

This is the equation to any straight line in terms of its intercepts on X and Y axes.

The equation $y = b$ is also in " intercept " form. It represents a line parallel to the X axis and making an intercept b with the Y axis. Similarly $x = a$ represents the line parallel to the Y axis and making an intercept a with the X axis.

Equation to a Straight Line in Perpendicular Form

Consider the diagram (Fig. 103). If you take a positive angle α, and a fixed distance p

along the bounding line of this angle, you reach the point N which is therefore fixed in terms of p and α. Thus any line at right angles to ON is also fixed, so that you must be able to find the equation to this perpendicular line (AB) in terms of p and α. This method of writing the general equation to a straight line is called the *perpendicular* or *normal* form. You can obtain the lengths of the intercepts OA and OB in terms of p and α from elementary trigonometry. Thus :

$$\begin{aligned} OA \cos\alpha &= p \text{ so that } OA = p\sec\alpha, \text{ and} \\ OB \cos\angle BON &= p. \quad \text{But} \\ \angle BON &= 90^\circ - \alpha, \text{ so that} \\ \cos\angle BON &= \sin\alpha, \\ \text{i.e.} \quad OB\sin\alpha &= p \text{ or } OB = p\operatorname{cosec}\alpha. \end{aligned}$$

We have already found the equation to any straight line in " intercept " form to be

$$\frac{x}{a} + \frac{y}{b} = 1,$$

and now $a = OA = p\sec\alpha$ and $b = OB = p\operatorname{cosec}\alpha$, so we can write this equation in the form

$$\frac{x}{p\sec\alpha} + \frac{y}{p\operatorname{cosec}\alpha} = 1$$

$$\text{i.e.} \quad \frac{x\cos\alpha}{p} + \frac{y\sin\alpha}{p} = 1$$

$$\text{or} \quad x\cos\alpha + y\sin\alpha = p \quad \ldots\ldots \quad (ii)$$

This is the *perpendicular* form for the equation of any straight line, because it expresses the relation between x and y in terms of p and α.

Example. Express the equation of the straight line given by $y = x - 1$ in "intercept" and "perpendicular" forms.

The form $y = x - 1$ shows that the slope is 1, i.e. it makes an angle of 45° with the X axis, and that the intercept on the Y axis is -1. Thus the line is as drawn in Fig. 104. Since the X— intercept is $+1$

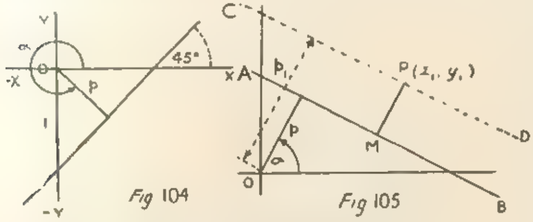


Fig. 104. Equation for a straight line $y = x - 1$ in " intercept " and " perpendicular " forms. Fig. 105. Finding length of perpendicular from from P on to line A B.

we may write the equation to the line in intercept form immediately as

$$\frac{x}{1} + \frac{y}{-1} = 1 \quad \ldots\ldots\ldots\ldots \quad (iii)$$

This could have been found directly from the original equation simply by making the " constant term " equal to 1.

As shown in the diagram $\alpha = 360^\circ - 45^\circ$, i.e. 315° and $p = 1 \cdot \cos 45^\circ = \frac{1}{\sqrt{2}}$. Thus in " perpendicular " form the equation becomes

$$x\cos 315^\circ + y\sin 315^\circ = \frac{1}{\sqrt{2}} \quad \ldots \quad (iv)$$

It is seen that (iii) and (iv) both reduce to the original equation $y = x - 1$; they are only different *forms* of this equation. Equation (iii) reduces to

$x - y = 1$, i.e. $y = x - 1$, and since $\sin 315^\circ = -\frac{1}{\sqrt{2}}$ and $\cos 315^\circ = +\frac{1}{\sqrt{2}}$, equation (iv) reduces to

$$\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\text{i.e.} \quad x - y = 1 \text{ or } y = x - 1.$$

Length of Perpendicular From a Point on to a Straight Line

Suppose, in the first instance, that the equation to the straight line is given in perpendicular form, i.e. p and α are known and the equation is

$$x\cos\alpha + y\sin\alpha = p \quad \ldots\ldots\ldots \quad (v)$$

You require the length of the perpendicular on to this line from the given point (x_1, y_1). In Fig. 105, AB represents the given line, P the given point. The given quantities are p, α, and x_1, y_1, the co-ordinates of the given point. You will find the perpendicular length PM in terms of these four quantities. Suppose a line CD is drawn through the given point P, parallel to AB. For this line α will be the same as for AB, but the perpendicular length from the origin will be greater than p. Let this length be p_1, as shown on the diagram. Then the equation to CD is

$$x\cos\alpha + y\sin\alpha = p_1 \quad \ldots\ldots\ldots \quad (vi)$$

and since the given point (x_1, y_1) lies on this line, these co-ordinates must satisfy this equation and thus you have

$$x_1\cos\alpha + y_1\sin\alpha = p_1 \quad \ldots\ldots\ldots \quad (vii)$$

This relation determines p_1, in terms of the known quantities x_1, y_1, α. You can now obtain the length you require (PM), for

$$\begin{aligned} PM &= p_1 - p \\ &= x_1\cos\alpha + y_1\sin\alpha - p \qquad \text{from } (vii) \end{aligned}$$

and everything on the R.H.S. is known.

This formula supposes that P is on the *opposite* side of AB from the origin. If P were on the *same* side, you would find that the distance is $-(x_1\cos\alpha + y_1\sin\alpha - p)$. Thus by calculating $x_1\cos\alpha + y_1\sin\alpha - p$ you can determine the distance of P from AB and find out on which side of AB the point P lies.

Example. Find the length of the perpendicular from the point $(3, -2)$ on to the line $x\cos 315^\circ + y\sin 315^\circ = \frac{1}{\sqrt{2}}$

You can rewrite the equation to the straight line as

$$x\cos 315^\circ + y\sin 315^\circ - \frac{1}{\sqrt{2}} = 0$$

and then, from relation (vii), all you require to do to find the length of the perpendicular from $(3, -2)$ on to this line is to substitute 3 for x and -2 for y in the

L.H.S. of this equation. Thus in this example the length of the perpendicular

$$= 3\cos 315^\circ - 2\sin 315^\circ - \frac{1}{\sqrt{2}}$$
$$= 3\cdot\frac{1}{\sqrt{2}} - 2\left(-\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}$$
$$= \frac{4}{\sqrt{2}}$$

The student should check this result by plotting the point (3, −2) and the given line and measuring the length of the perpendicular. Notice that (3, −2) is on the opposite side of the line from the origin.

The general equation of a line is $Ax + By + C = 0$, and it is useful to deduce the length of a perpendicular from a point (x_1, y_1) on to the straight line when it is given in this form. Comparing it with the perpendicular form you have

$$x\,.\cos\alpha + y\,.\sin\alpha - p = 0 \quad \ldots\ldots \quad (viii)$$
$$x\,.\,A + y\,.\,B + C = 0 \quad \ldots\ldots\ldots \quad (ix)$$

You cannot deduce from this that the coefficients of x in the two equations are equal, but you can say that there is a constant ratio between the coefficients of x, those of y, and the independent terms; thus

$$\frac{\cos\alpha}{A} = \frac{\sin\alpha}{B} = \frac{-p}{C} = k, \text{ say,}$$

therefore $\cos\alpha = Ak$ . . . . (x)
$\sin\alpha = Bk$ . . . . (xi)
and $-p = Ck$ . . . . (xii)

From (x) and (xi) you can find k, for squaring and adding

$$A^2k^2 + B^2k^2 = \cos^2\alpha + \sin^2\alpha$$
$$= 1$$
$$\therefore\ k^2(A^2 + B^2) = 1$$

i.e. $$k = \frac{1}{\sqrt{(A^2 + B^2)}}$$

so that $$\cos\alpha = \frac{A}{\sqrt{(A^2+B^2)}},\ \sin\alpha = \frac{B}{\sqrt{(A^2+B^2)}}$$

and $$-p = \frac{C}{\sqrt{(A^2+B^2)}}$$

where the sign of $\sqrt{(A^2 + B^2)}$ is chosen to make p positive.

Thus from (vii)

$$PM = x_1\cos\alpha + y_1\sin\alpha - p$$
$$= \frac{x_1A}{\sqrt{(A^2+B^2)}} + \frac{y_1B}{\sqrt{(A^2+B^2)}} + \frac{C}{\sqrt{(A^2+B^2)}}$$
$$= \frac{Ax_1 + By_1 + C}{\sqrt{(A^2+B^2)}} \quad \ldots\ldots\ldots \quad (xiii)$$

so that you have the length of the perpendicular from a point (x_1, y_1) on to the line $Ax + By + C = 0$.

Notice that if you take C and $\sqrt{(A^2 + B^2)}$ as positive, then $\dfrac{Ax_1 + By_1 + C}{\sqrt{(A^2+B^2)}}$ is positive if (x_1, y_1) is on the *same* side of the line as the origin and negative otherwise.

Examples. Find the lengths of the perpendiculars from

(1) (5, 6) on to the line $3x + 7y + 10 = 0$,
(2) (−3, −4 on to the line $x - 5y - 7 = 0$,
(3) the origin on to the line $3x - 4y + 20 = 0$

(1) Length of perpendicular (from xiii) from (5, 6) on to $3x + 7y + 10 = 0$

$$= \frac{3(5) + 7(6) + 10}{\sqrt{(3^2 + 7^2)}}$$
$$= \frac{67}{\sqrt{58}}$$

(2) We write this as $-x + 5y + 7 = 0$, to make 'C' positive, so that the distance from (−3, −4) is

$$\frac{-3(-1) - 4(5) + 7}{\sqrt{(1^2 + 5^2)}}$$
$$= \frac{10}{\sqrt{26}}$$

Thus the length required is $\frac{10}{\sqrt{26}}$ and (−3, −4) is on the opposite side of the line from the origin.

(3) Length of perpendicular from the origin on to $3x - 4y + 20 = 0$

$$= \frac{3(0) - 4(0) + 20}{\sqrt{(3^2 + 4^2)}}$$
$$= \frac{20}{5}$$
$$= 4$$

Regions

We have hitherto simply regarded the equation $Ax + By + C = 0$ as representing a straight line in a plane, but it is important to notice that this line will be the boundary between two *regions*, for one of which the function $Ax + By + C$ will be positive and for the other, negative. You can easily determine which is the positive and which the negative region by evaluating the function at a point definitely on one side or the other of the line $Ax + By + C = 0$.

Consider Fig. 106 (*a*); here we have drawn the line $x - 2 = 0$. To the right of this line

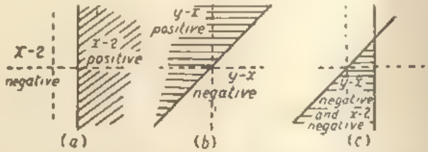


Fig. 106. Straight line dividing the plane into positive and negative regions.

(the shaded area), x is greater than 2, and therefore the function $x - 2$ is positive and to the left of $x - 2 = 0$, the function is negative. Thus $x - 2 = 0$ is the boundary line between $x - 2 > 0$ and $x - 2 < 0$. Similarly in Fig. 106(*b*), the line $y - x = 0$ is drawn. Evaluate $y - x$ for $x = 0$, $y = 5$ which represents a point definitely on the shaded side. With these values $y - x$ is $+5$, so that the shaded side indicates the region where $y - x$ is positive.

In Fig. 106 (*c*) we have combined the two previous cases, and the shaded area indicates

the region where *both* $y - x$ and $x - 2$ are negative.

Equation of a Circle

Now consider the equation of a circle in the simple case where the centre of the circle is at the origin. A circle is defined in geometry as the path traced out by a point which moves so that it is always a fixed distance from a fixed point. The fixed distance is the radius and the fixed point is the centre. To find the equation to the circle, you require to find a relation between the co-ordinates of any point on the circle which is true for *all* points of the circle, but which is *not* true for any points *not* on the circle.

Suppose a point P on the circumference of the circle has co-ordinates (x_1, y_1), then since the centre of the circle is at the origin, you may have that $x_1^2 + y_1^2 = r^2$ from the Theorem of Pythagoras, where r is the radius of the circle. This is the relation which exists between x_1 and y_1, the co-ordinates of a particular point on the circle, and therefore the equation to this circle will be given by

$$(x \text{ co-ordinate})^2 + (y \text{ co-ordinate})^2 = r^2$$

or more simply $x^2 + y^2 = r^2$

EXERCISES

(1) Draw the straight lines (i) $x + y = 0$, (ii) $2x + 3y = 1$, (iii) $y - 5 = 0$, (iv) $3y = 2 - x$, (v) $x + 2 = 0$. In each case check your drawing by choosing any point on the line, measuring its co-ordinates and verifying that they satisfy the equation.

(2) A is the point $(-2, 7)$ and O is the origin. Find the equation of OA.

(3) Find the equation of the line through $(-4, -2)$ parallel to the X axis.

(4) Find the equation of a line making intercepts of 3 and 2 on the X and Y axes respectively.

(5) Referring to Fig. 103, find the co-ordinates of A and B in terms of p and α, and hence obtain the equation to AB in perpendicular form by making both A and B satisfy the equation $y = mx + c$.

(6) Find the lengths of the perpendiculars from
- (i) (0, 8) on to the line $x \cos 30° + y \sin 30° = 3$,
- (ii) $(-3, 5)$ on to the line $3y + 4x + 7 = 0$,
- (iii) $(1, 2)$ on to the line $7x - 5y + 3 = 0$,
- (iv) (a, b) on to the line $x - y = 0$,
- (v) $(-3, 7)$ on to the line $x = 0$.

(7) Determine the single region in which $x + y - 7 < 0$, $y - 4 < 0$, $y > 0$ and $x > 0$ at the same time and find its area.

(8) Draw the circle $x^2 + y^2 = 9$. Choose any point on it, measure its co-ordinates, and check your drawing by verifying that these co-ordinates satisfy the equation of the circle.

ANSWERS TO EXERCISES IN LESSON 20

(1) $9\sqrt{2}$ (or 12·726) (2) $(2\frac{1}{2}, 1\frac{1}{2})$
(3) AB = $\sqrt{26}$, AC = $\sqrt{13}$, BC = $\sqrt{13}$. Area = $6\frac{1}{2}$ sq. units
(4) D $(1, -3)$; $5y = x - 3$ and $y = -5x + 2$; $(\frac{1}{2}, -\frac{1}{2})$
(5) (i) $y = 5x + 2$; (ii) $2y = x - 7$; (iii) $y = -3x$
(8) 40° 36′ or 139° 24′, (4, 4)
(9) $4y + x = 19$

LESSON 22

Lines and Conics

When we introduced the subject of algebraic geometry at the beginning of Lesson 20 we set out to discover the equation to the path traced out by a point moving in such a way that it was always the same distance from both the positive X and Y axes. The path traced out by a point moving under certain conditions is called its *locus* and in this example it was shown that the locus of the point was a straight line whose equation was $y = x$.

Again, when we found the equation to a circle having its centre at the origin, we were finding the equation of the locus of a point moving under the condition that it was always a fixed distance (the radius) from the origin.

This conception of a *locus* (plural, *loci*) is very important and we will consider here the general method of obtaining the equation of the locus of a point moving under any conditions. Suppose you imagine the point to be fixed, for the moment, at some particular point of its path, then in this position the point will have co-ordinates, say x_1 and y_1. Since the movement of the point is always subject to certain conditions, these will apply to x_1 and y_1 and so you may obtain from these given conditions the relation which must hold between x_1 and y_1. Thus you will have a relation between the co-ordinates of a particular point which satisfies the given conditions, but you require to find the equation of a curve such that *all* points on it satisfy the given conditions. To do this, you only need to generalise the relation between x_1 and y_1, i.e. you write x in place of x_1 and y in place of y_1, so that you have the equation of a curve such that *every* point on it satisfies the given condition.

To Obtain Equations of Bisectors of Angles between Two Straight Lines

We illustrate this method by an example which obtains a result of practical importance. Let

$$ax + by + c = 0$$
$$\text{and} \quad a_1x + b_1y + c_1 = 0$$

represent the equations to the lines AB, CD respectively (Fig. 107). If P (x_1, y_1) is a point on either of the bisectors of the angles between these two lines and if you draw perpendiculars PN and PM on to these lines, then PN = PM.

Thus each bisector is the locus of points which are equidistant from both AB and CD. We have already obtained (in Lesson 21) the length

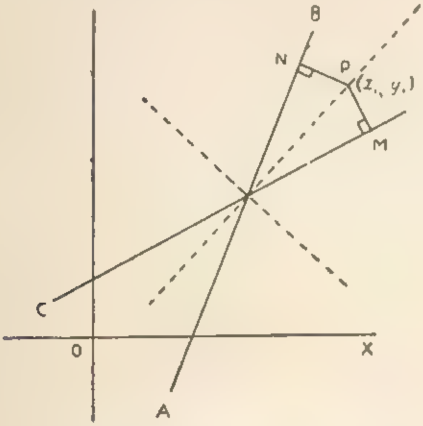


Fig. 107. Finding equations for bisectors of angles between straight lines.

of the perpendicular from a point on to a line whose equation is $Ax + By + C = 0$ so that, using this result,

$$PN = \pm \frac{ax_1 + by_1 + c}{\sqrt{(a^2 + b^2)}}$$

and $$PM = \pm \frac{a_1x_1 + b_1y_1 + c_1}{\sqrt{(a_1^2 + b_1^2)}}$$

If you equate these two lengths, you are finding the relation between x_1 and y_1 in order that P shall lie on a bisector, and if you replace x_1 and y_1 by x and y respectively, you obtain the equations of the lines which contain all such points, i.e. the equations of the bisectors are

$$\frac{ax + by + c}{\sqrt{(a^2 + b^2)}} = \pm \frac{a_1x + b_1y + c_1}{\sqrt{(a_1^2 + b_1^2)}} \quad . \; . \; . \; . \quad (i)$$

Notice that since each denominator contains a square root which can be taken positive or negative, there will be two distinct cases; either both terms will be of like sign (positive or negative) or the terms will be of different signs (− and +, or + and −). By taking each sign in turn you obtain the equations of the two bisectors of the angles between the original straight lines. Since a, b, c, a_1, b_1, c_1 are all constants, (i) must reduce to the form $Ax + By + C = 0$, i.e. it does correctly represent a straight line. The simplest way of deciding which sign in (i) gives which bisector is to draw a rough diagram and to see from this which bisector has a positive slope and which has a negative.

Example. Find the equations of the bisectors of the angles between the lines represented by $x + y = 4$ and $x = y + 1$.

You can write the equations of these two lines in the form

$$y + x - 4 = 0$$
and $$y - x + 1 = 0$$

so that the equations of the bisectors, from (i), are

$$\frac{y + x - 4}{\sqrt{(1^2 + 1^2)}} = \pm \frac{y - x + 1}{\sqrt{(1^2 + (-1)^2)}}$$

$$\frac{y + x - 4}{\sqrt{2}} = \pm \frac{y - x + 1}{\sqrt{2}}$$

This can be simplified by multiplying both sides by $\sqrt{2}$ and you obtain the two bisectors as

$$y + x - 4 = +(y - x + 1)$$
and $$y + x - 4 = -(y - x + 1)$$
i.e. $2x = 5$ and $2y = 3$

Thus the bisectors are the lines given by $x = 2\frac{1}{2}$ and $y = 1\frac{1}{2}$, i.e. they are parallel to the axes of X and Y in this case. Indeed this may be seen very easily from the fact that the slopes of the original lines are −1 and 1, i.e. the lines are inclined at 135° and 45° to the X axis respectively.

The student should verify the results obtained in this example by drawing the original lines and bisecting the angles between them geometrically.

General Equation of a Circle

We proceed to find the general equation of a circle by treating it as a locus problem. Let K (h, k) be the centre of the circle (Fig. 108)

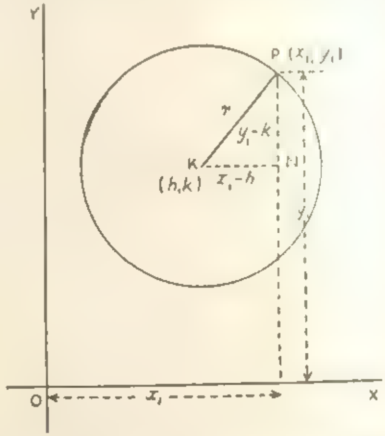


Fig. 108. The general equation of a circle of radius r, treated as a locus problem.

and P (x_1, y_1) any point on the circumference; then a circle will be traced out if P moves so that PK is constant. This constant distance is the radius of the circle (r) and you require the relation between x_1 and y_1. If KN is drawn parallel to the X axis, then KN is equal to the difference between the x co-ordinates of P and K, i.e. $KN = x_1 - h$. Similarly PN is the difference in the y co-ordinates of P and K, i.e. $PN = y_1 - k$ and since KNP is a right-angled triangle, you have from the Theorem of Pythagoras that $PK = \sqrt{(x_1 - h)^2 + (y_1 - k)^2}$ (the student should remember this formula for the distance between two points) and

$$(x_1 - h)^2 + (y_1 - k)^2 = r^2$$

This is the relation which exists between the co-ordinates of a particular point (P) on the circumference and hence the equation of the circle (which is to give a relation between the x and y co-ordinates of *any* point on the circumference) is

$$(x - h)^2 + (y - k)^2 = r^2 \quad \dots\dots\dots \quad (ii)$$

This is the general equation of a circle of radius r having its centre at the point (h, k). Working out the brackets, you can write (ii) as

$$x^2 - 2xh + h^2 + y^2 - 2yk + k^2 = r^2$$

or $$x^2 + y^2 - 2xh - 2yk + h^2 + k^2 - r^2 = 0 \quad (iii)$$

This is sometimes a more convenient form than (ii).

Example. Find the equation of the circle having its centre at the point $(-3, 5)$ of radius 4.

From (ii) you can write the equation to this circle as

$$(x + 3)^2 + (y - 5)^2 = 4^2$$

i.e. $$x^2 + 6x + 9 + y^2 - 10y + 25 = 16$$

or $$x^2 + y^2 + 6x - 10y + 18 = 0$$

Example. Find the centre and radius of the circle represented by $x^2 + y^2 + 2x + 4y - 4 = 0$.

You can write this equation as

$$x^2 + 2x + y^2 + 4y = 4 \quad \dots\dots\dots \quad (iv)$$

and in order to find the centre of the circle you require to arrange that the terms in x and in y are both perfect squares. You must add 1^2 to $x^2 + 2x$ and 2^2 to $y^2 + 4y$ to make each of these expressions a perfect square, and in order not to alter the value of (iv) you must therefore add 1^2 and 2^2 to the R.H.S. also, so that (iv) becomes

$$x^2 + 2x + 1 + y^2 + 4y + 4 = 4 + 5$$

i.e. $$(x + 1)^2 + (y + 2)^2 = 3^2$$

showing that the centre is at the point $(-1, -2)$ and the radius is 3.

It should be noticed from equation (iii) that the general equation of a circle must have the same coefficient of x^2 and y^2 and can contain no term in xy, so that you can write the general equation of a circle in the form

$$x^2 + y^2 + ax + by + c = 0.$$

Its centre is at $\left(-\frac{a}{2}, -\frac{b}{2}\right)$ and its radius is $\frac{1}{2}\sqrt{a^2 - b^2 - 4c}$.

Example. A point P moves so that its distance from (0, 1) is twice its distance from (4, 0). Find the locus of P.

Consider Fig. 109; let A and B represent the points (0, 1) and (4, 0) respectively so that P (x_1, y_1) is one position of the point which roughly satisfies the given condition. This condition is that AP = 2 PB.

The given condition is equivalent to

$$AP^2 = 4PB^2$$

or $$x_1^2 + (y_1 - 1)^2 = 4[(x_1 - 4)^2 + y_1^2]$$

which represents the relation between x_1 and y_1 in order that the given condition shall hold. Thus the equation of the locus of P is

$$x^2 + (y - 1)^2 = 4[(x - 4)^2 + y^2]$$

which simplifies into

$$x^2 + y^2 - 2y + 1 = 4(x^2 - 8x + 16 + y^2)$$

i.e. $$3x^2 + 3y^2 - 32x + 2y + 63 = 0 \quad \dots\dots \quad (v)$$

This is the equation you require; it represents a circle and you can proceed as in the previous example to obtain its centre and radius; thus dividing (v) by 3 you have

$$x^2 + y^2 - \frac{32}{3}x + \frac{2}{3}y + 21 = 0$$

or $$x^2 - \frac{32}{3}x + y^2 + \frac{2}{3}y = -21.$$

You need to add $\left(\frac{16}{3}\right)^2$ and $\left(\frac{1}{3}\right)^2$ to both sides in order to transform the L.H.S. into the sum of two perfect squares, then

$$\left(x - \frac{16}{3}\right)^2 + \left(y + \frac{1}{3}\right)^2 = \frac{256}{9} + \frac{1}{9} - 21 = \frac{68}{9}$$

showing that the locus is a circle centre $\left(\frac{16}{3}, -\frac{1}{3}\right)$ and radius $\frac{1}{3}\sqrt{68}$, i.e. $\frac{2}{3}\sqrt{17}$.

Conic Sections

The locus of a point which moves in a plane so that its distance from a fixed point (called the *focus*) bears a constant ratio to its distance from a fixed line (called the *directrix*) is called a *conic section*, since it can be constructed as the plane section of a right circular cone. Consider Fig. 110: AB is a fixed line, S a fixed point so that for the locus of any point P to be a conic SP/PN = a constant. This constant is called the *eccentricity* of the curve and is conveniently denoted by the letter e, so that

$$SP = e.PN \quad \dots\dots\dots\dots \quad (vi)$$

S is the focus and AB is the directrix. Three cases arise:

if $e = 1$, the curve is a *parabola*;
if $e < 1$, the curve is an *ellipse*;
if $e > 1$, the curve is a *hyperbola*.

Wherever you choose the X and Y axes, P will have some co-ordinates (say x_1, y_1), so that (vi) provides a condition which will give a relation between x_1 and y_1.

Thus you may use the same general method for obtaining the equation to a conic as you did for the circle on page 1256.

Equation of a Parabola

Referring again to Fig. 110, the curve will be a parabola if SP = PN, i.e. if $e = 1$. Thus

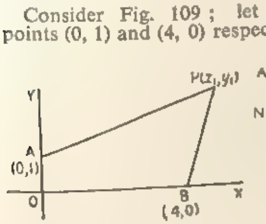


Fig.109

Fig. 109. Equation of the locus of P when AP = 2 PB.

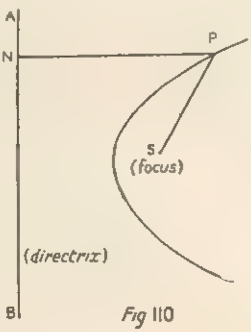
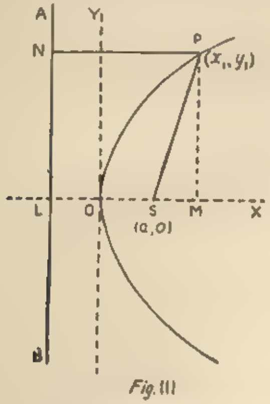
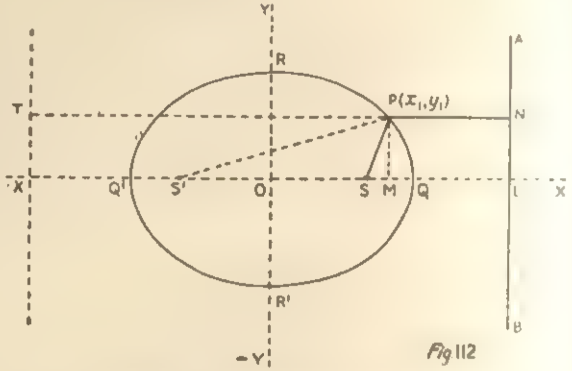


Fig 110

Fig. 110. Characteristics of a Conic.

if a perpendicular SL is drawn from the focus S on the directrix AB, the mid-point of SL must be a point on the parabola, for then this directrix (Fig. 112) and as for the parabola. Draw a perpendicular (SL) on to the directrix. Take the line containing SL as the X axis.





CONIC SECTIONS. Fig. 111. Constructing the equation for a parabola. Fig. 112. Calculating the equation for an ellipse with focus S and directrix AB.

point is equidistant from both S and L (Fig. 111). This is the most convenient point to take as the origin and if the direction of SL is taken as the X axis, then a line through O, parallel to the directrix will be the Y axis.

In addition, you must take some constant to represent the x co-ordinate of the focus S. Let this be a. Then S is the point (a, O), L is the point ($-a$, O), so that the equation to the directrix is

$$x = -a.$$

If P (x_1, y_1) represents any point on the parabola, you may obtain the relation

SP = PN in terms of x_1, y_1, and a.

If $\quad$ SP = PN

then $\quad$ $SP^2 = PN^2$ . . . . (*vii*)

If PM is drawn perpendicularly to the X axis, then

$$\begin{aligned} PN &= LM \\ &= LO + OM \\ &= a + x_1 \end{aligned}$$

and again,

since $\quad$ $MS = x_1 - a$ and $PM = y_1$, you have $PS^2 = (x_1 - a)^2 + y_1^2$

Thus (vii) becomes

$$(x_1 - a)^2 + y_1^2 = (x_1 + a)^2$$

This is the relation between x_1, y_1, and a in order that P shall lie on the parabola, therefore the equation of this parabola is given by

$$(x - a)^2 + y^2 = (x + a)^2$$

i.e. $\quad x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$, or

$$y^2 = 4ax$$

This, then, is the equation of the parabola when the focus is the point (a, O) and the directrix is the line $x = -a$.

Equation of an Ellipse

For an ellipse (by definition) the eccentricity is less than 1. Let S be the focus and AB the

There must be a point Q between S and L such that

SQ = e.QL . . . . (*viii*)

and this point will be nearer to S than to L because $e < 1$. There must also be a second point Q′ on LS produced such that

SQ′ = e.Q′L . . . . (*ix*)

so that the ellipse must cut the X axis in two points. Thus it is convenient to take the Y axis as the perpendicular bisector of the line joining these two points, i.e. the perpendicular bisector of QQ′, and so you have the origin (O) fixed. If you let OQ be represented by the letter a, then you may easily obtain the co-ordinates of S and L in terms of a and e.

Adding relations (viii) and (ix)

$$SQ + SQ' = e\,(QL + Q'L)$$

i.e. $\quad QQ' = e\,(OL - a + OL + a)$

i.e. $\quad 2a = e.2OL$

and therefore $\quad OL = \dfrac{a}{e}$

so that the directrix is the line

$x = \dfrac{a}{e}$ and L the point $\left(\dfrac{a}{e}, O\right)$

Subtracting (viii) from (ix) you obtain

SQ′ − SQ = e (Q′L − QL) . . . . (x)

Now $\quad SQ' = OQ' + OS = a + OS$

and $\quad SQ = OQ - OS = a - OS$

and therefore (x) becomes

$$a + OS - (a - OS) = e\,(QQ')$$

or $\quad 2.OS = e.2a$

i.e. $\quad OS = ae$

Thus the focus is the point (ae, O).

You can now obtain the equation to the ellipse by taking P (x_1, y_1) as any point on it and expressing the geometrical relation SP = e.PN in terms of x_1, y_1, a, e.

SM = OM − OS = $x_1 - ae$, PM = y_1 and PN = OL − OM = $\frac{a}{e} - x_1$.

The relation $\qquad SP = e.PN$

is equivalent to $\qquad SP^2 = e^2.PN^2$

i.e. $\qquad SM^2 + PM^2 = e^2.PN^2$

so that $\qquad (x_1 - ae)^2 + y_1^2 = e^2\left(\frac{a}{e} - x_1\right)^2 \quad \ldots \quad (xi)$

This is the relation between x_1, y_1, a and e, and therefore the equation to the ellipse is

$$(x - ae)^2 + y^2 = e^2\left(\frac{a}{e} - x\right)^2$$

or $\qquad x^2 - 2aex + a^2e^2 + y^2 = e^2\left(\frac{a^2}{e^2} - \frac{2ax}{e} + x^2\right)$

$$= a^2 - 2aex + e^2x^2.$$

Thus $\qquad x^2(1 - e^2) + y^2 = a^2(1 - e^2)$

or, dividing all through by $a^2(1 - e^2)$,

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1 - e^2)} = 1.$$

In this, you can for convenience write b^2 in place of $a^2(1 - e^2)$ so that the equation to the ellipse becomes

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \ldots\ldots\ldots\ldots\ldots \quad (xii)$$

You find the points where the ellipse cuts the Y axis by putting $x = 0$ in (xii). You then have $y^2 = b^2$, i.e. $y = \pm b$, giving the points R, R′.

You can show that the ellipse is a closed curve passing through $x = \pm a$ and $y = \pm b$ by writing (xii) in the form

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}.$$

Then if $x > a$, $\frac{x^2}{a^2}$ is greater than 1 so that the R.H.S. becomes negative, and the ellipse has no points (x, y) with $x > a$; similarly it has no points with $y > b$.

QQ′ (equal to $2a$) is called the *major* axis of the ellipse and RR′ (= $2b$) is the *minor* axis, so that a and b are termed the *semi-axes* of the ellipse. It is clear from the symmetry of the equation of an ellipse that it must have a second focus (plural *foci*) S′ at the point (− ae, O) and a second directrix (plural *directrices*) whose equation will be

$$x = -\frac{a}{e}.$$

Drawing an Ellipse

There is an important property of an ellipse which enables it to be drawn very quickly. Consider Fig. 112 again:

$$SP = e.PN$$

and $\qquad S'P = e.PT.$

Adding these two relations,

$$\begin{aligned} SP + S'P &= e\,(PN + PT) \\ &= e \times \text{distance between the two directrices} \\ &= e.2OL \\ &= e.\frac{2a}{e} \\ &= 2a, \text{ i.e. a constant.} \end{aligned}$$

Thus you may take a piece of string, stick drawing pins through it on to a piece of paper, and these two points will be the foci of an ellipse drawn by a pencil which keeps the string taut.

EXERCISES

(1) A circle, centre (4, 1) and radius 2, cuts the X axis in A and B. Draw the circle, measure AB, and check by calculation.

(2) The straight line $3x - 4y = 1$ touches a circle whose centre is (3, −3). Find the radius of the circle and confirm by drawing the line and circle.

(3) Draw the lines represented by $4x + 3y = 5$, $5x + 12y = 13$, measure the angle between them, and read off the co-ordinates of the point of intersection. Check your drawing by calculating these values.

(4) Find the equations of the bisectors of the angles between the lines given in question (3) and from these equations show that the bisectors are at right angles.

(5) Find the equation of the circle whose centre is the point (−2, −3) and which passes through the point (1, 1).

(6) Find the centre and radius of each of the following circles. Choose a point on each of the drawn circles, find its co-ordinates, and check the correctness of your drawing by seeing if the co-ordinates of the point satisfy the equation of the circle.

(i) $x^2 + y^2 + 2x - 2y = 0$
(ii) $4x^2 + 4y^2 - 12x + 8y = 3$

(7) A point moves so that its distance from the point (4, 0) is equal to its distance from the Y axis. Find the equation of its locus and plot the curve.

(8) Find the equation of the parabola whose focus is the point (h, k) and directrix the line $x = l$.

(9) Find the lengths of the semi-axes, the eccentricity, the co-ordinates of the foci, and the equations of the directrices of the ellipse $9x^2 + 25y^2 = 225$.

(10) Find the equation of the ellipse which has a focus at the point (1, 0), an eccentricity of $\frac{1}{2}$, and the directrix corresponding to this focus, the line $x = 2$. Find also the co-ordinates of the other focus.

Answers to Exercises in Lesson 21

(2) $2y + 7x = 0$ (3) $y + 2 = 0$
(4) $2x + 3y - 6 = 0$
(5) A (p sec α, 0), B (0, p cosec α)
(6) (i) 1, (ii) 2, (iii) 0, (iv) $\pm\frac{a - b}{\sqrt{2}}$, (v) 3
(7) A trapezium lying wholly in the first quadrant, area = 20 square units.

LESSON 23

Standard Curves: Graphical Solutions of Equations

In the last few Lessons on algebraic geometry we have obtained the equations to the straight line and a few important curves. Notice that in each case we started with the line or curve (or geometrical relation which defined it), and deduced, after fixing the co-ordinate axes, the algebraic equation representing the line or curve. In this Lesson we shall reverse the procedure, i.e. being given an algebraic equation, we shall obtain the shape of the curve that it represents.

In theory there is a very simple way of doing this: for instance, given the equation $y = f(x)$, you can substitute the values $x = 0, 1, 2, \ldots .$ $-1, -2, -3, \ldots .$ to obtain $f(0)$, $f(1)$, $f(2)$ and so on, which are the values of y at the corresponding values of x. Thus you may calculate a whole series of points lying on the curve $y = f(x)$ and by joining them up you will have the shape of the curve.

Although this method is partly useful it has the great disadvantage that there is in general no guide to the range of values of x which must be taken in order to be sure of obtaining an accurate idea of the shape of the curve. In addition, this method can be very laborious, and so we will develop some general methods which will enable us very rapidly to obtain an idea of the shape of the curve. This particular branch of study is known as *curve sketching* or *curve tracing*.

The Standard Curves

Begin by considering the graphs of simple functions in two groups:

(1) $y = x,\ x^3,\ x^5, \ldots \ldots$

and (2) $y = x^2,\ x^4,\ x^6, \ldots .$

(1) This group consists of the curves for which the corresponding function is raised to an odd power, and thus every one of these curves passes through the origin, and in addition it lies in the first and third quadrants for, if $y = x^n$ with n odd, then y is positive when x is positive and negative when x is negative. Notice also that all curves of this type pass through the points (1, 1) and $(-1, -1)$. Consider Fig. 113, where a few of these curves have been drawn: when x is numerically greater than 1, the curves are steeper as their power is increased because $x^3 > x$, $x^5 > x^3$ and so on. When x is numerically less than 1, $x^3 < x$, $x^5 < x^3$, i.e. the curves are lower.

(2) If the power of x is even, then, whatever the value of x, y is always positive so that these curves lie in the first and second quadrants. All the curves pass through three points: the origin, (1, 1), and $(-1, 1)$. The same remarks regarding the relative positions of members apply as in the first group. Fig. 114 shows two of these curves.

It should be noticed that if a constant appears in front of the term in x, e.g. $y = 3x^2$, this simply implies that the scale of y is increased threefold, but the general nature of the curve is otherwise unaltered.

From these standard forms it is a simple matter to deduce the shape of $y = -x^2$, $-x^3$, $-x^4$, ... You need only to draw the curves $y = x^2$, x^3, x^4 mirrored in the X axis.

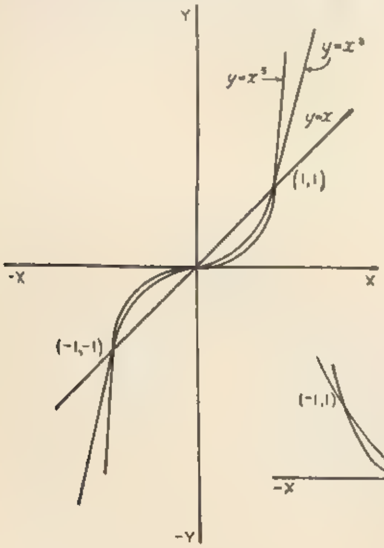


Fig. 113

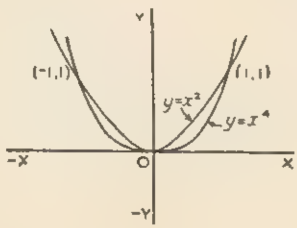


Fig. 114

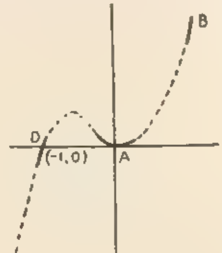


Fig 115

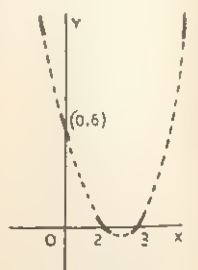


Fig. 116

STANDARD CURVES. Fig. 113. Curves of Group 1 $y = x^3, x^5$, etc. Fig. 114. Curves of Group 2 $y = x^2, x^4$ . . . . Fig. 115. Sketching the Curve represented by $y = x^2 + x^3$. Fig. 116. See text in p. 1261.

You can use the foregoing information directly to sketch composite curves such as $y = x + x^2$ or $y = x^2 - x^3$. Consider an actual example.

Example. Sketch the curve represented by

$$y = x^2 + x^3$$

You can easily discover what is the shape of the curve $y = x^2 + x^3$ for very small values of x and for very large values of x. By "small values" is meant values of x small compared with 1. When x is less than 1 then $x^2 > x^3$ and therefore for values close to $x = 0$ the x^3 term will be negligible compared to the x^2 term.

Thus $y = x^2 + x^3$ will approximate to $y = x^2$ for very small values of x. Therefore you can draw a short curve (A) representing $y = x^2$ at the origin (Fig. 115).

For values of x large compared with 1, the curve $y = x^2 + x^3$ will approximate to $y = x^3$ (because x^3 is very much greater than x^2 if x is very much greater than 1), i.e. you know that $y = x^2 + x^3$ goes off "to infinity" in a similar way to $y = x^3$. You may indicate by B and C these positions.

In addition, $y = x^2 + x^3$ can be put in the form $y = x^2\ (1 + x)$ and therefore $y = 0$ when $x = 0$ (twice) or $x = -1$. You know, then, that the curve crosses the X axis where $x = -1$ (indicated by the line D) and touches the X axis at the origin. Since $x^2 + x^3$ must always be finite for values of x between 0 and -1, the part D must curve round to link up with the part A. It is reasonable to expect that A links up with B, and C with D.

Thus you have a very rough approximation to the shape of the curve: later we shall give methods for examining the shape more exactly. We pass on now to the sketching of other types of curve.

Example. Sketch the curve represented by

$$y = (x - 2)\ (x - 3).$$

From the form of the quadratic function of x on the R.H.S. it is obvious that $y = 0$ when either $x = 2$ or $x = 3$. Since $(x - 2)\ (x - 3)$ is equivalent to $x^2 - 5x + 6$ it is also obvious that for values of x numerically large compared with 1, x^2 is the most important term and the curve will approximate to that of $y = x^2$, i.e. it goes "to infinity" in the first and second quadrants. Thus you have the information represented by the solid lines in Fig. 116, together with the fact that when $x = 0$, $y = 6$.

As before, you join up as indicated by the dotted lines. Notice that you may find the minimum value of this function by writing

$$\begin{aligned} y &= x^2 - 5x + 6 \\ &= \left(x - \frac{5}{2}\right)^2 + 6 - \frac{25}{4} \\ &= \left(x - \frac{5}{2}\right)^2 - \frac{1}{4} \end{aligned}$$

You have simply used the technique of "completing the square." Since the term $\left(x - \frac{5}{2}\right)^2$ can never be negative, the minimum value of y occurs when $x - \frac{5}{2} = 0$, i.e. when $x = 2\frac{1}{2}$, the minimum value of y then being $-\frac{1}{4}$.

You can imagine what the shape of the curve would have been if you had been drawing $y = (x - 2)\ (x - 2{\cdot}1)$, for instance. Then you would have had two points on the X axis very close together with only a tiny portion of the curve below the X axis. Obviously then, the curve for $y = (x - 2)^2$ will be similar in general shape to these first examples, but the two points where the curve crosses the X axis are coincident, i.e. they both occur at $x = 2$, so that the X axis is a tangent to the curve at this point.

It thus becomes apparent that if you draw the curve represented, in general, by $y = ax^2 + bx + c$ that it cuts the X axis in two points if $ax^2 + bx + c = 0$ has real roots, i.e. if $b^2 > 4ac$ (*see* Lesson 17), in two coincident points if $b^2 = 4ac$, and not at all if $b^2 < 4ac$, i.e. if the roots of $ax^2 + bx + c = 0$ are complex.

You can use the curve of

$$y = (x - 2)\ (x - 3)$$

to deduce the curve represented by

$$y = \frac{1}{(x - 2)\ (x - 3)}$$

and the method is important. It consists simply in noticing that when $(x - 2)\ (x - 3)$ is very large, then $\frac{1}{(x - 2)\ (x - 3)}$ must be very small, and vice versa.

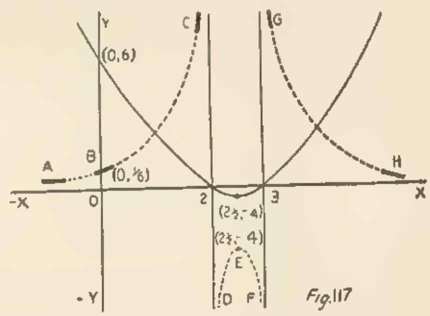


Fig. 117. How a reciprocal curve is deduced from that of the original function.

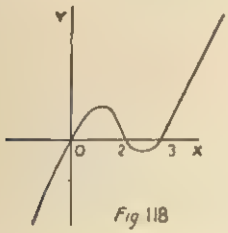
In Fig. 117 the curve of $y = (x - 2)\ (x - 3)$ has been drawn and also the two lines $x = 2$, $x = 3$. The dotted lines indicate the approximate shape of $y = \frac{1}{(x - 2)\ (x - 3)}$ and this shape is deduced as follows: A represents part of $\frac{1}{(x - 2)\ (x - 3)}$ which is positive but small, because the values of x in this position make $(x - 2)\ (x - 3)$ positive but large. When $x = 0$, $(x - 2)\ (x - 3)$ is $+ 6$ and therefore $\frac{1}{(x - 2)\ (x - 3)} = + \frac{1}{6}$ (the point of B).

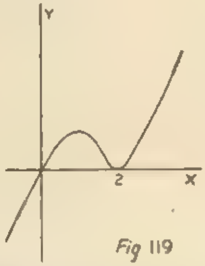
At a value of x just less than 2, $(x - 2)$ $(x - 3)$ is very small but positive, i.e. $\frac{1}{(x - 2)\ (x - 3)}$ is very large and positive

(this gives the part C). Since $(x-2)(x-3)$ is negative for all values of x between 2 and 3, $\dfrac{1}{(x-2)(x-3)}$ must also be negative, and so you deduce the loop DEF.

Notice that when x is just greater than 2, $(x-2)(x-3)$ is small and negative and therefore $\dfrac{1}{(x-2)(x-3)}$ is numerically large but negative. The point E $(2\frac{1}{2}, -4)$ is determined since when $x = 2\frac{1}{2}$, $(x-2)(x-3) = -\frac{1}{4}$. The part GH is deduced similarly to the part AC.

We can proceed to examine the shape of curves represented by a cubic polynomial in x.





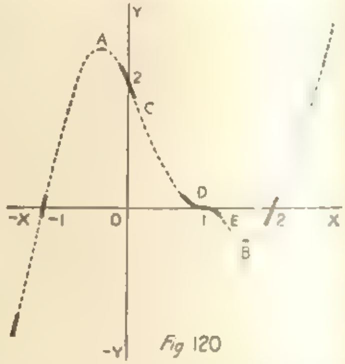


Fig. 118. Curve of cubic function of x. **Fig. 119.** Repeated root at $x = 2$. **Fig. 120.** Curve with treble factor $x - 1$.

Example. $y = x(x-2)(x-3)$.

You see immediately that $y = 0$ when $x = 0, 2,$ or 3, and when x is numerically large the curve approximates to $y = x^3$, so that you deduce the shape to be as indicated in Fig. 118.

Repeated Roots

If you wish to draw $y = x(x-2)^2$, the same situation arises as when dealing with quadratic functions of x; you have what is termed a *repeated root* at $x = 2$ so that the curve is as indicated in Fig. 119.

You can consider this question of repeated roots quite generally. Suppose you are sketching $y = f(x)$, and $f(x)$ contains a factor $(x-\alpha)^3$. Then when x is just less than α, this term is negative, but when x is just greater than α, this term is positive, i.e. the value of y must change sign when x passes through the value α, so that the curve crosses the axis of X as well as touching it at the point α, which is known as an *inflexional tangent*.

The same thing happens if you have a factor $(x-\alpha)^5$, $(x-\alpha)^7$, ... i.e. if $(x-\alpha)$ is raised to any odd power. If $(x-\alpha)$ is raised to an even power, the value of y does not change sign as x passes through α, so that in these cases the X axis is a tangent to the curve at the point $x = \alpha$, but the curve does not cross the X axis.

Example. Sketch the curve repre[illegible]ted by

$$y = (x+1)(x-1)^3(x-2$$

Here the factor $(x$ [illegible] 1) occurs [illegible]mes so that the curve touches the X axis and [illegible] it at the point $x = 1$. Also the curve crosses [illegible] axis when $x = -1$ and 2, and when [illegible] is lar[illegible] the curve approximates to that of [illegible] you have the important features indicated by [illegible]d lines in Fig. 120. Notice also that when [illegible] $y = 2$.

The highest point A is termed [illegible] *maximum* position, and you cannot yet find [illegible] maximum value of y nor the value of [illegible] which this maximum occurs. The point B [illegible] *minimum* position.

In addition, there are three p[illegible] on the curve where the tangents to the [illegible] change sides, i.e. C, D, and E; these points are known as *points of inflexion.*

Graphical Solution of Algebraic Equations

In cases where the roots of an equation cannot be determined accurately, it is often of great importance to be able to obtain approximate values or to be able to fix a range within which it can be stated definitely that a root occurs. There are some general methods which enable one to do this.

Suppose $f(x) = 0$ represents an algebraic equation, then if two neighbouring values of x can be found by trial such that the value of $f(x)$ at these two positions is of different sign, then there must be at least one root between the values of x. For instance, consider the equation :

$$x^3 - 10x + 1 = 0$$

If you write

$$f(x) = x^3 - 10x + 1,$$

then by trial

$$f(0) = +1, \text{ but } f(1) = -8.$$

These values indicate that if you plotted $y = f(x)$, then at $x = 0$, y would be negative and at $x = 1$, y would be positive, so that there must be a value of x between 0 and 1 for which $y = 0$, i.e. a point where the curve $y = f(x)$ crosses the X axis.

Thus there is a root of $x^3 - 10x + 1 = 0$ between $x = 0$ and $x = 1$. Since $f(1)$ is numerically greater than $f(0)$ it is probable that the root lies nearer to $x = 0$, so that you can then try $f(0{\cdot}2)$. The value of $f(0{\cdot}2)$ is negative, so that you now know that a root lies between $x = 0$ and $x = 0{\cdot}2$. By this trial and error method you can narrow the range within which x occurs until you establish the actual value of the root to any desired accuracy.

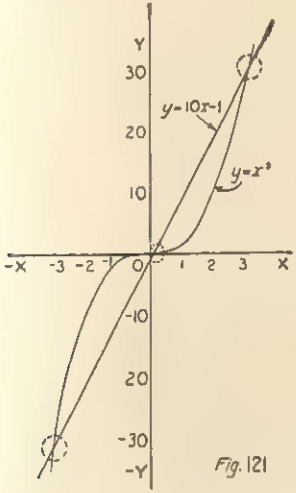


Fig. 121. Obtaining solutions of equations by graphical means.

There is another important method. In the section on algebraic geometry, we established the fact that the point of intersection of two straight lines is obtained by solving their equations simultaneously. This is true also of lines and curves or even of two curves. This fact is of use here. Consider the equation

$$x^3 - 10x + 1 = 0.$$

This may be written as

$$x^3 = 10x - 1$$

and if you draw the curve $y = x^3$ and the straight line $y = 10x - 1$, where they intersect will be points at which x^3 and $10x - 1$ have the same value, i.e. the $x-$ co-ordinates of the points of intersection will be such that $x^3 = 10x - 1$, i.e. these points provide approximate values of the roots.

Consider Fig. 121 : the curve $y = x^3$ is drawn, and superimposed on it is the straight line $y = 10x - 1$. You see that there are three points of intersection and as nearly as one can estimate, they occur where $x = -3{\cdot}15$, $+0{\cdot}15$, $+3{\cdot}18$. In the diagram the points of intersection have been ringed, and it will be observed that the scale of y has been taken to be five times that of x. This is only for convenience of drawing and naturally will not affect the results obtained.

EXERCISES

(1) Sketch the curve $y = x^2 - x^3$ and hence show that the equation $x^2 - x^3 = 1$ has one negative root and no other real roots.

(2) Solve the equation $x^2 - 3x - 5 = 0$ graphically. Check by calculation.

(3) Draw the graph of $y = x^3 - 3x + 5$ and hence show that $x^3 - 3x + 5 = 0$ has imaginary roots. Find the minimum value of $x^3 - 3x + 5$ and find for what value of x this occurs.

(4) Show that the curve represented by $y = 4x^2 + 4x + 3$ does not cut the axis of X.

(5) Roughly sketch the curves

(i) $y = x\,(x - 4)$
(ii) $y = (2 - x)\,(x - 1)$
(iii) $y = x^2\,(x - 3)$
(iv) $y = x\,(x - 3)^2$
(v) $y = x\,(x - 2)^3$

(6) Sketch the curves $y = x^2 + 2$ and $y = (x + 1)\,(2 - x)$. Hence deduce the curves representing $y = \dfrac{1}{x^2 + 2}$ and $y = \dfrac{1}{(x + 1)\,(2 - x)}$

(7) Draw the graph of $y = x^3$ for values of x from -5 to $+5$, and by its aid solve

(i) $x^3 = x + 20$
(ii) $x^3 - 10x - 10 = 0$

(8) Sketch the curve $y = (x^2 + 1)\,(x - 2)$. Hence deduce that the equation $(x^2 + 1)\,(x - 2) = 1$ has only one real root, and that its value is just greater than 2. Put $x = 2 + h$ and find the root more accurately. Confirm by writing the equation in the form $x^2\,(x - 2) = 3 - x$ and sketching $y = x^2\,(x - 2)$ and $y = 3 - x$.

Answers to Exercises in Lesson 22

(1) $2\sqrt{3}$ (2) 4

(3) 30° 31′ and 149° 29′, $\left(\frac{7}{11}, \frac{9}{11}\right)$

(4) $7y = 9x$, $77x + 99y = 130$

Slopes are $\frac{9}{7}$ and $-\frac{7}{9}$

(5) $x^2 + y^2 + 4x + 6y = 12$

(6) (i) Centre $(-1, 1)$, radius $\sqrt{2}$
(ii) Centre $(1\frac{1}{2}, -1)$, radius 2

(7) $y^2 = 8\,(x - 2)$

(8) $(y - k)^2 = (h - l)\,(2x - l - h)$

(9) $a = 5$, $b = 3$, $e = \frac{4}{5}$ Foci are at $(\pm 4, 0)$

Equations of directrices are $4x = \pm 25$

(10) $3x^2 + 4y^2 = 4x$, $(\frac{1}{3}, 0)$

LESSON 24

The Binomial Theorem Developed

In Lesson 19 it was shown that an expression of the form $(1 + x)^n$ where n is a positive whole number can be written in an alternative form in ascending powers of x, beginning with 1 and finishing with x^n. Now $(1 + x)^n$ has a meaning for values of n other than a positive whole number. For example:

$$\text{for } n = -1,\ (1 + x)^{-1} = \frac{1}{1 + x}$$

$$\text{for } n = -2,\ (1 + x)^{-2} = \frac{1}{(1 + x)^2}$$

$$\text{for } n = \tfrac{1}{2},\ (1 + x)^{\frac{1}{2}} = \sqrt{(1 + x)}$$

$$\text{for } n = -\tfrac{1}{2},\ (1 + x)^{-\frac{1}{2}} = \frac{1}{\sqrt{(1 + x)}}$$

The series which is the expansion of $(1 + x)^n$ for n a positive whole number, namely,

$$1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \ldots x^n$$

has no meaning as it stands for any value of n other than a positive integer, for the last term is x^n. If $n = \frac{1}{2}$, for instance, there could never be a final term $x^{\frac{1}{2}}$, since it could not occur anywhere in a development in ascending powers of x. The same would be true if n were -1 or -2, for there could be no term in x^{-1}, i.e. $1/x$ or x^{-2}, i.e. $1/x^2$.

Is one to conclude then that there is no expression in ascending powers of x for $(1 + x)^n$ if n is, say, a negative number or a fraction? That there is such an expansion can be seen from a particular case. Proceed as follows:

Multiply $1 - x$ by $1 + x + x^2 + \ldots . + x^r$.
You easily verify that the result is $1 - x^{r+1}$.

Thus

$$\frac{1 - x^{r+1}}{1 - x} = 1 + x + x^2 + \ldots . + x^r$$

Now if x is less than 1 in numerical value, irrespective of its sign, then the successive terms in this expression become smaller and smaller as r increases in amount. If, therefore, you make r indefinitely great, $1 - x^{r+1}$ gets as near as you like to 1, and the L.H.S. becomes, or *tends to* $\dfrac{1}{1 - x}$ or $(1 - x)^{-1}$.

The R.H.S. becomes an infinite series of powers of x, the successive terms of which get smaller and smaller. Therefore you are led to the conclusion that

$(1 - x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots ..$ without end . . . . , provided x is less than 1 in amount, apart from sign.

Here, then, is an expansion, and a very simple one, that has the form of a binomial expansion in the sense that it consists of ascending powers of x. It differs from it in that it does not terminate. But there is one significant point to be noticed.

Suppose the ordinary binomial expansion for $(1 - x)^n$ were valid for $n = -1$, what would the result have been? To find this, you have to insert $n = -1$ in the expression

$$1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

This gives

$$1 - (-1) \times x + \frac{(-1)(-2)}{2!}x^2 - \frac{(-1)(-2)(-3)}{3!}x^3 + \ldots$$
$$= 1 + x + x^2 + x^3 + \ldots$$

which is precisely what has been found otherwise. It follows that the Binomial Theorem can certainly be used in form at least for the case $n = -1$, provided x is less than 1.

This is in fact true generally, i.e. that provided x is less than 1 in value, irrespective of sign, then

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

where n may have any value, positive or negative. It must be noticed that only in the case where n is a positive whole number will the succession of terms come to an end. It would lead us beyond the scope of this Course to establish this theorem in its most general form for any value of n, but we shall indicate its truth for the case of n a negative whole number. It is required to show that

$$(1 + x)^{-n} = 1 - nx + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3 + \ldots . \quad (i)$$

where n is now a positive whole number. This has been obtained by replacing n by $-n$ in the original binomial expression; and, as has been remarked, the series of terms on the right run on indefinitely.

Let us write $S(n)$ for this expression. We have to show that in certain circumstances which we shall state,

$$S(n) = (1 + x)^{-n}.$$

Multiply each term on the R.H.S. of (i) by x, then

$$S(n) = 1 - nx + \frac{n(n+1)}{2!}x^2 - \frac{n(n+1)(n+2)}{3!}x^3 + \ldots (i')$$

$$xS(n) = x - nx^2 + \frac{n(n+1)}{2!}x^3 - \ldots .. \quad (ii)$$

Adding (i′) and (ii) we have

$$(1+x)S(n) = 1 - (n-1)x + \frac{(n-1)n}{2!}x^2 - \frac{(n-1)n(n+1)}{3!}x^3 + \ldots\ldots \quad (iii)$$

But the R.H.S. of (iii) is the same as that of (i′) provided in the latter n is replaced by $(n-1)$. Thus the R.H.S. of (iii) is necessarily represented by $S(n-1)$. Accordingly we have:

$$(1+x)S(n) = S(n-1) \ldots\ldots\ldots \quad (iv)$$

where n is a whole positive number.

Proof by Induction

We may now apply the method of proof by induction. That $S(1) = (1+x)^{-1}$ follows by replacing $-x$ by x in the infinite series for $(1-x)^{-1}$. If we suppose that $S(n-1) = (1+x)^{-(n-1)}$ for a particular value of n, then it follows from (iv) that $S(n) = (1+x)^{-n}$. This establishes formula (i).

If we replace n by $-n$ this means that

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots \quad (v)$$

is a valid expansion, where now n may be either a positive or a negative integer but x is restricted to take values between -1 and 1 if n is negative.

Example. To expand $(1+x)^{-4}$ in ascending powers of x. Here $n = -4$; inserting in (v)

$$(1+x)^{-4} = 1 - 4x + \frac{4 \times 5}{2!}x^2 - \frac{4 \times 5 \times 6}{3!}x^3 + \frac{4 \times 5 \times 6 \times 7}{4!}x^4 - \ldots$$

i.e. $$\frac{1}{(1+x)^4} = 1 - 4x + 10x^2 - 20x^3 + 35x^4 - \ldots$$

Check. You must take a value of x in the range $+1$ to -1. Take $x = 0{\cdot}1$ for instance, then R.H.S. $= 1 - 0{\cdot}4 + 0{\cdot}1 - 0{\cdot}02 + 0{\cdot}0035 - \ldots\ldots = 0{\cdot}683$

$$\text{L.H.S.} = \frac{1}{(1+x)^4} = \frac{1}{1{\cdot}1^4} = \frac{1}{1{\cdot}21^2} = \frac{1}{1{\cdot}4641} = 0{\cdot}683$$

You notice that had we taken a larger value for x, say $x = 0{\cdot}5$, we should have required to carry the terms on the R.H.S. a great deal further before the calculation would have shown with any accuracy the value to which the whole series was approaching, or *converging*.

Example. To expand $\frac{1}{(1-x)^2}$ in ascending powers of x. Here you have to expand $(1-x)^{-2}$, so that in (v) you replace x by $-x$ and n by -2. Hence

$$\frac{1}{(1-x)^2} = 1 + (-2)(-x) + \frac{(-2)(-3)}{2!}(-x)^2 + \frac{(-2)(-3)(-4)}{3!}(-x)^3 + \ldots$$
$$= 1 + 2x + 3x^2 + 4x^3 + \ldots\ldots$$

The form of the expression on the right has a simple structure, and the series can be at once written down without further calculation.

The Binomial Theorem as stated in equation (v) is also valid when n is a fraction positive or negative, provided again that x lies between +1 and −1.

The proof of this is beyond our scope here, but by simple examples it will be possible to check its truth.

Example. To expand $\sqrt{(1+x)}$, that is $(1+x)^{\frac{1}{2}}$ in ascending powers of x you insert $n = \frac{1}{2}$ in equation (v). Thus: $\sqrt{(1+x)}$

$$= (1+x)^{\frac{1}{2}}$$
$$= 1 + \tfrac{1}{2}x + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}x^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}x^3 + \ldots$$
$$= 1 + \tfrac{1}{2}x - \frac{1}{2^2.2!}x^2 + \frac{1.3}{2^3.3!}x^3 - \frac{1.3.5}{2^4.4!}x^4 + \ldots\ldots \quad (vi)$$

Again the structure of the successive terms on the R.H.S. is reasonably apparent.

Check. Let $x = 0{\cdot}1$, then

$$\sqrt{(1{\cdot}1)} = 1 + \frac{1}{2} \times 0{\cdot}1 - \frac{1}{8} \times 0{\cdot}01 + \frac{1}{16} \times 0{\cdot}001 - \frac{5}{128} \times 0{\cdot}0001 + \ldots\ldots$$
$$= 1 + 0{\cdot}05 - 0{\cdot}00125 + 0{\cdot}00006 - 0{\cdot}000004 \ldots\ldots$$
$$= 1{\cdot}048806 \ldots$$

the last figure after the decimal point being doubtful.

Actually, direct evaluation of the square root of 1·1 gives 1·048807 ....

It seems, therefore, that provided the conditions attached to formula (v) are adhered to when n is not a positive integer, it may be validly applied.

Arithmetical Application of the Binomial Theorem

We illustrate this directly by examples.

1. Required to evaluate the cube root of 65 to five places of decimals. You require $65^{\frac{1}{3}}$. The number nearest to 65 which is a perfect cube is 64, i.e. 4^3. Hence you write

$$(65)^{\frac{1}{3}} = (64+1)^{\frac{1}{3}} = (4^3+1)^{\frac{1}{3}} = \left[4^3\left(1+\frac{1}{64}\right)\right]^{\frac{1}{3}}$$
$$= 4\left(1+\frac{1}{64}\right)^{\frac{1}{3}}$$

It is legitimate to use the binomial theorem for a fractional value of n only when the x is less than 1, and the binomial form is $1 + x$.

You have to evaluate $\left(1+\frac{1}{64}\right)^{\frac{1}{3}}$. Accordingly

$$\left(1+\frac{1}{64}\right)^{\frac{1}{3}} = 1 + \frac{1}{3} \times \frac{1}{64} + \frac{\frac{1}{3}(\frac{1}{3}-1)}{2!}\frac{1}{64^2} + \frac{\frac{1}{3}(\frac{1}{3}-1)(\frac{1}{3}-2)}{3!}\frac{1}{64^3} +$$

$$= 1 + \frac{1}{192} - \frac{1}{9 \times 64^2} + \frac{10}{162 \times 64^3} \dots$$

and on multiplying by 4 you get

$$(65)^{\frac{1}{3}} = 4 + \frac{1}{48} - \frac{1}{9216} + \frac{10}{10616832}$$

The last term is of the order 10^{-6} and is in fact only 1 in the sixth decimal place. Now

$$\frac{1}{48} = 0{\cdot}0208333\dots \quad \frac{1}{9216} = 0{\cdot}0001085$$

Hence

$$\sqrt[3]{65} = 4{\cdot}03726\dots$$

2. To determine for what range of values of x it is legitimate to evaluate

$$\frac{\sqrt{(1+x)} - \sqrt{(1-x)}}{x} \text{ by using } 1 + \frac{1}{8}x^2$$

You note that if x lies between $+1$ and -1 then:

$$\sqrt{(1+x)} = 1 + \tfrac{1}{2}x - \frac{1}{2^2.2!}x^2 + \frac{1.3}{2^3\,3!}x^3 - \frac{1.3.5}{2^4.4!}x^4 + \frac{1.3.5.7}{2^5.5!}x^5 \dots$$

$$\sqrt{(1-x)} = 1 - \tfrac{1}{2}x - \frac{1}{2^2.2!}x^2 - \frac{1.3}{2^3.3!}x^3 - \frac{1.3.5}{2^5.4!}x^4 - \frac{1.3.5.7}{2^5.5!}x^5 - \dots$$

Thus

$$\frac{\sqrt{(1+x)} - \sqrt{(1-x)}}{x}$$

$$= 1 + \frac{1.3}{2^3.3!}x^2 + \frac{1.3.5.7}{2^4.5!}x^4 + \dots$$

$$= 1 + \frac{1}{8}x^2 + \frac{7}{128}x^4 + \dots$$

It follows then that if the given function is to be indistinguishable in value from $1 + \frac{1}{8}x^2$, then $\frac{7}{128}x^4$ must be negligible. The question asked, therefore, is not capable of being answered unless the degree of accuracy with which $1 + \frac{1}{8}x^2$ is to represent the function is stated in advance. Suppose, therefore, that it is required to tabulate the given function accurately to four places after the decimal point. This requires, roughly, that the term neglected, i.e. $\frac{7}{128}x^4$, must be less than 5 in the next decimal place, i.e. $\frac{7}{128}x^4 < 0{\cdot}00005$.

Thus $x^4 < 0{\cdot}0009$

i.e. $x^2 < 0{\cdot}03$ i.e. $-0{\cdot}17 < x < 0{\cdot}17$

and this is well within the range for which the original expansions were valid.

Check. Let $x = 0{\cdot}1$, then

$$[\sqrt{1{\cdot}1} - \sqrt{0{\cdot}9}]/0{\cdot}1 = (1{\cdot}048806 - 0{\cdot}94869)/0{\cdot}1 = 1{\cdot}0012$$

$$\text{Also } 1 + \tfrac{1}{8}x^2 = 1 + \tfrac{1}{8}(0{\cdot}1)^2 = 1 + 0{\cdot}01/8 = 1{\cdot}0012$$

both to four figures.

This example brings out yet another point to which we shall return when we are discussing the subject of limits. If you ask what is the value of

$$[\sqrt{(1+x)} - \sqrt{(1-x)}]/x$$

when $x = 0$, there is no answer because division by 0 is not a legitimate arithmetical operation. However, the expression above gets nearer and nearer to 1 as x gets near to 0, since we know it to be approximately equal to $1 + \frac{1}{8}x^2$ when x is small (but not zero). This is a particular case of the evaluation of a limit which is itself basic to an understanding of the Differential Calculus.

EXERCISES

(1) Expand $(1-x)^{\frac{1}{2}}$, $(1+x)^{\frac{1}{3}}$ and $(1-x^2)^{\frac{1}{2}}$ in ascending powers of x.

(2) Expand $\left(1 - \frac{x}{2}\right)^{\frac{1}{5}}$ in ascending powers of x as far as x^4.

(3) From (2) determine the 5th root of 0·75 to 3 decimal places.

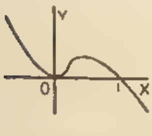
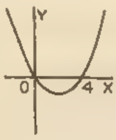
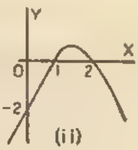


Fig.122

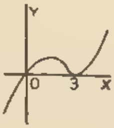


(i)



(ii)

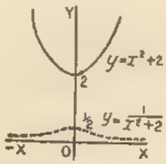
(iii)



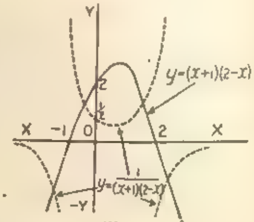
(iv)

(v)

Fig.123



(i)



(ii)

Fig. 124

STANDARD CURVES AND GRAPHICAL SOLUTIONS. **Figs. 122, 123, 124. Answers to Exercises 1, 5, and 6 in page 1263 in equations for standard curves and graphical solutions of algebraic equations.**

(4) Expand $(1 + x)^{-2}$ by the Binomial Theorem and show that on multiplying the expansion by $1 + 2x + x^2$ the result is 1.

(5) Expand $\frac{1}{2}\left[\frac{1}{1+x} + \frac{1}{1-x}\right]$ in ascending powers of x and verify that the same result is obtained by expanding $(1 - x^2)^{-1}$.

Why is this so?

(6) Show that $\left(1 + \frac{1}{x}\right)^{-1} = \frac{x}{x+1}$

Hence show that if x does *not* lie in the range -1 to $+1$, then

$$\frac{x}{x+1} = 1 - \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3} + \frac{1}{x^4} - \dots\dots\dots$$

(7) Determine $\sqrt{26}$ to 3 decimal places.

(8) Find for what range of x it is legitimate to use $2 - \frac{2x^4}{9}$ to calculate the value of $\sqrt[3]{(1 + x^3)} + \sqrt[3]{(1 - x^3)}$ to four decimal places.

ANSWERS TO EXERCISES IN LESSON 23

(1) *See* Fig. 122

(2) $x = 4{\cdot}19$, $x = -1{\cdot}19$

(3) Minimum value is $2\frac{3}{4}$ when $x = 1\frac{1}{2}$

(5) *See* Fig. 123 (i—v)

(6) *See* Fig. 124 (i and ii)

(7) (i) 2·84 (ii) −2·41, −1·15, 3·58

(8) 2·2, more accurately 2·175

LESSON 25

Elementary Statistics

WHEN it is stated that a town A has more sunshine than a town B it is not meant that on every day of the year A is sunnier than B. If it is said that the boys or girls in one class are older than those in another it is not meant that every individual in the one is older than every individual in the other. In both cases something is being stated about the group ; in the one case reference is to the total amount of sunshine or the *average* amount of sunshine in these towns, and in the other case to the total ages or perhaps simply the average age of the class of boys or girls. It will be explained here what the average really means, why it is used at all, and how much meaning it has.

These illustrations have been mentioned in order to bring out the fact that while in ordinary arithmetic one deals with things described by a single number, in the field of *statistics* one is concerned with measurements about a group, and therefore one has to deal with a group of numbers. In the case of a class of children, if you wish to consider the "age" of the class, you have to take into account the ages of all the members of the class.

The Average of a Set of Numbers

Suppose a class of ten boys shows the following ages, in years :

9, 12, 10, 9, 11, 10, 12, 11, 10, 12

The sum total of the ages is 106. There are in all 10 boys in the class. The average age is 106/10 = 10·6 years.

Notice that the average of a set of numbers is not a number of the set. Thus there is nothing absurd in saying, for example, that the average number of children in a family is 1·6 ; it is not asserted that *most* families have 1·6 children, but that the expected total number of children in, say, 10 families is 16.

More generally, if

$$a_1, a_2, a_3, \dots\dots a_n$$

be n numbers, the *mean* or *average* value is defined as

$$(a_1 + a_2 + a_3 + \dots + a_n)/n.$$

The average of a set is usually written by placing a horizontal bar above the typical letter. For instance in this case the above value would be written as $\bar{a}$.

Is the average of a set of numbers a good and sufficient description of that set ?

The first point to notice about an average is that it lies more or less centrally in the set. If you have two numbers, for instance 20 and 30, the average is $\frac{1}{2}(20 + 30) = 25$, which lies exactly midway between 20 and 30. If you have three numbers, say 11, 12, 16, the average is 13, which more or less lies centrally within the set of numbers.

Deviation

The *deviations* of a set of numbers, say $x_1, x_2, \dots\dots x_n$, from their mean or average are defined as the differences of each number of the set from the mean value of the set, i.e. $\bar{x}$. Thus

$$\bar{x} = (x_1 + x_2 + \dots + x_n)/n \qquad (i)$$

and the successive deviations are

$$x_1 - \bar{x},\ x_2 - \bar{x},\ \dots\dots x_n - \bar{x} \qquad (ii)$$

We shall now show that the value of $\bar{x}$ is such that the sum of the differences of all the numbers above it from the average is equal to the sum of the differences of all the numbers below it.

Example. 4, 6, 9, 9, 8, 6, 4, 5, 5, 4.

Average $\bar{x}=$

$(4 + 6 + 9 + 9 + 8 + 6 + 4 + 5 + 5 + 4)/10 = 6$

Deviations from the average :

$4 - 6 = -2$, $6 - 6 = 0$, $9 - 6 = 3$, $9 \quad 6 = 3$, $8 - 6 = 2$, $6 - 6 = 0$, $4 - 6 = -2$, $5 - 6 = -1$, $5 - 6 = -1$, $4 - 6 = -2$

The sum of the positive deviations is $3 + 3 + 2 = 8$. The sum of the negative deviations is $- 2 - 2 - 1 - 1 \quad 2 - \ -8$.

It is therefore in this sense that we say that the average lies centrally among a set of numbers.

Because of this it will very frequently be possible to refer to a set of experimental measurements of the same physical phenomenon by the average value obtained. For example, when you say that under certain circumstances the speed of sound is 1,190 ft. per sec., you mean that a number of measurements have been taken to obtain this result.

These differ from each other since the circumstances associated with each reading vary from one occasion to the next. Thus the measurements may range from 1,200 ft. per sec. to 1,180 ft. per sec. The value 1,190 is found by taking the average or mean of all the readings, and the deviations from the average are thought of as the experimental errors.

Can one say that the average is a sufficient description of a group of numbers ? Take a simple case :

The average of 10 and 90 is 50.
The average of 49 and 51 is 50.

Here are two pairs of numbers that differ very much from one another. The first pair has a *range* of 80, viz. 90 − 10, the second a range of 2, viz. 51 − 49, and yet they have the same average.

How are you to bring out this spread of the numbers ? The average itself does not succeed in doing so. You could, of course, besides the average, take the range, i.e. the difference between the largest and the smallest of the set, but even that is not satisfactory. For example, consider the two sets

(i) 1·0, 1·0, 1·0, 1·0, 1·4, 1·4, 1·4, 1·4
Average 1·2
Range 0·4

(ii) 1·0, 1·2, 1·2, 1·2, 1·2, 1·2, 1·2, 1·4
Average 1·2
Range 0·4

Here are two sets that differ very definitely in character, although they have the same average and the same range. In the first case the numbers occur at the two extremes, in the second case they are mainly bunched about the average. If they were intended to be readings of the same physical effect, you could not avoid saying that the second set is better since most of the readings actually coincide with the average.

The point to bear in mind is that you are trying to find a method of describing the collection or group of numbers. The average is valuable but it is not sufficient. The range may also be valuable but it may be misleading. We return therefore to the deviations from the average. In these cases they are

(i) −0·2, −0·2, −0·2, −0·2, 0·2, 0·2, 0·2, 0·2
(ii) −0·2, 0, 0, 0, 0, 0, 0, 0·2

Although the sum of the deviations is zero in both cases, the actual numbers are larger in the first case than in the second case. What you require is an over-all measure of the size of these deviations irrespective of their sign, for these are assuredly associated with their spread and the way in which the numbers are heaped up at the ends of the range in the first case, and at the middle of the range in the second case. For this purpose, therefore, we define a new quantity, the *variance*, which is the average value of the *squares* of the deviations. An equivalent and equally useful measure of the spread is afforded by the *standard deviation*, which is just the square root of the variance. You write σ (Greek *sigma*) for the standard deviation, so the variance is σ^2.

For example, in the above two cases the squares of the deviations are

(i) 0·04, 0·04, 0·04, 0·04, 0·04, 0·04, 0·04, 0·04
(ii) 0·04, 0, 0, 0, 0, 0, 0, 0·04

The average value of these numbers in each case is

(i) 0·04 (ii) 0·01

The standard deviation is the square root of these numbers. Thus

(i) $\sigma = 0{\cdot}2$ (ii) $\sigma = 0{\cdot}1$

Thus the standard deviation in the first case is twice that in the second, corresponding to the fact that in the first case the numbers were heaped up at the ends of the range, and in the second case they were crowded together in the centre.

Generally, therefore, if $x_1, x_2, x_3, \ldots x_n$ is a set of n numbers whose average is $\bar{x}$, then the squares of their deviations from the average is

$$(x_1 - \bar{x})^2, (x_2 - \bar{x})^2, \ldots (x_n - \bar{x})^2.$$

The average value of these numbers is

$$\frac{1}{n}\left[(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2\right]$$

and this is the variance. Thus

$$\sigma^2 = \frac{1}{n}\left[(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2\right] \quad \ldots (iii)$$

Suppose you had not taken the average value $\bar{x}$ from which to measure the deviations, but another number, say M. You would not

have found σ but a corresponding value, say S. Let us examine S.

$$S^2 = \frac{1}{n}\left[(x_1 - M)^2 + (x_2 - M)^2 + \ldots . (x_n - M)^2\right]$$

$$= \frac{1}{n}\left[(x_1-\bar{x}+\bar{x}-M)^2 + \ldots . (x_n-\bar{x}+\bar{x}-M)^2\right]$$

$$\frac{1}{n}\left[(x_1 - \bar{x})^2 + 2(\bar{x} - M)(x_1 - \bar{x}) + (\bar{x} - M)^2\right.$$
$$(x_2 - \bar{x})^2 + 2(\bar{x} - M)(x_2 - \bar{x}) + (\bar{x} - M)^2$$
$$+ \ldots \ldots \ldots$$
$$\left.+ (x_n - \bar{x})^2 + 2(\bar{x} - M)(x_n - \bar{x}) + (\bar{x} - M)^2\right]$$

Now, the sum of the first terms in each row is the sum of the squares required for the calculation of σ^2.

Again, every second term in each row has a factor $2(\bar{x} - M)$. If you add all these second terms, therefore, you would have $2(\bar{x} - M)$ multiplied by

$$(x_1 - \bar{x}) + (x_2 - \bar{x}) + \ldots . + (x_n - \bar{x})$$
$$= x_1 + x_2 + \ldots \ldots x_n - n\bar{x}$$
$$= n\bar{x} - n\bar{x} = 0.$$

It follows that the sum of the second terms contributes nothing to S^2.

Every third term is the same, viz. $(\bar{x} - M)^2$. Thus

$$S^2 = \frac{1}{n}\left[n\sigma^2 + n(\bar{x} - M)^2\right]$$

or $\quad S^2 = \sigma^2 + (\bar{x} - M)^2$ or $\sigma^2 = S^2 - (\bar{x} - M)^2$ (*iv*)

This shows at once that if you take the wrong mean, the value of S^2 found in this way would always be greater than the actual standard deviation. When M is chosen to be the mean, however, S^2 reaches its least value, viz., σ^2. Stated otherwise, the sum of the squares of the deviations is a minimum when the number from which the deviations are measured is chosen equal to the mean of the readings. This, then, is the property that links together the two characteristic numbers chosen, on other grounds, to represent the group.

The relation (iv) provides a convenient method of evaluating the standard deviation in most cases. For a set of numbers does not in general give a round or simple value for the average, so that if you use the direct definition of σ given by (iii), the calculation of σ may be very heavy. If you use (iv), and calculate S from a false but convenient "mean," M. the labour is enormously reduced.

Example. To find the mean and the standard deviation of the following set of numbers:

10, 9, 11, 14, 12, 12, 9, 10.

$$\text{Average} = \bar{x}$$
$$= \frac{1}{8}[10 + 9 + 11 + 14 + 12 + 12 + 9 + 10]$$
$$= \frac{1}{8} \times 87 = 10\tfrac{7}{8}$$

Instead of the true mean 10·875, let us choose 10. a false mean. Then the deviations are

0. −1. 1. 4. 2. 2. −1. 0

Thus

$$S^2 = \frac{1}{8}[0^2 + 1^2 + 1^2 + 4^2 + 2^2 + 2^2 + 1^2 + 0^2]$$
$$= \frac{1}{8} \times 27$$
$$\sigma^2 = S^2 - (\bar{x} - M)^2 = \frac{27}{8} - \left(10\tfrac{7}{8} - 10\right)^2 = \frac{27}{8} - \frac{49}{64}$$
$$= \frac{167}{64} \text{ i.e. } \sigma = 1{\cdot}615$$

Had you obtained each deviation from the true mean, viz. $10\frac{7}{8} = 10{\cdot}875$, every term would have involved decimals, and the calculation of their squares would have been much more laborious.

Studying the Frequency

The two measures chosen to represent a group of numbers suffice to describe certain aspects. In themselves they do not give a picture of a set of readings *as a group*. This can best be seen by drawing a *frequency* diagram. Here is a particular example.

Here are the marks of 16 candidates for an examination:

18, 24, 24, 25, 34, 45, 50, 54, 54, 55, 56, 70, 75, 77, 78, 88.

You divide them into equal ranges of marks, say, those that fall between 1–20, 21–40, 41–60, 61–80, 81–100.

Hence you have the following table.

Mark Range	1–20	21–40	41–60	61–80	81–100
Marks	18	24, 24 25, 34	45, 50 54, 54 55, 56	70, 75 77, 78	88
Number of Candidates in each range	1	4	6	4	1

By this means you have classified the candidates into five groups, the numbers 1, 4, 6, 4, 1 telling how many of the candidates fall into each group. These numbers therefore are called the frequencies.

To illustrate this by a diagram, proceed as in Fig. 125.

The horizontal axis is divided up into the equal ranges, and at the *centre* of each range a perpendicular line is erected equal in length to the frequency of the occurrence of candidates in that range.

When a horizontal line is drawn across the range at the top of each ordinate, the block diagram so obtained is called a *histogram*.

The average mark is then estimated by multiplying the central value of each range by the number falling in this range and dividing by the total number of candidates. This is equivalent to assuming that all the candidates that fall in that range have the same mark. While,

therefore, the method gives a picture of the performance of the whole group, it does so at the expense of slurring over the individual performances of the candidates.

The mid-values of the ranges are

$10\frac{1}{2}$, $30\frac{1}{2}$, $50\frac{1}{2}$, $70\frac{1}{2}$, $90\frac{1}{2}$

These have to be multiplied by

1, 4, 6, 4, 1

respectively, the results added, and divided by 16.

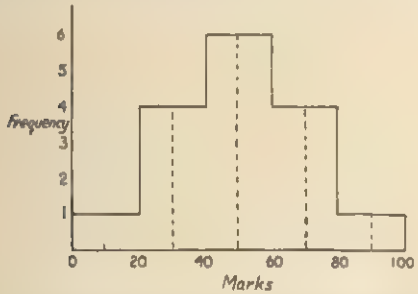
Thus, Average =

$$\frac{1 \times 10\frac{1}{2} + 4 \times 30\frac{1}{2} + 6 \times 50\frac{1}{2} + 4 \times 70\frac{1}{2} + 1 \times 90\frac{1}{2}}{16}$$

$$= \frac{1}{16}[10\tfrac{1}{2} + 122 + 303 + 282 + 90\tfrac{1}{2}]$$

$$= 50{\cdot}5$$

The standard deviation can be calculated directly by taking this average, thus avoiding the occurrence of fractions.



FREQUENCY DIAGRAM. Fig. 125. This histogram shows the mean and standard deviation for the marks obtained by sixteen candidates.

Deviations from the mean and the frequencies with which they occur are as follows.

Deviations from mean	$10{\cdot}5-50{\cdot}5 = -40$	$30{\cdot}5-50{\cdot}5 = -20$	$50{\cdot}5-50{\cdot}5 = 0$
Frequencies	1	4	6
Deviations from mean	$70{\cdot}5-50{\cdot}5 = 20$	$90{\cdot}5-50{\cdot}5 = 40$	
Frequencies	4	1	

Hence

$$\sigma^2 = \frac{1 \times (40)^2 + 4(20)^2 + 6(0)^2 + 4 \times (20)^2 + 1 \times (40)^2}{16}$$

$$= \frac{1600 + 1600 + 1600 + 1600}{16}$$

$= 400$. Hence $\sigma = 20$.

In what sense can one now say that $\sigma = 20$ gives a measure of the spread or dispersion of the marks about the mean, i.e. 50·5? Let us reply by asking a counter-question. How many candidates fall within a range of ($\pm$) 20 of the average? This ranges from $50{\cdot}5 - 20 = 30{\cdot}5$ to $50{\cdot}5 + 20 = 70{\cdot}5$. The number 30·5 falls exactly midway in the interval 21–40 within which 4 candidates fall. You take 2 of these, therefore, as falling within the band $\pm\sigma$ about the average. In addition, 6 fall in the range 41–60 and for the same reason as before you must add 2 from the range 61–80, since 70·5 falls midway. Thus altogether $2 + 6 + 2 = 10$ candidates fall within the band $\pm\sigma$ about the average—or a proportion of 10/16 of the total. Thus for this set of candidates 62·5 per cent. of the candidates fall in the range $\pm\sigma$. If in this type of examination this batch of 16 candidates is typical, you have now a measure of what to expect in future cases.

In fact, by keeping a record of the performances of candidates in the past you can draw a histogram and obtain a measure σ and an average. You can state what proportion is to be expected to fall within the range $\pm\sigma$ for future batches and hence can determine whether such a batch is a normal collection or not.

Application to Engineering

This method of analysis is of great importance in relation to standardised engineering production. In place of candidates and their marks you have engineering material that is being produced to standardised size. For a whole variety of reasons the individual items differ slightly among themselves. The average should be the specified size if the system is working correctly and stably. The value of σ can be calculated from the material which has been produced in the past or estimated from samples. Later batches can then be tested to determine whether they are consistent with this. It is not our purpose here to enter into this field of engineering statistics which goes by the name of *Quality Control*, as it has now become a separate field of study and of engineering practice. It must suffice to give this indication of the lines along which the analysis would proceed.

EXERCISES

(1) Find the mean of each of the two following sets of numbers:

(i) 4·2, 4·6, 5·9, 7·3, 8·5

(ii) 4·2, 4·6 (twice), 5·9, 7·3 (five times), 8·5 (eight times). Explain why the mean of (ii) is much larger than the mean of (i).

(2) 30 engines were examined for defects, with the following results:

1 engine had 0 defect.
4 engines had 1 defect.
10 engines had 2 defects.
8 engines had 3 defects.
5 engines had 4 defects.
2 engines had 5 defects.

(i) How many defects were there in all?

(ii) What was the average number of defects per engine?

(3) The lengths of 30 rods are given in inches as follows:

28	18	24	27	20	17	23	34	26	19
13	16	29	24	23	25	22	26	11	24
20	19	25	25	23	24	21	25	26	24

Find the average length of rod. Group the measurements in the ranges 10—15, 15—20, 20—25, 25—30, 30—35 inches, and draw the histogram.

(4) The ages (in years) of a group of 6 boys are 12, 13, 14, 17, 15, 16. Find the mean and variance of their ages directly. Calculate the average of the squares of the deviations of each age from 12, 13, 14, 16, 17 in turn, and show that the average of these squared deviations in each case is greater than that taken from the mean.

Show that the true variance may be obtained by using relation (4) by calculating it in turn from false means of 12, 17, 14.

Answers to Exercises in Lesson 24

(1) (i) $1 - \frac{1}{4}x - \frac{3}{32}x^2 - \frac{7}{128}x^3 - \frac{77}{2048}x^4 - \ldots\ldots$

(ii) $1 + \frac{1}{4}x - \frac{3}{32}x^2 + \frac{7}{128}x^3 - \frac{77}{2048}x^4 + \ldots\ldots$

(iii) $1 - \frac{1}{4}x^2 - \frac{3}{32}x^4 - \frac{7}{128}x^6 - \frac{77}{2048}x^8 - \ldots\ldots$

(2) $1 - \frac{1}{10}x - \frac{1}{50}x^2 - \frac{3}{500}x^3 - \frac{33}{20{,}000}x^4 - \ldots\ldots$

(3) 0·944 (7) 5·099 (8) $-0{\cdot}39 < x < 0{\cdot}39$

LESSON 26

The Trigonometry of the Triangle

We now propose to link up the subjects of geometry and trigonometry much more closely. It is true that we defined the trigonometrical ratios by reference to a right-angled triangle, but we will now obtain relations between the sides and angles of *any* triangle.

It is convenient to adopt a standard notation when dealing with this part of the work. We denote the magnitude of the angles of any triangle ABC by the letters A, B, and C, and the lengths of the sides opposite to these angles by a, b, c respectively. The radius of the inscribed circle of the triangle ABC is represented by r, while R denotes the radius of the circumscribed circle of the triangle ABC. The area of this triangle is conveniently represented by the symbol Δ.

The Sine Rule

The sine rule states that for any triangle ABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

Consider Fig. 126 (a). Here we have drawn any triangle ABC and constructed a perpendicular AD on to the side BC. Since this perpendicular might fall on the base BC produced we must consider whether this fact will affect our proof. Thus we need to consider the two possible cases which may arise, indicated by Figs. 126 (a) and (b). In (a) it is clear that since both ADC and ADB are right-angled triangles, then AD = c sin B or alternatively, b sin C. In (b), from the triangle ABD, AD = c sin B as before, but a difference apparently occurs in the triangle ADC. Here AD = b sin $\angle$ACD = b sin (180° − C), but since sin (180° − C) = sin C we have exactly the same relation in both cases, namely

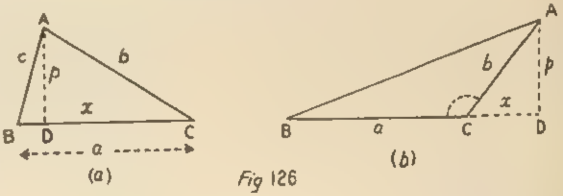
$$b \sin C = c \sin B$$

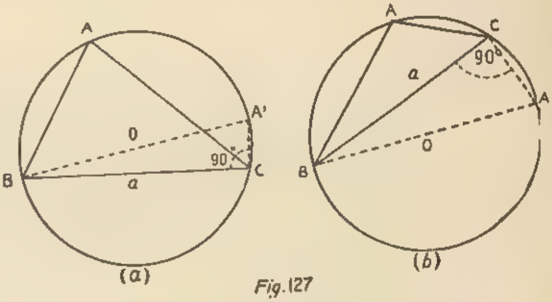
i.e.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Similarly, we may draw perpendiculars from either of the two other vertices, when either case (a) or case (b) must apply, so that we obtain a general relation, true of all triangles, that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$





Figs. 126 (a) and (b). The sine rule applied to any triangle ABC. Figs. 127 (a) and (b) demonstrate that either of the ratios a/sin A = b/sin B = c/sin C is equal to 2R (radius of the circumscribed circle of the triangle ABC).

To prove that either of these ratios is equal to 2R, consider Figs. 127 (a) and (b). We have drawn two triangles ABC, together with their circumscribed circles, the difference in the two cases being that in (b) ABC lies wholly in a semicircle. In each case join B to the centre of the circle, and produce to meet the circumference in A′. Join CA′. We now need to use some of the geometrical facts obtained

in Lesson 6; first, that the angle in a semicircle is a right angle. Thus in each case

$$a = A'B \cos \angle A'BC$$
i.e. $$a = 2R \cos \angle A'BC \quad \ldots\ldots \quad (i)$$

since $A'B$ is a diameter of the circumscribed circle. Secondly, in both cases $\angle BA'C = 90° - \angle A'BC$, i.e. $\cos \angle A'BC = \sin \angle BA'C$. Considering (a) only, the angle $BA'C = A$ since by a geometrical theorem the angles in the same segment of a circle, standing on the same chord, are equal. Thus as regards Fig. 127 (a), relation (i) becomes

$$a = 2R \sin \angle BA'C$$
$$= 2R \sin A.$$

For Fig. 127(b), $\angle BA'C$ (i.e. A') $= 180° - A$, since the opposite angles of a cyclic quadrilateral together add up to 180°, and therefore in this case relation (i) becomes

$$a = 2R \sin (180° - A)$$

But since

$$\sin (180° - A) = \sin A$$

we have also in this case that

$$a = 2R \sin A$$

so that in *all* cases

$$\frac{a}{\sin A} = 2R$$

There is another important theorem which has no particular name but states that there are three relations between the sides and angles of any triangle of the form

$$a = b \cos C + c \cos B.$$

This can be proved by again considering Figs. 126 (a) and (b).

For (a)
$$a = BD + CD$$
$$= c \cos B + b \cos C$$

and for (b),
$$a = BD - CD$$
$$= c \cos B - b \cos \angle ACD$$
$$= c \cos B - b \cos (180° - c)$$
$$= c \cos B + b \cos C$$
because $\cos (180° - C) = -\cos C$

By drawing perpendiculars from the other two vertices we obtain the other two relations of this type, which are

$$b = a \cos C + c \cos A$$
and $$c = a \cos B + b \cos A$$

The Cosine Rule

The cosine rule is the name given to three relations of the form

$$c^2 = a^2 + b^2 - 2ab \cos C$$

which may be proved by the application of the Theorem of Pythagoras. Refer again to Figs. 126 (a) and (b), and let AD be denoted by p and CD by x. Then, for (a),

$$c^2 = AD^2 + BD^2$$
$$= p^2 + (a - x)^2$$
$$= p^2 + a^2 - 2ax + x^2$$
$$= b^2 + a^2 - 2ax \quad \ldots\ldots\ldots \quad (ii)$$

because $p^2 + x^2 = b^2$. Now $x = b \cos C$.

so that (ii) becomes

$$c^2 = b^2 + a^2 - 2ab \cos C$$

For the case where C is obtuse, Fig. 126 (b), we have

$$c^2 = p^2 + (a + x)^2$$
$$= p^2 + a^2 + 2ax + x^2$$
$$= b^2 + a^2 + 2ax \quad \ldots\ldots\ldots \quad (iii)$$

as before; but here

$$x = b \cos \angle ACD$$
i.e. $$x = b \cos (180° - C)$$
$$= -b \cos C, \text{ so that (iii) becomes}$$
$$c^2 = b^2 + a^2 - 2ab \cos C \quad \ldots\ldots \quad (iv)$$

proving the result generally. The two other relations of this type are

$$a^2 = b^2 + c^2 - 2bc \cos A$$
and $$b^2 = c^2 + a^2 - 2ac \cos B$$

and it should be noticed that they provide the means of obtaining the cosine of each of the angles of *any* triangle in terms of the sides only, for (iv) may be written

$$\cos C = \frac{b^2 + a^2 - c^2}{2\,ab} \quad \ldots\ldots\ldots\ldots \quad (v)$$

Example. Find the largest angle of the triangle whose sides are 6, 7, 8 ins.

The largest angle is that opposite the longest side; in this case it is the angle opposite to the side of length 8 ins. Thus

$$8^2 = 6^2 + 7^2 - 2.\ 6.\ 7 \cos C$$

(if we take $c = 8$), so that, re-arranging,

$$\cos C = \frac{6^2 + 7^2 - 8^2}{2.\ 6.\ 7}$$
$$= \frac{21}{84} = \frac{1}{4}$$

and, from tables,

$$C = 75° 31'$$

Example. Prove that in any triangle ABC, if m represents the median drawn from A on to BC, then $4m^2 = 2b^2 + 2c^2 - a^2$.

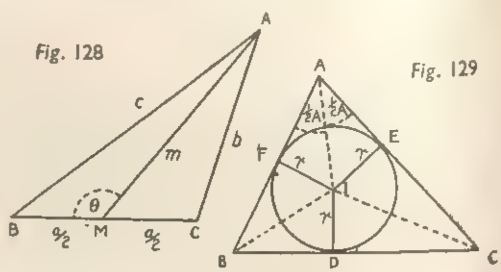


Fig. 128. **Cosine rule applied to two triangles.**
Fig. 129. **Radius of inscribed circle of triangle ABC.**

In Fig. 128 ABC is the triangle and AM the median. Let $\angle AMB = \theta$, so that applying the cosine rule to the triangle AMB,

$$c^2 = m^2 + \frac{a^2}{4} - 2 \,.\, m \,.\, \frac{a}{2} \cos \theta \quad \ldots\ldots \quad (vi)$$

Also, by applying the cosine rule to the triangle AMC, we obtain

$$b^2 = m^2 + \frac{a^2}{4} - 2 \cdot \frac{a}{2} \cdot m \cos (180^\circ - \theta)$$

$$= m^2 + \frac{a^2}{4} + 2 \cdot \frac{a}{2} \cdot m \cos \theta \quad . \; . \; . \quad (vii)$$

Adding relations (vi) and (vii),

$$b^2 + c^2 = 2m^2 + \tfrac{1}{2}a^2$$

i.e. $$4m^2 = 2b^2 + 2c^2 - a^2$$

This is best remembered as

$$AB^2 + AC^2 = 2AM^2 + 2BM^2$$

The cosine rule has enabled us to write the cosine of any angle of a triangle in terms of the sides alone, and therefore, using the fundamental identity $\sin^2 A + \cos^2 A = 1$, we may obtain an expression for the sine of any angle of a triangle in terms of the sides only. Thus

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

therefore

$$\sin^2 A = 1 - \cos^2 A$$

$$= 1 - \left(\frac{b^2 + c^2 - a^2}{2bc}\right)^2$$

We can simplify the expression on the R.H.S. by noticing that it is the difference of two squares, and so we may write

$$\sin^2 A = \left(1 + \frac{b^2 + c^2 - a^2}{2bc}\right)\left(1 - \frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$\frac{1}{4b^2c^2}(2bc + b^2 + c^2 - a^2)\ (2bc - b^2 - c^2 + a^2)$$

by putting each term on the R.H.S. over its common denominator. We may further simplify the R.H.S. thus:

$$\sin^2 A =$$

$$\frac{1}{4b^2c^2}(b^2 + 2bc + c^2 - a^2)\ (a^2 - (b^2 - 2bc + c^2))$$

$$= \frac{1}{4b^2c^2}[(b + c)^2 - a^2]\ [a^2 - (b - c)^2],$$

and now each factor on the R.H.S. is itself the difference of two squares, i.e.

$$\sin^2 A = \frac{1}{4b^2c^2}(b + c + a)\ (b + c - a) \times$$

$$(a - b + c)\ (a + b - c) \quad . \; . \; . \quad (viii)$$

These factors on the R.H.S. are of common occurrence in work of this nature and it is convenient to write $2s = a + b + c$. The R.H.S. may then be written much more simply in terms of s, the *semi-perimeter*, for

$$b + c - a = b + c + a - 2a, \text{ i.e. } 2s - 2a.$$

similarly $a - b + c = a + b + c - 2b = 2s - 2b$

and so (viii) may be written

$$\sin^2 A = \frac{1}{4b^2c^2} 2s \cdot (2s - 2a)\ (2s - 2b)\ (2s - 2c)$$

$$= \frac{4}{b^2c^2} s\ (s - a)\ (s - b)\ (s - c)$$

i.e.

$$\sin A = \frac{2}{bc}\sqrt{(s\ (s - a)\ (s - b)\ (s - c))} \quad . \; . \; . \quad (ix)$$

Refer again to Figs. 126 (a) and (b). In both cases the area of the triangle ABC = $\tfrac{1}{2}$ AD. BC, and since AD was shown to be equal to $b \sin C$ in both cases, and BC = a, we have

$$\Delta = \tfrac{1}{2}a \cdot b \sin C$$

By analogy with relation (ix)

$$\sin C = \frac{2}{ab}\ \sqrt{(s\ (s - a)\ (s - b)\ (s - c))}$$

and therefore the area of the triangle ABC is given by

$$\Delta = \tfrac{1}{2}ab \cdot \sin C$$

$$= \sqrt{(s\ (s - a)\ (s - b)\ (s - c))} \quad . \; . \; . \; . \; . \quad (x)$$

From this relation you can obtain the area of *any* triangle, knowing the lengths of the three sides.

Radius of Inscribed Circle of a Triangle

Let ABC (Fig. 129) be any triangle and D, E, F the points where the inscribed circle touches the sides. Join A, B, and C to I, the centre of the inscribed circle. Then ID = IE = IF = r and each of these radii is perpendicular to the sides of the triangle, so that

area Δ ABC = area Δ AIB + area Δ BIC + area Δ CIA

$$= \tfrac{1}{2}r \cdot AB + \tfrac{1}{2}r \cdot BC + \tfrac{1}{2}r \cdot AC$$

$$= \tfrac{1}{2}r\ (AB + BC + AC)$$

$$= \tfrac{1}{2}r\ (a + b + c)$$

$$= \tfrac{1}{2}r \cdot 2s$$

$$= rs$$

so that $\Delta = rs$

or $r = \dfrac{\Delta}{s}$

Summarising, we have the following group of formulae for any triangle:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R,$$

$$\Delta = rs = \tfrac{1}{2}bc \sin A = \tfrac{1}{2}ca \sin B$$

$$= \tfrac{1}{2}ab \sin C = \sqrt{s\ (s - a)\ (s - b)\ (s - c)}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = c^2 + a^2 - 2ca \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

The "Half-Angle" Formulae

From the same diagram (Fig. 129) we obtain an important relation concerning $\tan \tfrac{1}{2}A$. The triangles AIF, AIE are congruent since AF = AE (tangents from an external point to a circle are equal) so that

$$\angle FAI = \tfrac{1}{2}A$$

Thus $\tan \tfrac{1}{2}A = \dfrac{r}{AF}$ . . . . (xi)

and we can obtain AF in terms of s. From

the geometrical theorem just quoted, AE = AF, CE = CD, BF = BD and therefore

$$\begin{aligned} 2s &= a + b + c \\ &= a + (CE + AE) + (BF + AF) \\ &= a + CE + BF + 2AF \\ &= a + CD + BD + 2AF \\ &= 2a + 2AF \end{aligned}$$

i.e. $\quad AF = s - a$

so that (ii) becomes

$$\tan \tfrac{1}{2}A = \frac{r}{s-a}$$

and since

$$r = \frac{\Delta}{s} \text{ we have}$$

$$\begin{aligned} \tan \tfrac{1}{2}A &= \frac{\Delta}{s(s-a)} \\ &= \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s(s-a)} \\ &= \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \quad \ldots\ldots\ldots \quad (xii) \end{aligned}$$

From this relation for $\tan \frac{1}{2}A$ can be deduced the corresponding relations for $\cos \frac{1}{2}A$ and $\sin \frac{1}{2}A$.

Now $\quad \sec^2 \frac{1}{2}A = 1 + \tan^2 \frac{1}{2}A$

$$\begin{aligned} \therefore \quad \sec^2 \tfrac{1}{2}A &= 1 + \frac{(s-b)(s-c)}{s(s-a)} \\ &= \frac{s(s-a) + (s-b)(s-c)}{s(s-a)} \\ &= \frac{2s^2 - sa - sb - sc + bc}{s(s-a)} \\ &= \frac{2s^2 - s(a+b+c) + bc}{s(s-a)} \\ &= \frac{2s^2 - s \,.\, 2s + bc}{s(s-a)} \\ &= \frac{bc}{s(s-a)} \end{aligned}$$

$$\begin{aligned} \text{i.e.} \quad \cos^2 \tfrac{1}{2}A &= \frac{1}{\sec^2 \frac{1}{2}A} \\ &= \frac{s(s-a)}{bc} \end{aligned}$$

$$\text{or} \quad \cos \tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}$$

Also, since $\tan \frac{1}{2}A = \dfrac{\sin \frac{1}{2}A}{\cos \frac{1}{2}A}$

$$\begin{aligned} \text{we have} \quad \sin \tfrac{1}{2}A &= \cos \tfrac{1}{2}A \,.\, \tan \tfrac{1}{2}A \\ &= \sqrt{\frac{s(s-a)}{bc}} \,.\, \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \\ &= \sqrt{\frac{(s-b)(s-c)}{bc}} \end{aligned}$$

In each case there are corresponding expressions for the sine, cosine, and tangent of $\frac{1}{2}B$ and $\frac{1}{2}C$.

EXERCISES

(1) Find the angles A and B and the side c of the triangle for which $a = 3''$, $b = 5''$, $C = 60°$.

(2) Show that there are two triangles having $b = 3$, $c = 4$, $B = 40°$ and find the angle A in each case.

(3) Show that there is no triangle with $b = 5$, $c = 20$, $B = 40°$.

(4) In a triangle $\sin^2 C = \sin^2 A + \sin^2 B$. Prove that the triangle is right-angled.

(5) If l is the length of a diagonal of a parallelogram which makes angles A, B with the sides, show that the lengths of the sides are $l \sin A/\sin (A + B)$ and $l \sin B/\sin (A + B)$.

(6) Given $c = 10$, $a = 12$, $B = 35°$, find the length of the median which bisects BC.

(7) In a triangle ABC prove that

$$\tan \frac{B - C}{2} = \frac{b - c}{b + c} \cot \frac{A}{2}$$

(8) The lengths of the sides of a cyclic quadrilateral are represented by a, b, c, d and the angle between a and b is θ. Draw the diagonal opposite to θ and express its length by applying the cosine rule to each of the triangles so formed. From these two expressions prove that

$$\cos \theta = \frac{a^2 + b^2 - c^2 - d^2}{2(ab + cd)}$$

(9) In a triangle $a = 13$, $b = 14$, $c = 15$ ft. Find R, r, Δ.

(10) A tower, of height T, standing on a hill sloping at an inclination α to the horizontal, throws a shadow of length S directly downhill. Show that the inclination θ of the sun's rays to the hillside is given by

$$\cot \theta = \tan \alpha + \frac{S}{T \cos \alpha}$$

If T = 100 ft., $\alpha = 10°$ and S = 120 ft., determine θ.

(11) Prove that in any triangle
(i) $4 R^2 (\sin^2 A + \sin^2 B + \sin^2 C) = a^2 + b^2 + c^2$
(ii) $a \cos A + b \cos B + c \cos C = R (\sin 2A + \sin 2B + \sin 2C)$.

Answers to Exercises in Lesson 25

(1) (i) 6·1 (ii) 7·28
The greater frequency of the larger numbers in (ii) increases the mean, compared with (i).

(2) (i) 78 (ii) 2·6

(3) Mean = 22·7 ins., frequencies are 2, 6, 13, 8, 1.

(4) Mean = 15 years, σ^2 = 2[illegible]
S^2 (from 12) = 11[illegible]
S^2 (from 13) = S^2 (from 17) = 6[illegible]
S^2 (from 14) = S^2 (from 16) = 3[illegible]

LESSON 27

The Calculus: Speed, Acceleration, and Rate of Growth

The work already done now provides us with the trunk from which emerges a new and most important branch of mathematics—the *Calculus*. This is concerned with *rates of change*, and is basic in applied mathematics. Here, for example, are some typical illustrations:

At what rate does the level of water fall in a tank if a tap is opened at its lowest point? Notice that the rate of flow through the tap will depend on the height of water in the tank, and vice versa.

At what rate will the speed of a truck increase if it runs freely down a given gradient under gravity and against friction only? At what rate will the gas in a cylinder diminish in volume if the pressure on the piston is itself increased at a given rate? At what

rate will a block of ice melt if heat is supplied to it at a given rate? At what rate will a bomb increase in speed if dropped from a given height?

Rates of Change

It will be seen at once that rate of change is a fundamental concept in mathematics. Here is a simple example.

Suppose the side of a uniformly expanding cube at any moment is x inches. Its volume is then $V = x^3$ cub. in. What is the rate of change of volume?

Suppose x increases from x to $x + h$. Then the volume will increase from x^3 to $(x + h)^3$.

The symbol δ (Greek *delta*) stands for the words *small change in*, so δV means not $\delta \times V$, but "small change in volume." Hence in the case,

$$\delta V = (x + h)^3 - x^3 = 3hx^2 + 3h^2x + h^3.$$

The rate at which V is changing with increase h in length of side, will be the ratio of δV to h. Thus,

$$\frac{\delta V}{h} = 3x^2 + 3hx + h^2$$

The R.H.S. gets closer and closer to $3x^2$ as h gets smaller and smaller. We say that it *tends* to $3x^2$ as h *tends* to 0 and write this

$$\lim_{h\to 0} (3x^2 + 3hx + h^2) = 3x^2$$

We then *define* the rate of change of V as $\lim\limits_{h\to 0} \dfrac{\delta V}{h}$. Notice that the rate of change of volume depends on the length of the side of the cube (at the instant of time when the rate of change is being measured), although the sides are expanding uniformly with time.

Thus the rate of change of $V = x^3$ with x is $3x^2$. For example, a cube of side 100 in. has a volume of $10^6 = 1{,}000{,}000$ cub. in. and at that stage it will increase in volume by $3 \times 100^2 = 30{,}000$ cub. in. per inch increase in x, a side.

If δV stands for a small change in V, you can replace h by δx, a small change in x.

Now consider this more generally. Suppose $y = Ax^n$, where A and n are any numbers. You inquire at what rate y will change with x. From what we have just done it is clear that a small change in y, viz. δy, will be produced by a small change δx in x, where

$$\delta y = A(x + \delta x)^n - Ax^n.$$

Now

$$(x + \delta x)^n = x^n + nx^{n-1}\delta x + \frac{n(n-1)}{2} x^{n-2} (\delta x)^2 + \dots$$

so that

$$\delta y = An\, x^{n-1}\, \delta x + \frac{An(n-1)}{2!} x^{n-2} (\delta x)^2 + \dots$$

so

$$\frac{\delta y}{\delta x} = An\, x^{n-1} + \frac{An(n-1)}{2!} x^{n-2}\, \delta x + \dots$$

On the right, as $\delta x \to 0$, all the terms except the first also tend to 0, so that $\dfrac{\delta y}{\delta x} \to Anx^{n-1}$.

You write $\dfrac{dy}{dx}$ for $\lim\limits_{\delta x\to 0} \dfrac{\delta y}{\delta x}$, so that

$$\text{if } y = Ax^n \text{ then } \frac{dy}{dx} = An\, x^{n-1}$$

$\dfrac{dy}{dx}$ is called the *derivative* or the *differential coefficient* of y with respect to x, and measures the rate of change of y with x at the position x.

Note that when $n = 3$ this verifies the result already obtained.

The student is advised to study the foregoing definitions very carefully. He should particularly remark (i) that there is no question of "evaluating" $\dfrac{0}{0}$; what is evaluated is the *limit* of the expression for $\dfrac{\delta y}{\delta x}$ as δx *tends* to 0; (ii) that it is the *symbols* δy, δx, $\dfrac{\delta y}{\delta x}$, $\dfrac{dy}{dx}$ which are curious, not the ideas they represent; thus $\dfrac{dy}{dx}$ is the limiting form of $\dfrac{\delta y}{\delta x}$ and is not some mysterious dy divided by an equally mysterious dx.

Examples. The differential coefficients of

$$x^4,\ x^7,\ \frac{1}{x},\ \frac{1}{x^3},\ x,\ x^{\frac{1}{3}},\ 5, \text{ are}$$

$$4x^3,\ 7x^6,\ -\frac{1}{x^2},\ -\frac{3}{x^4},\ 1,\ \frac{1}{3x^{\frac{2}{3}}},\ 0$$

If a function is composed of the sum of a series of separate terms, then the whole function will increase as a result of the sum of the increases in the separate terms. It follows that the derivative of such a function will be the sum of the derivatives of each of the separate terms. Thus, for example,

if $\quad y = A + Bx + Cx^2 + Dx^3$

then $\quad \dfrac{dy}{dx} = B + 2Cx + 3Dx^2$

You can express the rule for differentiating a sum by writing

$$\text{if } y = u + v, \text{ then } \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \dots \quad (i)$$

Similarly,

$$\text{if } y = uv, \text{ then } \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} \dots \quad (ii)$$

and

$$\text{if } y = \frac{u}{v}, \text{ then } \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \dots \quad (iii)$$

Examples. Let $y = x^2(x + 3)$. By rule (ii), $\dfrac{dy}{dx} = x^2 \cdot 1 + 2x\,(x + 3) = 3x^2 + 3$. This can be checked by using rule (i).

Let $y = \dfrac{1 + x}{x^2}$ By rule (iii), $\dfrac{dy}{dx} = \dfrac{x^2 \cdot 1 - (1 + x) \cdot 2x}{x^4}$ $= \dfrac{-2x - x^2}{x^4} = -\dfrac{2}{x^3} - \dfrac{1}{x^2}$. This again can be checked by using rule (i)

Consider the function

$$y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

an infinite series of terms. We shall assume that for every value of x this function has a definite value and that the derivative of the function is obtained by differentiating term by term; thus

$$\frac{dy}{dx} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

which is the original function. Thus in this case the function increases at a rate equal to its own value. It is called the *Exponential Function.* It can be shown that the series given above can be represented in the form e^x where e is the number 2·718 . . . approximately. Hence the rate of change of e^x is e^x. A function of this type applies to any population whose rate of growth is proportional to its size, a feature of many forms of propagation at the animal and vegetable level. For a similar argument to that above shows that if $y = e^{ax}$ then $\frac{dy}{dx} = ae^{ax}$, so that the rate of growth of y is proportional to y. It occurs also in cases of mechanical instability in which a disturbance increases at a rate proportional to its own size. Interest on capital continuously accumulating will also increase according to such a law.

Now consider a fluctuating function such as that given by $y = A \sin nx$. Can one discover the rate at which this changes with x? If x increases to $x + \delta x$ then y increases to $y + \delta y$. Hence

$$\delta y = A \sin n(x + \delta x) - A \sin nx$$

$$= 2A \cos n\left(x + \frac{\delta x}{2}\right) \sin\left(\frac{n}{2}\,\delta x\right)$$

$$\frac{\delta y}{\delta x} = 2A \cos n\left(x + \frac{\delta x}{2}\right) \sin\left(\frac{n}{2}\,\delta x\right)/\delta x$$

$$= An \cos n\left(x + \frac{\delta x}{2}\right) (\sin \theta)/\theta$$

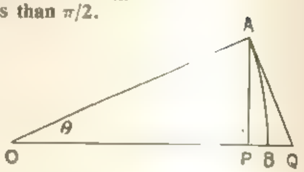
$$\text{where } \theta = \frac{n}{2}\delta x$$

As we propose to make $\delta x \to 0$ this implies that $\theta \to 0$. In that case $\cos n\left(x + \frac{\delta x}{2}\right) \to \cos nx$, and it remains to evaluate the limit of $\frac{\sin \theta}{\theta}$ as $\theta \to 0$.

In Fig. 130 θ is the angle AOB (in radians). The point P is the foot of the perpendicular AP on OB. The arc AB has its centre at O, so that OA = OB = r, say. AQ is the tangent to this arc at A, therefore AQ is perpendicular to OA.

AP = $r \sin \theta$, and since $\frac{AQ}{OA} = \tan \theta$, AQ = $r \tan \theta$. Since the area of the triangle OAB is less than the area of the sector OAB, which is itself less than the area of the triangle OAQ, it follows that:

Fig. 130. Sin θ is less than θ and less than tan θ if θ is in radians and is less than $\pi/2$.



area △ OAB < area sector OAB < area △ OAQ

i.e. $\frac{1}{2}$. OB . AP $< \frac{1}{2}r^2\theta <$ $\frac{1}{2}$OA . AQ

i.e. $\frac{1}{2}r \cdot r \sin \theta < \frac{1}{2}r^2\theta < \frac{1}{2}r \cdot r \tan \theta$

or $\quad \sin \theta < \theta < \tan \theta$ . . . .

provided θ is less than a right angle.

Dividing all along by $\sin \theta$, which is positive, we have

$$1 < \frac{\theta}{\sin \theta} < \frac{1}{\cos \theta}.$$

This is a relationship of inequality which sandwiches the value of $\frac{\theta}{\sin \theta}$ between 1 and $\frac{1}{\cos \theta}$. Now $\cos \theta$ is itself always less than 1, and, when θ is small, $\cos \theta$ is very nearly 1. Thus, as $\theta \to 0$, $\frac{1}{\cos \theta}$ tends to 1, and therefore $\frac{\theta}{\sin \theta}$ must also tend to 1; it immediately follows that $\frac{\sin \theta}{\theta}$ also tends to 1.

Returning now to the expression for $\frac{\delta y}{\delta x}$ for the case where $y = A \sin nx$, it is seen that the limit of $\frac{\delta y}{\delta x}$ as $\delta x \to 0$ is given by

$$\frac{dy}{dx} = An \cos nx$$

Example. If $y = \sin x$, the rate of change of y with x is

$$\frac{dy}{dx} = \cos x$$

It can be shown in the same way (or it can be deduced from the foregoing) that if

$$y = A \cos nx \text{ then } \frac{dy}{dx} = -An \sin nx.$$

Illustration.

Suppose an electrical conductor is being charged through a wire with a fluctuating charge Q given by $Q_0 + Q_1 \sin pt$ where Q_0, Q_1, and p are constants, and t is the time in seconds. Thus at time $t = 0$ the charge is Q_0. It rises to $Q_0 + Q_1$ after $\pi/2p$ secs., falls to Q_0 again after π/p secs, and drops to a minimum of $Q_0 - Q_1$ after $3\pi/2p$ secs. What is the strength of the current flowing along the wire?

Since the charge flows to and fro along the wire, the current in the latter is measured simply by the

rate of change of the charge. Thus current $= \frac{dQ}{dt}$ $= Q_1 p \cos pt$. Thus the current is a maximum when the charge is a minimum, and vice versa. The maximum current is $Q_1 p$, and its period is $2\pi/p$.

Graphical Meaning of Differentiation

Consider the graph of $y = f(x)$ illustrated in Fig. 131. Here P is the point (x, y) that is

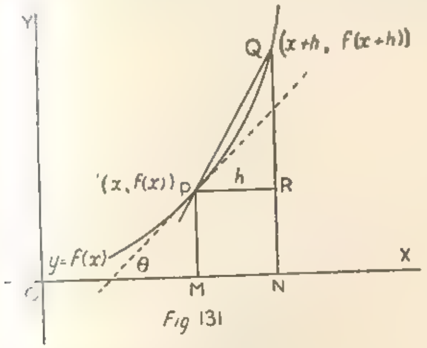


Fig 131

$(x, f(x))$. A neighbouring point Q is therefore $(x + h, f(x + h))$.

$$RQ = NQ - MP = f(x + h) - f(x).$$

Since PR $= h$, the slope of the chord PQ is RQ/PR $[f(x + h) - f(x)]/h = \delta y/\delta x$ if $h = \delta x$.

It follows that as δx tends to zero, $\frac{\delta y}{\delta x}$ tends to the slope of the limiting position of the chord PQ as Q tends to P along the curve. Hence $\frac{dy}{dx}$ is the slope of the tangent at the point (x, y) on the graph of $y = f(x)$; this is also called the *gradient* of the curve at (x, y).

Example. To find the equation to the tangent at the point (1, 1) on the parabola $y = x^2$. The derivative of $y = x^2$ is $\frac{dy}{dx} = 2x$, and hence the tangent at (1, 1) has a slope of $2 \times 1 = 2$. Since the line through (1, 1) is

$$y - 1 = m(x - 1)$$

and $m = 2$ the required tangent is given by

$$y - 1 = 2(x - 1)$$

or $$y = 2x - 1$$

Maxima and Minima

By studying the curves given in Lesson 23 and at the end of Lesson 24, the student will see that the points of a curve at which it is a maximum or minimum are points where the tangent is parallel to the X axis. Thus to find maxima or minima you must find the points of $y = f(x)$ at which $\frac{dy}{dx} = 0$. Again consulting the curves drawn, e.g. in Fig. 123, you see that at a maximum the gradient goes from positive to negative, while at a minimum the gradient goes from negative to positive. In other words, the rate of change of $\frac{dy}{dx}$ is negative at a maximum and positive at a minimum. The rate of change of $\frac{dy}{dx}$ is called the *second derivative* of y and is written $\frac{d^2y}{dx^2}$.

Example. Find the maxima and minima of $x^3 - 9x^2 + 15x + 10$

If $y = x^3 - 9x^2 + 15x + 10$, then $\frac{dy}{dx} = 3x^2 - 18x + 15$, and $\frac{dy}{dx} = 0$ if $x^2 - 6x + 5 = 0$, i.e. $x = 5$ or 1

Differentiating again, $\frac{d^2y}{dx^2} = 6x - 18$, which is positive for $x = 5$, and negative for $x = 1$.

Thus $x^3 - 9x^2 + 15x + 10$ has a maximum at $x = 1$ and a minimum at $x = 5$.

The student is advised to check this by sketching the curve.

Example. Prove that a rectangle of given perimeter encloses the greatest area when it is a square.

If the perimeter is c units and one side of the rectangle is x units, the other is $\frac{c}{2} - x$. Thus the area y is given by

$$y = x(\frac{c}{2} - x) = \frac{cx}{2} - x^2$$

Then $$\frac{dy}{dx} = \frac{c}{2} - 2x = 0 \text{ if } x = \frac{c}{4}$$

and $$\frac{d^2y}{dx^2} = -2, \text{ so that } x = \frac{c}{4} \text{ gives a maximum};$$

but $x = \frac{c}{4}$ means that the rectangle is a square of side $\frac{c}{4}$

Kinematic Meaning of Differentiation

If a body is in motion, its speed is measured by the rate at which its distance from a fixed point increases or diminishes. Hence if s, the distance passed over, is plotted against t, the time taken, the slope of the tangent to this curve will give the speed of the body at that time and place. If you know the equation to the distance–time graph you can derive the formula for the speed by direct differentiation. For instance, if a body is projected vertically upwards against gravity it is found that the height s (in feet) reached t secs. after projection, is given by a relation of the form

$$s = ut - 16t^2$$

where u is the initial speed of projection.

The speed at any time t, viz. $\frac{ds}{dt}$ is therefore found by differentiating this, i.e.,

$$\frac{ds}{dt} = u - 2 \times 16t = u - 32t.$$

At the starting point, $t = 0$, so the starting speed is u. If you write v for the velocity at time t, then $v = u - 32t$ ft. per sec.

Again, the acceleration is the rate of change of speed, hence if you differentiate v you obtain the acceleration. Thus

$$\frac{d^2s}{dt^2} = \frac{dv}{dt} = -32 \text{ ft. per sec. per sec.}$$

Thus a body projected vertically upwards is *retarded* with a constant acceleration (or deceleration) of 32 ft. per sec. per sec.

Oscillating Motion

If the distance of a body from its starting point is given by $s = A \sin nt$ then, since $\sin nt$ oscillates between $+1$ and -1 in value, this will represent a swinging motion of amount A on each side of the initial position. The speed, by differentiation, will be

$$v = \frac{ds}{dt} = A\, n \cos nt, \text{ and the acceleration}$$

$$\frac{d^2s}{dt^2} = \frac{dv}{dt} = -A\, n^2 \sin nt = -n^2 s$$

Thus in this case the acceleration is always proportional to the displacement s from the starting point ; and since there is a negative sign it will act in opposition to the direction of the displacement. A motion following this law is called *Harmonic Motion*, and it is found in the vibration of a tuning fork, a plucked wire, a pendulum, and an extended elastic string when released. It is the motion usually set up in any stable system when slightly disturbed from its equilibrium position.

Integration

The *Integral Calculus* poses the inverse problem to that posed in the differential calculus. It says: If you know the rate of change—can you find the total change? If you know the speed, varying as it does from second to second, can you find the total distance gone? If you know the acceleration, can you find the speed? You ask, in fact, from what function did this given differential coefficient come? Examine the relation in this way.

Fig. 132 shows the graph of $y = f(x)$ from any given position LM up to PQ where OQ $= x$. Thus P is *any* point on the curve. Let A = area of LMQP, and let us seek the rate at which this area changes as x increases. Accordingly, we let x increase to $x + \delta x$, that is Q changes to Q′ and P to P′. The increase in area, viz. δA, is then PP′Q′Q, which is approximately PQ × QQ′, i.e. $f(x) \times \delta x$.

$$\text{Thus } \delta A = f(x) \times \delta x$$

(Actually PP′Q′Q is not exactly a rectangle, but the error in this formula for δA is small compared with δx.)

It follows that $\dfrac{dA}{dx} = f(x)$. We write this relation in the alternative form

$$A = \int f(x)dx$$

(where $\int$ is an elongated S for sum) and say that A is the integral of $f(x)$.

Example. If $f(x) = 2x$ then, since the differential coefficient of x^2 is $2x$, it follows that A = x^2. Notice, however, we have not stated our result in sufficiently general terms. For $2x$ is also the differential coefficient of $x^2 + C$ where C is any number. Hence if

$$\frac{dA}{dx} = 2x$$

$$A = \int 2x \,.\, dx = x^2 + C$$

The constant C will be found by specifying from which point M the area is to be measured. For example, if M coincides with the origin, then A = 0 when $x = 0$. Since A = x^2 + C it follows that C = 0. Thus in this case A = x^2.

Example. To find the area between the parabola $y = x^2$ and the x-axis as far as $x = 1$.

$$A = \int x^2\, dx = \frac{1}{3}x^3 + C$$

When $x = 0$, A = 0 ; hence C = 0. Thus A = $\frac{1}{3}x^3$ up to any value of x. When $x = 1$ therefore A = $\frac{1}{3}$, which is the required area.

How the Calculus is Used

1. To determine how the level of water falls in an open rectangular tank as the contents are discharged through an orifice in the base.

Let the water level be H ft. initially above the base, the area of the surface A sq. ft., and of the orifice a sq. ft. If y ft. is the height at any time t,

the rate of fall is $-\dfrac{dy}{dt}$, and the rate of loss of volume is $-A\dfrac{dy}{dt}$.

Thus, if v is the speed through the orifice, this rate of loss is also av.

$$\text{Hence} \qquad -A\frac{dy}{dt} = av$$

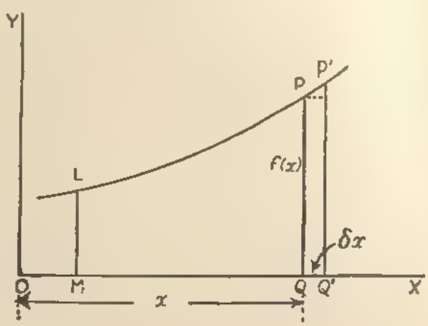


Fig. 132. Rate at which area changes with x.

The energy of this discharged water is that derived from falling a height y.

$$\text{Hence } \frac{1}{2}v^2 = gy \text{ or } v = \sqrt{(2gy)}$$

$$\text{Thus } -A\frac{dy}{dt} = a\sqrt{(2g)} \, . \, y^{\frac{1}{2}}$$

Had you taken a small horizontal slice of the liquid, of thickness δy, and considered what happens to it in time δt, you would get the approximate equation

$$-A\frac{\delta y}{\delta t} = a\sqrt{(2g)} \, . \, y^{\frac{1}{2}}$$

i.e.

$$-y^{-\frac{1}{2}} \, . \, \delta y = \frac{a}{A}\sqrt{(2g)} \, . \, \delta t$$

Summing up the elements on both sides,

$$-\int y^{-\frac{1}{2}}\, dy = \frac{a}{A}\sqrt{(2g)} \int dt$$

or

$$-2\, y^{\frac{1}{2}} = \frac{a}{A}\sqrt{(2g)} \, . \, t + C$$

where C is a constant.

Since $y = H$ when $t = 0$, this means that $C = -2H^{\frac{1}{2}}$.

$$\text{Hence finally } -2y^{\frac{1}{2}} = \frac{a}{A}\sqrt{(2g)} \, . \, t - 2H^{\frac{1}{2}}$$

or

$$y = \left(\frac{4at}{A} - \sqrt{H}\right)^2$$

It follows that the tank will empty, i.e. $y = 0$ when $t = \dfrac{A\sqrt{H}}{4a}$ secs.

2. To find the volume of a right circular cone the radius of whose base is a, and height of cone h.

Take the origin at the vertex, and the X axis along the axis of the cone. A plane at right angles to the axis at distance x from the origin will cut the cone in a circle of radius xa/h. The area of this circle is therefore $\pi a^2x^2/h^2$. If now the cone is cut by a parallel plane at distance $x + \delta x$ from the origin, the volume of the small cylinder between the two planes will be $\delta x \times \dfrac{\pi a^2x^2}{h^2}$. This is the increase in volume V of the cone when the height of the cone is raised from x to $x + \delta x$.

Thus

$$\delta V = \frac{\pi a^2}{h^2}\, x^2 \delta x$$

Summing up these elements, that is, integrating:

$$V = \frac{\pi a^2}{h^2} \int x^2 dx = \frac{\pi a^2}{3h^2}\, x^3 + C$$

When $x = 0$, $V = 0$. Hence $C = 0$. Thus volume of a cone of height x is $\pi a^2x^3/3h^2$. Hence when $x = h$, for the whole cone, the volume is $\pi a^2h^3/3h^2 = \pi a^2h/3$.

3. To find the volume of a hemisphere of radius a. Take the origin at the centre of the sphere. At a point x on the X axis, slice the sphere by a plane at right angles to this axis. It will cut the sphere in a circle of radius $\sqrt{(a^2 - x^2)}$, and therefore of area $\pi(a^2 - x^2)$. Slice the sphere by another plane parallel to the first through the point distant $x + \delta x$ from the origin. These two planes cut off a volume which is approximately a cylinder of volume $\pi(a^2 - x^2)\, \delta x$. Thus if V is the volume from the parallel plane through the origin to the plane through x, then

$$\delta V = \pi(a^2 - x^2)\, \delta x$$

Summing these elements up, that is, integrating,

$$V = \int \pi(a^2 - x^2)dx = \pi a^2 \int dx - \pi \int x^2 dx$$
$$= \pi a^2 x - \pi x^3/3 + C$$

Since $V = 0$ when $x = 0$, therefore $C = 0$.

$$\text{Hence } V = \pi a^2 x - \pi x^3/3$$

When $x = a$, V becomes the volume of the hemisphere. Thus

$$V = \pi a^2 \times a - \pi a^3/3 = 2\pi a^3/3$$

Hence total volume of sphere $= 4\pi a^3/3$

Note.—Volume of a spherical cap cut off by a plane at distance x from the origin is $2\pi a^3/3 - \pi a^2 x + \pi x^3/3$, or, if the depth of the cap is h, so that $h = a - x$, the volume of the cap is:

$$2\pi a^3/3 - \pi a^2(a - h) + \pi(a - h)^3/3 = \pi h^2(a - h/3)$$

EXERCISES

(1) Differentiate:

$3x^6$, $\dfrac{1}{x^3}$, x^3+x, $\sin 2x$, $7x^4+5x$, $3x^{\frac{1}{2}}-x$, $9x^{-3}+4x+7$, $6x^2 + 4e^x + \cos x$

(2) Find the speed of a particle whose distance s at time t is given by

$$\frac{1}{t},\ (2t^2 + 3)/t,\ 4 - 2t - 5/2t$$

(3) By expanding each of the following expressions in powers of x, find dy/dx.

$$y = (x^3 - 1)(1 + 2x),\ y = (x + 2)^3$$
$$y = (x^3 + 2x + 1)/x^2.$$

Check by using rules (ii) and (iii).

(4) Find the area between the curve, the axis of x and the ordinates at $x = 0$ and $x = 1$ in each of the following cases:

$$y = x^4,\ y = \sin \pi x,\ y = \cos(\pi x/2)$$

(5) Find the tangent to the curve $y = x(x^2 - 4)$ at the point (2, 0).

(6) Find the maxima and minima of $\frac{1}{2}x^2 + 5 + \dfrac{1}{x}$

Answers to Exercises in Lesson 26

(1) A = 36° 34′, B = 83° 26′, c = 4·359″

(2) A = 81° 1′ or 18° 59′ (6) 6·14

(9) R = 8·125 ft., r = 4 ft., Δ = 84 sq. ft.

(10) θ = 35° 39′

LESSON 28

What is Mathematics?

We have progressed in a series of steps, from the use and meaning of numbers. or integers, to the preliminary study of the integral calculus. In that progress we have provided the student, if he has on his part given it adequate and serious attention, with the apparatus for his own further development in the fascinating mental discipline that mathematics affords.

Arithmetic, algebra, plane and co-ordinate geometry, trigonometry, logarithms and the slide rule, the calculus—all these fields have been opened with indications of the practical uses and applications of the operations described in them.

The student is strongly advised, as in all departments of science and technics, to make certain of his ground before each fresh advance. This he can do by checking his knowledge and grasp with the help of the exercises provided with each Lesson. At this stage he can proceed with further studies, in general or in particular branches of mathematics, by selecting appropriately from the list of books given at the end of the Course.

Mathematics and History

Having now acquired a basis of mathematical knowledge, he can usefully consider some of its philosophical and historical aspects and ask "What is mathematics?", a question better answered at this stage than at the beginning of our studies.

Mathematics has developed in response to a variety of needs. Engineering and physical science, for example, have thrown up certain problems at critical periods in their history that have called urgently for solution. Thus at the end of the 16th century and the beginning of the 17th the growth of merchant trade and with it the need for overseas communication gave a new stimulus to shipbuilding. and turned attention to a study of tides and of the movements of stars and planets. so important as an aid to navigation.

Many questions of physical and engineering science pushed their way to the fore at that time—problems concerned with the resistance of bodies in their motion through water and air, efforts to predict the position and motion of planets involving therefore a recognition of law and order among the heavenly bodies. problems on the effect of the Earth's rotation and of the attraction of the sun and the moon on the height of the tides. All these, involving as they do the setting up of theories, demanded also the creation of a mathematical technique to assist in analysing these theories and drawing from them conclusions of practical value.

Period of Discovery

The scope of the subject already dealt with in the earlier Lessons of this Course sufficed for Newton and his contemporaries to handle these issues. They formulated the Three Laws of Motion that connect forces in operation with the speeds and accelerations produced by their action. They formulated the laws of planetary motion, and the Law of Gravitation, and showed that from the assumption that any two bodies in space attract each other with a force inversely proportional to the square of the distance between them, it followed mathematically that the Earth moved round the sun in an ellipse with the sun at a focus, that the radius joining the Earth to the sun swept out equal areas in equal times, and that for all the planets the squares of the times taken to make a complete circuit of the sun were proportional to the cubes of the distances of these planets from the sun.

This was the period when a new lease of life was given to the study of algebra, trigonometry, and similar branches of mathematics. Above all it was the period of the discovery and development of the differential and the integral calculus. For the latter, concerned as it is with rates of change and the calculation of distances travelled under conditions of varying speeds, provided precisely the mathematical tools to solve the complex problems of astronomy and physics that had pushed their way to the front.

The Logic of Development

But a topic like mathematics does not develop only in response to this kind of social pressure. The subject itself has an inner logical drive which inevitably poses mathematical problems calling for an answer. For example, the study—even the very definition—of rate of change required the introduction of the notion of limit.

Out of this emerged the new theory of limits, and in this way not only were new developments possible into the differential and the integral calculus, but other fields such as the study of infinite series were opened out. Thus when society requires to solve practical problems of a novel nature, it tends to evoke new forms of mathematical technique; these in their turn force a reconsideration of the

logical basis of the subject, and when this has been successfully completed, other branches of mathematics tend to be opened up that have not necessarily any immediate practical application. The scope of mathematics thus extends both by external and by internal pressure.

As a further example, it is important to remember that Einstein founded his relativity theory on a branch of mathematics (Tensor Calculus) which had been invented without such an application in view.

How Numbers Arose

This pattern of development can be seen even in the earliest stages of arithmetic. It is easy enough to understand how the marks arose that correspond to the integers. The figures or shapes were in fact pictures, words, or symbols, to stand for groups of various sizes. In this setting there would be no place for the symbol 0, as it does not stand for a group at all. Nevertheless early Indian traders appear to have found it necessary to have some mark to represent the fact that a debt had been paid off, or that nothing was outstanding.

It is believed that the invention of the zero mark arose out of the needs of early commerce. Once that had been accomplished, it became possible, by giving a meaning to the order in which numbers were written down, to use the ten symbols, and no more than these, to represent a group of any size. The symbol 0 took its appropriate place in this system by asserting that where it appeared there were no members of the particular group which corresponded to the place occupied by 0.

From that moment arithmetic advanced rapidly. Since it was now possible to write down any number of the infinite set of integers with no more than the ten different symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, the whole field of integers could be examined for its properties. Rules were discovered for the finding of factors. It was proved that there were an unlimited number of prime numbers, and many new discoveries about such numbers have been laid bare. Some have been discovered, but have not been proved.

For example, it is always possible in any particular case to show that an even number can be written as the sum of two prime numbers, but no one has succeeded in establishing this simple proposition for *any* even number. Arithmetic grew under the drive of its own logical necessities once practical needs had called it forth.

Symbols and Abstract Thought

Mathematical symbols, and the logical process they express in a mathematical argument when written on paper, are really an objective way of describing a process of reasoning. Anyone who has written down a mathematical argument can hand the paper to someone else, and if he will follow it, the latter can trace again the reasoning through which the writer went.

The significant point about this lies in the refined or abstract nature of the thinking involved. In most other situations one's thoughts, the connexion between thoughts and the weighing of the relative importance of one argument against another, are profoundly influenced by one's feelings and one's past experience. In general, thinking and feeling are very closely interwoven, as is evident in the extreme case where a person decides to do something—that is, decides upon his course of action—in a fit of temper. It is not possible to evade a mathematical conclusion whatever one's state of feeling. One does not say that the three angles of a triangle are equal to 190° just because one is angry. The whole setting and the ideas involved are so abstract that the logic is compelling. Feeling enters only in registering agreement with the steps of the argument, and in the aesthetic sense of pleasure that arises in appreciating the meaning of a proposition and its manner of proof.

Nature of General Propositions

The nature of the abstraction involved in mathematical reasoning is worth closer study, for in it lies the peculiarity of mathematics that makes possible the stating of propositions of very wide generality. When you draw a triangle on a sheet of paper, and by means of it you prove some general proposition, such as that the three angles together amount to 180°, how is it that from this very particular triangle you can deduce something of such wide generality?

The point is that the triangle you have drawn is really an object—a mathematical object composed of fine particles of graphite and paper. If you look, you can easily find a stone or a piece of wood from which, by forgetting everything except the shape, you "abstract" the idea of a triangle. A triangle is an idea, not an object. It is a mental image drawn from the real world. It is an image composed of the idea of three straight lines, each of which is itself an idea drawn from the real world.

In order to discuss these ideas it is convenient to have an object from which to abstract them when necessary, but it would not be convenient to carry around with you pieces of wood and stone for this purpose. So the mathematician makes his own object, a much simpler one, a set of marks on a piece of paper, and this he calls a triangle. It is of course a misnomer, because the triangle he discusses is really the

idea and not the mathematical object, which is a material triangle.

If you can see this distinction between the ideas which are discussed and the mathematical objects that are used to help in the abstraction, it becomes at once clear why the mathematician is able to deduce general propositions from particular figures. He does not really deal with these figures in one sense but with ideas derived from them ; it is about the latter that he reasons, and the figure is just an aid to thought, and far more special than the idea it inspires.

Along such lines, therefore, can one come to an appreciation of the basic notions of mathematics and the forces that have stimulated its growth. In itself it represents one of the finest efforts of human thought.

Answers to Exercises in Lesson 27

(1) $18x^5$, $-\frac{3}{x^4}$, $3x^2 + 1$, $2\cos 2x$, $28x^3 + 5$, $x^{-2/3} - 1$, $-27x^{-4} + 4$, $12x + 4e^x - \sin x$

(2) $-1/t^2$, $2 - 3/t^3$, $-2 + 5/2t^2$

(3) $6x^2 + 2x - 2$, $3(x + 2)^2$, $-\frac{4}{x^3} - \frac{3}{x^4}$

(4) $1/5$, $2/\pi$, $2/\pi$

(5) $y = 8x - 16$

(6) Minimum of $6\frac{1}{3}$ at $x = 1$; maximum of $3\frac{2}{3}$ at $x = -1$

BOOK LIST

General. *Mathematics for the Million*, L. Hogben (Allen & Unwin) ; *Mathematician's Delight*, W. W. Sawyer (Penguin) ; *Elementary Mathematics*, H. Levy (Nelson) ; *Introduction to Mathematics*, A. N. Whitehead (Butterworth) ; *Pure Mathematics*, G. H. Hardy (C.U.P.) ; *Mathematics in Action*, O. G. Sutton (Bell) ; *Mathematical Recreations and Essays*, W. W. Rouse Ball (Macmillan) ; *What is Mathematics?* R. Courant and H. Robbins (O.U.P.).

Geometry and Trigonometry. *Analytical Geometry*, B. C. Molony (Bell) ; *Foundations of Geometry*, G. de B. Robinson (O.U.P.) ; *Modern Geometry*, C. V. Durell (Macmillan) ; *Geometrical Conics*, C. V. Durell (Macmillan) ; *Analytical Geometry of the Conic Sections*, E. H. Askwith (Black) ; *Elementary Trigonometry*, Prescott and Lowry (Longman).

Algebra. *Higher Algebra*, W. L. Ferrar (O.U.P.) ; *Theory of Equations*, H. W. Turnbull (Oliver & Boyd) ; *Higher Algebra*, S. Barnard and J. M. Child (Macmillan).

Calculus. *Elementary Calculus*, F. Bowman (Longman) ; *Infinitesimal Calculus*, H. Lamb (C.U.P.) ; *Elementary Treatise on the Calculus*, G. A. Gibson (Macmillan) ; *Calculus Made Easy*, S. P. Thompson (Macmillan).

Statistics. *Elementary Statistics*, H. Levy and E. Preidel (Nelson) ; *Facts from Figures*, M. J. Moroney (Penguin) ; *Introduction to the Theory of Statistics*, G. U. Yule (Griffin).

History of Mathematics. *Mathematics for the Million*, L. Hogben (Allen & Unwin) ; *The Great Mathematicians*, H. W. Turnbull (Methuen) ; *Men of Mathematics*, E. T. Bell (Penguin) ; *Mathematics, Queen and Servant of Science*, E. T. Bell (Bell).

ECONOMIC GEOGRAPHY

THIS *Course is concerned with the influence of geographical factors on commerce; with climate and soil as they affect man's food and drink and clothing and shelter; with products on which human life depends, and products for man's pleasure, with fuel and power; and ways and means whereby products are transferred from their places of origin to where they are used. The student should refer also to the Courses on* REGIONAL GEOGRAPHY, *in Vol. 2, and* PHYSICAL GEOGRAPHY, *in Vol. 4.*

16 LESSONS

LESSON 1

World Transport

When civilization had advanced beyond the stages of huntsman, nomadic herdsman, and primitive agriculturist, there arose the need for a method of transporting commodities for the purpose of trade. At first, pack animals and human porters were used ; and these have not wholly disappeared even from areas in which mechanical transport preponderates. The sledge (dog or reindeer) is used in the extreme north of Asia and America ; the camel, assisted by the horse, in the great low-latitude deserts (Sahara, Arabia, and Persia).

The yak in Tibet, the elephant in south-eastern Asia, and the llama of the Central Andes are other beasts of burden. Human transport belongs mainly to the tropics, especially where dense forests make roads almost impossible ; but still in China, where the building of roads for modern transport was a late development, there are millions of human porters, many of whom trundle wheelbarrows with exceptionally large wheels.

Transport by Water

Water transport was an easy, and consequently an early, form to be adopted, by river and by sea. Some of the oldest of the world's great cities are on the banks of important rivers or on the sea coast, and some are on both, i.e. at the mouth of a river. Examples are London, Babylon, Baghdad, and Cologne. Water transport was in a highly developed and well-organized condition on such rivers as the Nile, Tigris, Euphrates, Ganges, and Yangtze before a start had been made with the construction of roads in the countries concerned. To-day the Great Lakes function in the same way for the industrial community of North America ; with their connecting waterways they constitute the largest fresh-water navigable waterway in the world. Nearly twice as much tonnage passes through the Sault Ste. Marie Canal, connecting Lakes Superior and Huron, as passes through the Panama and the Suez Canals combined. From Montreal, where the route begins, to Duluth at the head of the lakes is 1,340 miles.

Europe also has a system of inland waterways. France, for example, has rivers and connecting canals extending to most parts of the country. Russia has developed her system of waterways to such an extent that vessels of the capacity of a light cruiser can move freely between the Baltic, White, Caspian, Azov, and Black Seas, with Moscow the centre of the system. The Imperial Canal in China, constructed in the 13th century, is an important water link between the Peiho at Tintsin and the Yangtze river at Nanking. The canal system of Britain is principally in the area between the rivers Humber and Thames, although there is the important Manchester Ship Canal, and the Aire and Calder Navigation that links the east and the west. To-day there are about 3,000 miles of canal in the British Isles.

Ocean transport is the mode of navigation presenting the greatest combination of advantages. Besides cheap haulage for low speeds offered by navigable water generally, the ocean offers a free way to all nations, traversable in all directions, a way on which it is possible to increase almost indefinitely the size of vessels, the limiting factor being mainly accommodation available at ports and the dimensions of such canals as the Panama and the Suez.

As long ago as 4000 B.C. the Egyptians built ships capable of carrying 50 passengers, mainly for use on the Nile, though later on they shared with Greeks, Phoenicians and Romans in trading voyages to all parts of the Mediterranean. For centuries nothing which could be called an ocean voyage was undertaken, with the result that until the time of Columbus (*c.* 1451–1506) hardly any ships more than 100 feet long were built. During the next 300 years little progress was made in the matter of size. The *Great Harry*, the first English two-decker, built by Henry VIII in 1514, had a tonnage of only 1,000. Nowadays ordinary cargo steamers (ocean tramps) are of 5,000 to 9,000 tons, and the average liner is more than 10,000 tons. The *Queen Elizabeth* is 83,673 tons gross, the *Queen Mary* 81,237, the *United States* 53,329, the *Liberté* 51,839.

Compass, Steam, and Oil

Ocean navigation was greatly facilitated by the magnetic compass, first used about the year 1300, enabling the mariner to steer a fairly constant course although out of sight of land. Columbus worked out the variation of the compass, making it more useful and accurate. The astrolabe, used by the Greeks and Arabs and in medieval Europe, precursor of the modern sextant, allowed the navigator to determine his latitude simply (although rather roughly) from the Pole Star. The determination of longitude came in 1736, following John Harrison's (1693–1776) invention of the compensation pendulum.

Steam navigation, which effected such a revolution in sea-carriage, originated, as did steam railways, in the 19th century. By about

the middle of that century steamers began somewhat rapidly to displace sailing vessels, and in the 1860s iron was increasingly substituted for wood as the building material. Early in the 20th century motor ships were introduced as an alternative to steam ships. An oil-tanker registered in 1911, the *Volcanus*, was the first ship to be driven by a Diesel oil-engine running on heavy crude oil. In 1951 steamships still outnumbered motor ships by three to one, but new ships launched in that year were in the reverse proportion. The largest motor ship afloat in the 1950s was the *Britannic*, 27,666 tons gross.

It is interesting to note the extent to which oil has replaced coal as fuel for shipping ; in 1914 some 97 per cent. of the total steam and motor tonnage depended on coal fuel, in the early 1950s over 80 per cent. was dependent on oil fuel. At the turn of the half-century, of the total tonnage of merchant shipping fleets (excluding the U.S.S.R.) the U.S.A. had 34 per cent. (including vessels trading on the Great Lakes) ; the U.K. 21 per cent. ; Norway 7 per cent. ; Panama, France, and the Netherlands each 4 per cent.; Italy and Japan each 2 per cent.

Railway Areas

What may be called the railway areas cover all the more populated regions of the globe. In North America, with about 40 per cent. of the total world mileage of track, the pattern is densest in the east, especially in the vicinity of the Great Lakes, where railways supplement the water-borne traffic. Trunk lines connect Chicago with the eastern seaboard ports of New York, Philadelphia, and Baltimore. The coal lines, exemplified by the Chesapeake and Ohio, are primarily engaged in handling coal from the Appalachian coalfield ; other lines carry goods north and south from the Gulf of Mexico. The transcontinental lines do not all (as the name would imply) connect the two oceans ; most of the lines so classified connect Chicago with the Pacific coast and, via the trunk lines, with the east coat. Only one transcontinental line in the U.S.A., the Southern Pacific, running from New Orleans to Los Angeles, fails to reach Chicago. The Canadian railways—the Canadian Pacific and Canadian National—are truly transcontinental, running from Quebec and Montreal to Vancouver, B.C., and Prince Rupert.

In South America, with 8 per cent. of the total world mileage of track, the largest railway area corresponds to the chief cattle-raising district of the Central Argentine. A smaller network of rails serves the coffee belt of Brazil. A transcontinental line connects the La Plata cities (Buenos Aires, Rosario, and Santa Fé) with Valparaiso in Chile. Chile has a trellis pattern of lines extending from the north of the country to Puerto Montt in the extreme south.

The railway networks of the Indian sub-continent and of Japan are the brightest spots in the transportation map of Asia. The former, with over 40,000 miles of track, owes its great railways system (more than double the mileage of all British Railways) to a far-seeing British administration. It will be understood, however, that in such an extensive region as the sub-continent of India railways are actually few and far between, so that millions of the natives never see a train, and the ox is still the chief carrier of burdens. In Japan a network of narrow-gauge lines covers most of the main island ; there the mountainous character of the land is the greatest handicap.

Russia and China

The Soviet railway system amounted to 57,500 miles of track in mid-20th century. The heaviest traffic in the Soviet Union is in the south between the Donbas coalfield and the neighbouring industrial districts, especially Kharkov and the Dneiper bend region. A network of rails radiates from Moscow like the spokes of a wheel, reaching Leningrad, Arkhangelsk, Sverdlovsk, Gorki, Astrakhan, Kiev, and Minsk. Of the lines which join western Russia with the east the best known is the Trans-Siberian, running from Leningrad to Vladivostok on the Pacific ; this line, 6,000 miles long, is double-tracked and of great strategic importance, but long stretches provide little revenue. Another railway, the "Turk-Sib," connects southern Turkestan and the Caspian Sea with the Trans-Siberian Railway.

Because of popular prejudice there against the foreigner and all his ways, railways were a late arrival in China, and then mainly as the result of Western enterprise. The first, built to connect Shanghai with its outport Woosung, was not opened until 1876, and in the following year it was bought in order that it might be torn up. Its destruction was a last effort to keep railways out of that country. In 1952 China had 14,500 miles of track, including those of Manchuria, which is well supplied with lines as a result of former Russian and Japanese interests. An important line providing communication from south to north connects Canton, Hankow, Peking, and Mukden. New tracks are being laid to connect western interior provinces with those farther east.

Australia and Europe

The railway pattern of Australia illustrates the peculiar distribution of her population ; railways are in the south-western corner around Perth and Fremantle, and in the largest settled area in the east extending along the coast from Adelaide to beyond Brisbane. The two

sections of the rail network are joined by a transcontinental line across an extensive, virtually uninhabited, desert region. There is rail communication between the five state capitals, but the regrettable differences in gauge on the railways of some of the states is a great drawback. Australia and New Zealand between them have only 4 per cent. of the total world mileage of railway, Africa has 3 per cent.

Europe has about a third of the railway mileage of the world. It may be noted that, in general, the trucks, coaches, and locomotives of European railways are lighter than those of, for instance, America, where average distances between large cities are much greater.

The Motor Lorry

Most goods transported by road move over relatively short distances, either to markets or terminals. In Britain the motor lorry, by providing economical and fairly rapid door-to-door transport of small loads, has made great inroads into the traffic formerly carried by the railways. In countries less well covered by a rail-net, and in countries such as the "pioneer belts" of Africa, where the railway has not yet arrived, the motor lorry is of great help in promoting economic development. Rough roads that will carry a lorry are far more cheaply and quickly constructed and much less trouble to maintain than are railways. The use of six wheels instead of four and the mounting of vehicles on caterpillar tracks have overcome the difficulty of rough pioneer roads.

WORLD AIR ROUTES. **Main transcontinental air lines of the land hemisphere.**

The world total of motor cars and commercial vehicles in use in mid-20th century was nearly 74 million—a quarter of them coming within the latter category. The U.S.A. had the largest number, 51½ million, or more than two-thirds of the total.

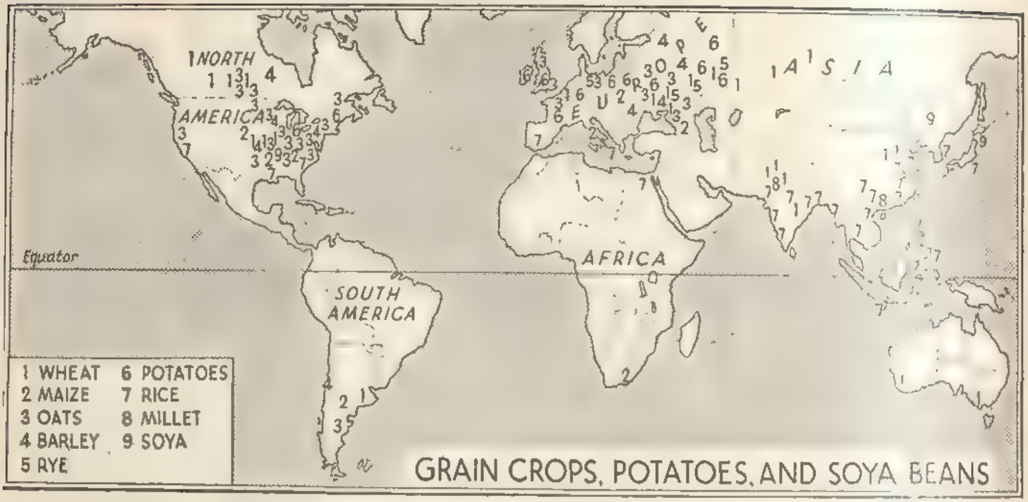
The Aeroplane

The most striking development in 20th-century transport was the coming of the aeroplane. Between the two world wars the world was covered by a network of regular air

services, transcontinental and transoceanic as well as national. British, French, Dutch, Belgian, and American companies were prominent in developing long-distance routes. When the Second World War broke out (1939) the principal British company, Imperial Airways, was operating regular services, mostly with flying boats, linking up with the Middle East, India, Burma, Siam, Singapore, Hong Kong, and Australia, as well as with West, Central, and South Africa. In 1940 Imperial Airways was merged with British Airways into a single government-owned, non-profit-making corporation, known as British Overseas Airways Corporation (B.O.A.C.). This was used in the Second World War in maintaining lines of communication, opening new routes, transporting military personnel, etc. As a result of the Civil Aviation Act of 1946, B.O.A.C. was made responsible for long-distance Commonwealth services and for the routes to Canada and the U.S.A. pioneered during the war. Only the Corporation or its associates can operate scheduled services; but private companies may run charter services, and some private companies operate scheduled air services as associates of British European Airways (B.E.A.).

International as well as national controls have been set up. Following an international conference in Chicago at the end of 1944, the Chicago Convention came into force in 1947 and by the end of that year had been ratified by 46 states. The convention provided for an International Civil Aviation Organization (I.C.A.O.), of which there are some 60 member states. Its chief task is to develop international rules to govern civil air operations in the interests of the safe and orderly development of civil flying. Companies operating from I.C.A.O. member states flew 24,400 million passenger miles in 1952, of which U.S.A. aircraft were responsible for 63 per cent., U.K. 5 per cent., Australia 4 per cent., France 4 per cent., Canada 3 per cent., the Netherlands 3 per cent., Norway, Sweden, and Denmark each 2 per cent., India 1 per cent., others 11 per cent.

IN THE WATER HEMISPHERE air transport forms a swift link between widely separated lands.



THE WORLD'S FOOD. Principal sites of production of various grains and other food crops.

LESSON 2

Grains of Temperate Lands, and Potatoes

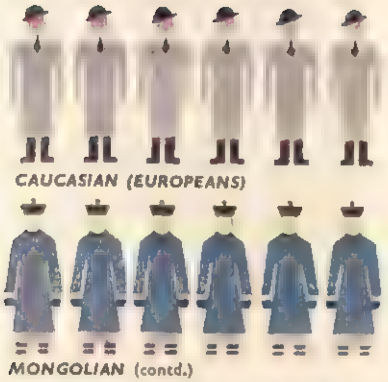
Wheat is the leading bread grain and basic food for human consumption within the temperate zones. It ranks with rice as the most important of all crops. Although varieties of wheat suited to all manner of climates and purposes have been produced, wheat itself has become one of the most important commodities entering world trade. As much as one-fifth of the world's output may enter into international trade with some European and Asiatic countries partly dependent on imports (the U.K., for example, relies on overseas supplies for three-quarters of its requirements). Canada and Australia, two leading producers, rely on exports for one-third of their sales. The U.S.S.R. was the largest exporter of wheat up to 1914, but has now virtually withdrawn from this market ; its place as principal exporter has been taken by the U.S.A.

Wheat is an annual or biennial plant grown from seed. From each seed, cultivation produces four to eight or more stems, on each of which is a head with many kernels—the grain or corn of commerce. From the kernels the miller obtains flour and bran. The simpler wheats of little commercial value are spelt varieties, grown on dry land in Europe and mainly used as food for animals. Emmer, a hard, two-grained spelt, is also unimportant commercially. Club wheat, which has short, stout straw stems and holds its kernels long after they are ripe, is grown in Turkestan, China, Chile, and western North America. Among wheats proper, durum, a hard wheat grown for macaroni, is widely cultivated in southern Europe, northern Africa, and the United States. The most obvious distinction between the different kinds of wheat is that between red and white. The red varieties are somewhat hardier but the grain is much inferior to the white.

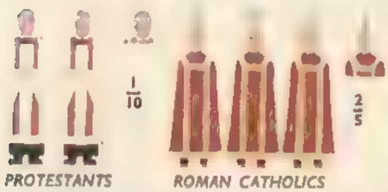
In Britain winter wheat is sown as soon as the ground has been ploughed in autumn, and it is reaped the following year ; spring wheat is sown on ground ploughed in the autumn, directly the winter frost has thawed out of it, and it is reaped the same year as it is sown. Three-quarters of the crop is autumn sown.

Mechanisation of Wheat Production

It was the development of cheaper and faster forms of transport like the ocean-going steamer and the extension of the railway network to the interior of North America and the U.S.S.R. that brought the new wheat lands in distant countries within reach of importing countries. New production and trade were stimulated, but the great increase in wheat output since 1870 reflects the triumph of modern science and farm technology. New and more productive strains which are earlier maturing and resistant to insect and disease attacks are continually being developed. Selective weed killers, the greater application of superphosphates and other fertilizers, and increasing efficiency in sowing and harvesting, have combined to achieve steep rises in yield per acre (e.g. yield in Britain rose by 50 per cent. between 1870 and 1952).



RACE-GROUPS OF THE WORLD



RELIGIONS OF THE WORLD IN

WORLD FACTS AND FIGURES
peoples, and their resources. Top
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Where the farmer cultivates his own ground and grows wheat on a small scale for his own uses, the principal limits are climatic ; the annual rainfall should be between 15 and 30 inches, and should occur generally least during the harvesting period, and the growing period should be relatively long. Where cultivation is extensive, both in area over large tracts of fairly level country and in bulk intended for sale in the great wheat markets, the principal limits tend to be commercial rather than climatic. The crop must, on the average, pay over a series of years. To this end wheat production has been mechanised, all the operations—ploughing, drilling, harvesting, threshing, and transporting to storehouses—being done more cheaply by machines which reduce the number of hands employed. Consequently, the three main limits are suitability of the land to the use of machines, nearness to markets to reduce the cost of transport, and adequate supplies of human labour at a wage which makes its employment commercially practicable.

Average annual world wheat production at mid-20th century was 191,702,000 tons, of which the U.S.S.R. produced 26 per cent., U.S.A. 16 per cent., China 11 per cent., Canada 6 per cent., France 4 per cent., Italy 4 per cent., India 3 per cent., Australia 3 per cent., Argentina 3 per cent., Pakistan 2 per cent., Germany 2 per cent., Spain 2 per cent., others 18 per cent. About two-thirds of world wheat exports come from North America—the U.S.A. 43 per cent., Canada 24 per cent.

North American Wheat

In the United States the chief area is the wheat belt, roughly halved by the upper Mississippi. West of this river the crop is grown mainly for consumption elsewhere ; south of the line from Chicago to Omaha winter wheat tends to be more frequent than spring wheat. Chicago wheat market (the famous " pit ") is the most important in the world. Transport is by numerous railways and the Great Lakes waterways.

Canada grows wheat chiefly in the centre of the country in the prairies. The somewhat scanty rainfall occurs mostly during the growing season. The high latitudes give 16 hours of sunshine per day during the ripening period, and growth is rapid during a shortish summer. Hard spring wheats are most common. Trade in wheat is so important that it has produced cities such as Winnipeg, notable as a wheat inspection and collecting centre. Most of the railway lines of the area, and such a density of population, would have been impossible but for the production of wheat. It has also contributed to the development of the canal system of the Great Lakes, new transcontinental railway lines, a port on Hudson Bay, and that tremendous project, the St. Lawrence seaway, one object of which is to enable ocean-going ships to reach the Great Lakes from the Atlantic Ocean.

South American Wheat

Exports from the southern hemisphere, while smaller than from North America (23 per cent. of world exports compared with 67 per cent.) are still important, especially as the crop is harvested at a different time. Australia grows wheat in the south-east, where the rainfall is at least 20 inches annually. The ground is never frost-bitten as in Canada. The farms are large, labour tends to be scarce, and the climate is uncertain, so that yield per acre and total yield for the area fluctuate. The harvest occurs in December and January.

In Argentina wheat is grown west of Buenos Aires ; from the 30° S. parallel, where durum wheat is cropped, south through areas of semi-soft wheat suitable for bread-making and for export, to semi-hard wheat near the 40° S. parallel is the range of an extensive production, much of which is exported.

Maize

One of the most important cereal crops is maize, commonly known in England as Indian corn, in America as corn or sweet corn, in Africa as mealies. It is the seed of an annual plant, *Zea mays*, native to Central America and cultivated in most warm climates. Maize is used for human food as a vegetable in various forms—as sweet corn and, ground into flour, as cornflour and as hominy. It is also used as food for livestock. The plant requires for its cultivation a longer and a hotter and wetter summer than wheat, and no frosts, and the soil should not become saturated with water. It grows, therefore, in the warmer lands where winter wheat is also grown. About 59 per cent. of the world's maize production is in the U.S.A., south of the spring wheat belt. The next large producers are the U.S.S.R. and China, which each yield about one-twelfth of the U.S. crop.

Oats, Barley, and Rye

Oats are hardier than wheat. Chief producers of this cereal are the U.S.S.R., Poland, and Germany. It crops well in a climate that is both moister and cooler than that best suited to wheat. Most European countries, except those on the Mediterranean, produce more oats than wheat because oats are easier to grow. Of the exports (small compared with wheat), western Europe takes the greater parts—Canada, Argentina, Australia, and the U.S.A. are the chief exporters.

The hardiest of cereals is barley. Its production is scattered throughout the wheat-

growing areas, though it will grow in districts too cold or too wet for wheat or too dry for maize. The main purpose of the crop is for animal feeding, but considerable quantities of higher-grade barley are used to provide malt for brewing, especially in Britain and Germany. Barley as a rule produces a heavier crop to the acre than wheat. The chief producers are the U.S.S.R., China, the U.S.A., Canada, and India.

Rye is the bread grain of eastern Europe. It will flourish in indifferent climates and on poor soil. It is the most important grain crop in northern Germany, Poland, and the U.S.S.R., in areas where the soil is poor and sandy.

Potatoes

A crop which often serves as a substitute for grain in the diet in temperate zones is the potato, native to west regions of North and South America, introduced by the Spaniards into Europe in the latter half of the 16th century. approximately a thousand varieties have been produced from the original "wild" stock. Heaviest yields per acre occur in cool (not cold) climates. It will grow on almost any soil, but deep, rich, sandy loam gives the best results. Occasionally, particularly in the case of early potatoes, the crop is grown continually on the same soil ; but the main potato crop normally forms one of a three- or four-course rotation.

Under good conditions potatoes can produce more food per acre than any of the cereals. This was an important factor in the rapid increase in population of North-West Europe in the 19th century. To-day in central and eastern Europe the potato is the staple food for millions of people. In some parts of Europe as much as 40 per cent. of the crop is sometimes used for feeding livestock. Potatoes are also a source of industrial alcohol and potable spirits (especially in the U.S.S.R. and eastern Europe), and of starch and flour. The average world production in mid-20th century was 230 million tons, of which the U.S.S.R. accounted for 31 per cent., Germany 15 per cent., Poland 13 per cent., France 6 per cent., the U.S.A. 5 per cent., the U.K. 4 per cent.

LESSON 3

Production Areas of Meat and Milk

Meat and milk enter into world trade in response to universal demand for food and the profitable use of vast expanses of grassland in less populated areas. The great grasslands of the world are of two kinds ; both lack sufficient moisture for the growth of trees—the one having a snow-cover in winter, the other having mild but arid winters. Both occur on a dry side of a forest region ; the cold grasslands, known as prairie and steppe, are on the warm equatorial side of the cold coniferous and deciduous forests, and consequently penetrate into the forest clearings ; the warm grasslands—savanna, veld, pampas, and downs—lie on the cool side of the equatorial forest, between the forest and the hot arid desert.

The Ranches

Until comparatively recent times the prairie, the pampas, and the downs, in North America, South America, and Australia respectively, were open spaces with extremely small populations. On ranges, haciendas, and sheep-runs, settlers let animals roam wild. On the ranges, cattle were preferred to sheep ; on the downs, sheep were preferred to cattle. Grazing rights, water rights, access to trails to market, and other circumstances led to the delimitation of sections of grassland as the property of a stock-owner ; and the land became property with a value as a potential feeding-ground for so many head of animals. Much grew to more ; and property in land, and the animals which it sustained, led to property in animal produce. This resulted in the fencing of the ranches and improvement in the breed of the animals.

Canning and Refrigeration

The advance was accompanied by an extension of railways and shipping facilities, for while animals can travel "on the hoof" to market, modern requirements demand a regular supply, which hoof transit cannot maintain, coupled with a quality of condition on arrival which the driven animal cannot show. Further, the change involved reorganization to maintain a regular demand for a regular output. This meant the use of devices for dealing with a surplus or glut of supplies, with the consequence that most of the transport is now in the form of meat and not in that of living animals ; this development was made possible by the invention of mechanical methods of canning and preserving meat. Meat packing and refrigeration both originated in the United States. An extension of this business is the trade in tinned milk, meat extracts, etc., whereby bulk is reduced in proportion to nutriment.

Great Britain imports quantities of chilled and frozen meat from Australia and New Zealand, and from Argentina and other South American countries. The United States, which at the beginning of the 20th century was the largest supplier of meat to Great Britain, has

ANIMAL AND DAIRY PRODUCTS. Main areas of production of three important foodstuffs.

become an importer of meat from Australia and South America. The establishment of arable farming on land formerly used for cattle rearing caused changes in methods, e.g. in Argentina; where the cattle used to roam over great ranches, much meat is now raised on smallish farms which grow fodder. The milk product travels long distances as condensed or dried milk, butter, and cheese. Britain, Germany, Belgium, and Luxembourg are importers.

Sheep, Pigs, and Dairy Produce

On the whole, cattle are suited to a wider range of conditions than sheep; for example, cattle thrive in the warm grasslands of Brazil, in the cold grasslands of the American prairie, in the humid cool land of Denmark, and in Switzerland (with its tinned milk and milk chocolate industries). Difference in the range of sheep and cattle is very evident in Australia, where sheep are excluded by excessive heat from northern districts and from the wetter eastern coasts where dairy cattle flourish.

Consideration of the distribution of sheep involves attention to wool as well as mutton. The predominant importance of the 11 million sheep in Australia rests on wool; but the 34½ millions in New Zealand, and also those of Britain, are for mutton as well as for wool. There are some 48 million sheep in Argentina, mainly for meat, 34 millions in South Africa mainly for wool. The imported mutton supplies of Britain and western Europe come almost entirely from the southern hemisphere; very little from the 50 million sheep of the U.S.A.

Production of, and trade in, pork and bacon, etc., is immense. At mid-20th century world pig population was 277,100,000, distributed roughly as follows: China 25 per cent.; U.S.A 21 per cent.; U.S.S.R. 9 per cent.; Brazil 8 per cent.; W. Germany 4 per cent.; France, Poland, Mexico, Canada, Hungary, and Italy, each 2 per cent. Great Britain imports 82 per cent. of all bacon in international trade; the major exporters are Canada 24 per cent., Poland 13 per cent., Denmark 4 per cent.

European dairying and egg production depend largely on importation of cattle-cake for winter feed, and of grain for poultry; any restriction of those imports reduces the maintainable number of cattle and poultry, and therefore the output of their products. Pigs are similarly reduced in numbers when there is a restriction in dairy products, the whey and separated milk left over from butter- and cheese-making being important items in their feeding.

LESSON 4

Deep Sea Fisheries

During many centuries throughout Europe all except a few cattle, sheep, and pigs kept for breeding were killed off and the meat salted down as winter approached, because of the impossibility of feeding many animals at that season. The result was a great desire to supplement the meagre diet of meat, which became less and less appetising as winter wore on, with more fish, and the increasing demand at last induced fishermen to venture in their small boats farther into the Atlantic for larger catches. After the discovery in the 16th century of the fishing grounds (the Grand Banks) off Newfoundland, wealthy merchants and

others engaged in and fostered the enterprise with tremendous zeal. The Grand Banks are still the greatest cod-fishing grounds in the world. Another notable major fishery is the Dogger Bank, in the North Sea.

Both are due largely to the presence of extensive submarine continental shelves (or plateaux) and the mixing of warm and cold waters. Fish food, which consists mainly of minute organisms (plankton), thrives abundantly in warm waters; food fish, mainly cold-water creatures, abound where the meeting of warm and cold ocean currents brings about the conditions which favour both the fish and their food. Fishing is a seasonal business, and the height of the fishery season varies in different areas. The herring fishery maximum comes down the North Sea as summer advances, and feverish activity prevails at British fishery centres from Wick (Caithness, Scotland) down to Yarmouth (Norfolk, England) in succession.

Strategy of the Fisheries

The tons of fish which it is necessary to catch to make the fisheries pay are not encountered by chance. The fish must be sought when and where they occur in gigantic shoals, or they must be awaited in areas which they habitually frequent at certain times. Some of the bigger fishing vessels now go on a six-week voyage and travel up to 5,000 miles. Some are equipped with radio, some with an echo-sounding device which enables them to locate big shoals of fish.

Fish often reach the land in gluts, or spasms, and fishery organizations must be adaptable. The landed catch must be handled quickly. Labour must be available to prepare the fish for market, including gutting and salting preliminary to packing in barrels for long-distance transport; packing in ice; curing or smoking for preservation; or canning for preservation more or less indefinitely. The port must be in easy touch with markets. Sea, rail, or road transport must be ready for seasonal and intermittent rush traffic.

British fisheries depend in the main upon herring, cod, mackerel, halibut, sole, and plaice. Cod-liver oil and halibut-liver oil have long been valuable products. Herrings are sold in various forms. A herring cured and smoked is called a red herring or bloater; split open and smoked, it is a kipper. Herrings are also pickled. There is great trade in sardines canned in olive oil, and fish pastes potted, tinned, or glassed. The sturgeon, caught in the N. Atlantic and the Black and Caspian Seas, and the bigger rivers entering these, and off the coast of Britain, is a food fish and a source of caviare (the roe prepared for table) and isinglass.

The fish oil industry has always included whale oil as well as oil from cod, halibut, and herring. Whales are sought usually in the colder oceanic waters. Fish surplus to market requirements are converted into fertilizers for the land, and into meal for cattle, pigs, and poultry. Even fish scales are used in the manufacture of some kinds of artificial pearls.

Resources of the Pacific

Fisheries developed in the North Pacific Ocean along the eastern and western shores in comparatively similar localities to those of the North Atlantic Ocean; for off Japan and British Columbia the oceanic waters are related to the warm current of the Pacific Ocean—the Kurosiwo or Black Stream of Japan. Herring, salmon, cod, and other food fishes are taken off Japan. On the American coast, in Alaska, in British Columbia, and in the neighbouring areas of the United States, salmon in great numbers frequent the seas and the rivers; the Fraser and Columbia rivers and Kodiak Island are important sources of salmon, tinned in immense quantities for market. Oyster, crab, and lobster fisheries were originally local; the canning industry has found a vastly wider market.

In mid-20th century the annual total fish catch was 25,097,000 tons. The U.S.S.R., China, the U.S.A., Japan were each responsible for 10 per cent., Norway and the U.K. each 5 per cent., Canada 4 per cent. The U.K. (21 per cent.), U.S.A. (18 per cent.), and Germany (17 per cent.) were the main importers of fish and fish products. Norway (28 per cent.), Canada (20 per cent.), and Iceland (12 per cent.), were the main exporters.

LESSON 5

Staple Foods of the Orient

PADDY fields—the term conjures up a romantic picture of the Orient. One sees a flat valley floor laid out chequer-board style, with squares or rectangles of water-covered mud separated by lines of dry mud. Coolies, lightly clad, with heads shielded from the sun, bend over young rice plants as they work ankle-keep in the mire. Paddy, or padi, is rice in the husk.

The rice of commerce is the seed of an annual grass, *Orzra sativa*, growing to 6 ft. in height, and aquatic in habit. It is rich in starch but has little protein and fat. In the husk are salts and vitamins, and when this is removed (in milling)

a prolonged rice diet may result in the disease called beri-beri. By-products of rice include saké, the Japanese national beverage, and starch; cigarette papers and thin tissue used in confectionery are made from the straw.

The average yearly world production of rice has been estimated at about 150 million tons. China produces 30 per cent., India 22 per cent., Pakistan 8 per cent., Japan 8 per cent., Indonesia 6 per cent., Siam 4 per cent., Burma 3 per cent., Indo-China 3 per cent., Korea 2 per cent. Burma, Siam, the U.S.A., Egypt, Formosa, and Indo-China normally produce an exportable surplus, most of which goes to India, Malaya, Ceylon, and Japan. But a very large part of the rice crop is consumed in the areas in which it is grown. In India rice is the staple food of about one-third of the population. In Japan, the Philippine Islands, parts of the East Indies, Indo-China, and parts of China, the great bulk of the people live mainly on it. The amount of rice consumed in Europe is comparatively small, and the international trade, except between the countries of Asia, is therefore not great. Great Britain imports rice principally from India, Spain, and Australia.

Rice Production in Bengal

The Bengal region of the Indian sub-continent is a typical rice-producing area. At least half the cropped land in this area is devoted to paddy. There are 100,000 square miles of stoneless soil unbroken by any elevation —practically treeless black alluvium of superabundant fertility, continually enriched by riverine floods. In Bengal the needed water is supplied by the monsoon rains, which come with a burst usually in mid-June, and continue in a succession of cyclones, penetrate the Ganges lowland from the Bay of Bengal, and reach a maximum in July and August, falling away in September and finishing in October; during this period a soaking soil steams between the downpours under a scorching sun.

The rains may be localised in the area and patches may be dry, so that in September, which is a critical time, the rice plants may lack water and shrivel in the increasing dryness of the air; if water cannot be supplied from irrigation channels, the crop fails. The main crop, so-called winter rice, is reaped from November to February; a smaller crop of early rice is cut between July and September, and a coarse spring rice is obtained in April. This cropping is related to the three seasons.

Other Areas of Production

The hot season, March to June, is useless; the wet season, June to October, supplies the water; the cool season, November to February, is the main reaping time. An inferior rice is grown on terraced, irrigated hill-sides in India. In Japan, where low land is strictly limited, the hill-sides are used for crops of superior quality, and Japan exports some of this rice in exchange for larger quantities of inferior rice.

The preparation of rice is not dissimilar from that of wheat. The first process is the separation of the grain from the stalk. The winnowed clean paddy is then husked; the husk is used in Burma as a fuel and for the manufacture of producer gas. Bran is removed from the rice by a process of skinning, the bran being used for cattle food. The skinned rice is white; and for the European market it is polished.

Other Cereals, Pulses, Millets

Rice is not the sole or even the chief food grain of the East; the gigantic crops which are grown in the suitable areas of the Indian sub-continent, for example, are insufficient for the teeming millions of the flood-swept alluvial plains of the Ganges and the Yangtze. In Bihar state, a neighbour of West Bengal, most of the people live on other cereals—maize, wheat, and barley—or on pulses and millets. The millets are rainy-season crops, and the pulses, such as gram (chick-peas), are cold-season crops.

In the Punjab the chief food grains are wheat, gram, baira (spiked millet), and jowar (great millet); rice comes after these. In Madras, where about four-fifths of the cultivation is devoted to the food grains, rice, though the chief cereal grown, is only a quarter of the supply. Next comes millet, followed by red ragi (raggee). For the Japanese, soya beans are a staple food.

Sago, a starchy substance made from the pith of a palm tree native to the E. Indies and New Guinea, is very nutritious. It is familiar in Britain in the form of pearly grains (pearl sago), and is used to make milk puddings.

China and its Food Problems

Detailed information about China's food crops for home consumption is not available. But rice does not enter into the economy of northern China, the land of the Yellow river; there wheat and millets, with the inevitable soya bean—sometimes called the "universal food provider," so rich is it in nutriment—are the chief food sources. In central and southern China, the lands of the Yangtze and Si rivers, possibly two-thirds of the people rely almost entirely on rice; cultivation is of the most intensive character, every square foot of available soil being used. In some of the rice-eating areas there is a population density of over 6,000 per square mile, and these folk live on what they grow. That explains why, even in time of peace, crop failure means famine to such densely peopled areas.

SUGAR. Principal regions of sugar production.

LESSON 6

Sugar's Place in World Economics

THERE is competition between the growers of the sugar cane, which is a cultivated tropical grass, and the sugar beet, a root-crop of temperate regions, to supply the world's demand for sweetness. The sugar cane requires great heat and considerable moisture in the soil, though less moisture than is needed by rice. The stems, or canes, grow to about 20 feet high, and it is from these that cane-sugar is extracted. The roots are perennial, throwing up each year fresh stems to replace those cut down at the previous harvest. Under favourable conditions the plants may be productive for 30 years, though replanting is usual after five years. Golden syrup, black treacle, molasses, and rum are by-products of cane-sugar.

In contrast, sugar beet must be sown every year. It can be grown where potatoes yield well, and it is a welcome addition to the harvests on the poorer soils of western Europe. An important by-product of sugar beet is stock-feed, made from the pulp after the sugar has been extracted from the swollen roots; and the refuse is used as a fertilizer.

Varieties of Sugar

The production of sugar for the European markets is a highly organized business. Its varieties, in form, are loaf or granulated; in colour it is white, yellow, or brown. It involves factory operations; the processes of extraction and treatment of the juice require operative skill and expert chemical knowledge. There is a vast difference between a modern sugar factory and the crude arrangements which prevail in some parts of southern India, where sugar canes are a feature of the landscape. There a pair of bullocks attached to a pole amble in a wide circle to cause a rotation of rollers which press the juice from the canes, in the open air. This native industry produces raw sugar, country sugar, and molasses, mainly for local consumption, and the quantity is difficult to assess.

Rise of Beet-sugar Industry

Development of the beet-sugar industry owes its rise to the Napoleonic and other wars of the early 19th century when the British blockade impeded the import into the Continent of tropical sugar from overseas. By 1820 the manufacture of sugar from beet was an established industry in several European countries, but it was not till the end of the 19th century that beet-sugar became a real rival of cane-sugar. In England beet-sugar production became an established industry in 1912.

Normal average annual consumption of sugar in the U.K., Australia, New Zealand, and the U.S.A. is over 100 lb. a head, less in continental Europe. The eating of sweetmeats —candy in America, chocolate and boiled sweets in Europe; the increased popularity of sweetened tea and coffee as beverages; the making of jams and marmalade; developments

in the preservation of fruits in tins and bottles; these are all factors of importance in sugar economics.

The U.S.S.R. is the largest producer of beet-sugar, India of cane-sugar. Total world production of beet-sugar in mid-20th century was 86,250,000 tons, made up as follows: U.S.S.R. 26 per cent.; U.S.A. 11 per cent.; Germany 11 per cent.; France 10 per cent.; Poland 6 per cent.; U.K. 5 per cent.; Czechoslovakia 5 per cent.; others 26 per cent. Total world production of cane-sugar in the same period was 235,935,000 tons, made up as follows: India 21 per cent.; Cuba 18 per cent.; Brazil 13 per cent.; Puerto Rico, Java, Madeira, Colombia, and Mexico, each 4 per cent.; others 28 per cent.

On a lesser scale, sugar is obtained from some species of palm trees and from the sap of American maple trees.

LESSON 7

Fruit—Origin and Distribution

BRITISH orchards and market gardens produce soft bush and ground fruits, and tree fruits such as apples, pears, plums, and cherries. But the supply is insufficient for home consumption, and Britain is probably the greatest importer of fruit in the world.

The citrus fruits, native to Asia, of which orange, lemon, and grapefruit are the best known, are of enormous commercial importance. Tangerine, mandarin, Californian seedless, Jaffa, and navel are commercial types of orange. Cultivation is on a large scale in California, Florida, the West Indies, Brazil, and S. Africa.

Soft tree fruits, such as peaches and apricots (native to Asia), were at first rare except in the countries of their origin; canning made it possible for American and other produce to be extensively marketed. Pineapples come whole, or in tins, cubed, chunked, or sliced.

Some of the largest banana plantations are in Central and South America, Jamaica, and the Canary Islands. For export, the fruit is cut when green, and the unripe bananas are shipped in vessels whose holds are equipped to maintain a temperature of 53 degrees F. The importer stores them for a few days in ripening rooms heated to about 65 degrees F., and when yellow they are distributed to the retailers. Canary Island bananas, however, are commonly shipped in crates, without refrigeration.

With other fruit, dehydration is successful. Apples, apricots, etc., can be deprived of their water content for transit; the purchaser plumps them up by soaking them in water.

The following summary gives the chief commercial sources of common edible fruits.

Apples. Worcestershire, Herefordshire, Devon, Nova Scotia, Tasmania, Washington, British Columbia, S. Africa. Cider is fermented apple juice: Devon, Hereford, Brittany, Normandy.

Apricots. Canada, France, California, S. Africa, Australia.

Bananas. Brazil, India, Tanganyika, West Indies, Canary Islands.

Cherries. England, Germany, Netherlands, Belgium, Italy, U.S.A.

Currants. Red, white, black: northern Europe (England), America. Dried (small grapes): Greece, Spain, U.S.A., Australia, Turkey, Persia, S. Africa.

Dates. Persia, Iraq, Arabia, N. Africa, Mexico, California.

Figs. Italy, Greece, Syria, Algeria, Portugal, California.

Grapefruit. Florida, California, W. Indies, Israel, S. Africa.

Grapes. Spain, France, Netherlands, Italy, U.S.A., Greece, S. Africa.

Lemons. Spain, Italy, U.S.A., Egypt, S. Africa, Australia, New Zealand.

Melons (water-melons). India, China, U.S.A., S. Africa, Egypt.

Oranges. California, Spain, S. Africa, Brazil, Israel.

Peaches. U.S.A., Argentina, Italy, S. Africa, Netherlands, Spain, Hungary, Australia, Canada.

Pears. U.S.A., S. Africa, Canada, Belgium, France, Australia, Britain. Perry is fermented pear juice: Worcester, Gloucester, Hereford, Devon, Somerset, Normandy, Brittany, Germany.

Pineapples. Florida, Hawaii, S. Africa, Australia, Malaya.

Plums. England, France, U.S.A., S. Africa.

Prunes (dried plums). California, France, Italy, Portugal.

Raisins (dried grapes). U.S.A., Australia, Turkey, Persia, Greece.

Sultanas (dried seedless grapes). U.S.A., Australia, Turkey, Persia, Greece.

LESSON 8

Tea, Coffee, and Cocoa

THE beverage tea is made (with boiling water) from the dried and prepared leaves of a species of evergreen tree, chiefly *Thea sinensis*, native to Asia. It was first drunk by the Chinese, hundreds of years (the date is uncertain) before it was known in Europe. It reached England in about 1660, but chiefly because of the prohibitive price of the prepared leaf (as much as £3 per lb.) its use was on a very small scale until greatly increased production gave it a vastly wider market in about the middle of the 18th century.

LUXURY COMMODITIES. Production figures for tea, coffee, and cocoa are given in this Lesson; those for beer, wine, and tobacco, in Lesson 9. The map above shows the principal sources of all six products.

China was the source of the world's tea supply until the 19th century. Indian tea reached England in 1839. Ceylon entered the tea market in about 1876. By the 1920s tea was being grown in Kenya, Tanganyika, Uganda, and Nyasaland. At mid-20th century the total average annual world production was 845,000 tons, of which China produced 36 per cent., India 32 per cent., Ceylon 17 per cent., Japan 4 per cent., Indonesia 3 per cent., Pakistan 3 per cent., U.S.S.R. 2 per cent., others 3 per cent.

Brands of tea differ in flavour according to country of origin, the quality of leaf, the manufacturing process, and the subsequent blending. On a tea plantation the trees are kept to about 5 feet in height (by regular pruning) so that the leaves are within easy reach of the pickers. The trees are picked-over several times a year, the tips of young shoots only being taken. Over 3,000 tips go to a pound of tea ; expert pickers can pluck about 30,000 tips in a day. The tea plant is hardy, though frost diminishes the yield. It needs rich soil, copious regular rains during the growing period, and good drainage. Hill slopes are suitable, provided the soil meets with the plant's requirements.

Coffee

Coffee is prepared from the roasted and ground seeds of evergreen trees of the genus *Coffea*, native to Abyssinia, where it was first used as a beverage. The coffee plant was carried across the Red Sea to Yemen, a province of Arabia. The climate of Yemen is peculiarly suited to its growth : a thick mist rises on almost every morning of the year and protects the bushes during the hottest part of the day. Coffee was introduced into England in the 17th century ; and by the Dutch into Java and Ceylon. Then it spread to all the countries of tropical America, and eventually to East Africa. Arabia's present production of coffee, most of which goes to Turkey and Egypt, has little commercial significance, though Mocha (a coffee once shipped in great quantities from the port of that name) is still considered to be one of the best. Production in Brazil far outstrips that in any other country, amounting to nearly half the world total of 2,152,000 tons.

At mid-20th century the chief producing countries and their percentages of annual production were Brazil 49 per cent. ; Colombia 16 per cent. ; El Salvador 3 per cent. ; Guatemala 3 per cent. ; Mexico 3 per cent. ; Angola 2 per cent. ; Venezuela 2 per cent.

In the early stages of growth the coffee plants need shade, and this is supplied in coffee plantations by interplanting with certain other plants, such as the banana, which are better able to withstand full sunshine. For easy cultivation the coffee trees are restricted (by regular pruning) to about 10 feet in height. The deep scarlet, cherry-like fruits each contain two seeds. The fruit (berries) are gathered when ripe, and the extracted seeds are soaked in water, allowed to ferment for a short time, then washed, dried, and packed for export.

During the first part of the 19th century coffee production increased in Ceylon until it became the principal export of the island. But in 1869 a leaf fungus appeared in the plantations and so impaired the productiveness of the trees that the cultivation of coffee in Ceylon seriously declined, and eventually tea plantations took the place of the coffee plants.

Cocoa

Cocoa, as a beverage, is prepared from the seeds (beans) of an evergreen tree, *Theobroma cacao*, height about 20 feet, native to the West Indies and tropical America. The drink was

introduced into England from South America in 1656. The tree needs much moisture, a deep soil, and greater heat than coffee. The seeds number about 60 per pod ; the pods measure up to about 10 inches long and 3½ inches in diameter. The seeds, after separation from their pulp, are fermented slightly, then dried, roasted, and ground. Cocoa-butter, a vegetable fat extracted during the manufacture of the cocoa powder, is used in making chocolate. At mid-20th century the total average annual world production of cocoa was 764,700,000 tons, made up as follows : Gold Coast (British West Africa) 32 per cent. ; Brazil 18 per cent. ; Nigeria 13 per cent. ; French West Africa 7 per cent. ; French Cameroons 5 per cent. ; Dominican Republic 4 per cent. ; Ecuador 3 per cent. ; others 18 per cent.

LESSON 9

Beer, Wine, and Tobacco

BEER and wine are typically European commodities. One of the oldest beverages, beer is an intoxicating liquor made from barley or other malted grain, hops, sugar, and yeast. Varieties of beer include bitter, mild, pale ale, old ale, lager, porter, stout. Beer had for long been a common drink in England when duty was first imposed on it, in 1660. At mid-20th century the average annual production of beer in Great Britain was 25 million bulk barrels (a bulk barrel is 36 gallons), and excise duty in the U.K. was over £243,000,000. Most of the barley grown in Britain is used for malting. Hops are grown in England (21,000 acres) in Kent, Sussex, Hampshire, Hereford, and Worcester ; imports are from the U.S.A., Hungary, and France.

Climate and the Grape

The grape vine, from the fruit of which wine is made, will grow in most temperate and warm temperate climates ; but only from grapes grown in areas where the September temperature in the northern hemisphere (in the southern, the March temperature) averages 60° can wine of a commercial quality be produced. Before the wine trade developed, the vine used to be grown in England and wine made from its grapes ; nowadays, except possibly here and there as a hobby, vine-growing for wine-making has ceased in England.

A sunny climate without excess of rain, and warm soil that retains moisture, are most suitable to the cultivation of the grape vine. It roots deeply, and this enables it to thrive where the summers are dry (e.g. in the Mediterranean area), also where, provided the summers are warm and long enough, it has to survive winter frost (e.g. in south Russia).

The Wine-producing Countries

Most of the wine produced is consumed in the country of origin. France, largest wine producer in the world, is also the largest consumer. In normal times the average annual consumption in France is 14 gallons per head, and though France exports a proportion of her fine wines she imports something like 25 per cent. in quantity of her own production, in inferior wines, chiefly from Algeria.

Local conditions of climate, aspect, and character of the soil have considerable effect upon the kind of grape grown, and therefore the type of wine produced. Burgundy and champagne originate in the old French provinces of those names. Claret, or Bordeaux wine, comes from the Gironde district of France ; hocks and moselles from the Rhine and Moselle vineyards of Germany ; sherry from the neighbourhood of Jerez de la Frontera in Spain ; port from the Douro valley of Portugal. Chianti comes from the Chianti hills of Italy, and barolo from Piedmont. In many wine-making districts there are vintages of first-rate quality quite unknown outside the area of growth, either because they do not travel well or because they are too small in quantity to be commercially valuable.

Viticulture flourishes in South Africa (the wines of the Constantia district of Cape Province were already well known in England in the 17th century) and Australia (South Australia, Victoria, and New South Wales). Preferential customs treatment gave an impetus to the import of Commonwealth wines. But the consumption of wine in Great Britain is low, averaging only 0·6 gallons per head per annum. The total annual world production of wines in mid-20th century was 18,100,000 tons, accounted for as follows : France 29 per cent. ; Italy 20 per cent. ; U.S.A. 8 per cent. ; Algeria 8 per cent. ; Spain 7 per cent. ; Argentina 6 per cent. ; Portugal 5 per cent. ; others 17 per cent.

Tobacco

Tobacco had its origin in the New World, for the plant from the leaves of which it is made is native to America. The custom of smoking was popularised in England by (according to tradition) Sir Walter Raleigh, in about 1586. At first a luxury, it came to be regarded by millions of men and women as a necessity, and its cultivation spread to all parts of the world having a suitable climate, though the country producing the largest quantity for export is still the United

States. The best leaf is obtained from plants grown in warm temperate regions; but tobacco will grow anywhere in the tropics, and it is grown on a small scale, for private use, in England. Tobacco imported annually into Britain in the 1950s was over 280 million lb.

The tobacco plant is an annual. Two species most commonly grown are *Nicotiana tabacum* and *N. rustica*. Height ranges from four feet to about six feet. The leaves are green when harvested. Processes before the leaf is ready for factory include drying, fermenting, and maturing. Even slight differences of soil and climate cause a difference in the character of the crop. Tobaccos produced for export are of several types and each type has a flavour of its own. The most common commercial distinctions are between Virginia and Balkan types, and between cigarette, cigar, and pipe tobaccos. Flue-cured Virginian tobacco is the most popular type for cigarettes. Cuban cigar leaf and Sumatra cigar wrapper are special tobaccos. Snuff, for inhaling through the nose, is a powdered form of tobacco made from the stalks of the plant.

LESSON 10

Textiles of the World

Until the beginning of the 20th century the textiles of the world were made from natural fibres, of which the chief were two vegetable (cotton and linen), and two animal (wool and silk). The 20th century saw the rapid commercial development of rayon (or artificial silk), invented in 1883, which is replacing silk, cotton, and linen in many of their uses besides being woven into mixtures of those materials and of wool. Rayon gives to fabrics a silk-like texture and lustre at a cost much below that of silk.

All the natural textiles are of great antiquity. Manufactured flax fibres have been found among the remains of the Bronze Age lake dwellings of Switzerland; cotton fabrics date back to the earliest times of which there are record; wool was used by the Romans, and before; the spinning and weaving of silk began, according to Chinese legend, about 2700 B.C.

Linen is made from the fibres of the stems of the flax plant, *Linum usitatissimum*, an annual up to 3 ft. in height. The plant flourishes in very different climates—it will grow well in India, and in the colder parts of Russia. But the flax of India is grown mainly for its seed, which yields linseed oil, used in making paints, printer's ink, linoleum, patent leather; the residue (oil-cake) is used for feeding cattle. Flax for linen is grown in cooler climates. The largest producer in mid-20th century was Russia, accounting for 80 per cent. of the world total of 884,000 tons. The best-quality thread comes from flax grown in Belgium; Northern Ireland grows flax to make fine linen.

Linen making is an important industry in Northern Ireland (centre, Belfast) and Scotland (centres, Dundee and Dunfermline) in the British Isles, also in Germany, the Netherlands, Czechoslovakia, and Belgium.

NATURAL TEXTILES Regions of production of the four chief raw materials.

The woolly fibres which cover the seeds of the cotton plant are the cotton of commerce. The species generally cultivated is native to India, Indo-China, and the East Indian islands. It needs a long summer; sufficient but not excessive moisture; a warm and even but not excessive temperature; and bright sunshine. It is grown as an annual, and reaches a height of about 6 ft.

Cotton Production in the U.S.A.

The United States produces the largest crop of cotton in the world—42 per cent. of the average annual world production (mid-20th century) of 6,546,000 tons. Texas, Mississippi, Arkansas, Alabama, Georgia, South Carolina are some of the important cotton-producing states. In the islands along the coast of Georgia and South Carolina, and in northern Florida, Sea Island cotton—the best of all—is cultivated. It produces fibres up to 2½ in. long; the average length is 1·6 in. Slightly saline soil and slight salinity in the atmosphere seem to be important factors in its successful cultivation. Sea Island cotton is also grown in the West Indies, in Egypt, and some of the Pacific islands. In the U.S.A. the greatest production is of cheap, medium fibre-length (staple) cottons.

Cotton in the U.S.S.R., India, Egypt

Next to the United States in quantity of output comes the Soviet Union, whose average (mid-20th century) annual production is 17 per cent. of the world total. The main producing areas in the U.S.S.R. are Soviet Central Asia, south of the Aral Sea, west of the Caspian Sea, and South Ukraine. In the Indian subcontinent (7 per cent. of world production) cotton (mainly the coarse and short varieties) is grown on the so-called black cotton-soil of the Deccan, in Uttar Pradesh, and in the Punjab (W. Pakistan). Egypt (6 per cent. of world production) produces the finest long staple cotton (up to 1¾ inches long), and this commands higher prices than the average American or Indian product. China uses almost all her own crop (6 per cent. of world production).

Cotton Manufacture

In the manufacture of cotton, Lancashire, England, was long pre-eminent. The first inventions for machine spinning and weaving of cotton were made by Englishmen in the early days of the British industrial revolution, and these, employed in a climate naturally suited to cotton spinning, gave Lancashire an initial advantage which she maintained for many years. Lancashire's manufactures still rank among the finest in the world; but she has lost most of her former immense trade in coarse and cheap cottons to India and Japan. The United States has developed an extensive cotton manufacturing industry; the states of Georgia and North and South Carolina consume most of the American cotton.

Wool Production and Consumption

Sheep's wool is the wool of commerce. The merino sheep, which bears the best wool, seems to have originated in North Africa, whence it was introduced into Spain. For centuries Spain produced the best wool. During the Middle Ages wool was far and away the most important of England's products, and she exported great quantities, noted for length of staple, to the woollen manufacturing centres of Flanders. Towards the close of the 18th century the merino sheep was introduced into Australia, and there it has flourished exceedingly. Parts of Victoria and New South Wales produce a wool of unequalled softness and lustre, which is also long in staple. The highest prices in the London market are paid for the wools of Australia.

The woollen industry of Great Britain absorbs most of the wool from Australia and South Africa. The United States, in spite of her own large production, imports wool for manufacture from India, Argentina, Great Britain, and elsewhere. Wool from Argentina helps also to maintain the woollen industries of the U.K., Germany, France, Italy, and Belgium. The average annual wool production in mid-20th century was 1,704,000 tons, accounted for as follows: Australia 28 per cent; Argentina 11 per cent.; New Zealand 10 per cent; U.S.A. 7 per cent.; U.S.S.R. 7 per cent.; South Africa 6 per cent.; Uruguay 4 per cent.; Spain, China, the U.K., each 2 per cent.; others 21 per cent.

Silk Production and Consumption

Silk is made by uniting a number of the fine threads spun by caterpillars as a wrapping for themselves when they reach the cocoon stage. The greater part of the silk of commerce comes from the cocoon of the silkworm, the caterpillar of a moth, *Bombyx mori*, which feeds on the leaves of the mulberry tree. Silkworms are reared in quantity in China (the original home of silk), in Japan, and in India. Italy, France, and Spain have a comparatively small silk-rearing industry, Italy producing about three-quarters of the silk grown in Europe. A silk farm at Lullingstone Castle, in Kent, produced silk for the 1953 coronation robes of Queen Elizabeth II.

As an early stage in the preparation of silk for market, threads from several cocoons are wound off together; the finer grades have 5 to 7 threads, the coarser from 11 to 20. The threads have a slight stickiness which makes them cling together readily. It takes about

11 lb. of cocoons to produce 1 lb. of reeled silk. In addition to the cultivated silk, China and India both produce " wild " silk (tussore), made from the collected cocoons of caterpillars of other moths, and of the mulberry moth in its wild state. The wild silk of China is about a quarter of her whole output. Some parts of Africa also produce wild silk.

Countries Manufacturing Silk

Besides being one of the great silk producers of the world, Japan is also a big manufacturer of silk goods, of which she exports more than any other country. Most of the silk of China and of India is made up on the spot, very largely for home consumption, though China exports a certain amount of silk goods of a better quality than the Japanese. The United States has a big silk manufacturing industry, based on imported raw silk. Silk goods of the finest type have for many years been made in France, in the Lyons area. Krefeld, in Germany, and Zürich and Basle, in Switzerland, are other European centres of silk manufacture. In the U.K. the silk-spinning industry is relatively small; Bradford and Macclesfield are chief centres. A drastic decline in the silk trade followed the Second World War; this was due mainly to competition of stronger synthetic fibres, especially rayon.

Rayon

The name rayon was adopted in the U.S.A. in 1924, and has become established in Europe, for filaments made by drawing or pressing cellulose in solution through tubes of very fine bore, and solidifying the resulting threads. These filaments are so fine that, as with real silk, a number of them—10 to 20—are united to form a thread of "silk." Cotton waste, sawdust, or wood pulp is the usual material from which the cellulose solution is made; and these materials are chemically the same as those used by the silkworm in the natural process. Threads of sufficient tensile strength and elasticity to stand weaving and working-up can be produced at low cost from numerous waste or semi-waste natural materials.

The Second World War, by interrupting the supply of natural silk, greatly stimulated the production and improvement of the artificial substitutes, particularly in connexion with war uses (e.g. for parachutes), where expense of experiment and production counted for little.

In mid-20th century the annual world production of rayon filament yarn was 860,000 tons, made up as follows: U.S.A. 50 per cent.; U.K. 10 per cent.; West Germany 7 per cent.; Italy 6 per cent.; Japan 5 per cent.; France 5 per cent.; Netherlands 3 per cent.; Canada 2 per cent.; Brazil 2 per cent.; others 10 per cent.

Rayon is produced by three different chemical processes: viscose, acetate, and cuprammonium; in mid-20th century 85 per cent. of world output was of the viscose type. Viscose rayon dyes readily and is widely used for apparel and in special industrial applications, e.g. in rubber tyres. Acetate, which is the " silkier " rayon, is used where appearance and " handle " are important. Cuprammonium rayon production is comparatively unimportant. World production of rayon is increasing rapidly, as the following table shows.

PERCENTAGE DISTRIBUTION OF FIBRE CONSUMPTION
(Based on Production by Weight)

	1920	1930	1940	1950
Cotton	85	82	74	68
Wool	15	15	13	13
Rayon	—	3	13	19

The consumption of silk is now less than 1 per cent.

Other Synthetic Fibres

Nylon, terylene, orlon, and glass fibre are man-made fibres, synthesised from by-products of coal carbonisation and petroleum refining. At mid-20th century, nylon was much the most important of these, but their combined production (76,000 tons) was far less than that of rayon. The U.S.A. produces about 85 per cent. of the total of these synthetic fibres, the U.K. 5 per cent., Canada and France each 3 per cent. World capacity is expanding rapidly; it trebled between 1950 and 1953.

LESSON 11

Rubber, Natural and Synthetic

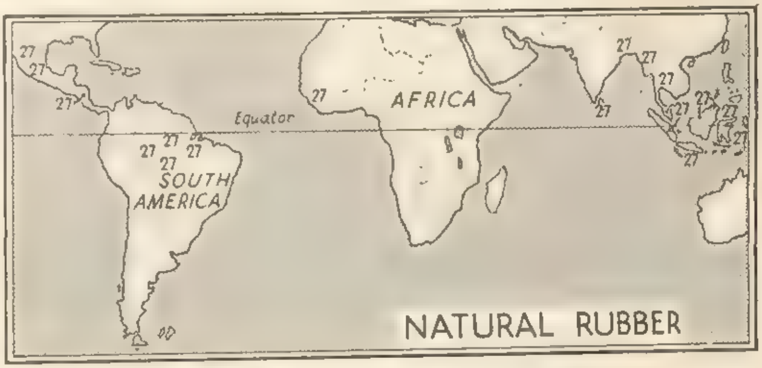
It is difficult to imagine a world without rubber. Yet this valuable substance played a negligible part in world economy until the invention of the pneumatic tyre by J. B. Dunlop (1840–1921) in 1888.

Columbus, in 1493, observed the natives of Haiti playing with a ball of a black resilient substance; and Juan Torquemada, in 1615, referred to a material from a Mexican tree with which the Spaniards made their cloaks waterproof. In 1736 La Condamine read to the French Academy a paper on caoutchouc, the South American name by which rubber first became known in Europe. It was first brought to England in 1770, and was used to rub out pencil marks (hence "india-rubber"). A

process for waterproofing cloth with dissolved rubber was patented by Charles Macintosh (1766–1843) in 1823, and the name still survives for a waterproof material.

The development of the process for hardening rubber by the addition of sulphur (vulcanisation)—Charles Goodyear (1800–60) patented a method in 1844—made rubber available for a number of other purposes. Even so, the amount used was small; in 1850 much less than 500 tons was imported into the U.K. The 20th century saw a great growth of rubber output from 50,000 tons to 1¾ million tons a year, mainly through the development of highly capitalised estates. The estates produce internationally standardised grades by large-scale methods and increase output through the introduction of higher-yielding types.

Natural rubber is obtained from a milky liquid called latex which is produced by certain tropical trees; but the rubber of commerce comes almost entirely from the tree *Hevea brasiliensis*. It is native to the forests of the Amazon valley, and produces a less resinous rubber than any other known plant. In natural conditions it will grow to a height of 100 feet, with a trunk 12 feet in circumference. Latex forms in a layer of cells between bark and wood, and is drained off through cuts in the bark. *Hevea* latex usually contains 30 to 40 per cent. of rubber in the form of globules suspended in the liquid. When suitable chemicals are added, the globules coagulate in a mass at the top of the fluid.



RUBBER. Sources are indicated by the figure 27.

"Wild" Rubber from Brazil

Until the turn of the 20th century, rubber was obtained almost exclusively from trees and plants growing wild. Most of it came from Brazil, from the *Hevea brasiliensis*. The collecting point was Manáos, at the confluence of the river Amazon and the Rio Negro, and it was exported from Para—from which latter circumstance *Hevea* rubber got its name of Para rubber. The native collectors in a very short time exhausted the resources of the more accessible parts of the forests by overtapping, so that the trees failed to recover, and even by deliberately killing trees in order to drain every drop from the cells.

These reckless methods led to increased difficulty, danger, and expense in collection as it became necessary to penetrate deeper and deeper into the Amazon forests, and helped in the displacement of wild rubber from the market in favour of plantation-grown rubber. Similar destructive methods of collection almost exterminated various indigenous rubber-producing plants in Nigeria and the Belgian Congo, where in the early years of the 20th century wild rubber formed an important item of export—small and inferior though it was in comparison with Brazil's output.

Cultivation of Rubber

The possibility of growing rubber on a commercial scale in the East was suggested as early as 1834, but nothing was done for a number of years: the forests of the Amazon supplied the world's needs. But in 1873 some 2,000 *Hevea* seeds were sent to England—to the Royal Botanic Gardens, Kew, Surrey. Only a dozen of these germinated, and six of the young plants, sent to the Royal Botanic Gardens at Calcutta, failed to flourish. Seedlings from another consignment of *Hevea* seeds from the Amazon raised at Kew and sent to Ceylon in 1876 did much better; and seedlings were then sent to other countries. The falling-off in the production of wild rubber and the increased demand for the commodity led many planters to take up its cultivation in the early 1900s, and Singapore became a centre of seed distribution to Malaya and the Indies.

More and more plantations were established in Malaya, the Netherlands Indies, and Ceylon. In Ceylon, for instance, about 300 acres were planted with *Hevea brasiliensis* in 1890; by 1927 the area had risen to 400,000, by 1940 to 604,000 acres. For a year or two before the First World War (1914) when rubber prices were high, the value of the rubber exported by Ceylon exceeded that of tea. Plantations were also established in India, Sarawak, North Borneo, Indo-China, Siam, parts of Africa and later Brazil itself.

Hevea brasiliensis will flourish on even poor soil if there is a rainfall of 70–75 inches,

well distributed through the year, and a temperature of 75° to 95°, but it grows most rapidly and gives the best yield in moist, rich, alluvial, well-drained soil immediately after this has been cleared of virgin forest. Low-growing plants are planted between the young rubber trees, to prevent soil from being washed away by heavy rains, and also to shade the young trees in the first stages of their growth. They are ready for tapping at five years old.

Tapping on a rubber estate is carried out very systematically. Cuts are made in the bark, not too deep and not too shallow, and as the latex flows sluggishly out it is collected in cups fastened below the cuts. Even with expert tapping there comes a time when the trees cease to be commercially valuable and the estate has to be replanted. In favourable circumstances the yield per tree is 20–30 lb. a year.

Smallholder Rubber Production

Since the Second World War native smallholders, mostly relying on their own efforts, have been producing increasing amounts. Smallholder production now accounts for over one-third of the total output of rubber. The annual production of natural rubber at mid-20th century was 1,623,000 tons, accounted for as follows: Malaya 42 per cent.; Indonesia 32 per cent.; Siam 6 per cent.; Ceylon 6 per cent.; British Borneo 4 per cent.; Indo-China 3 per cent.; U.S.S.R. 2 per cent.; Brazil 1 per cent.; others 4 per cent.

The American motor industry uses half the rubber consumed in the world, mainly in the form of tyres and pneumatic tubes for her cars (70 per cent. of the world total). The U.K., the U.S.S.R., France, Germany, and Japan are also large consumers. The rubber market periodically experiences violent fluctuations in turnover and prices, and changes in the fortunes of the producers are equally great.

Growth of "Synthetic" Industry

A new element of instability was occasioned by the American synthetic rubber industry, which became a major source of supply during the Second World War. Now nearly a third of the total world rubber supply is produced synthetically. In the early 1950s the United States produced 798,566 tons of synthetic rubber in a year, Canada 74,272 tons, Western Germany 4,931 tons. The U.S.S.R. is also an important producer. Though not at present cheaper than natural rubber, synthetic rubber supplies half the American total requirements; a minimum consumption of synthetic in main products is prescribed by law.

Increasing Use of Latex

Latex is liquid rubber with excess water removed. In contrast to rubber sheets which are, in effect, solid latex smoked and preserved, liquid latex can be used for a large number of new purposes such as the manufacture of foam rubber (for cushions, mattresses, upholstery, etc.). The use of latex for purposes other than transportation has expanded rapidly.

Consumption of natural rubber latex in the U.K. increased from 2,123 tons in 1946 to 15,539 tons in 1952. In the United States consumption increased from 5,724 tons to 53,567 tons over the same period.

The cutting off from the Allied nations of normal sources of supply of natural rubber in the Second World War led to increased production of rubber from other natural sources. The Amazon forests were worked over again, and the *Hevea* plantations of Central and South America were extended. A shrub, *Parthenium argentatum*, natural to the semi-arid areas of Mexico and south-west U.S.A., was extensively cultivated; it yields what is known as guayule rubber. The whole plant is pulled up and processed. The latex is much more resinous than *Hevea* rubber, and cultivation is worth while only when *Hevea* rubber is dear or scarce. *Kok saghyz* is a rubber-bearing plant of the dandelion family which will grow in temperate regions, and it is of some importance in the U.S.S.R.

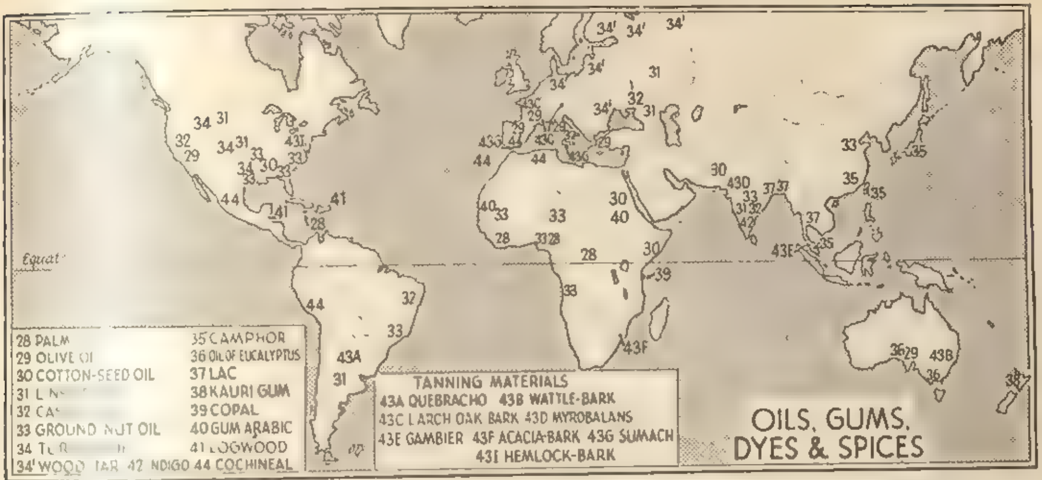
Average annual world consumption of natural and synthetic rubber in mid-20th century was 2,162,592 tons; the U.S.A. consumed 53 per cent.; the U.K. and the U.S.S.R. about 10 per cent. each; France about 5 per cent.

LESSON 12

Vegetable Oils, Gums, Dyes, and Spices

Rubber far exceeds in importance any other substance of similar origin, but a number of much older products derived from similar sources are of considerable importance in commerce. These are the vegetable oils, resins, and gums, which form the basis of many commodities in daily use.

Oils are either fixed, when they decompose at high temperatures, or essential, when they can be distilled and concentrated. The fixed vegetable oils of commerce include palm olive, linseed, castor, cotton-seed, colza or rape, arachis, sesame, and soya. Allied to the oils are the more viscous butters, such as those obtained from the cacao bean and from copra, the dried kernel of the coconut.



VEGETABLE OILS AND OTHER PRODUCTS. The map indicates the chief areas of origin.

Palm oil is obtained from the Guinea oil-palm, a native of the Guinea coast of Africa, and occurring generally in West Africa between 10° N. and 10° S. It is also cultivated in the East Indies. The oil-palm yields two types of oil ; one from the fleshy part of the fruit, and used in the manufacture of margarine, soap, candles, high explosives, and tin plate (to prevent oxidation of the iron sheets before tinning). The other type of oil comes from the kernels (strictly, palm-kernel oil), and is used as a substitute for the more expensive olive oil, which is derived from the fruit of the olive tree.

Groves of olives are characteristic of the Mediterranean region, particularly of Italy, Spain, Portugal, Greece, and Turkey, where olive oil is an important item in the dietary of the people. The tree thrives in other winter rain regions, but, except in California and South Australia, the production of olive oil elsewhere is insignificant.

Cotton-seed, Linseed, Colza Oils

Cotton-seed oil is a valuable by-product of the cotton harvest. No other oil seeds are so extensively produced. The oil is used as an adulterant of, and a substitute for, olive oil. Linseed (flax seed) oil, produced from flax specially grown and not, like cotton-seed oil, a by-product of the fibre, comes from Canada, India, the U.S.A., and Argentina. It is used in mixing paints and varnishes, and is the basis from which oilcloth and linoleum are manufactured. The oil-cake left after the oil has been expressed from cotton and linseed is an important winter feed for cattle ; it is imported by the big dairying countries (e.g. Denmark, the Netherlands, England).

Rape oil or colza oil is obtained from rape seed ; the plant is cultivated in India and China (for food oils and lubricants), and in northern and central Europe (for illuminating oil). Colza cake is inferior in quality to cotton and linseed cake. Sesame, an annual plant grown in India and China, produces seeds which yield a tasteless oil; the meal is eaten by human beings and by cattle. Castor oil is pressed from the seeds of a small tree, *Ricinus communis*, which is indigenous to tropical Africa and is cultivated in most tropical and warm temperate climates ; India, Brazil, Ukraine, and California are chief sources of supply. Castor oil is used in soaps, paints, plastics, medicines, and as a lubricant. Arachis (ground nut, peanut, or monkey nut) oil is produced in India, West Africa, China, U.S.A., and Indonesia. It is used as food and in soap making, and comes from a leguminous plant, *Arachis hypogaea*, which has the peculiarity that after pollination the flower-stalk lengthens and curves to the ground, in which it buries the developing pods. Soya-bean oil is produced in China, Manchuria, Java, and the U.S.A. It has so many uses that the soya bean is the world's most important oil seed crop. All the oils enumerated are fixed oils, yielding vegetable fats of the same general character, and they can to a considerable extent be substituted for one another after suitable treatment.

Volatile Oils

Volatile, essential, or ethereal oils, as they are variously called, are in the main associated with perfumes (clove, rose petals, jasmine blossoms, lavender, etc.). Turpentine, chief of the essential oils, has more commonplace uses. It is distilled from the resin of certain coniferous trees, and is used as a solvent of resins, in the manufacture of varnish and paint, and for cleaning. It comes mainly from the U.S.A. From the coniferous forests of Europe comes wood-tar,

from which creosote and pitch are obtained. Camphor is obtained from the wood of the camphor laurel, which grows in Japan, Formosa, central China, and Malaya. Oil of eucalyptus comes from a tree indigenous to Australia ; apart from its medicinal use, it enters into the manufacture of soap, varnish, and perfumes. A species of mint, *Mentha piperita*, cultivated in England, Europe, the U.S.A., China, and Japan, yields oil of peppermint, of medicinal value ; from another mint, *Mentha arvensis*, menthol is obtained.

Resins and Gums

One of the important resinous substances of commerce is lac, used in making shellac, lacquer, paints, and varnishes. It is formed on the twigs and young branches of certain trees by the action of a minute scale insect, *Tachardia lacca*. It is removed from the wood, washed, melted, strained, and allowed to set into flakes. Bengal, Siam, and Assam are sources of production. The insect white wax of China is of somewhat similar origin.

Kauri gum, used in making varnish and linoleum, is the resin of a New Zealand pine, *Agathis australis*, obtained chiefly not from the living tree but from the ground where pine forests once grew. Copal, another important resin, is obtained from certain trees in Sierra Leone (West Africa) and the Philippines ; in the Congo and in Zanzibar it is dug from the ground where producing trees formerly grew.

Amber, found in the sands of Baltic shores, is the fossil resin of extinct conifers. Dragon's blood is a red resin from various plants of the Far East. Gamboge, the gum resin of a tree indigenous to Cambodia, Siam, and South Vietnam, is used as a pigment, for colouring varnishes, and as a cathartic. Gum arabic, from a species of acacia, is used in medicines and in various industries ; it comes from the Sudan and Senegal. Gum tragacanth, used in calico printing, comes from Ismir (Smyrna) in Asia Minor. Frankincense, from trees native to Africa, southern Arabia, and India, and myrrh, from the same regions, are resins used from time immemorial for burning as incense.

Dyes and Stains

Many dyes and stains are of vegetable origin, although there are now synthetic substitutes. The chief tree dye is logwood, common in Mexico, though British Honduras and the West Indies are chief sources of the commercial supply ; the wood is dark red in colour and yields an extract from which blue, brown, and black dyes can be produced. From fustic, a tree of the mulberry family native to Central America, a yellow extract is produced which is used in wool dyeing, generally combined with other dye-wood extracts to produce browns, olives, and similar colours ; it is obtained mainly from Nicaragua. Brazil-wood, a dense, compact wood of reddish-brown colour, yields a bright red dye ; it grows in South America and the West Indies.

Sanderswood, from tropical Asia, and barwood, from the Guinea Coast, West Africa, produce dull red-browns ; camwood, also from the Guinea Coast, produces a violet-red. Madder plants of various species yield red and yellow dyes ; they were extensively cultivated in southern Europe and in India, but have been almost completely ousted by synthetic dyes. Indigo, a blue colouring matter from the stems and leaves of a plant, has its synthetic counterpart. Cochineal, obtained from the dried bodies of scale-insects native to Mexico and Peru, is used for dyeing, and for colouring confectionery and toilet preparations. The insect which produces lac yields a dye somewhat similar to cochineal. The natural indigo and cochineal, like madder, have suffered from the competition of synthetic dyes.

Tanning Materials

Until the beginning of the 19th century tanning in Great Britain was done exclusively with home-grown oak bark. Then various chemicals, and other natural products, came into use. The natural products include quebracho, from the forests of Argentina, of which it is the most valuable single product ; sumach, a preparation of the dried leaves of a south European tree ; gambier, from the leaves of a shrub native to Malaya and the Eastern Archipelago ; the bark of the hemlock-spruce from the U.S.A.; wattle bark, and extracts of it, from Australia ; and acacia bark from Natal.

Spices and Condiments

Spices were of great importance to Europe in the Middle Ages when there was no means of keeping food fresh. To be preserved, the food had to be dried or pickled, and spices were needed to make it palatable. The urge to find new ways to the Spice Islands of the Far East was one of the main incentives that sent explorers out on hazardous journeys which led to the discovery of the American continent and the passages round the Cape of Good Hope and Cape Horn. The chief spices and condiments in use to-day, and their sources, are as follows :

Allspice or PIMENTO. The unripe aromatic berries, dried and ground, of an evergreen tree, *Pimento officinalis*, native to tropical America, cultivated in the West Indies. Contains the flavours of cinnamon, nutmeg, and cloves, hence " allspice."

Aniseed. Aromatic seed of a perennial plant, *Pimpinella anisum*, native to Mediterranean countries ; cultivated in Spain, South Russia, etc.

Capers. The unopened flower buds of a tree, *Capperis spinosa*, native to Mediterranean countries.

Caraway. The seeds of a biennial plant, *Carum carvi*, 2 ft. high, native to Europe and Central Asia, cultivated in Britain.

Cassia, the bark of a tree of the laurel family. The main supply comes from China. It is used as a flavouring in cooking and for liqueurs, also medicinally.

Cayenne Pepper. A pungent powder made from the dried pod-like fruits of species of capsicum plants native to South and Central America.

Cinnamon. The powdered bark of an evergreen tree, *Cinnamomum zaylanicum*, native to Ceylon and the sub-continent of India.

Cloves. The dried, unopened flower buds of an evergreen tree, *Eugenia carryophyllata*, native to the Moluccas and Indonesia.

Coriander. Small, aromatic seeds of an annual plant, *Coriandrum sativum*, 18 in. high, native to South-east Europe and West Asia.

Ginger. The dried underground stem (rootstock) of a perennial plant, *Zingiber officinale*, 4 ft. high, native to the East Indies; cultivated in India, China, West Africa, West Indies, Central America.

Mace. The aromatic, dried, fibrous covering of the nutmeg (*q.v.*).

Mustard. Made from the pungent seeds of annual plants of the *Brassica* genus, native to Asia, North Africa, Europe (Britain), and widely cultivated.

Nutmeg. Seed of an evergreen tree, *Myristica fragrans*, native to the East Indies.

Pepper. The powdered berries (peppercorns) of an evergreen climbing plant, *Piper nigrum*, native to the East Indies, cultivated in Sumatra, Borneo, Southern India, West Indies, the Philippines, etc.

Pimento. Another name for Allspice (*q.v.*).

Turmeric. A yellow, peppery powder made from the rhizome of *Curcuma longa*, a tropical plant of the ginger family, cultivated in the Indian sub-continent.

Vanilla. Obtained from the dried seed pods (6-10 in. long) of a climbing plant, *Vanilla planifolia*, native to Central America, cultivated in Madagascar, Mexico, East Indies, West Indies, etc. A substitute is commonly made from coal-tar and some other substances.

LESSON 13

The World's Timber

The forests of the world, covering nearly one-third of the earth's surface, are of two kinds—temperate and tropical. The temperate forests fall into two categories—the deciduous forests of oak, elm, maple, beech, birch, and such temperate hardwood trees; and the coniferous forests of fir, pine, larch, and other evergreen trees which produce softwoods.

Most of central and northern Europe was once covered with forest, and to-day in England there are still considerable stretches of woodland. But over the great part of Europe the forests have gradually been cut down and cleared to make way for agriculture, pastoral life, and industry. Only in those parts of the European continent subject to severer climates—Scandinavia, northern Russia, and Finland—do forests remain covering large tracts. Much of the forest land that existed in North America when it was inhabited by roving Indian tribes has also been cleared, but vast stretches of forest still remain, in both Canada and the United States.

Softwoods in North America

Softwoods constitute 80 per cent. of the timber production of the world. A large proportion of the softwoods felled is turned into wood-pulp, whose main use is to make paper, particularly newsprint, and in forest areas where water power is available—notably Canada, near Niagara Falls, Norway, and Sweden—wood-pulp and paper factories are on the spot where the raw material grows. The great forest regions of the temperate zone all lie in areas having a heavy winter snowfall, and the lumberman's cycle begins in the winter, when the trees are felled and dragged to the riverside, there to be floated to sawmill and factory when the snows melt and the rivers are in spate.

Forest still covers nearly a third of the whole area of the U.S.A. The total annual cut and destruction amounts to about 13,400 million cubic feet, while annual growth amounts to about 11,000 million cubic feet. This is a serious discrepancy, since nature unassisted takes 50 to 60 years to regenerate a temperate forest area even in the more reproductive regions; the U.S.A. government therefore practices systematic reafforestation.

The forests of the Pacific coast supply most of the U.S.A. timber exports, the supply of large trees elsewhere having been nearly exhausted. Oregon is the biggest producer, and it is estimated to contain about a quarter of all the mature timber in the U.S.A. Washington comes next in importance; Michigan, Minnesota, Wisconsin and California have considerable output. Alaska is also a substantial producer.

Over a third of Canada is forested, and over half of that is productive and accessible. About a third of the accessible area contains mature timber. More than half the total estimated production of pulp wood in Canada comes from British Columbia, though Quebec's output of pulp and paper has a higher value. Lumbering is British Columbia's second most important industry. Vancouver is the main centre. Ontario is another important timber-producing area.

Softwoods in Europe and Russia

The chief timber-exporting country of Europe is Sweden, with over 50 per cent. of the country's area forested; timber and wood-pulp provide 40 per cent. of her exports. Timber, wood-pulp,

SOURCES OF TIMBER. Temperate forests supply 98 per cent. of the world's wood.

and paper are, together with dairy produce, the chief exports of Finland, which has 37 per cent. of its total area forested. Timber, wooden goods (including doors and window frames), and wood-pulp constitute 25 per cent. of the value of Norway's exports. About a quarter of Norway's forest area, which covers 23 per cent. of the country, is productive, her annual output being some 367 million cubic feet, which goes for the most part into paper. Her enormous water-power development—the largest per head of any country in the world—greatly facilitates her paper-making industry.

Russia possesses large natural reserves of softwood; 41 per cent. of the total area is forested. It is all under government control, but nearly 73 million acres have been granted free to the peasants for their use. European Russia includes 405 million acres, Asiatic Russia (chiefly Siberia) 300 million acres. The 18,000,000 acres of forest in the Caucasus contain many kinds of valuable timber.

Temperate Hardwoods

Some 18 per cent. of the world's timber production consists of temperate hardwoods. These are more easily worked than the hardwoods of the tropics, but are used for similar purposes. The chief hardwood is oak, used for furniture, panelling, shipbuilding, etc. It does not grow anywhere south of the equator, and the chief sources of supply are Central Europe, Canada, the U.S.A., and Japan. Great Britain grows a type called "brown oak," much in favour in Europe and America for fine furniture and panelling.

Other temperate hardwood timbers are ash, a tough resilient wood used in the manufacture of axe-handles, oars, hockey sticks, the bodywork of carriages and motor cars, etc.; beechwood, for furniture and other indoor uses; elm, very resistant to water and used in the past for water pipes, now chiefly for coffins; hickory, an American wood with uses similar to those of ash, and specially useful for the shafts of golf clubs because, with the toughness and resilience of ash, it combines greater stiffness; maple for floor blocks; bird's-eye maple with a beautiful grain that makes it sought after for furniture; birch, another tough wood valued by carriage builders and furniture makers.

Tropical Hardwoods

Only 2 per cent. of the world's timber requirements comes from the forests of the tropics, but the hardwoods of the tropics include some of the most beautiful and valuable cabinet woods as well as woods of great endurance. The forests of the tropics are far more mixed than those of the temperate zone. Mahogany, native to tropical America, is exported by Cuba, Jamaica, Mexico, British Honduras, and Haiti, which produces the best quality. Some red woods of West Africa from trees having no relation to mahogany are commonly exported as African mahogany. The hard black wood called ebony is the timber of any one of a number of closely related trees native

to Mauritius, India, and Ceylon. Rosewood and cedarwood grow in Central America and the West Indies. The cedarwood of Cuba is used in the manufacture of cigar boxes for Cuba's cigar trade. (The so-called cedar from which pencils are made comes from trees of the juniper family, and is a product of the temperate zone, particularly North America.)

Teak is probably the most valuable economically. Hard and durable as oak, it contains an oil which helps to preserve iron, and is used in shipbuilding, railway-carriage building, and heavy construction generally. Burma, Siam, India, and Java are the chief sources of teak. The redwoods of the eucalyptus trees, peculiar to the south-west of Western Australia, are used for street paving blocks, furniture, etc.; one of them, jarrah, is also valuable for making piles to be sunk in water.

Of the total world production of hard and soft timber at mid-20th century the U.S.S.R. accounted for 40 per cent.; North and Central America 26 per cent.; Europe 14 per cent.; South America 7 per cent.; Asia 6 per cent.; Africa 5 per cent. About 60 per cent. of the total fellings is absorbed by industry; the rest is used as firewood. Sawlogs and veneers are the chief industrial uses, taking more than half of all felled wood. The largest single industrial user is the pulp and paper industry, based on the coniferous forest belt of the northern hemisphere.

LESSON 14

Noble Metals and Precious Stones

The noble metals are so called because they do not lose their lustre when exposed to the air. They are few in number—gold, silver, and the six metals of the platinum group.

Uses of Gold

Gold was probably the first metal observed by prehistoric man; it occurs in nature already exhibiting the yellow colour that is its most noticeable superficial characteristic. Nearly all rocks contain it, though generally in minute quantities. The waters of the ocean carry gold in suspension, in amounts estimated at from ·03 to 1 gram per ton in different areas, and people have endeavoured (without success) to find a profitable way of extracting it from sea water. Gold seems to be present even in vegetation, for it is found in the coal of the Cambrian coalfield of Wyoming and in lignite deposits in Japan.

Pure gold is the most malleable and ductile of all metals, and one of the softest. Its two chief uses are as a medium of exchange, in the forms of coinage and bullion, and for ornament in the forms of jewelry, gilding with gold leaf, porcelain gilding, decoration with gold wire, etc. In use gold is nearly always alloyed with some other metal, to give it sufficient hardness. British standard gold coinage, for instance, was made of 22-carat gold containing 91·67 per cent. of gold with 8·33 per cent. of copper. Standard gold in the U.S.A. contains 10 per cent. of copper. The so-called "white gold" of the jeweller is an alloy of gold and platinum or palladium, containing up to 25 per cent. of the second metal. Gold is otherwise of little practical value, though it is used to a small extent in dentistry, and chloride of gold is used in photography and medicine.

The ancients found gold in Egypt, Asia Minor, and Transylvania. The gold of Peru was the strongest incitement to the Spanish exploitation of that country. Peru still produces gold, but in small quantities. All previous gold production was eclipsed when gold was discovered in California in 1848, and in Australia a few years later. The goldfields of the Rand, in the Transvaal, discovered in the 1880s, put even California and Australia into the shade, and the Rand has remained the most productive area, with no apparent likelihood of exhaustion. The Yukon district of Canada has been eclipsed by the Porcupine and Kirkland Lake goldfields of that country. The U.S.S.R., thought to be second in world gold production, has deposits in the Urals which have been worked for many years, and richer deposits of more recent discovery in Siberia.

Types of Gold

What is known as alluvial gold occurs in alluvial deposits, and it can be recovered by washing the soil. These are the easy fields to work, where a man with luck can pick up a fortune in a short time. The gold of the Yukon was, in the main, of this kind. Reef gold is embedded in quartz and other hard rocks, and can be recovered only with the help of crushing machines. The gold of the Transvaal is of this type, and in some places it is worked to a depth of 7,000 feet. The successful working of the Rand mines has been made possible only by the presence near by of deposits of easily mined coal, and by heavy capital expenditure on the necessary plant.

The total annual production of gold ore (metal content), excluding the U.S.S.R., at mid-20th century was 27,733,000 fine troy ounces, accounted for as follows: South Africa 49 per cent.; U.S.A. 9 per cent.; Australia

4 per cent. ; Gold Coast 3 per cent. ; Southern Rhodesia 2 per cent. ; Mexico 2 per cent. ; Colombia 2 per cent. ; Belgian Congo 1 per cent. ; Philippines 1 per cent. ; others 27 per cent.

Silver

Brilliant white in colour, silver has been known from very early times. It is the most malleable and ductile of the metals except gold. It is sometimes found free, more often in association with other metals. Originally silver was used chiefly for coinage and jewelry ; now most of it is put to industrial use. The pure metal is so soft that it is generally hardened by alloying it with a small amount of copper. The current British " silver " coinage contains nickel and copper but no silver The chief silver production areas are Mexico, the U.S.A., Canada, South America, Australia ; also there are silver mines in Austria, Germany, and Spain.

The Platinum Group

Platinum, palladium, iridium, osmium, rhodium, and ruthenium are generally found, two or more together, in association, and are known as the platinum group of metals. They are of rare occurrence. Platimum itself is a very heavy steel-grey metal, soft when pure, but in use generally alloyed with iridium to harden it. Because of its resistance to heat and acids it is the best material for making crucibles and vessels for certain purposes. Other uses of platinum are as a catalyser in various chemical processes ; for surgical instruments, such as needles for hypodermic syringes ; and for photographic prints, platinum prints being of great beauty and absolutely permanent. About a third of the world's small output of platinum finds its way into jewelry, about a third is wanted for dental purposes, and the remainder is used in the electrical, chemical, and other industries.

Formerly most of the production of platinum was from gravels in the U.S.S.R. ; now the famous nickel ores of Sudbury, in Ontario, Canada, yield much of the world's supply. The total annual world production of platinum in mid-20th century was 623,000 troy oz., of which Canada produced 50 per cent., South Africa 24 per cent. U.S.S.R. 16 per cent., U.S.A. 5 per cent., Colombia 4 per cent.

Palladium is always present to some extent in platinum ores, and is sometimes found in osmiridium (a native alloy of osmium and iridium) or combined with gold. It is also present in appreciable amounts in most nickel ores. It was first isolated as a distinct metal in 1802. On account of its unalterability and comparative lightness it is used as the basis of graduated surfaces in astronomical and other precision instruments ; also for surgical instruments, in dentistry, to make hairsprings for watches, for coating silver goods, and sometimes for depositing on glass to produce permanent mirrors. Iridium, identified in 1804, is found in small amounts in ordinary platinum but is generally obtained from osmiridium. It plays an important part in industry in platinum alloys. A very fine black is produced on china and porcelain with iridium. Osmium, the second partner in the native alloy osmiridium, was isolated at about the same time. It is used for filaments in electric lamps. Rhodium (isolated in 1804) is used for plating silver, to prevent tarnishing. Ruthenium (isolated in 1828) has so far no practical uses.

Precious and Semi-precious Stones

Of the precious stones, diamonds are the most important—and the most useful. Diamond is the hardest material known, and the most imperishable and brilliant of minerals. Diamond powder, made from cloudy or otherwise imperfect stones, and from fragments too small to be cut as gems, is the only material with which the diamond itself can be ground and polished.

Diamonds are found in single crystals, cloudy and faintly coloured stones being more common than colourless ones. India produced diamonds from the earliest times to the close of the 19th century; to-day India's production is negligible. Diamonds were discovered in Brazil in 1725, and they have been an important item of production ever since. But far surpassing any other known deposits are those of Kimberley in South Africa, and of Lüderitz in South-West Africa. Diamonds that are too poor for gems but suitable for industrial use come mainly from the Belgian Congo. They are used for drilling, for finishing accurate turning work, and for fine engraving such as that required on scales. They are also used in cutting and drilling glass and porcelain, for drilling in dentistry, and for rock drilling. Diamonds, sapphires, and rubies are all used for watch bearings.

Emeralds of a clear green were known to the ancients, who obtained them from Upper Egypt. The Peruvians possessed large numbers when the Spaniards reached their country, but the source of these gems was never discovered. Colombia, the Ural mountains, and New South Wales are the chief producers now.

Rubies (red) and sapphires (blue) are transparent coloured varieties of corundum, a very hard material which in its massive impure state is the prosaic emery. Rubies come from Upper Burma ; sapphires come from Ceylon, Siam, Upper Burma, Kashmir, Madagascar, Queensland, Victoria, New South Wales, North Carolina, and Montana. The value of these stones has declined since it has been found possible

to make them artificially in a way that renders them indistinguishable from the natural rubies and sapphires.

Semi-precious stones, such as the opal (Hungary, New South Wales, Queensland, Japan, Mexico, and Nevada), the cat's eye (different types of which come from Ceylon, Bavaria, and Griqualand West), aquamarines (Russia), amethysts (Brazil, Uruguay, and Russia) and turquoises (Persia and Turkestan) are made into ornaments, mostly as jewelry, but have little value and no practical use.

LESSON 15

The Base Metals

The base metals are those which, unlike the noble metals, are affected in greater or lesser degree by contact with the air and moisture. They are the metals of everyday life, from which many useful things are made.

Aluminium

Aluminium is a bright white metal, valuable for its lightness. Although aluminium is not found in a metallic state in nature, in combination it is one of the most widespread of the elements, and it has been estimated to constitute one-thirteenth of the earth's crust. It was first isolated, as a powder, in 1827. To-day it is obtained by electrolysis mainly from bauxite, a clay-like mineral discovered in 1821 at Les Baux, near Arles, in the south of France. Surinam (26 per cent.), British Guiana (21 per cent.), the U.S.A. (16 per cent.), and France (10 per cent.) are the largest producers of bauxite. The U.S.A. is the greatest producer of aluminium. World annual average production of aluminium in mid-20th century was 1,510,000 tons; the U.S.A. produced 43 per cent., Canada 24 per cent., the U.S.S.R. 12 per cent., West Germany and France each 5 per cent.

Main uses of aluminium are in aircraft construction, and in the manufacture of motor-cars and railway carriages. Its light-reflecting quality makes it important as paint, and also as leaf, in which form it has almost entirely replaced silver leaf. It is much used in the electrical industry, and, since aluminium salts are non-poisonous and the metal therefore requires no coating, for cookery utensils. Its resistance to nitric acid makes its valuable in the processes of chemical and explosives manufacture. Owing to the large amount of electric power required in its preparation, aluminium factories are generally sited in situations where water power is readily available: e.g. at Niagara Falls, on the Saguenay river in Canada, in south-east France, in Switzerland, and in Scotland at Kinlochleven.

Antimony

Antimony, a silvery white, brittle metal with a high lustre, is occasionally found in a pure state, but it occurs chiefly as the sulphide, stibnite. Alloyed with lead and tin and a small quantity of copper or zinc, it becomes type metal; alloyed with tin, it is the basis of Britannia metal. Antimonial lead, containing about 12 per cent. of antimony, is used in the manufacture of bullets and other ammunition, and also of storage batteries. Antimony oxide enters into the production of enamels and pigments; the sulphide is used in rubber manufacture. Antimony also has medicinal uses. The antimony used in America and Great Britain is from ores obtained principally from Bolivia and Mexico.

Chromium

Chromium, which is never found free in nature, was isolated in 1859. The chromium compounds used so much in industry are derived principally from the mineral chromite which occurs in chrome ore. It is a steel-white metal, harder than iron, will take a brilliant polish, and does not tarnish readily. These last two qualities have given it prominence in the form of chromium plating, particularly of the exposed metal parts of motor cars; it is also the element which, alloyed with steel, produces stainless steel. It figures in a number of ferrous and non-ferrous alloys used in engineering and industry; alloyed with cobalt, tungsten, and sometimes also molybdenum, it is used for high-speed cutting tools. Turkey and South Africa are the largest producers of chrome ore, followed by Southern Rhodesia, the Philippines, and the U.S.S.R.

Cobalt

Cobalt, a silver white metal, magnetic at all temperatures up to 1,150 C., is closely allied to iron and nickel. It occurs free in nature, but in very small quantities, and it is generally produced as a by-product of copper mining, silver mining, and gold mining. It is used in steel alloys, particularly those required as permanent magnets for telephony, in jet engine parts, etc. It forms about 50 per cent. of alloys from which non-ferrous high-speed cutting tools are made. It is used also in connexion with atomic weapons. Smalt, a permanent pigment of an intense blue, is produced by roasting impure cobalt oxide with quartz and potassium carbonate, and cobalt enters into the

production of a number of pigments used in art and in the china trade. The Belgian Congo, Northern Rhodesia, and French Morocco are chief sources of the ores from which cobalt is extracted.

Copper

Copper is salmon-pink in colour, strong and malleable. It occurs in a pure state, and the largest known deposits are in Michigan near Lake Superior. It is also, and more widely, found in a number of different ores. An excellent conductor of electricity, copper figures largely in all branches of the electrical industry. Other important uses are in connexion with brewers' and distillers' plant, ammunition, ship-building, and plates with which to print textiles. Alloys include bronze (with tin) and brass (with zinc). The total world annual production of copper in mid-20th century was just under 2½ million tons. Producers of copper ore (metal content) were as follows: U.S.A. 32 per cent.; Chile 15 per cent.; Northern Rhodesia 12 per cent.; Canada 10 per cent.; U.S.S.R. 10 per cent.; Belgian Congo 7 per cent.; Mexico, Japan, Yugoslavia each 2 per cent.; South Africa 1 per cent.; others 7 per cent.

Iron

Iron is not found native, but has to be extracted from ores by smelting. The resulting pig-iron is brittle and for most purposes has to be converted into wrought (or malleable) iron by ridding it of the impurities it contains. Cast-iron is pig-iron remelted and cast into moulds. Wrought iron, or purified pig-iron, is converted by the addition of a small proportion of carbon (from ·3 to 22 per cent.) into steel, a tenacious, flexible, elastic, very hard metal. Steel is used in machinery, ships, bridges, railways, buildings, and tools of many kinds. It has virtually replaced wrought iron, and is replacing cast iron in some uses.

Steel Alloys

There has been great development of special steel alloys to meet special purposes. Nickel-steel, containing about 3 per cent. of nickel, is much tougher and stronger than ordinary steel, and is used as armour plating for war-ships. Chrome steel, containing about 2 per cent. of chromium, is very hard and very elastic, and is used to make armour-piercing projectiles. Manganese steel, containing up to 14 per cent. of manganese, is the toughest material known, and is used where the metal has to withstand immense strains as, for instance, in ship- and bridge-building. High-speed steel, containing some 5 per cent. of chromium and some 18 per cent. of tungsten, remains hard at high temperatures, and was for long the best material available for high-speed cutting tools which, owing to friction, develop intense heat in working.

Britain, Pioneer in Iron and Steel

The iron and steel industry developed in Great Britain long before it did elsewhere, owing to the facts that Britain was in the van of the general industrial development that took place in the 19th century, and that Britain's iron was conveniently situated near deposits of the coal required for its working. Great Britain's iron and steel industry is still a very important one, coming fourth in annual value of her industries, but she is now obliged to import vast quantities of iron ore and scrap to meet requirements. In mid-20th century average annual steel production totalled 187 million tons, to which countries contributed as follows: U.S.A. 45 per cent.; U.S.S.R. 16 per cent.; Great Britain 9 per cent.; West Germany 6 per cent.; France 5 per cent.; Japan 3 per cent. The chief exporters of iron ore are Sweden, France (to neighbouring countries only), the U.S.A., Algeria, Chile, Canada (Newfoundland).

Lead

Metallic lead is of rare occurrence in nature; galena is the principal ore from which it is extracted. Pure lead is a bluish-white metal, heavy, of feeble lustre, very soft and plastic and having little elasticity. The development of plumbing and of the domestic use of gas during the latter half of the 19th century necessitated miles of small pipes and led to a great increase in the consumption of lead.

Nearly one-third of the world's production of lead is used for storage batteries, about half that amount for cable covering, and about 10 per cent. of the metal is used as "white lead." Other uses are in buildings, ammunition, solder, and anti-knock petrol. In mid-20th century average annual lead ore (metal content) production was 1,608,000 tons, made up as follows: U.S.A. 23 per cent.; Mexico 14 per cent.; Australia 12 per cent.; U.S.S.R. 11 per cent.; Canada 9 per cent.; Yugoslavia 5 per cent.; Peru 4 per cent.; Morocco 3 per cent.; West Germany 3 per cent.; others 16 per cent.

Manganese and Mercury

Manganese is slightly brown in colour and of considerable hardness. It was first isolated as a separate metal in 1774. About 95 per cent. of the manganese consumed is used in the manufacture of iron and steel. Ores of the metal are widely distributed, but the manganese ores used commercially are produced mainly by the U.S.S.R. (46 per cent.), the Gold Coast (13 per cent.), India (12 per cent.), South Africa (10 per cent). Average annual world production in mid-20th century was 3,013,000 tons.

Mercury, or quicksilver, is unique among metals in being fluid at ordinary temperatures. It is silvery white in colour. Its chief uses are in the electrical industry for mercury-arc rectifiers, automatic switches for refrigerators, oil-burners, and in the columns of thermometers and barometers. It also has an important place in medicine. Average annual production of mercury in mid-20th century was 4,533 tons, of which Spain produced 40 per cent. ; Italy 38 per cent. ; Yugoslavia 10 per cent. ; U.S.A. 5 per cent. ; Mexico 4 per cent.

Molybdenum

Molybdenum is a silver-white metal, malleable, softer than steel. It is not found free in nature but always chemically combined with other elements. It is used in the making of special steels. The U.S.A. is by far the largest producer —91 per cent. of 14,800 tons (mid-20th century annual average). Chile produced 7 per cent.

Nickel

Nickel is a silvery white metal, malleable, and ductile to a high degree. It occurs widely as kupfernickel (arsenide), but the chief sources of commercial nickel are the hydrated silicate of nickel and magnesia found in New Caledonia and the nickeliferous iron pyrites of Sudbury, Canada. It is used for coinage, for domestic utensils and crucible plating, and in some important alloys: e.g. "Monel" (nickel, copper, iron, and manganese) ; German silver ; "Invar" and other nickel steels ; "Constantan" (nickel and copper). The total annual output of nickel in mid-20th century was 148,333 tons, of which Canada (Ontario) produced about 78 per cent., the U.S.S.R. 17 per cent., New Caledonia 4 per cent.

Tin

Tin is a white metal. It occurs free in nature, but the tin of commerce is mostly obtained from tinstone (or cassiterite). It is used in alloys (such as solder, pewter, and bronze), and in tin-plate for the canning industry and domestic pots and pans. The tin of the Cornish mines attracted Phoenician traders three thousand years ago, and Cornish tin was exported to Italy after, if not before, Caesar invaded Britain. For long, Britain was the chief source of supply of this metal. There are immense deposits in Malaya and the islands of Banka and Billiton in Indonesia, and in China and Bolivia.

Tin is also mined in Canada, the U.S.A., Mexico, British Honduras, Australia, New Zealand, Nigeria, and the Belgian Congo. In mid-20th century the average annual production of tin ore was 164,667 tons. Malaya produced 34 per cent. ; Bolivia 20 per cent. ; Indonesia 19 per cent. ; Belgian Congo 8 per cent. ; Siam 6 per cent. ; Nigeria 5 per cent. ; China 2 per cent.

Tungsten

Tungsten (wolfram) is a dull white metal, very heavy ; its name is Swedish for heavy stone. It occurs in nature in the form of tungstates of iron, manganese, and calcium, the first two forming the wolfram group, the third being known as scheelite. It is used for electric-lamp filaments, and in some industrial alloys. Tungsten carbide is the main constituent in cemented carbides, a group of materials used to make high-speed cutting tools of hardness unchanged up to temperatures of 900° C. (compared with 400° C. for high-speed steel). Average annual production of tungsten ore (WO_3 content) in mid-20th century was 18,437 tons (excluding the U.S.S.R.). China produced 39 per cent. ; Portugal 12 per cent. ; the U.S.A. 10 per cent. ; Bolivia 8 per cent. ; Australia 5 per cent. ; Spain 4 per cent. ; Siam 4 per cent. ; Burma 3 per cent.

Zinc

Zinc has a bluish white colour. It does not occur free in nature, the chief ores from which it is extracted being zinc-blende or sphalerite, and zinc-spar or calamine. It was first known in Europe as an import from China and India.

It is used as a constituent in industrial alloys, e.g. brass and solders ; for galvanising iron and other metals to prevent corrosion ; for roofing, batteries, etc. In mid-20th century the average annual world production of zinc ore was 2,032,000 tons. The U.S.A. produced 28 per cent. ; Canada 14 per cent. ; the U.S.S.R. 10 per cent. ; Australia 10 per cent. ; Mexico 9 per cent. ; Poland 4 per cent. ; Peru 4 per cent. ; Italy 4 per cent. ; West Germany 3 per cent. ; Spain 3 per cent. ; Belgian Congo 3 per cent.

LESSON 16

Fuel and Power

About one-fifth of the world's energy supplies are still obtained from primitive sources such as fuel-wood, peat, dung, and agricultural wastes. These domestic fuels are important in Asia, Africa, and parts of Europe. The sources of nearly half the world's commercial energy are coal and lignite ; in Europe this proportion is about 80 per cent.

Coal did not begin to come into fairly general use in Great Britain until the 17th century. The

total annual fuel consumption in 1660 was only about two-fifths of a ton per head of the population. Now it is equivalent to over 4 tons of coal per head (17 tons in the U.S.A., less than ½ ton per head in Asia).

Constitution of Coal

Coal is fossilised vegetable matter which, buried and sealed from the air in long-past ages, has undergone various chemical changes that have reduced the hydrogen and oxygen content, and increased the relative carbon content, of the original material. It varies in type from anthracite (which contains 90–94 per cent. of carbon, compared with little more than 50 per cent. in ordinary wood fibre) to lignite, a brown coal which contains about 70 per cent. of carbon.

The bulk of the world's coal lies in the northern hemisphere, and the U.S.A. has the largest resources. The estimated world reserves of bituminous coal and anthracite total 4,263,000 million tons, made up as follows: U.S.A. 31·6 per cent. ; China 23·3 per cent. ; U.S.S.R. 23·0 per cent. ; South Africa 4·7 per cent. ; Germany (incl. Saar) 4·2 per cent. ; U.K. 4·0 per cent. ; Poland 3·9 per cent. ; India 1·4 per cent. ; Canada 1·1 per cent. ; others 2·8 per cent.

Uses of Coal

Coal is used directly as a fuel, and also for the production of thermal electricity and manufactured gas. Coal, as a fuel for steam-raising, provided the basis of the Industrial Revolution. Coal output rose rapidly in the U.K. during the 19th century with the change-over to factory production and the introduction of railways and steamships, of blast furnaces, gasworks, and electrical installations, and a parallel expansion in the U.S.A. and in western Europe continued into the 20th century. The problem of coal consumption varies throughout the world, but in the U.K. in mid-20th century it was as follows : domestic 30 per cent. ; iron and steel 7 per cent. ; other industry 32 per cent. ; railways 12 per cent. ; collieries 9 per cent. ; others 10 per cent.

By-products of Coal Gas

The production of coal gas by distillation produces important by-products whose number and value are increasing. First of all there is coke, the solid burnable residue. Less bulky but more valuable by-products include tar (from which are derived benzene, naphtha, naphthalene, creosote, pitch, the phenols, dyes, and saccharin), sulphur, ammonia, and vitamin B_1, a by-product from the preparation of sulphate of ammonia. The elements required to produce some types of synthetic rubber and plastics are also derivable from coal.

The world's annual total production of coal in mid-20th century was 1,388,000,000 tons. Of this total the U.S.A. produced 34 per cent. ; the U.K. 16 per cent. ; U.S.S.R. 15 per cent. ; West Germany 8 per cent. ; Poland 6 per cent. ; France 5 per cent. ; Japan 3 per cent.

In general the thickest seams of best-quality coal in Western Europe are nearly exhausted and the coalfields are becoming more difficult to work. Some coal can be obtained by opencast mining, but most coal comes from deeper mines ; the deeper the mine, the more costly are operations like pumping, ventilation, and winding the coal to the surface. The extent of mechanisation—coal cutters, conveyer belts, etc.—varies widely and is greatest in the U.S.A. and in Western Europe.

Petroleum Production

Petroleum is the name given to oils which flow freely, or can be pumped, from the earth. Similar products can be expressed from oil shale and produced by hydrogenation from coal (particularly lignite and other low-grade kinds). Any picture of the world's production of petroleum is likely to remain true for but a short time, as old sources tend to give out and new sources are discovered. But the main source of supply of petroleum has been and remains the U.S.A. The average annual world production of crude petroleum in mid-20th century was 536,000,000 tons, of which the U.S.A. produced 50 per cent. ; Venezuela 15 per cent. ; U.S.S.R. 7 per cent. ; Saudi Arabia 5 per cent. ; Persia 5 per cent. ; Kuwait 4 per cent. ; Mexico 2 per cent. ; Indonesia 1 per cent. ; Iraq 1 per cent. ; others 10 per cent.

Great Britain appears to be almost entirely lacking in petroleum, though intensive search has been made ; but she has certain oil-shale deposits from which a limited amount of oil is extracted. Britain has also developed methods of producing oil from coal ; one method leaves as residue a smokeless fuel called coalite. Germany, which has petroleum wells in Hanover, also produces oil from her immense lignite resources. Consumption is greatest in the U.S.A. and it has outstripped domestic production. Great Britain, the U.S.S.R., Germany, and France are also big consumers, and the use of petrol is increasing all over the world with the increase in motor and air transport. The introduction of the gas turbine, however, has increased the demand for paraffin rather than for the special petrol required for piston-engined aircraft.

Distribution of Petroleum

The crude oil requires refining to a greater or less extent before use. The process of refining produces several oils of varying degree : kerosene for lighting, heavier oils for heating and for diesel engines, still heavier oils for lubrication,

as well as petrol of different degrees of fineness for internal combustion engines ; and by-products which are constituents for artificial rubber; bitumen or asphalt for making roads; paraffin wax for candlemaking ; vaseline and medicinal paraffin oil ; oils used in softening flax and other tough fibres, etc.

Oil Pipe-Lines

The crude oil is pumped from holes bored in the ground and carried by pipe to ports or direct to refineries. Some of these pipe-lines are very long and have played a part in international politics. Russia has pipe-lines from Baku on the Caspian to Batum on the Black Sea ; from Makhach-Kala on the Caspian to Grozny ; from Grozny to Armavir and Tuapse ; from Armavir to Trudovaya ; from Chapayev (Gurev) to Orsk ; and some shorter lines, totalling 2,616 miles.

The Persian oilfields at Maidan-i-Sulaiman are connected with the head of the Persian Gulf, and a line runs from the Mosul wells of Iraq to Tripoli and Banias in Syria. In Rumania, Ploesti is connected with Constanta on the Black Sea and also with Giurgiu island. The longest pipe-line in the world is in the U.S.A., between Longview, Texas, and New York City —a distance of some 1,400 miles. A close second is the Canadian pipe-line from Edmonton, Alberta, to Superior, Wisconsin, which is 1,130 miles long and was laid in about 5 months in 1950. Spectacular increases in crude-oil production have taken place in Alberta since the opening of the Leduc, Redwater, and other fields. The world's reserves of crude petroleum were estimated to be 14,350 million tons on Jan. 1, 1952.

Electricity

Electricity can be generated from coal, oil, and gas, and from the movement of falling or rapidly flowing water. The harnessing of water to produce electric power has given an impetus to industry in countries where lack of coal prevented industrial development during the 19th century ; and the possession of abundant water power has influenced the siting of certain industries (e.g. aluminium smelting). Switzerland, with very little coal, has an abundance of water power, which has been steadily developed since 1894 ; to-day nearly all her factories run on electricity, and so do her 3,250 miles of railway. Norway, not much better provided with coal, has an immense reserve of water power, developed to a greater extent per head of the population than in any other country. Sweden and Japan, too, have little coal but much water power.

But development of water power is not limited to countries with little coal. Canada and the U.S.A. are rapidly developing their natural water power—for instance, from the falls on the Hudson River, the falls of St. Anthony on the Mississippi, Niagara Falls (which serve both Canada and the U.S.A.), and the Spokane falls in Washington. Water power provides over 80 per cent. of the Canadian output of electric power.

France, where the name " white coal " for this source of power originated, has considerable installations at Grenoble and on the Romanche, Drac, and Rhône rivers, depending on the waters coming down from the Alpine heights. Germany also has considerable natural water-power resources. The natural water power of Scotland has been harnessed at Kinlochleven, at Kirkcudbright, and near Fort William ; and so has that of North Wales. England is not well supplied with natural water-power resources. In addition to the utilisation of natural water power, great dams have been built in some parts of the world to create artificially the force required to generate electricity from water power. Examples are given in the following table.

SOME OF THE WORLD'S LARGEST DAMS

Name	Location	Finished	Height (Feet)	Amount of Water Held (Thousand Millions of Gallons)
Hoover (Boulder)	Arizona-Nevada	1936	726	10,543
Fort Peck	Montana	1940	250	6,325
Cauvery-Mettur	India	1934	214	5,978
Grand Coulee	Washington	1941	550	3,160
Shasta	California	1944	602	1,466
Gatun	Panama	1912	115	1,437
Assuan	Nile, Egypt	1933	174	1,322
Hume	Australia	1934	180	651
Kingsley	Nebraska	1941	162	651
Roosevelt	Arizona	1911	280	462
Dnieper	U.S.S.R.	1932	140	291
Chambon	France	1934	443	—
Burrinjuck	Australia	1927	247	125

Cheap hydro-electric power is used in large quantities by aluminium and wood pulp industries, also for the electrification of railways and for heating industrial furnaces. Dependence on the water power of rivers and falls for the generation of electricity is subject to one disadvantage : the possibility of drought, and the consequent reduction in the volume of water flow.

In mid-20th century the annual world production of hydro-electricity was 298,974,000,000 Kwh (kilowatt-hours). The U.S.A. produce 32 per cent. ; Canada 18 per cent. ; Japan 12 per cent. ; Italy 7 per cent. ; Norway and Sweden each 6 per cent. ; France 5 per cent. ; Switzerland 4 per cent.

Of Africa's enormous potential natural water power—41 per cent. of a world total—only 0·4 per cent. has been tapped. Immense reserves of natural water power exist untapped in all the other continents. The amount that could be created by engineering feats appears to be limitless.

Water power, though the cheapest, is not at present the largest total source of applied electrical energy. In mid-20th century the U.S.A., for instance, generated three times as much electricity from coal, oil, and gas as from water power. Lignite and peat supply a great part of Germany's electricity; coal supplies almost all that of England. There is an appreciable loss of energy in using coal to generate electricity instead of using it more directly, but electricity can be easily transmitted over hundreds of miles, thus making possible a much wider dispersal of industry without involving the corresponding transport of coal. Power in the form of electricity can be applied with great smoothness and steadiness; and, used as the agent of propulsion on railways, it gives more rapid acceleration and is much cleaner than steam traction.

Nuclear Fuels

The bombardment of uranium, thorium, or certain other elements, with neutrons is known as nuclear fission. Such nuclear fission causes the nucleus of uranium to "split" with the release of enormous amounts of energy, and the production of chain reaction affecting neighbouring nuclei until the entire mass of fissionable matter is involved. Estimates suggest that the disintegration by nuclear fission of an ounce of uranium or thorium yields energy equivalent to that produced by the burning of more than 100 tons of coal. The atomic bomb represents the application of nuclear fission to destructive purposes, but nuclear fuels can be of great benefit to mankind as a source of power for peaceful ends. The enormous power available from nuclear fission could allow important advances in industry, medicine, and transport, and also projects like vast schemes of irrigation.

BOOK LIST

General. *Chisholm's Handbook of Commercial Geography*, L. D. Stamp and S. C. Gilmour (Longmans); *Fundamentals of Economic Geography*, N. A. Bengtson and W. van Royen (Prentice-Hall); *Economic Geography*, C. F. Jones and G. G. Darkenwald (Macmillan); *World Resources and Industries*, E. W. Zimmermann (Harper); *Economic and Social Geography*, E. Huntington, F. E. Williams, S. van Valkenburg (Chapman-Hall); *Intermediate Commercial Geography*, L. D. Stamp (Longmans); *Oxford Economic Atlas of the World* (Oxford Univ. Press); *Industrial and Commercial Geography*, J. R. Smith and M. O. Phillips (Constable).

British Isles. *An Economic Geography of Great Britain*, W. Smith (Methuen); *The British Isles: a geographic and economic survey* (Longmans, Green); *Economic Geography*, W. S. Thatcher (English University Press).

GERMAN

THIS Course in German can be worked through without a teacher in from four to six months. As in our other Courses on modern languages, the basis of study is the Vocabulary of Essential Words. Used in accordance with the grammar, and supplemented by the methods of German word-building described, this Vocabulary provides a remarkable instrument for the purpose of reading. The reading matter given in the Lessons progresses from simple anecdotes and stories from Grimm to a passage by H. G. Wells and an extract from Einstein's "Relativity." FOREIGN LITERATURE, *German section, in Vol. 4, provides suggestions for further reading.*

Courses on other modern languages are in Vol. 2 (French), Vol. 4 (Spanish, Italian) and Vol. 5 (Russian and Portuguese). The Course on PHILOLOGY *in Vol. 5 is complementary to the study of all languages.*

15 LESSONS

LESSON 1

The Alphabet and Pronunciation

This Course consists of 15 lessons. Some are a little longer than others, but the average time required by the learner without a teacher to master a lesson is about ten days—that is, by devoting at least half an hour daily to study. For self-tuition, one hour daily is advisable. On this basis, the whole Course can be worked through in from four to six months. With a teacher, much more rapid progress can be made ; and a teacher should be found to help with pronunciation.

The chief difficulty of German for the beginner is the rather complicated system of the " mechanics " of the language : those forbidding declensions, the separable and inseparable prefixes, and the order of words in a sentence. While it is necessary on a first perusal of the Course to master the declensions of the articles and pronouns, there is no need to keep grinding away at the declensions of nouns until all the inflexions and irregularities are fully known. Let the learner be content for the time being with a knowledge of the general principles —indeed, this applies to all the grammar.

Vocabulary

This is far more important. German differs from French in that a much wider vocabulary is required to read the average German book than is necessary to read the average French book. And the goal of the learner should be to reach the reading stage as quickly as possible, for, when German can be read with some ease, speaking and understanding the spoken language will quickly follow, providing there are opportunities for hearing and speaking.

Grammar

The grammar provided in this Course is all that is likely to be required, unless the student aims at becoming an expert. The vocabulary of essential and " root " words, together with the rules for word-building, more than cover the requirements of everyday life.

Keep a notebook. Learn something new *every day*. For at least three months, never let a day pass without application to the language, if only by revision of grammar or by learning at least *ten* new words.

Let not the beginner lose heart if he thinks that he is not making sufficient progress. The moment he feels that he is bewildered or in difficulties, let him pause, abandon for the moment the desire to push ahead, and concentrate on the revision of what has been covered. Write down every different rule, and go over it again and again until it is known. Read again and again the German text of the exercises, until it can be read with complete understanding. By following these simple rules, no person of average intelligence can fail to make progress.

While the foregoing suggestions apply specially to this German Course, the student will be well advised to keep in mind the general principles in the Lesson " How to Learn a Language," in Vol. 2, page 666.

The Alphabet

The alphabet is the same as in English, but German may be printed in Gothic or in Latin characters ; and there is a special script for writing. The beginner is advised not to worry with either Gothic type or script during the first weeks of learning. It is surprising how soon one becomes quite at home with the Gothic ; facility in reading comes almost without effort, once a little headway has been made with the language. The handwriting should be avoided until these lessons are well known.

For purposes of reference, here is the Gothic alphabet with the Latin equivalents ; and the names of the letters underneath :

𝔄 𝔞	𝔅 𝔟	ℭ 𝔠	𝔇 𝔡	𝔈 𝔢	𝔉 𝔣	𝔊 𝔤	ℌ 𝔥
A a	B b	C c	D d	E e	F f	G g	H h
ah	*beh*	*tsay*	*day*	*eh*	*eff*	*gay*	*hah*
ℑ 𝔦	𝔍 𝔧	𝔎 𝔨	𝔏 𝔩	𝔐 𝔪	𝔑 𝔫	𝔒 𝔬	𝔓 𝔭
I i	J j	K k	L l	M m	N n	O o	P p
ee	*yot*	*kah*	*ell*	*em*	*en*	*oh*	*pay*
𝔔 𝔮	ℜ 𝔯	𝔖 ſ 𝔰[1]	𝔗 𝔱	𝔘 𝔲	𝔙 𝔳	𝔚 𝔴	
Q q	R r	S s	T t	U u	V v	W w	
koo	*airr*	*ess*	*tay*	*oo*	*fow*	*vay*	
𝔛 𝔵	𝔜 𝔶	ℨ 𝔷					
X x	Y y	Z z					
ix	*ipsilon*	*tsett*					
Ch, 𝔠𝔥	𝔠𝔨	𝔖𝔠𝔥, 𝔰𝔠𝔥	ß	ſſ	tz		
ch	ck	sch	sz[2]	ss	tz		

Modified Vowels : 𝔄̈ 𝔞̈ ; 𝔒̈ 𝔬̈ ; 𝔘̈ 𝔲̈.

[1] 𝔰 is used only at the end of a word or syllable.

[2] ß or ss is always used instead of sz in Latin type.

There are no "accents" as in French, for example, but only the " umlaut " (¨) which is used to " modify " the sounds of the vowels A, O, U and the diphthong AU when written Ä, ä, Ö, ö, Ü, ü, Äu, äu, to be pronounced in the manner now to be described.

Vowel Sounds

A vowel is *long* when it is doubled (*aa, ee, oo*) : Das Boot, *boat* ; when it comes before a single consonant followed by a vowel (eben, *even*) ; or when it is followed by the letter *h* (Der Sohn, *son*). Also before a single final consonant (gut, *good*) ; ß (*sz*) counts as one consonant, even if spelt ss.

A vowel is *short* when it comes before two or

more consonants (besser, *better* ; Licht, *light*). The letter e at the end of a word and in the endings *-el*, *-en*, *-er*, is always short (Katze, *cat* ; Singer, *singer*).

All German vowels are pure, like those in Scots pronunciation of English.

Long **A** is pronounced like English **a** in *father* : Der Vater, *the father*.

Long **Ä** and long **E** are pronounced like **a** in *gate*, or before **r** like e in *fête* : Das Mädchen, *the girl*; Der Esel, *the donkey*.

Long **I** is pronounced like **i** in *machine* : mir, *me*; **i** is always long before *e* (Das Fieber, *fever*).

Long **O** is pronounced like **o** in *no*: Ofen, *stove*.

Long **Ö** is rather like an exaggeration of the vowel sound in the word *earth* ; or exactly like the **eu** in French *le feu*. Das Öl, *oil*.

Long **U** is like the **u** in English *rude*: Blut, *blood*.

Long **Ü** has no equivalent in English, but is like the French **ü** in *sûr* (English **ee** with the lips pursed as for whistling) : Übung, *practice*.

Short **A** is like English **a** in *fat*: Der Mann, *man*. (Or better, like the Scotch *man*.)

Short **Ä** and short **E** are like English **e** in *net*, *bet* : wenn, *when*: Hände, *hands*.

Short **I** is like English **i** in *miss*, *sit*: Das Kind, *the child*.

Short **O** is like English **o** in *not*, *pot*: Das Korn, *corn*.

Short **Ö** is the same sound as long **ö** pronounced more sharply : Die Hölle *hell*.

Short **U** is like English **oo** in *soot*, *foot*: Null, *naught* ; Das Pfund, *pound*.

Short **Ü** is the same sound as long **ü** pronounced more sharply : Der Müller, *the miller*.

NOTE.—The letter **Y** is a vowel which follows exactly the pronunciation of **I**.

Diphthongs

AI, **AY**, **EI**, **EY** are almost like English **ei** in height : Der Kaiser, *the kaiser*; Bayern, *Bavaria*; kein, *no* ; Meyer, a surname ; Reich, *rich* (or *Empire*).

AU is like English **ou** in *house* (but broader, like ah-oo); Das Haus, *house*.

ÄU and **EU** are like **oy** in *boy* : neu, *new*; Die Häuser, *houses*.

IE strictly is not a diphthong but follows the sound of long **I**.

Consonants

Unless mentioned below, these are pronounced as in English.

B at the end of a word is clipped sharply almost to resemble **p**: Der Dieb, *thief*.

ck is like *k*.

CH has no equivalent in English but, after **a**, **o**, **u**, **au** is a guttural like Scots **ch** in *loch*; Das Buch, *book* ; Auch, *also*. Otherwise it is softer and a palate sound : ich, *I*; Mädchen, *girl*.

D at the end of a word is clipped sharply like **t** : Pfund, *pound*.

G is always hard as in *go*, *get*; gehen, *to go*.

-NG is like the English **-ng** in *sing*; Der Ring, *the ring*.

-NGER is always like **-nger** in English *singer*, and never like *-nger* in English *finger* (which is *fing-ger*). The German word **Finger** rhymes with the English *singer* and *springer*.

H is mute after any vowel, otherwise it is strongly voiced : hundert, *hundred*.

J is like English **y** : Der Jäger, *huntsman*; jung, *young*.

K is pronounced before *n* in words like : Der Knabe, *boy*; Das Knie (Pr. *k-nee*), *knee*.

PF, both letters are sounded.

QU is like English **kv**; Die Quelle, *the well*.

R is quite unlike English **r**. It is always trilled or vibrated.

S before and between vowels is like English **z**: Die Rose, *rose*; Der Sohn, *son*.

-s final is always like English **ss** : Das Haus, *house*.

-sz final (in Gothic printed ß, in Latin ß) always like **ss** : Der Kuß, *the kiss*.

SCH is like English **sh**: Schmidt, *Smith* (*-dt* final like **t**).

SP and **ST** at the beginning of a word are like English **shp**, **sht**: sprechen, *to speak*; stehen, *to stand*.

-TI in **-TION** is like English **tsee**: Nation, *nation* (Pr. *Nahtseeohn*).

V is like English **f**: Der Vater, *the father*.

W is like English: **v** Der Wein, *wine*; Das Wasser, *water*.

Z is like **ts** in *rats*, cats: schwarz, *black* (Pr. *shvarts*) ; **tz** (ß) is similarly pronounced. Die Katze, *the cat*.

Orthography

All nouns are written with a capital letter, whatever their position in a sentence : **Singt der Mann ?** *Does the man sing?*

All adjectives, even those of nationality, are written with small letters. **Es ist deutsch.** *It is German.*

Except at the beginning of a sentence **ich**, *I*, is written with a small letter.

Sie and **Ihr**, meaning *you* and *your*, are written with capitals, to distinguish them from **sie**, **ihr**, *they*, *their* (or *she*, *her*).

Every German word is written as it is pronounced and pronounced as written.

The rules of punctuation are the same as in English.

Accentuation

One syllable of every German word is stressed more heavily than the others. This is the "*root*" syllable and, as a general rule, it is the *first*. In compound words the stress is usually on the first component. This accentuation must be learnt from a teacher or by hearing.

German pronunciation is more definite, more vigorous, more distinct than English. Syllables are clear-cut. There is no mumbling, slurring, or drawling such as we have in "Oxford" English. To English people Germans make what appears to be an exaggerated use of tongue, lips and vocal apparatus generally.

NOTE. *It should be clearly understood that the equivalents for pronunciation given above are makeshifts.* Strictly, every letter or combination of letters should be considered to represent a sound or sounds which have no exact equivalents in English. Good pronunciation can be learnt only from good native speakers. Gramophone records are helpful. German wireless broadcasts provide opportunities for learning the

sounds of the language. Take comfort in the fact that few foreigners pronounce German so well as to pass for natives. The goal should be to understand and be understood.

Declension

German is what grammarians call a synthetic language. By this they mean that it prefers to make compound and derivative words and to use endings and inflexions instead of making a liberal use of particles as the "analytic" languages do—English, for example. As will be seen later, articles, nouns, adjectives, and pronouns change their endings to convey different meanings, and this process is called *Declension*; it hardly exists in English.

Articles, nouns, adjectives, and pronouns have four "cases" in the singular and the same in the plural. All except nouns have changes for the three genders (masculine, feminine, and neuter).

The four cases are: Nominative, Genitive, Dative, and Accusative.

1. The *Nominative* names the *Subject*, or initiator of action, speech, etc.

2. The *Genitive* indicates the *Possessor* of something.

3. The *Dative* indicates the *Recipient* of something—the Indirect Object of action or speech.

4. The *Accusative* is used for *Direct Object*, the person or thing which directly receives the action.

THUS: *I gave John's hat to Tom.*

I is **Nominative.** Answers the question: *Who?*
John's is **Genitive.** Answers the question: *Whose?*
Tom is **Dative.** Answers the question: *To Whom?*
hat is **Accusative.** Answers the question: *Whom, What?*

It is impossible to make progress in German until the simple use of the cases is understood. It is pardonable to make mistakes in the declensions of nouns (many Germans trip up on them) but the remainder—articles, adjectives, pronouns—need not cause great trouble. It is a question of **repetition** until they are mastered. In the pages which follow, the minimum necessary to express the most frequently recurring ideas will be given. It should be regarded as a basis upon which to build.

LESSON 2

The Declension of Articles and Nouns

THE words *the* and *a* are called articles, the former the definite, the latter the indefinite article. The former is declined in German, in both singular and plural; the latter, by its nature, only in the singular. The **definite article** is declined thus:

	Singular Masc.	Singular Fem.	Singular Neuter	Plural all genders	English equivalent for all
Nom.	DER	DIE	DAS	DIE	*the*
Gen.	DES	DER	DES	DER	*of the*
Dat.	DEM	DER	DEM	DEN	*to the*
Acc.	DEN	DIE	DAS	DIE	*the*

The following words* are similarly declined: **DIESER**, *this, these*; **JENER**, *that, those*; **WELCHER**, *which, what*; **SOLCHER**, *such, such a*; **JEDER**, *each, each one*; **MANCHER**, *many a*. **ALLE**, *all, every* (as in "alle Welt" pl.).

For example:

	Masc.	Fem.	Neuter	Plural	English
N.	DIESER	DIESE	DIESES	DIESE	*this, these*
G.	DIESES	DIESER	DIESES	DIESER	*of this, these*
D.	DIESEM	DIESER	DIESEM	DIESEN	*to this, these*
A.	DIESEN	DIESE	DIESES	DIESE	*this, these*

* *See also* p. 1330, Demonstrative Pronouns.

Note carefully:

	N.	*G.*	*D.*	*A.*
Masculine endings	-ER,	-ES,	-EM,	-EN
Feminine "	-E,	-ER,	-ER,	-E
Neuter "	-ES,	-ES,	-EM,	-ES
Plural "	-E,	-ER,	-EN,	-E

These should be noted because they are the endings used not only in the words given in the previous paragraph, but in the "Strong" declension of *all* adjectives.

The **indefinite article** is declined thus:

	Masc.	Fem.	Neuter	English
N.	EIN	EINE	EIN	*a*
G.	EINES	EINER	EINES	*of a*
D.	EINEM	EINER	EINEM	*to a*
A.	EINEN	EINE	EIN	*a*

Certain words are declined like the indefinite article in the singular and like the definite article in the plural: **MEIN**, *my*; **DEIN**, *thy*; **SEIN**, *his*; **KEIN**, *no, not a*.

For example:

	Masc.	Fem.	Neuter	Plural	English
N.	MEIN	MEINE	MEIN	MEINE	*my*
G.	MEINES	MEINER	MEINES	MEINER	*of my*
D.	MEINEM	MEINER	MEINEM	MEINEN	*to my*
A.	MEINEN	MEINE	MEIN	MEINE	*my*

Different Usage of Articles in German and English

As a general working rule the beginner should assume that the use of the article in German corresponds to that in English. But note these exceptions. *In German use the article before:*

Days of the week.
Months of the year.
Names of seasons.
Names of countries when preceded by an adjective.

For example: **Des Sonntags**, *on Sunday*; **Im (in dem) Winter**, *in winter*; **Im Dezember**, *in December*; **Das schöne Deutschland**, *beautiful Germany*.

The article is omitted: after **ALLE** (*all*) and **BEIDE** (*both*). **Alle Fische**, *all the fishes*. **Beide Knaben**, *both the boys*. Also when speaking of rank or profession: **Er ist Hauptmann geworden**, *He has become a captain*. **Er ist Matrose, Arzt**, *He is a sailor, a doctor*.

Contractions of the Article. With the following prepositions the definite article is contracted:

AN, *on*; **ZU**, *to, toward*; **BEI**, *by or near*; **DURCH**, *through*; **FÜR**, *for*; **IN**, *in*; **ÜBER**, *over*; **UM**, *round*. Thus: **AN DEM**, becomes **AM**: **AN DAS** becomes **ANS**: **BEI DEM** becomes **BEIM**. And so on: **DURCHS, FÜRS, IM, INS, ÜBERS, UMS, ZUM.** Note the feminine **ZUR** for **ZU DER.** All the others are masculine and neuter, and dative and accusative singulars.

Note the following idiomatic usages: **Einmal am Tage**, *once a day*; **Drei Mark das Pfund**, *three marks a pound*; **Eine halbe Stunde**, *half an hour*; **Gefahr laufen**, *to run a risk*.

EXERCISE ON ARTICLES

Different Usage of Article in German and English

Der **Dezember war kalt.**
December was cold.
Zwei Kinder spielten im Garten des Hauses in *der* **Königsstrasse.**
Two children played in the garden of the house in King Street.
Ihr Vater war Gärtner.
Their father was a gardener.
Sie fanden einen Vogel.
They found a bird.
Er schien zu sagen: gebt mir Brot.
It seemed to say: give me some bread.
Beide Kinder fütterten ihn.
Both the children fed him.
Der Vogel kam alle Tage wieder und wartete auf *das* **Frühstück.**
The bird came again every day, and waited for breakfast
Auch *an* **Sonntagen.**
Also on Sundays.
Aber als *der* **Frühling kam, flog er davon.**
But when spring came, he flew away.
Die **Schule ist aus.**
School is over.
Ist *das* **Frühstück fertig?**
Is breakfast ready?

CONTRACTIONS OF THE ARTICLE

Im **Sommer gehen die Menschen** *aufs* **Land oder** *ans* **Meer.**
In summer people go to the country or to the sea.
Einmal *am* **Tage schwimmen die Kinder** *im* **See.**
Once a day the children swim in the lake.
Eine halbe Stunde ist genug für sie.
Half an hour is enough for them.
Dann laufen sie *übers* **Gras zum Dorf.**
Then they run over the grass to the village.
Sie kaufen Butter *fürs* **Frühstück** *beim* **Bauern.**
They buy butter for breakfast from the farmer.
Die Butter kostet eine Mark das Pfund.
The butter costs one mark a pound.
Am **Abend gehen sie** *ins* **Haus zurück.**
In the evening they go back to the house.
Zum **Abendessen gibt es frische Milch.**
There is fresh milk for the evening meal.
Die Kinder gehen früh *ins* **Bett.**
The children go early to bed.

Nouns

A Noun is a word used for naming some person or thing.

Here we are concerned with the genders, inflexions and declensions of nouns. In the following Lesson we give classified lists of essential nouns, providing a considerable and practical vocabulary.

Gender. There are three genders in German: masculine, feminine, and neuter, but they do not follow the simple English rules of gender, as will be seen.

General Rule.—Nouns denoting males are masculine, and those denoting females are feminine.

But note: **Das Weib**, *the woman*; **Die Waise**, *the orphan* (male or female); **Die Schildwache**, *the sentinel*.

The feminine of many male nouns is formed by adding **-IN** to the masculine: **Der König**, *the king*; **Die Königin**, *the queen*.

Hints Worth Noting and for Reference

MASCULINE ARE: Names of seasons, months, days of the week, winds, points of the compass. **Der Stein**, *the stone*, and all other stones; nouns ending in **-M, -LING, -ICH, -IG**: nouns derived from verbs and ending **-ER** and **-EL** (**Der Flügel**, *wing*: from **fliegen**, *to fly*); nouns of one syllable formed from roots of verbs: **Der Biss**, *bite*: from **beissen**, *to bite*. Exceptions are those ending in **-T, Die Tat**, *deed*.

NOTE.—**Der Mond**, *the moon*.

FEMININE ARE: Names of most trees, flowers, small animals, insects, etc.: (**Die Maus**, *the mouse*; **Die Ratte**, *the rat*; **Die Mücke**, *the midge*); nouns ending in **-EI, -IN, -HEIT, -KEIT**, and **-SCHAFT, -UNG, -IE, -IK, -ION, -TÄT** (the last four often of foreign origin); nouns ending in **-E** derived from adjectives or verbs: (**Die Länge**, *the length*: from **lang**, *long*; **Die Höhe**, *height*: from **hoch**, *high*); all nouns ending **-T** derived from verbs: (**Die Macht**, *might*: from **mögen**, *to be able*; **Die Schrift**, *writing*: from **schreiben**, *to write*.)

NOTE.—**Die Sonne**, *the sun*.

NEUTER ARE: Names of countries and places (**Deutschland, Berlin**), except **Die Schweiz**, *Switzerland*; names of metals (**Das Gold**); all verbs used as nouns (**Das Singen**, *singing*); most nouns ending in **-TUM, -SAL, -SEL**; but note **Die Mühsal**, *trouble*; all diminutives in **-CHEN** and **-LEIN**: (**Das Mädchen**, *the girl*).

Compound Nouns. These take the gender of the last component (**Die Tasche**, *the pocket*; **Das Tuch**, *cloth, rag*; **Das Taschentuch**, *the pocket-handkerchief*). Remember, however, that those compounds of **Der Mut**, *the spirit*, which denote the kindlier and gentler qualities are *feminine*: **Die Sanftmut**, *gentleness*; **Die Anmut**, *grace*; **Die Grossmut**, *generosity*. But the viler qualities remain *masculine*: **Der Kleinmut**, *cowardice*; **Der Unmut, Der Missmut**, *ill-temper*; **Der Hochmut, Der Übermut**, *haughtiness*.

It is best to memorise the gender of each noun as it is met. What is given above is merely a series of rough pointers to the gender of German nouns. **Declension often depends upon gender,** hence the latter is important.

Declension of Nouns. The beginner should take this gently, and avoid mental indigestion, which will inevitably trouble him if he attempts to memorise too quickly the pages immediately

following. Let him read them through a few times in order to grasp the general principles of the German declensions: and return to them later, mastering them a little at a time. He should, however, refer to them continually while learning the list of essential nouns in pages 1323–1325.

Ten Golden Rules for Nouns

1. In the plural of *all* nouns *all* cases are the same, excepting the dative which invariably ends in **-N**.
2. *All* cases of the singular of feminine nouns are the same.
3. Most *monosyllables* with the vowels **a, o, u** and the diphthong **au,** modify them into **ä, ö, ü, äu,** in the plural.
4. The genitive singular of *all* neuter nouns, and of most masculines, excepting those ending in **-E**, ends in **-ES**, or **-S**.
5. The accusative singular of *all* neuters and of most masculines except those ending in **-E** is the same as the nominative.
6. Masculine nouns with nominative ending in **-E** add **-N** for *all* other inflexions, singular and plural.
7. Masculine and neuter nouns ending in **-EL, -ER, -EN** and the diminutive endings **-CHEN** and **-LEIN** add **-S** in the genitive singular and the first two add **-N** in the dative plural. They have no other inflections, but often modify **a, o, u, au** into **ä, ö, ü, äu**, in the plural.
8. Foreign nouns usually follow the rules for pure German nouns, but those not naturalised are irregular in that they follow the rules of their own language.
9. In *all* compound nouns only the final component is declined.
10. Proper names add **-S** in the genitive singular, otherwise do not change.

These rules are "golden," because the exceptions to them are comparatively rare.

Model for the First Declension

	Singular	*Plural*	*English*
N.	**DER DIENER**	**DIE DIENER**	*the male-servant, -s*
G.	**DES DIENERS**	**DER DIENER**	*of the male-servant, -s*
D.	**DEM DIENER**	**DEN DIENERN**	*to the male-servant, -s*
A.	**DEN DIENER**	**DIE DIENER**	*the male-servant, -s*

Like **DIENER** are declined:

(i) All masculine and neuter nouns ending **-EL, -EN, -ER** and the diminutives in **-CHEN** and **-LEIN.**

(ii) All collective nouns beginning **GE-** and ending **-E** (**Das Gebirge,** *the mountains*).

(iii) The words **DIE MUTTER** and **DIE TOCHTER** (*mother, daughter*), and **DER KÄSE,** *cheese.* (Plural, **Die Mütter, Die Töchter**; no changes in the singular.)

The following essentially masculine nouns follow this declension, but modify in the plural:

APFEL, *apple*	**NAGEL,** *nail*
BRUDER, *brother*	**OFEN,** *stove*
GARTEN, *garden*	**SCHADEN,** *damage*
HAFEN, *harbour*	**SCHWAGER,** *brother-in-law*
HAMMER, *hammer*	**VATER,** *father*
LADEN, *shop, store*	**VOGEL,** *bird.*

There is another small group of words following this declension in the plural without modifying, and in the singular following the word **Name,** name, as:

1a. Sing.: **DER NAME, DES NAMENS, DEM NAMEN, DEN NAMEN.**
Plural: **Die, Der, Den, Die NAMEN**

Such are:

DER BUCHSTABE, *letter of the alphabet*; **DER FRIEDE** or **FRIEDEN,** *peace*; **DER FUNKE(N),** *spark*; **DER GEDANKE,** *thought*; **DER GLAUBE,** *faith*; **DER HAUFE(N),** *heap*; **DER SAME,** *seed*; **DER WILLE,** *will.*

Note.—**Das HERZ,** *heart,* **Des HERZENS, Dem HERZEN, Das HERZ.** (Plural, **Die HERZEN.**)

NOTE.—These are exceptions to Golden Rule No. 6.

Model for the Second Declension

Sing.: **DER SOHN, DES SOHNES, DEM SOHN(E), DEN SOHN,** *the son.*

Plural: **DIE SÖHNE, DER SÖHNE, DEN SÖHNEN, DIE SÖHNE.**

For the genitive of monosyllables and words ending in sibilants (**-s, -z, -sch, -st**) add **-ES.** Otherwise only **-S** is added.

Like **DER SOHN** are declined:

All masculine nouns of one syllable, except those mentioned below (third and fourth declension)

Many neuter monosyllables ending **-R** and **-L**.

Most masculine nouns ending in **-IG, -ICH, -ING, -LING, -AT** (unaccented).

Foreign masculine nouns ending **-AL, -AT, -AN, -AR, -AST, -IER.** (**Der General,** *general.* Plural, **Die Generäle,** etc.)

All neuters ending **-NIS, -SAL.**

Nouns beginning **GE-**: **Das Gefühl,** *feeling.*

Most masculine monosyllables modify the vowel in the plural. Most *feminine monosyllables* take their plural according to this declension, and modify their root vowel: **Die Hand, Die Hände,** but are unchanged throughout singular. (Golden Rule No. 2.)

Model for the Third Declension

Sing.: **DAS KIND, DES KINDES, DEM KINDE, DAS KIND.**

Plural: **DIE KINDER, DER KINDER, DEN KINDERN, DIE KINDER.**

This declension includes most neuters, especially monosyllables not hitherto mentioned. Also *all* nouns ending **-TUM,** and some masculines of one syllable: **GEIST,** *spirit*; **GOTT,** *god*; **LEIB,** *body*; **MANN,** *man*; **WALD,** *forest*; **WURM,** *worm.*

All nouns belonging to this declension modify vowel **a, o, u, au** in the plural. **DAS HAUS, DIE HÄUSER,** *house, houses*; **DER IRRTUM, DIE IRRTÜMER,** *error, -s.*

Model for the Fourth Declension

Sing.: **DER MENSCH, DES MENSCHEN, DEM MENSCHEN, DEN MENSCHEN,** *man, mankind.*

Plural: **DIE, DER, DEN, DIE MENSCHEN.**

Like **MENSCH** are declined:

Most masculines ending in **-E.**

All feminines of more than one syllable take the *plural* from this declension, but remain unchanged throughout the singular. This also applies to foreign nouns ending in **-ANT, -AT, -ENT, -GRAPH, -IE, -IK, -ION, -TÄT.** Also **Der Bär,** *bear*; **Graf,** *count*; **Held,** *hero*; **Hirt,** *shepherd*; **Fürst,** *prince.*

Referring back to p. 1319, it will be remembered that **-EI, -HEIT, -KEIT, -SCHAFT, -UNG, -IN, -IE, -IK, -ION** and **-TÄT** are feminine endings, and remembering "Golden Rule No. 2" (in left col.) that feminines remain *unchanged* throughout the singular, we may generalise as follows:

The majority of polysyllabic feminine nouns add **-EN** in the plural, but monosyllables add **-E, -E, -EN, -E** and modify; and that is the end so far as feminine declension is concerned.

NOTE.—Feminines ending in **-IN** double the N and add **-EN** to it in the plural: **Die Fürstin,** *the princess.* Plural, **Die Fürstinnen.**

The word **DER HERR** is so important (meaning *master, gentleman,* and *Mr.*) that it stands alone: **DER HERR, DES HERRN, DEM HERRN, DEN HERRN.** Plural, **DIE HERREN,** etc. The letter **E** is added in the plural to distinguish it from the singular.

Model for the Fifth Declension

Sing.: **DER DOKTOR, DES DOKTORS, DEM DOKTOR, DEN DOKTOR,** *the doctor.*

Plural: **DIE DOKTOREN,** etc.

A small group of masculine nouns follows this "mixed" declension: **BAUER,** *peasant*; **DORN,** *thorn*; **MUSKEL,** *muscle*; **NACHBAR,** *neighbour*; Foreign words in **-OR**: **PROFESSOR,** *professor*; **SCHMERZ,** *pain*; **SEE,** *lake*; **STAAT,** *state*; **VETTER,** *cousin*; **ZINS,** *interest.*

And the following neuters: **DAS AUGE,** *eye*; **BETT,** *bed*; **ENDE,** *end*; **HEMD,** *shirt*; **LEID,** *grief*; **OHR,** *ear.*

Summary of the Inflexions of Nouns

DECLENSION

		I	Ia	II	III	IV	V
Sing.	*N.*	—	—	—	—	—	—
	G.	-S	-NS	-(e)S	-eS	-eN	-(e)S
	D.	—	-N	-(e)	-(e)	-eN	—
	A.	—	-N	—	—	-eN	—
Plural	*N.*	*	-N	-E*	-ER*	-eN	-eN
	G.		-N	-E	-ER	-eN	-eN
	D.	-N	-N	-EN	-ERN	-eN	-eN
	A.		-N	-EN	-ER	-eN	-eN

* With or without modifications, through plural.

Decline like

I. Masculines and neuters ending **-EL, -EN, -ER.** Also diminutives in **-CHEN** and **-LEIN.**

Ia. The group of nouns like **NAME.**

II. Most masculine and feminine mono-syllables, but the latter take the plural endings only and always modify.

III. Most neuters, especially monosyllables; all **-TUM** nouns.

IV. Masculines ending **-E.** All feminines not of one syllable (invariable in singular).

V. The group like **DOKTOR.**

Know the ten golden rules and the above summary before proceeding to learn the essential nouns in Lesson 3.

EXERCISE ON NOUNS

Gender

Insert correct article, applying rules.

D —— Hund und d—— Katze spielen zusammen.
The dog and the cat play together.
D—— Mücke und d—— Vogel fliegen durch d—— Luft.
The midge and the bird fly through the air.
D—— Nebel hindert d—— Sicht (from sehen).
The fog obstructs the view.
D—— Raum ist groß.
The room is large.
D—— Traum ist schön.
The dream is beautiful.
D—— Baum ist hoch.
The tree is high.
D—— November ist kalt.
November is cold.
D—— Frühling kommt.
Spring comes.
E—— Apfel hängt am Baum.
An apple hangs on the tree.
E—— Eiche steht im Wald.
An oak stands in the wood.
D—— Rose ist eine Blume.
The rose is a flower.
Ich messe d—— Größe und d—— Höhe.
I measure size and height.
D—— Krone ist golden.
The crown is golden.
D—— Sonne und d—— Mond scheinen auf die Menschheit herab.
The sun and the moon shine upon mankind.
D—— Rechnung für d—— Zeitung kommt morgen.
The bill for the newspaper comes to-morrow.
D——Gebirge enthält Silber.
The mountain contains silver.
D—— Gewicht ist schwer.
The weight is heavy.
D—— Schicksal ist günstig.
Fate is favourable.
D—— Geschäft geht gut.
Business goes on well.
D—— Sänger und d—— Sängerin singen.
The singers sing.
D—— Mädchen spricht.
The girl speaks.
D—— Weib schläft.
The woman sleeps.
D—— Wand ist weiß.
The wall is white.
D—— Zimmer ist groß.
The room is large.
D—— Zimmerwand ist weiß.
The wall of the room is white.
D—— Anmut der Königin ist berühmt.
The grace of the Queen is famous.

Declension of Nouns

Golden Rules 1, 2, 3, page 1320

Er ist mein Gast.
He is my guest.
Wir haben Gäste.
We have guests.
Der Diener bringt den Gästen Tee.
The servant brings tea for the guests.
Sie trinken den Tee im Garten.
They drink their tea in the garden.
Im Garten gibt es Blumen und Bäume.
There are trees and flowers in the garden.
Die Äste der Bäume tragen Früchte.
The branches of the trees bear fruits.
Es sind Äpfel.
They are apples.
Die Gäste essen die Äpfel.
The guests eat the apples.

Golden Rules 4, 5, 6, *page* 1320

Im Garten des Königs steht ein Turm.
In the garden of the king stands a tower.
Auf dem Turm lebt ein Vogel.
On the tower lives a bird.
Der König hat einen Sohn.
The king has a son.
Der Hund des Sohnes ist der Feind des Vogels.
The son's dog is the enemy of the bird.
Der Hund jagt den Vogel.
The dog chases the bird.
Der Vogel flieht in einen Busch.
The bird escapes into a bush.
Der Hund trinkt das Wasser des Baches.
The dog drinks the water of the brook.
Der Ruhm des Riesen ist groß.
The fame of the giant is great.

Wir wollen Frieden.
We want peace.
Ich gab dem Boten einen Brief.
I gave the messenger a letter.

Golden Rule 7, *page* 1320

Das Alter des Mädchens ist bekannt.
The age of the girl is known.
Die Mäntel hängen an den Nägeln.
The coats are suspended from the nails.
Die Hämmer sind aus Eisen.
The hammers are iron.
Ich gab die Schüssel den Brüdern.
I gave the keys to the brothers.

Golden Rules 9, 10, *page* 1320

Karls Taschentuch ist verloren.
Charles' handkerchief has been lost.
Die Farbe des Taschentuches war weiß.
The colour of the handkerchief was white.
Ich gebe Karl das Tuch zurück.
I give the cloth back to Charles.
Ich traf Otto und Anna.
I met Otto and Anna.

Now try to work out which case each of these nouns represents.

First Declension

Der Regen schadet den Äpfeln im Garten meines Bruders.
The rain damages the apples in my brother's garden.
Im Laden gibt es Hämmer und Nägel.
In the shop are hammers and nails.
Sie liegen auf einem Haufen.
They lie in a heap.
Der Ofen hat Schaden erlitten.
The stove has suffered damage.
Der Glaube erfüllt die Herzen.
Faith fills the hearts.
Wir bewundern die Funken des Feuers.
We admire the sparks of the fire.

Second Declension

Die Hunde bellten im Hof.
The dogs barked in the courtyard.
Der Arzt untersuchte Arme und Beine, Hände und Füße und den Hals meines Freundes.
The doctor examined the arms, legs, hands, feet, and neck of my friend.
Es gab Brot, Fische und Bier zum Essen.
There was bread, fish, and beer for dinner
Er sammelte Vorräte.
He gathered stocks (supplies).
Das Schicksal der Generäle ist unbekannt
The fate of the generals is unknown.
Der König sah das Ereignis.
The king saw the event.

Third Declension

Geister leben im Wald.
Ghosts live in the wood.
Die Kinder kauften Kleider, Bücher und Bilder.
The children bought dresses, books, and pictures.
Die Männer sangen ein Lied.
The men sang a song.
Die Dächer des Dorfes sind rot.
The roofs of the village are red.
Der Wurm gräbt ein Loch.
The worm digs a hole.

Fourth Declension

Die Frau des Grafen ist Gräfin.
The wife of the count is a countess.
Der Held erzählte der Dame seine Geschichte
The hero told his story to the lady.
Die Uhren an den Wänden schlagen die Stunden.
The clocks on the walls beat the hours.
Alle Parteien haben ihre Zeitungen.
All parties have their newspapers.
Die Fabriken brauchen Baumwolle.
The factories need cotton.

Fifth Declension

Ich habe Schmerzen im Ohr.
I have pains in my ear.
Ich zeige mein Auge dem Professor.
I show my eye to the professor.
Der Bauer wohnt am See.
The peasant lives by the lake.
Ich gab dem Nachbar die Hemden.
I gave the shirts to the neighbour.
Der Staat zahlt Zinsen.
The state pays interest.

Now form the singular of each noun where the plural is given, and vice versa.

LESSON 3

The Essential Vocabulary of Nouns

For practical purposes fewer than 750 words which differ in German and English have to be memorised. In this Course the essential vocabulary consists of 235 nouns similar in both languages and 322 dissimilar; 85 similar verbs and 220 dissimilar; 58 similar adjectives and 140 dissimilar. There are also 130 invariable words. Used in accordance with the grammar this will give a working vocabulary of several thousand words.

But these German words have this additional merit: they are mostly "root" words which, in accordance with the methods of word building to be described are used to form derivatives and compounds. Thus an incalculable vocabulary for reading will soon be brought within the scope of the student who masters the lists given in these Lessons.

The similar essential words can be assimilated very rapidly. The dissimilar words can be memorised at the rate of about 15–30 per hour. After a few hours' practice, when the student has become fairly familiar with the language, 40 or 50 can be learnt in an hour. In view of the remarkable capacity of German for forming derivatives and compounds, the mastery of such a "key" vocabulary is much more than half the battle in learning the language.

ESSENTIAL NOUNS OR NAMES OF THINGS

Section 1. Nouns which are alike or almost alike in German and English.

Section 2. Nouns which differ in the two languages.

Section 3. Days of the week, months, and seasons.

Section 4 Proper nouns.

NOTE. Hundreds of nouns are the same in German and English, or almost identical in sound or spelling. They can be recognized immediately in reading or when heard. Only those considered to be essential are given in the pages which follow, and they are given because they must be learnt. For it is of the utmost importance that the German words necessary to express the most frequently recurring ideas should be on the tip of the tongue. It is one thing to recognize **Fisch** as the equivalent of *Fish* : but it is quite another thing to be able to think of **Fisch** when the idea of *Fish* has to be expressed.

Before proceeding to Section 2 the list of similar nouns should be mastered. Then Section 2 will be found to be easier. And so on.

As each word is being memorised an image of the thing or an association of the idea should be in the mind. This not only makes the learning process more interesting, and therefore easier, but it accustoms the learner to think in German.

Thus, when **Das Meer** is being memorised, think of *the sea ;* and imagine that you are bathing in it on a pleasant summer's day. So on through the lists.

Know all the words **both ways** : **Die Zukunft,** *the future;* *the future,* **Die Zukunft.** Know them by **sight** and by **sound** : and know how to pronounce without hesitation.

Section 1. NOUNS ALIKE OR ALMOST ALIKE IN GERMAN AND ENGLISH

NOTE. When learning the nouns read them always with the appropriate **Der, Die, Das.** Thus: **Der Acker, Die Asche, Das Bett,** etc.

(a) MASCULINE NOUNS

Acker, *acre, field*
Apfel, *apple*
Arm, *arm*

Ball, *ball*
Berg, *berg, hill, mount*
Bruder, *brother*
Bund, *bundle, league*
Bürger, *burgher, citizen*
Busch, *bush*

Charakter, *character*

Dieb, *thief*
Doktor, *doctor*
Donner, *thunder*
Dorn, *thorn*
Durst, *thirst*

Engel, *angel*

Finger, *finger*
Fisch, *fish*
Freund, *friend*
Fuß, *foot*

Garten, *garden*
Gast, *guest*
Gott, *God*
Grad, *grade, degree*
Grund, *ground, cause*

Hafen, *harbour, port*
Hammer, *hammer*
Humor, *humour*
Hund, *hound, dog*
Hunger, *hunger*
Hut, *hat*

Kaffee, *coffee*
Kanal, *canal*
König, *king*
Kuß, *kiss*

Mann, *man*
Mantel, *mantle, cloak*
Markt, *market*
Marsch, *march*
Meister, *master*
Monat, *month*
Mond, *moon*
Mord, *murder*
Morgen, *morning*
Muskel, *muscle*

Nachbar, *neighbour*
Nagel, *nail*
Name, *name*

Ofen, *oven, stove*
Onkel, *uncle*

Park, *park*
Plan, *plan*
Platz, *place*
Preis, *price*
Punkt, *point*

Raum, *room, space*
Regen, *rain*
Rest, *rest, remainder*
Ring, *ring*

Sand, *sand*
Schattan, *shade, shadow*
Schnee, *snow*
Schuh, *shoe*
Sohn, *son*
Soldat, *soldier*
Sommer, *summer*
Staat, *state*
Stab, *staff*
Stahl, *steel*
Stein, *stone*
Stern, *star*
Stock, *stick*
Stoff, *stuff, material*
Strom, *stream, current*
Stuhl, *stool, chair*
Sturm, *storm*

Tabak, *tobacco*
Tag, *day*
Tee, *tea*
Traum, *dream*
Tropfen, *drop*

Vater, *father*

Wagen, *wagon, car*
Weg, *way*
Wein, *wine*
Wert, *worth, value*
Wille, *will*
Wind, *wind*
Winter, *winter*

Zucker, *sugar*
Zweig, *twig, branch*

(b) FEMININE NOUNS

Achse, *axle*
Adresse, *address*
Asche, *ash*

Bank, *bank, bench*
Bibel, *bible*
Brücke, *bridge*
Brust, *breast, chest*
Butter, *butter*

Cousine, *female cousin*

Dame, *dame, lady*

Erde, *earth*

Familie, *family*
Faust, *fist*
Feder, *feather, pen*
Figur, *figure*
Flamme, *flame*
Flasche, *flask, bottle*
Flotte, *fleet*
Form, *form, shape*
Frucht, *fruit*

Gruppe, *group*

Halle, *hall*
Hand, *hand*
Hilfe, *help*
Hölle, *hell*
Hütte, *hut*

Jacke, *jacket*

Kammer, *chamber*
Karte, *card, map*
Kasse, *cash, till*
Katze, *cat*
Kirche, *church (kirk)*
Klasse, *class*
Kohle, *coal*
Krone, *crown*
Küste, *coast*

Lampe, *lamp*
Linie, *line*
Lippe, *lip*

Macht, *might, power*
Mahlzeit } *meal*
Mahl }
Maschine, *machine*
Masse, *mass*
Milch, *milk*
Minute, *minute*
Mitte, *middle*
Mühle, *mill*
Münze, *mint, coin*
Musik, *music*
Mutter, *mother*

Nacht, *night*
Nadel, *needle*
Nase, *nose*
Nation, *nation*
Natur, *nature*
Not, *need, want*
Note, *note*
Nummer, *number*

Ordnung, *order*

Partei, *party*
Pause, *pause, interval*
Periode, *period*
Person, *person*
Pfeife, *pipe, whistle*
Pflanze, *plant*
Post, *post*
Presse, *press*

Religion, *religion*

Schule, *school*
Schulter, *shoulder*
Schwester, *sister*
See, *sea* (**Der See,** *lake*)
Seele, *soul*
Seite, *side, page*

Section 1. NOUNS ALIKE OR ALMOST ALIKE IN GERMAN AND ENGLISH (*continued*)

(*b*) Feminine Nouns (*continued*)

Sekunde, *second*
Sonne, *sun*
Straße, *street*
Stufe, *step*
Summe, *sum*
Sünde, *sin*

Tat, *deed, act*
Tochter, *daughter*
Tür, *door*

Ware, *ware, goods*
Weise, *wise, manner*
Woche, *week*
Wolle, *wool*
Wunde, *wound*

(*c*) Neuter Nouns

Bett, *bed*
Bier, *beer*
Blut, *blood*
Boot, *boat*
Brot, *bread*
Buch, *book*

Ding, *thing*

Einkommen, *income*
Eis, *ice*
Ende, *end*

Feld, *field*
Fest, *festival*
Feuer, *fire*
Fieber, *fever*
Fleisch, *flesh, meat*

Gas, *gas*
Glas, *glass*
Gold, *gold*
Grab, *grave*
Gras, *grass*
Güter (Die) (plural) *goods*
(Gut means *property*)

Haar, *hair*
Haus, *house*
Heim, *home*
Herz, *heart*

Insekt, *insect*
Interesse, *interest*

Jahr, *year*

Korn, *corn, grain*
Kreuz, *cross*
Kupfer, *copper*

Land, *land, country*
Leder, *leather*
Licht, *light*

Mädchen, *maid, girl*
Material, *material*
Metall, *metal*

Netz, *net*

Ohr, *ear*
Öl, *oil*

Paar, *pair, couple*
Papier, *paper*
Publikum, *public*

Recht, *right*

Salz, *salt*
Schiff, *ship*
Schwert, *sword*
Segel, *sail*
Silber, *silver*
System, *system*

Tal, *dale, valley*
Theater, *theatre*

Wasser, *water.*
Weib, *wife, woman*
Werk, *work, works*
Wetter, *weather*
Wort, *word*

Section 2. NOUNS WHICH DIFFER IN THE TWO LANGUAGES

(*a*) Masculine Nouns

Abend, *evening*
Anfang, *beginning*
Angriff, *attack*
Anlaß, *cause*
Anspruch, *claim*
Anstand, *decency*
Anzug, *dress, suit*
Arzt, *physician*
Ast, *branch*
Auftrag, *order*
Augenblick, *moment*
Ausgang, *exit, way out*

Bach, *stream, brook*
Bahnhof, *railway station*
Bauer, *peasant*
Baum, *tree*
Bedarf, *demand*
Beruf, *profession*
Bleistift, *pencil*
Blitz, *lightning*
Boden, *ground*
Bote, *messenger*
Brief, *letter*
Brunnen, *well, fountain*
Buchstabe, *letter* (*of the alphabet*)

Dampf, *steam*
Dichter, *poet*
Diener, *manservant*
Draht, *wire*

Eid, *oath*
Eifer, *zeal*
Eingang, *way in, entrance*
Eltern (plural), *parents*
Erbe, *heir*
Erfolg, *success*
Esel, *donkey*

Faden, *thread*
Fehler, *fault, mistake*
Feind, *enemy*
Fluch, *curse*
Fluß, *river*
Friede, *peace*
Frühling, *springtime*
Funke, *spark*

Gedanke, *thought*
Gegensatz, *contrast*
Gegenstand, *object* (*concrete*)
Gegenstand, *subject* (*abstract*)
Geist, *spirit, ghost*
Geschwister (plural), *brother and sister*

Hals, *neck*
Handel, *trade*
Haufen, *heap*
Herbst, *autumn*
Himmel, *heaven, sky*
Herr { *master, lord*; *gentleman*; (address) *Mr., Sir.*
Hof, *court, yard*
Hügel, *hill*

Inhalt, *contents*

Kampf, *fight*
Kaufmann, *shopkeeper, merchant*
Knabe, *boy*
Knochen, *bone*
Knopf, *button*
Kopf, *head*
Körper, *body*
Kreis, *circle*
Krieg, *war*
Kunde, *customer, client*

Laden, *shop*
Lärm, *noise*
Leute (Die) (plural only) *people*
Löffel, *spoon*

Magen, *stomach*
Mangel, *lack*
Mensch, *man* (*general*)
Mittag, *noon*
Mund, *mouth*
Mut, *courage, spirit*

Nebel, *fog*
Neid, *envy*

Ort, *place, spot*

Rand, *edge*
Rat, *advice, counsel, counsellor*
Rauch, *smoke*
Riese, *giant*
Rücken, *back*
Ruf, *call, fame*
Ruhm, *glory, fame*

Same, *seed*
Satz, *sentence*
Schaden, *damage*
Scherz, *joke*
Schlüssel, *key*
Schmerz, *pain*
Schmutz, *dirt*
Schneider, *tailor*
Schwager, *brother-in-law*
See, *lake*
Sieg, *victory*
Sinn, *sense, mind*
Spiegel, *mirror*
Staub, *dust*
Strich, *line, dash*

Tausch, *exchange, barter*
Teil, *share, part*
Tisch, *table*
Tod, *death*

Ton, *sound, clay*
Topf, *pot*
Trotz, *spite, defiance*
Turm, *tower, steeple*

Umstand, *circumstance*
Unfall, *accident*

Verdacht, *suspicion*
Verein, *association*
Verlust, *loss*
Vertrag, *treaty, contract*
Vetter, *cousin* (*male*)
Vogel, *bird*
Vorrat, *stock, supply*
Vorteil, *advantage*

Wald, *forest*
Wirt, *host, landlord*

Zahn, *tooth*
Zeuge, *witness*
Zins, *interest*
Zoll, *toll, customs, duty inch*
Zufall, *chance*
Zug, *train, draught*
Zustand, *condition*
Zweck, *purpose*
Zweifel, *doubt*

(*b*) Feminine Nouns

Absicht, *intention*
Angst, *fear*
Ansicht, *view*
Arbeit, *work*
Art, *kind*
Aufgabe, *task*
Auskunft, *information*
Ausnahme, *exception*
Auswahl, *choice*

Bahn, *course*
Baumwolle, *cotton*
Bedingung, *conditions*
Blume, *flower*

Ecke, *corner*
Ehe, *marriage*
Ehre, *honour*
Eiche, *oak*
Eifersucht, *jealousy*
Eisenbahn, *railway*
Ernte, *harvest, crop*

Fabrik, *factory*
Fahne, *flag*
Farbe, *colour, paint*
Flucht, *flight, escape*
Frage, *question*
Frau, *woman, wife, Mrs.*
Freiheit, *freedom, liberty*
Freude, *joy, pleasure*

Gabel, *fork*
Geburt, *birth*

Section 2. NOUNS WHICH DIFFER IN THE TWO LANGUAGES (*continued*)

(*b*) Feminine Nouns (*continued*)

Geduld, *patience*
Gefahr, *danger*
Gegenwart, *presence*
Gelegenheit, *occasion*
Geschichte, *story, history*
Gesellschaft, *society, company*
Gestalt, *shape*
Gewalt, *force*
Gewohnheit, *habit, custom use*
Grenze, *limit, frontier*
Gunst, *favour*

Heimat, *home*
Hose, *trousers*

Insel, *island*

Jugend, *youth*

Kartoffel, *potato*
Kette, *chain*
Kraft, *strength*
Küche, *kitchen*
Kugel, *ball, bullet*
Kunst, *art*

Last, *load, burden*
Lücke, *gap*
Luft, *air*
Lüge, *lie*

Mauer, *wall*
Menge, *quantity*
Miete, *rent*
Mühe, *trouble, pains*

Nachricht, *news*
Nacht, *night*
Neugier, *curiosity*
Niederlage, *defeat*

Pflege, *care, nursing, cultivation*
Pflicht, *duty*
Predigt, *sermon*
Probe, *trial*

Quelle, *source, spring, well*

Rechnung, *bill*
Regel, *rule*
Regierung, *government*
Reihe, *row*
Richtung, *direction*
Rücksicht, *regard*
Ruhe, *quiet*

Schlacht, *battle*
Schranke, *barrier*
Schraube, *screw*
Schuld, *guilt, blame, debt*
Seife, *soap*
Sitte, *custom, morals*
Sorge, *sorrow, trouble*
Spannung, *tension*
Speise, *food, dish*
Spitze, *point, tip*
Stadt, *town, city*
Steurer, *tax*
Stimme, *voice, vote*
Stirn, *forehead*
Strafe, *penalty*
Stunde, *hour*

Tasche, *pocket, bag*
Tatsache, *fact*
Träne, *tear*
Treppe, *stairs*
Tugend, *virtue*

Uhr, *clock, watch*
Ursache, *cause, origin*

Vereinigung, *association*
Vernunft, *reason, sense*
Vorsicht, *caution*

Waffe, *weapon, arms*
Wahrheit, *truth*
Wand, *wall*
Welle, *wave*
Welt, *world*
Wirtschaft,[1] *economy (political)*
Wolke, *cloud*
Würde, *dignity*
Wut, *rage*

Zahl, *number*
Zeile, *line*
Zeit, *time*
Zeitung, *newspaper*
Zukunft, *future*
Zunge, *tongue*

[1] Wirtschaft also means *Housekeeping* and *Public-house*

(*c*) Neuter Nouns

Alter, *age*

Amt, *office*
Auge, *eye*

Bein, *leg, bone*
Beispiel, *example*
Bild, *picture*
Blatt, *leaf*

Dach, *roof*
Dorf, *village*

Ei, *egg*
Eisen, *iron*
Erbe, *inheritance*
Ereignis, *event*

Fach, *compartment, profession*
Fenster, *window*
Flugzeug, *airplane*
Fräulein, *Miss*
Frühstück, *breakfast*

Gebäude, *building*
Gebiet, *district*
Gebirge, *mountain range*
Gefühl, *feeling*
Geld, *money*
Geräusch, *noise*
Gericht, *court (of law)*
Gerücht, *rumour*
Geschäft, *business*
Geschenk, *present*
Geschlecht, *sex*
Gesetz, *law*
Gesicht, *face*
Getreide, *corn*
Gewehr, *rifle*
Gewicht, *weight*
Gewissen, *conscience*
Gift, *poison*
Glied, *limb, member*
Glück, *fortune, good luck*

Haupt, *head*
Heer, *army*
Hemd, *shirt*
Holz, *wood*

Kind, *child*
Kleid, *dress, gown*

Leid, *grief*
Lied, *song*
Loch, *hole*

Mass, *measure*
Meer, *sea*
Mehl, *flour*
Messer, *knife*
Mitleid, *pity*
Mittel, *medium, means, remedy*
Muster, *model, sample*

Pferd, *horse*

Rad, *wheel*
Reich, *empire, kingdom*

Schicksal, *fate*
Schlid, *signboard*
Schloß, *castle, lock*
Steuer, *steering wheel*
Stück, *piece*

Tier, *animal*
Tor, *gate*
Tuch, *cloth*

Ufer, *bank, shore*
Urteil, *judgment*

Werkzeug, *tool*
Wunder, *miracle*

Zeichen, *sign, token*
Ziel, *aim*
Zimmer, *room*

Section 3. DAYS OF THE WEEK, MONTHS, SEASONS

Days of the Week

Sonntag, *Sunday*
Montag, *Monday*
Dienstag, *Tuesday*
Mittwoch, *Wednesday*
Donnerstag, *Thursday*
Freitag, *Friday*
Samstag[1] } *Saturday*
Sonnabend[2] } *Saturday*

[1] In South Germany, Austria, and Rhineland.
[2] Elsewhere in Germany.

Months of the Year

Januar (**Jänner** in Austria), **Februar**, **März**, **April**, **Mai**, **Juni**, **Juli**, **August**, **September**, **Oktober**, **November**, **Dezember**

Seasons

Der Frühling, *Spring*
Der Sommer, *Summer*
Der Herbst, *Autumn*
Der Winter, *Winter*

Section 4. PROPER NOUNS

England, **Engländer**, *England, Englishman*; **Grossbritannien**, *Great Britain*.

Frankreich, **Franzose**, *France, Frenchman*; **Deutschland**, **Deutscher**, *Germany, German*.

Österreich, **Österreicher**, *Austria, Austrian*; **Polen**, **Pole**, *Poland, Pole*.

Italien, **Italiener**, *Italy, Italian*; **Die Schweiz**, **ein Schweizer**, *Switzerland, a Swiss*.

Russland, **ein Russe**, *Russia, a Russian*; **Spanien**, **Spanier**, *Spain, Spaniard*.

Die Vereinigten Staaten, **Amerika**, **ein Amerikaner**, *The United States, America, an American*.

Ungarn, **Ungar**, *Hungary, Hungarian*; **Die Tschechoslowakei**, *Czechoslovakia*; **Tschechoslowake**, *Czechoslovakian*.

Nordsee, **Ärmelkanal**, *North Sea, English Channel*; **Köln**, *Cologne*; **München**, *Munich*; **Wien**, *Vienna*; **Genf**, *Geneva*.

LESSON 4

Adjectives and Numbers

THE student should now have a substantial vocabulary of nouns in addition to a knowledge of the articles. We now turn to the adjectives and to numbers.

An adjective is a word used to describe the quality of a noun.

Declension

1. Adjectives used *alone* after a verb, to describe a state or quality, are invariable: **Er ist gut, sie ist gut, Es ist gut.** *He, she, it is good.* Otherwise they are inflected.

2. When preceded by the definite article (**Der, Die, Das**) or by **Dieser, Jener, Jeder, Mancher, Welcher, Derselbe,** the adjective (or adjectives) take the following terminations:

	Masc.	*Fem.*	*Neuter*	*Plural, all genders*
N.	-E	-E	-E	
G.	-EN	-EN	-EN	-EN
D.	-EN	-EN	-EN	-EN
A.	-EN	-E	-E	

DEM GUTEN WEIN, *to the good wine*; DES GUTEN VATERS, *of the good father*; DIESER SCHÖNEN BLUMEN, *of these nice flowers*; JENE GUTEN MÄNNER, *those good men.*

3. When preceded by the indefinite article (**ein, eine, ein**) or by its negation **kein, keine, kein,** *no*; or by any of the possessive adjectives (*see* next column), the full inflexions are:

	Masc.	*Fem.*	*Neuter*	*Plural, all genders*
N.	-ER	-E	-ES	
G.	-EN	-EN	-EN	-EN
D.	-EN	-EN	-EN	-EN
A.	-EN	-E	-ES	

4. When the adjective is used before a noun, and is not preceded by any of the above, it is declined like **dieser**, except that in the genitive singular of the masculine and neuter the ending **-en** usually takes the place of **-es**, though the latter may be used:

				Plural
N.	GUTER	ALTER	WEIN	GUTE
G.	GUTEN	ALTEN	WEINES	ALTE
D.	GUTEM	ALTEM	WEIN	WEINE,
A.	GUTEN	ALTEN	WEIN	etc.

When two or more adjectives precede a noun, both follow the rules given in 2, 3, 4, above.

Adjectives can often be used as nouns: **der Alte,** *the old man*; **ein guter Alter.**

Adjectives ending -EL and -EN (but not -ER) usually drop the -E in inflexions: **edel,** *noble*; **der edle Mann,** *the noble man.*

HOCH drops C before all vowels: **das hohe Haus,** *the high house.*

With all numerals except **ein,** adjectives take the endings of **dieser**: **sieben gute alte Männer.**

Comparison

To form the comparative add **-(e)r** } as in
To form the superlative add -(e)st } English.

Monosyllables modify **a, o, u** into **ä, ö, ü** (but **au** is not modified).

ALT, *old*	ÄLTER, *older*
LANG, *long*	LÄNGER, *longer*
LAUT, *loud*	LAUTER, *louder*

ÄLTEST, *oldest*
LÄNGST, *longest*
LAUTEST, *loudest*

To form the superlative if *no* noun is expressed use: **am (an dem)** and add **-en** to the regular superlative: **Es ist am besten,** *It is best.* This is also an adverb (*see* p. 1343).

Comparatives and superlatives are declined like other adjectives.

THAN: expressed by **als**: **er ist grösser ALS ich,** *He is bigger than I.*

AS . . AS: **Er ist SO gross WIE ich.** *He is as big as I.*

NOT SO .. AS: **Er ist NICHT SO gross WIE ich.**

Mehr, Weniger, Besser .. Als, *more, less, better .. than.*

Irregular Comparison. The following are of frequent occurrence:

Positive	*Comparative*		*Superlative*
BALD, soon	EHER, sooner	DER, DIE, DAS	EHESTE
GERN, willingly	LIEBER, rather		LIEBSTE
GUT, good	BESSER, better		BESTE
HOCH, high	HÖHER, higher		HÖCHSTE
NAHE, near	NÄHER, nearer		NÄCHSTE
WENIG, little	WENIGER, less		WENIGSTE

NOTE.—**Höchstens,** *at most*; **nächstens,** *at an early date*; **wenigstens,** *at least.*

Possessive Adjectives. Mein, dein, sein, are declined like **ein.** Plural **-e, -er, -en, -e.**

Ihr (*her*), **unser, euer, ihr** (*their*) and **Ihr** (polite *your*) are declined like **Dieser.**

Indefinite Adjectives. Alle (plural only), all; **jeder,** *every, each*; **mancher,** *many a,* are declined like **Dieser.**

Kein, *no, not a, not any*; decline like **Mein, ein.**

Viel, *much,* and **Wenig,** *little,* are undeclined, unless preceded by the definite article, and then they are declined like adjectives.

NOTE.—**Einige,** *some, a few*; **mehrere,** *several*; **viele, manche,** *many*; and **wenige,** *a few.* These words follow the fourth method of declining adjectives.

EXERCISE ON ADJECTIVES

Declension

Es ist ein warm*er* Tag.
It is a warm day.
Die weiß*en* Wolken im blau*en* Himmel bewegen sich nicht.
The white clouds in the blue sky do not move.
Viele selten*e* Blumen stehen auf dem groß*en* Feld.
Many rare flowers stand on the large field.
Sie sind bunt, blau und gelb.
They are coloured, blue and yellow.
Weiß*e* Lämmer stehen im ho*hen* Gras.
White lambs are standing in the high grass.
Ein jung*er* Hirte sitzt auf dem schmal*en* Weg.
A young shepherd sits on the narrow path.
Er singt ein froh*es* Lied.
He sings a cheerful song.
Drei lus*tige* Mädchen kommen aus dem hoh*en*, dunk*len* Wald.
Three jolly girls come out of the high, dark wood.
Sie haben lang*e* schwarz*e* Haare.
They have long black hair.
Sie begrüßen den fremd*en* Sänger.
They greet the strange singer.
Er wirft seinen alt*en* Hut in die stil*le* Luft und lächelt den nett*en* Mädchen zu.
He throws his old hat into the still air and smiles at the nice girls.
Sie wünschen ihm gut*en* Tag.
They bid him good day.

Comparison

Nach dem Tod des Löwen wählten die Tiere einen neuen König.
After the death of the lion the animals chose a new king.
Der Tiger sagte : Ich bin *der beste* König.
The tiger said : I am the best king.
Ich bin *mutiger als* andere Tiere.
I am braver than other animals.
Das Pferd sagte : Laßt mich *lieber* König sein.
The horse said : Let me rather be king.
Ich bin nicht so mutig wie der Tiger aber *schöner* und *edler*.
I am not so brave as the tiger but more beautiful and nobler.
Aber der Fuchs sagte : Ich bin *das* klügste Tier und klug sein ist *besser* als schön.
The fox, however, said : I am the cleverest animal and it is better to be clever than beautiful.

VOCABULARY OF ESSENTIAL ADJECTIVES

Section 1. ADJECTIVES ALIKE OR ALMOST ALIKE IN GERMAN AND ENGLISH

all, *all, every, any*
alt, *old*
aufrecht, *upright*
äußere, *outer*

besser, *better*
best, *best*
bitter, *bitter*
blau, *blue*
blind, *blind*
braun, *brown*
breit, *broad*

dick, *thick, big*
dünn, *thin*

eben, *even*
ernst, *earnest, serious*

falsch, *false, wrong*
fein, *fine*
fett, *fat*
flach, *flat*
frei, *free*
frisch, *fresh*

grau, *grey*
grün, *green*
gut, *good*

hart, *hard*
hoch, *high*
hohl, *hollow, empty*

innere, *inner*

jung, *young*

kalt, *cold*
klar, *clear*

lang, *long*
laut, *loud, noisy*
leicht, *light, easy*
letzt, *last*

mächtig, *mighty*
meist, *most*
mild, *mild*

natürlich, *natural*
neu, *new, recent*

offen, *open*

recht, *right*
reich, *rich*

reif, *ripe*
richtig, *right, correct*
rot, *red*
rund, *round*

sauer, *sour*
scharf, *sharp*
still, *still, quiet*

teuer, *dear, expensive*
tief, *deep*
treu, *true, faithful*

unter, *under, lower*

voll, *full*

warm, *warm*
weiß, *white*
weit, *wide, distant*

Section 2. ADJECTIVES DISSIMILAR IN THE TWO LANGUAGES

ähnlich, *similar*
allgemein, *general*
allmählich, *gradual*
andere, *other, different*
angenehm, *agreeable*
anstädig, *decent, respectable, considerable*
anwesend, *present*
arm, *poor*

bedeutend, *considerable*
bekannt, *known*
bequem, *convenient, comfortable*
bereit, *ready*
berühmt, *famous*
bescheiden, *modest*
besonders, *particular*
bestimmt, *certain*
billig, *cheap*
blaß, *pale*
bloß, *bare, mere*
böse, *bad, evil*
bunt, *coloured*

deutlich, *clear*

dicht, *dense, close*
dumm, *stupid*
dunkel, *dark*

echt, *genuine*
eckig, *angular*
ehrlich, *honest*
eigen, *own, proper*
eilig, *hasty, urgent*
einfach, *simple, single*
einig, *united*
einsam, *lonely*
endgültig, *final*
erheblich, *considerable*
ewig, *eternal*

fähig, *able*
faul, *lazy, rotten*
fern, *far*
fertig, *ready*
fest, *firm*
feucht, *damp*
fleißig, *industrious*
fremd, *strange*
froh, *glad*
fromm, *pious*
früh, *early*

ganz, *all, whole*
geboren, *born*
geheim, *secret*
geistig, *mental, intellectual*
geistlich, *religious, clerical*
gelb, *yellow*
genau, *exact*
gerade, *even, straight*
gerecht, *just*
gering, *little, small*
gesund, *healthy*
gewaltig, *powerful, mighty*
gewiß, *certain, sure*
gewöhnlich, *usual, common*
glatt, *smooth*
gleich, *like, same*
glücklich, *fortunate, happy*
grausam, *cruel*
groß, *great, large, tall*
gründlich, *thorough*
günstig, *favourable*

häßlich, *ugly*
häufig, *frequent*
heimlich, *secret*
heiß, *hot*

hell, *bright*
hinter, *back*
höflich, *courteous, polite*
hübsch, *pretty*

klein, *little, small*
klug, *clever, sensible*
kräftig, *strong*
krank, *ill, sick*
künftig, *future*
kurz, *short, brief*

langsam, *slow*
leer, *empty*
leise, *low, soft*
lieb, *dear*
link, *left*
lustig, *gay, cheerful*

mäßig, *moderate*
merkwürdig, *remarkable*
möglich, *possible*
müde, *tired*
mutig, *courageous*

nahe, *near*

Section 2. ADJECTIVES DISSIMILAR IN THE TWO LANGUAGES *(continued)*

naß, *wet*
nett, *nice neat*
niedrig, *low*
nötig, *necessary*

ober, *upper*
öffentlich, *public*

rein, *pure, clean*
ruhig, *quiet*

sanft, *gentle*
satt, *satisfied*
sauber, *clean*
schlecht, *bad*
schlimm, *bad, evil*
schmal, *narrow*
schnell, *quick, fast*
schön, *beautiful, fair*
schwach, *weak*
schwarz, *black*
schwer, *heavy, difficult*
schwierig, *difficult, hard*
selten, *rare*
sicher, *sure, safe*
spät, *late*
stark, *strong*
stolz, *proud*
streng, *stern*
stumm, *dumb*
süß, *sweet*

tot, *dead*
traurig, *sad*
trocken, *dry*
tüchtig, *able, efficient*

üblich, *usual*
ungefähr, *approximate*

verkehrt, *wrong*
vernünftig, *reasonable*
verschieden, *different, various*
viel, *much, many*
volkommen, *perfect*
vorder, *front*
vortrefflich, *excellent*

wahr, *true*
wahrscheinlich, *probable*
weich, *soft*
wenig, *little, few*
wichtig, *important*
wirklich, *real*

zufrieden, *content satisfied*

ADJECTIVES OF NATIONALITY

englisch, *English* , **französisch**, *French* ; **deutsch**, *German* ;
österreichisch, *Austrian*; **polnisch**, *Polish*; **italienisch**, *Italian* ;
russisch, *Russian* ; **spanisch**, *Spanish* ; **amerikanisch**, *American*.

NOTE.—Always written with small letters in German, except when first word of sentence.

NUMBERS

Cardinals

1	**ein, eins**	18	**achtzehn**
2	**zwei**	19	**neunzehn**
3	**drei**	20	**zwanzig**
4	**vier**	21	**ein und zwanzig**
5	**fünf**	30	**dreissig**
6	**sechs**	40	**vierzig**
7	**sieben**	50	**fünfzig**
8	**acht**	60	**sechzig**
9	**neun**	70	**siebenzig, siebzig**
10	**zehn**	80	**achtzig**
11	**elf**	90	**neunzig**
12	**zwölf**	100	**hundert**
13	**dreizehn**	101	**hundert und ein, eins**
14	**vierzehn**	200	**zweihundert**
15	**fünfzehn**	1,000	**tausend**
16	**sechzehn**		
17	**siebzehn**		

10,000 **zehntausend**
1,000,000 **eine Million**, *a million*
1,000,000,000 **eine Milliarde**

The word **EINS** is used only if not followed by another number: **zweihundert und eins. Hundert** and **Tausend** used as nouns take -E in nominative and -EN in dative plural. (HUNDERTE VON MENSCHEN, *hundreds of men*.)

Ordinals.

Ordinals are formed from cardinals by adding **-te** from 2 to 19, and **-ste** from 20 upwards: **der zweite**, the second ; **der dreiszigste**, the thirtieth ; **der hundertste**, the hundredth , **der vier-und-zwanzigste**, the twenty-fourth, etc. They are written with a full stop after the number : **Der 1. Januar**, the 1st of January.

Irregulars : **der erste**, the first ; **der dritte**, the third ; **der achte**, the eighth.

Ordinals are declined like ordinary adjectives, but only the last number of a compound is inflected : **Den fünfzehnhundertneunundzwanzigsten.**

Miscellaneous. In German a comma is used to indicate the decimal point, a full-stop to indicate thousands : 6,25 (English 6·25) and 1.000 (English 1,000).

ein Drittel (*n.*) a third ; **ein Zwanzigstel** (*n.*) a twentieth part ; **ein Viertel** (*n.*) a fourth ; **ein Hundertstel** (*n.*) a hundredth part, etc.

These are formed from the ordinals with the suffix **-tel**, from **der Teil**, part, portion.

Anderthalb, one and a half ; **zweieinhalb**, two and a half, etc. ; **Halb**, half ; **Die Hälfte**, half.

einfach, simple ; **zweifach, doppelt**, double; **dreifach**, threefold, etc. **Einerlei**, of one kind, **zweierlei**, of two kinds, **dreierlei**, etc., are indeclinable.

einzeln, individual, separate. **DER EINZELNE DEUTSCHE**, the individual German.

einzig, single, only. **KEIN EINZIGER MANN**, not a single man.

einmal, once ; **zweimal**, twice ; **dreimal**, three times, etc.

erstens, in the first place ; **zweitens**, secondly ; **drittens**, etc. ; **zuerst**, at first, first of all.

DIFFERENT USAGE IN GERMAN

Nouns denoting weight and measure are [illegible] in the singular : **DREI GLAS WASSER**, *three [illegible] of water*. **SIEBEN FUSS HOCH**, *seven feet h[illegible]*

What time is it? **WIE SPAT IST ES ?**

Es ist ein, zwei, drei Uhr. *It is one, two, three o'clock.*

Es ist zwanzig Minuten nach sechs. *It is 6.20*

Es ist (ein) Viertel nach zwei. *It is a quarter- past two.* Or, **Es ist Viertel drei.**

Es ist halb drei. *It is half-past two.*

Est ist drei Viertel drei. *It is a quarter to three.*

What is the date ? **DEN WIEVIELTEN HABEN WIR HEUTE ?**

Den dritten. Den siebenten. *The third. The seventh.*

Vor acht Tagen, *a week ago.*

In vierzehn Tagen, *a fortnight hence.*

Im Jahre neunzehnhundertdreiunddreissig, *in the year 1933.*

EXERCISE ON NUMBERS

Hunde*rte* kamen nach London.
Hundreds came to London.
Das Jahr hat zwölf Monate.
The year has twelve months.
Der Tag hat vierundzwanzig Stunden.
The day has twenty-four hours.
Er hat Tausen*den* geholfen.
He helped thousands.
Der er*ste* dieses Monats ist ein Freitag.
The first of this month is a Friday.
Mit Hilfe zwei*er* Männ*er* holten sie den Baum.
With the help of two men they brought the tree.
(But : **Mit Hilfe** *von zwölf* **Männ***ern*).
Vor acht Tagen war der zwölfte.
A week ago was the 12th.
Am erst*en* Januar fiel Schnee.
Snow fell on the first of January.
Der einundreißigste Dezember ist der dreihundertfünfundsechzigste Tag des Jahres.
31st December is the 365th day of the year.
In vierzehn Tagen wird kein *einziger* **Mensch hier sein.**
A fortnight hence not a single man will be here.
Ich habe ihn zweimal gesehen.
I have seen him twice.
Zuerst aß er anderthalb Äpfel und trank *zwei Glas Milch* **dazu.**
First he ate an apple and a half and drank two glasses of milk with it.
Einmal um halb drei, das zweite Mal um drei viertel acht.
Once at half past two, the second time at a quarter to eight.

LESSON 5

Pronouns

A PRONOUN is a word which is used instead of a noun or of a noun equivalent.

In German IT takes the gender of the noun to which it refers: **Wo ist das Mädchen?** *Where is the girl?* **Es ist hier,** *She is here.* **Wo ist mein Stock?** (*masc.*) *Where is my stick?* **Er ist nicht hier,** *It is not here.* **Haben Sie die Blume?** *Have you the flower?* **Ich habe sie.** *I have it.*

Note the idiomatic usage: **Wer ist es?** *Who is it?* **Ich bin es,** *It is I.* **Waren Sie es?** *Was it you?* **Ich war es,** *It was I.* **Er ist es,** *It is he.* **Er ist müde und seine Schwester ist es auch.** *He is tired and so is his sister.* **Es war einmal ein Mann.** *There was once a man . . .*

The dative and accusative of **ES** is translated by **DA** (**DAR** before vowels) with certain prepositions, and compounded with them: **davon,** *of it, thereof*; **damit,** *with it*; **dadurch,** *through it*; **darin,** *in it, therein*; **daraus,** *thereout, out of it.*

Reflexive Pronouns

The accusatives of the first and second persons in the table of personal pronouns are used as *reflexive* pronouns—that is, when it is necessary to refer back an action to the subject (*nominative*) of the sentence:

He killed himself: himself is a reflexive pronoun.

I wash myself: myself is a reflexive pronoun.

The word **sich** is used for all genders, singular and plural, of the third person and also as reflexive for **Sie,** *you.*

Examples:

ICH WASCHE MICH, I wash myself.
ER TÖTETE SICH, He killed himself.
SIE IRREN SICH, You make a mistake, are mistaken (lit. you err to yourself).

The word **SELBST** (self) is used to emphasize any of the personal pronouns, and must not be confused with the reflexives

ICH SELBST, I myself. **SIE SELBST,** you yourself.
ER HAT ES SELBST GETAN, He has done it himself.

And **SELBST** placed in *front* of any noun or pronoun means *even*:

SELBST ICH, Even I.
SELBST SEINE MUTTER LIEBT IHN NICHT, Even his mother does not love him

Relative Pronouns

A relative pronoun is one which connects the noun or pronoun to which it refers with the part of the sentence which follows. Thus: The man *whom I* know. The house *that* Jack built. *Whom* and *that* are relative pronouns. In English the relative is often omitted, e.g., *The house Jack built,* which would be quite as correct as *The house that Jack built,* and would probably be used in conversation.

In German relatives may *never* be omitted, so if you wish to say, "The woman I saw with you to-day," it must be **Die frau, die ich heute mit Ihnen sah.**

There are two main relatives in German: **der, die, das,** and **welcher, welche, welches.** In speaking *always* use **der, die, das.**

Their declension must be specially learnt, as it is slightly irregular (see below),

	Masc.	*Fem.*	*Neuter*	*Plural of all genders*
N.	**DER**	**DIE**	**DAS,** who, which, that	**DIE**
G.	**DESSEN**	**DEREN**	**DESSEN,** whose, of which	**DEREN**
D.	**DEM**	**DER**	**DEM,** to whom, to which	**DENEN**
A.	**DEN**	**DIE**	**DAS,** whom, which, that	**DIE**
N.	**WELCHER**	**WELCHE**	**WELCHES,** who, which, that	**WELCHE**

A TABLE OF PERSONAL PRONOUNS

Person	Nominative	Genitive	Dative	Accusative
1.	**ICH,** *I*	**MEINER,** *of me*	**MIR,** *to me*	**MICH,** *me*
2.	**DU,** *thou*[1]	**DEINER,** *of thee*	**DIR,** *to thee*	**DICH,** *thee*
3	**ER,** *he* (it)	**SEINER,** *of him* (it)	**IHM,** *to him*	**IHN,** *him*
	SIE, *she* (it)	**IHRER,** *of her*	**IHR,** *to her*	**SIE,** *her*
	ES, *it*	**SEINER,** *of it*	**IHM,** *to it*	**ES,** *it*
Plural				
1	**WIR,** *we*	**UNSERER,** *our, of our*	**UNS,** *to us*	**UNS,** *us*
2.	**IHR,** *you*[1] (ye)	**EUERER,** *of you*	**EUCH,** *to you, ye*	**EUCH,** *you, ye*
3.	**SIE,** *they*	**IHRER,** *of them*	**IHNEN,** *to them*	**SIE,** *them*
NOTE:	**Sie,** *YOU*	**Ihrer,** *OF YOU*	**Ihnen,** *TO YOU*	**Sie,** *YOU*

[1] English *YOU* can be translated in three ways: By **DU** when speaking to the Deity, intimate friends, relations, children, or animals. **IHR** is the plural of **DU**, and is used also in poetry, sermons, and conventional literary works. **SIE** is the ordinary word for *YOU* and should always be used until the language and people are well known. It is written with a capital (Sie) to distinguish it from sie, *they*, but takes the same part of the verb: **Sie haben,** *You have*; **sie haben,** *they have*, etc.

G. Masc.	Fem.	Neuter	Plur. of all genders
DESSEN	DEREN	DESSEN, whose, of which	DEREN
D.			
WELCHEM	WELCHER	WELCHEM, to whom, to which	WELCHEN
A.			
WELCHEN	WELCHE	WELCHES, whom, which, that	WELCHE

Agreement: The relative agrees in *gender* and *number* with the noun to which it refers:

DER MANN, DER (or **WELCHER**) *The man who*

DIE FRAU, DIE (or **WELCHE**) *The woman who*

DAS KIND, DAS (or **WELCHES**) *The child who*

But the relative takes its *case* from its own clause: **Die Frau, deren Buch verloren ist,** *The woman whose book is lost.*

WORD ORDER.—It will be seen in the last example that the word order differs from English. The rule is that in the part of a sentence introduced by a relative pronoun, the verb is placed at the *end*, with past participle or infinitive before auxiliary. Hence, **Der Mann, welcher den Garten gekauft hat,** *The man who has bought the garden.* Word order will, however, be treated later (*see* p. 1348).

CONTRACTION WITH PREPOSITIONS.—**Wo** (**wor** before a vowel) is used with prepositions for both the above relatives, when the clause preceding is not a personal one: **Das Haus worin** (or **in dem** or **in welchem**) **ich wohne ist sehr warm,** *The house in which* (*wherein*) *I live is very warm.*

Other contractions are: **Woran,** *at which, what*; **worauf,** *upon which, what*; **woraus,** *from which*; **wobei,** *at, near which*; **wodurch,** *through, by which*; **wofür,** *for which*; **womit,** *with which*; **worüber,** *over which*; **worunter,** *under, among which*; **wovon,** *of which*; **wozu,** *to which, for what.*

All these words are also used as interrogatives. (*See* below.)

THAT WHICH, WHAT are often translated by the word **was**—after **alles,** *all*; **nichts,** *nothing,* and certain neuter adjectives used in the superlative as **das erste,** *the first*; **das letzte,** *the last*; **das beste,** *the best*;

Alles was Sie gesagt haben, *All that you have said.*

Interrogatives

These are the pronouns which ask questions: *Who goes there? What are you doing? Which one do you prefer? WHO, WHAT, WHICH,* are interrogative pronouns.

WER? *Who?*	WAS? *What?*	WELCHER? *Which one?*
N. WER, *Who?*	WAS, *What?*	(WELCHER is declined like DIESER. See p. 1318.)
G. WESSEN, *Whose?*	WESSEN, *Of what?*	
D. WEM, *To whom?*	WAS, *To what?*	
A. WEN, *Whom?*	WAS, *What?*	WELCHER, WELCHE, WELCHES, etc.

A useful interrogative is: **Was für EIN . . . ?** *What sort of a . . . ?*

The **Ein** is declined in the ordinary manner:

Was für ein Mann ist Ihr Vater? *What sort of man is your father?*

Was für eine Frau ist Ihre Mutter? Was für ein Mädchen ist Ihre Schwester?

Was für leute sind . . . ? *What kind of people are . . . ?*

CONTRACTIONS.—**Was,** *What?* preceded by a preposition is contracted as explained above: **Wovon sprechen Sie?** *What are you speaking of?* And so on.

OF WHICH, when it is *not* possessive, is translated by **Von** with the dative: **Von welchen dieser Bücher sprachen Sie?** *Of which of these books did you speak?*

Demonstratives

These are pronouns which point out or demonstrate something.

The two principal demonstratives are **dieser** and **jener,** of which the declension has already been given with the articles (*see* page 1318). It should now be revised.

DIESER, *this, these,* is also used for *the latter*; and **JENER,** *that, those,* for *the former.*

DIES and **DAS** translate This and That when there is no noun: **Dies ist gut,** *This is good.* **Das ist schlecht,** *That is bad.* **Was ist dies?** *What is this?* **Dies** and **das** is this sense really mean, *This one* and *That one.*

CONTRACTIONS: **Da** (**dar** before vowels) translates THIS and THAT with prepositions, *see* above. **Darin, daraus,** *in that, out of that*; **darüber,** *over that,* etc.

Other demonstratives are **derjenige** (**diejenige, dasjenige:** plural, **diejenigen**) which translates THAT OF in such sentences as: **Mein Hut und derjenige meines Freundes,** *My hat and that of my friend.* Usually **derjenige** is contracted into **der** (**die, das**). Note also: **derselbe** (**dieselbe, dasselbe**), *the same*; **der andere** (**die andere, das andere**), *the other.*

SUCH followed by a noun is **solcher, solche, solches,** etc. SUCH followed by an adjective, use **So**: **Ein solcher Mann, eine solche Frau, ein solches Kind.** *Such a man, woman, child.* But: **Ein so kleines Kind,** *Such a small child.*

Possessive Pronouns

The genitives of the personal pronouns shown in the Table are used as possessive pronouns: **meiner, deiner, seiner, ihrer, seiner,** but note **unsrer, euer, ihrer, Ihrer.** They are all declined like **dieser** (*see* p. 1318).

M. MEINER, MEINES, MEINEM, MEINEN
F. MEINE, MEINER, MEINER, MEINE
N. MEIN(E)S, MEINES, MEINEM, MEIN(E)S
(like DIESER)

Pl., all genders: MEINE, MEINER, MEINEN, MEINE.

Wessen Hut ist dies? *Whose is this hat?* **Es ist meiner.** *It is mine.* **Es ist seiner, Ihrer.** *It is his, yours.* Or, **Es ist der meine,** etc.

OWNERSHIP.—The usual way to translate ownership is by using the verb **gehören** (to belong to), with the dative of the personal pronoun. Thus: **Dieser hut gehört mir,** *This hat is mine* (*belongs to me*); **die häuser gehören uns,** *The houses are ours.*

Indefinite Pronouns

There is a group of words which may conveniently be called indefinite pronouns. They are of very frequent occurrence, and should be known thoroughly.

Man is used to express ONE, THEY, PEOPLE (French *On*); **Man sagt . . .** They say . . . (On dit . . .). **Wenn man müde ist,** *When one is tired.*

Man is a most useful word. By its use you may, for practical purposes, avoid learning the whole of the passive voice of verbs. Thus, instead of translating I HAVE BEEN ASKED, you can say ONE HAS ASKED ME: **Man hat mich gefragt.** And so on, as will be seen later.

Note the following, which are invariable: **Einander,** one another; **selbst,** self (**Ich habe es selbst getan,** I have done it myself); **Nichts,** nothing; **etwas,** something.

Jedermann, everybody; **Jemand,** somebody; **niemand,** nobody. These words take -s in the genitive singular, but are otherwise invariable.

EXERCISE ON PRONOUNS

Personal Pronouns

Sind Sie traurig?
Are you sad?
Ich bringe Ihnen ein Buch.
I bring you a book.
Ich habe es ihm gegeben.
I have given it to him.
Ich erwarte Sie.
I expect you.
Ich erwarte dich.
I expect you
Du gehst nach Berlin.
You are going to Berlin.
Ich antworte Euch.
I answer you.
Ich erwarte Euch.
I expect you.
Sie haben die Schraube verloren.
They have lost the screw.
Gib sie mir.
Give it to me.
Sie gehört ihnen.
It belongs to them.
Wir waschen uns.
We wash ourselves.
Ich wünsche dir eine glückliche Reise.
I wish you a happy journey.

Reflexive Pronouns

Ein Mädchen saß an einem Bach.
A girl was sitting by a brook.
Er hatte klares Wasser.
It had clear water.
Das Mädchen trank *davon* **und wusch** *sich darin.*
The girl drank from it and washed herself in it.
Es **betrachtete sein Bild im Wasser.**
She looked at herself in the water.
Dann legte es *sich* **ins Gras.**
Then she lay down in the grass.
Es bot *selbst* **ein hübsches Bild.**
She was herself a pretty picture.
Selbst **die Vögel auf den Bäumen bewunderten es.**
Even the birds on the trees admired her.

Relative Pronouns

Es war einmal ein Kind, *dessen* **Vater und Mutter gestorben waren.**
There once was a child whose father and mother had died.
Er hatte keinen Menschen, *den* **er liebte, nichts,** *was* **ihm Freude machte.**
He had nobody whom he loved, nothing that gave him pleasure.
Da verließ er die Welt, *worin* **niemand ihn kannte und ging zum Himmel,** *an* **dem der Mond stand.**
So he left the world in which nobody knew him and went to the sky where the moon stood.
Der Mond aber war nur ein Stück Holz.
The moon, however, was only a piece of timber.
Das Kind *dessen* **Augen sich mit Tränen füllten, ging zur Sonne, die lustig lachte.**
The child, whose eyes filled with tears, went to the sun, which laughed gaily.
Aber die Sonne war nur eine stolze Sonnenblume.
The sun, however, was only a proud sunflower.
Dann ging das Kind zu den Sternen, *die* **am Himmel standen.**
Then the child went to the stars, which stood in the sky.
Die Sterne aber waren nur goldene Mücken, *die* **auf Dornen steckten und starben.**
The stars, however, were only golden midges, which stuck upon thorns and died.
Dann ging das Kind zur Erde zurück, *die* **er verlassen hatte.**
So the child went back to the earth which he had left.
Die Erde aber war nur eine Büchse, worauf sich das Kind nieder setzte und weinte.
The earth, however, was only a box, on which the child sat down and cried.
Denn er war sehr einsam.
For he was very lonely.

Interrogatives

Was brauchst Du für die Reise?
What do you need for the journey?
Wem geben Sie das Geld?
To whom do you give the money?
Wen besuchen Sie?
Whom are you going to visit?
Was werden Sie in Berlin sehen?
What will you see in Berlin?
Was für eine Frau ist Ihre Schwester?
What sort of a woman is your sister?
Wessen Geschäft ist es?
Whose business is it?
Wovon lebt er?
On what does he live?
Wozu nehmen Sie den Stock?
What are you taking the stick for?
Welchen Zug nehmen Sie?
Which train do you take?

Demonstratives

Es war ein so schöner Tag.
It was such a beautiful day.
Solches Wetter brauchen wir.
We need such weather.
Wir lachen *darüber.*
We laugh at that.
Jener große Mann hat eine so kleine Frau.
That tall man has such a small wife.
Sie sprachen über dies und das.
They talked about this and that.
Ich nahm meine Uhr und *die* **meines Freundes.**
I took my watch and that of my friend.

Possessive and Indefinite Pronouns

Wir fahren mit dem Wagen nach London.
We go by car to London.
Er nimmt seinen und wir nehmen unseren.
He takes his and we take ours.
Wessen Hut ist dies?
Whose hat is this?
Es ist meiner.
It is mine.
Hat jemand etwas vergessen?
Has anybody forgotten anything?
Niemand?
Nobody?
Dies Haus gehört uns.
This house is ours.
Wohnt jemand darin?
Does anybody live in it?
Ja, wir wohnen selbst darin.
Yes, we live in it ourselves.
Man lebt hier sehr billig.
One lives here very cheaply, i.e. *living here is very cheap.*
Jedermann ist höflich zum andern.
Everybody is polite to the other (i.e. to everybody else).
Wir unterstützen einander.
We support one another.

LESSON 6

Verbs: Tenses and Moods

A VERB is a word used for saying something about some person or thing.

The German verb appears formidable, but it is by no means as bad as it looks. The student will soon find that very often it resembles the English verb. It is not essential to know every form, every subtlety and idiomatic usage. But what follows in these pages (unless expressly stated to be for reference only) should be mastered. Mastery can be recognized when that part of a verb required to state an idea comes to the mind without hesitation.

Parts of the Verb which Must Be Known

For practical purposes the parts of the German verb required are:

(*a*) The *Infinitive* ("that part of a verb which names the action, without reference to the doer, and is therefore not limited by person or by number"). Thus **LOBEN**, *to praise*; **ESSEN**, *to eat*; **SCHREIBEN**, *to write*.

(*b*) The *Present Tense Indicative*, which corresponds to the three English forms, *I speak, I do speak, I am speaking*, all of which are translated by one form in German: **ICH SPRECHE**. Similarly **ICH LOBE**, *I praise, I do praise, I am praising*, etc.

(*c*) The simple *Past Tense Indicative* (also called the *Imperfect*) corresponding to the English *I praised, I was praising, I did praise*, etc., all of which are represented in German by one form, **ICH LOBTE**.

(*d*) The simple *Future Indicative*, corresponding to the English *I shall praise*, etc., and formed in German with the auxiliary, **WERDEN**. **ICH WERDE LOBEN**, *I shall praise*.

(*e*) The *Past Participle*, which is used to form compound tenses, and sometimes as an adjective, *I have praised, sung, spoken*: **ICH HABE GELOBT, GESUNGEN, GESPROCHEN**. (**Gelobt, gesungen, gesprochen** are past participles.)

If the Infinitive, the Past, and the Past Participle are known, ALL other parts can be formed by the "Ten Golden Rules" given in the opposite page.

Classification of Verbs, etc.

All infinitives of German verbs end in **-en** or **-n**. That part of a verb which remains when this **-en** or **-n** is cut off is called the "root" (or stem). All parts of a verb consist of this "root" plus an *ending* or inflexion. **Ich lobE**, *I praise*; **Ich lobTE**, *I praised*. **Lob-** is the root, **-e** and **-te** are endings. In addition to the ending, most past participles have a prefix **ge-**: **ge-lob-t**, *praised*.

Verbs are said to be *regular* (or "*weak*") when, as in **loben**, the past tense is formed by adding **-te** (or **-ete**) to the "root," and the past participle by prefixing **ge-** and adding **-t** to the "root."

Verbs are said to be *irregular* (or "*strong*") when the "root" vowel in the past differs from that of the infinitive: **geben,** *to give* (root, **geb-**), Past tense, **Ich gab.** (Note English: *to give, I gave.*)

The past participle of "strong" verbs is usually formed by prefixing **ge-** and adding the ending **-en**, but the vowel may differ from that of the root of the infinitive. **Singen,** to sing. Past participle, **gesungen,** sung. **Sprechen,** to speak; **ich sprach,** I spoke; **ich habe gesprochen,** I have spoken. These are examples of strong verbs. Also: **tragen,** *to carry, to wear*; **ich trug,** *I wore*; **ich habe getragen,** *I have worn.* Here **getragen** retains the root vowel of the infinitive.

The "strong" or irregular verbs are best mastered in groups, in accordance with the changes of the root vowels. (*See* pp. 1340 and 1341.)

The present participle of all verbs ends **-end,** corresponding to English **-ing,** and it is very often used as an adjective: **loben,** *to praise*; **lobend,** *praising*; **lieben,** *to love*; **liebend,** *loving*; **eine liebende Mutter,** *a loving mother.* It is often also used as a noun: **reisen,** *to travel*; **reisend,** *travelling*; **der Reisende,** *the traveller* (**ein Reisender**). And as an adverb: **lachen,** *to laugh*; **lachend,** *laughing*; **lachend sagte ich . . .** *laughingly (or laughing) I said . . .*

Auxiliary Verbs

Auxiliary verbs are so called because they are used not only alone in their original meanings, but as helps to form the compound tenses of *all* other verbs. They are the most frequently recurring verbs in the language and must be known thoroughly. In German there are three auxiliaries: **haben,** *to have*; **sein,** *to be*; **werden,** *to become.* They are conjugated in the Table in p. 1335. **Haben** is an example of a "weak" verb; **sein** of one that is irregular; and **werden** of a straightforward "strong" verb. It will be observed that in the compound tenses of **sein** it is used itself: **ich bin gewesen,** *I have been.* In English we always use *to have* in the formation of compound tenses, but in German many verbs take **sein,** as will be seen later.

To Form the Tenses of All Regular or "Weak" Verbs

The verb **LOBEN** in the Table (p. 1335) is given as a model of a fully conjugated regular or "weak" verb. It is given for reference and practice. All regular verbs—that is, ALL VERBS NOT GIVEN WITH THE "STRONGS" AND IRREGULARS below—follow the model of **LOBEN.** All tenses of

regular verbs can be formed by following these rules :

Ten Golden Rules for Verbs

1. Verbs with roots ending in **-D, -T, -M, -N,** preceded by a consonant, retain **E** of the infinitive throughout. Verbs with roots ending **-S, -Z, -SCH** retain this E only in the second person of the present indicative: **ANTWORTEN,** *to answer* ; **Ich antworte, Du antwortEst, er antwortEt,** *I answer*, etc. **Ich antwortETE,** *I answered*, etc. Similarly **REGNEN,** *to rain* ; **ÖFFNEN,** *to open, etc.*

2. To form the present tense indicative : Drop **-EN** of infinitive (i.e. find the root) and add **-E -ST, -T, -EN, -T, -EN.** (**Ich lobe,** etc.)

3. The past or imperfect indicative is formed by adding to the root : **-TE, -TEST, -TE, -TEN, -TET, -TEN.** (**Ich lobTE,** etc.)

4. To form the future : use the present of **WERDEN** with the infinitive of the verb of which the future is required : **Ich werde loben,** etc. This applies to *all* verbs, regular and irregular.

5. To form the conditional : use imperfect subjunctive of **WERDEN** with the infinitive : **Ich würde loben,** *I should praise*, etc. This applies to *all* verbs, regular and irregular.

6. To form the present subjunctive : add to the root **-E, -EST, -E, -EN, -ET, -EN.** (**Ich lobe, du lobest, er lobe,** etc.)

7. The imperfect subjunctive is the same as the imperfect indicative : **Ich lobte,** etc. In strong verbs there is usually modification of the vowel : **Ich sänge.**

8. To form the imperative : add the pronoun **SIE** to the infinitive. **Loben Sie !** *Praise!* (This is only for the second person plural, which is the one most frequently used.)

9. All verbs ending **-IEREN** are regular. They require no **GE-** in the past participle. **REGIEREN,** *to rule* ; **MARSCHIEREN,** *to march* ; **STUDIEREN,** *to study* ; **REGIERT, MARSCHIERT, STUDIERT.**

10. Verbs ending **-ELN, -ERN** drop **E** before **L** and **R** in the first person singular of the present tense : **HANDELN,** *to handle, act* ; **Ich handle,** *I act.*

THE SUBJUNCTIVE MOOD
(For Reference)

In most grammars the subjunctive mood is given a position of importance, and for fine shades of meaning it is very necessary. The student who wishes to know his German thoroughly cannot afford to neglect it. But it is not *essential* for the expression of the common ideas of everyday life. One should, however, be able to recognize it. It is usually formed as follows. Present subjunctive : add to the "root" **-E, -EST, -E, -EN, -ET, -EN.** **Ich habe, du habest, er habe, wir haben, ihr habet, sie haben. Ich lobe, Ich werde, Ich singe,** etc.

The past or imperfect is the same as that of the indicative (i.e. given in the Table), but usually **a, o, u** become **ä, ö, ü,** and **E-** is added to the singular : **Ich spräche, ich würde, ich wäre, ich könnte** (from **können,** *to be able*), **ich trüge** (from **tragen,** *to bear, carry*).

The Imperative, or How to Give Commands. This is formed by placing the pronoun after the plural of the present tense : **Loben Sie ihn !** *Praise him!* **Gehen wir,** *Let us go.* It may also be formed with the verb **Lassen** (*to let, allow*) : **Lassen Sie uns hinausgehen,** *Let us get out of here* ; **Lassen Sie ihn kommen,** *Let him come,* etc.

To Use the Verb Interrogatively. Merely put the personal pronoun after : **Haben Sie einen Hut ?** *Have you a hat ?* **Rauchen Sie ?** *Do you smoke ?* **Spricht er ?** *Does he speak ?* **Sind Sie müde ?** *Are you tired ?* **Was sagten Sie ?** *What did you say ?* Similarly the noun, with all attributes, is put after the verb : **Ist meine Schwester zu Hause ?** *Is my sister at home ?*

The Negative of Verbs. Use the word **nicht,** *not.* In simple tenses, direct statements, it is placed immediately after the verb : **Ich lobe nicht,** *I do not praise* ; **Ich spreche nicht,** *I am not speaking.* In compound tenses always *before* infinitives or past participles. Otherwise **nicht** stands *after* a *direct object* or an *adverb of time* ; *before* an *adverb of place* or *manner*, a *preposition*, or an *emphatic word*.

Examples : **Ich habe ihn nicht gefunden,** *I have not found him* ; **Ich will nicht sagen,** *I will not say* ; **Ich gab ihm den Wein nicht,** *I did not give him the wine* ; **Er ist nicht hier,** *He is not here* ; **Er ist nicht mit uns gekommen,** *He has not come with us,* etc.

To Form the Passive of Verbs. The passive of all verbs is formed with **Werden.**

Thus : **Ich werde gelobt,** *I am praised* (or *being praised*) ; **Ich wurde gelobt,** *I was praised* ; **Ich werde gelobt werden,** *I shall be praised,* etc.

The past participle of **Werden** used in this sense is **Worden : Ich bin gelobt worden** (not **geworden**), *I have been praised.*

NOTE.—**Man** is often used instead of the passive form (*see* p. 1331).

Reflexive Verbs. A "reflexive verb" is one in which the action performed is suffered by the subject. Thus : *I wash myself.* In German there are many other verbs which are reflexive —that is, they are conjugated with two pronouns instead of one : **sich freuen,** *to be glad* :

ICH FREUE MICH, DU FREUST DICH, ER FREUT SICH, WIR FREUEN UNS, IHR FREUT EUCH, SIE FREUEN SICH. } *I am glad,* etc.

Past : **Ich freute mich.** *Future :* **Ich werde mich freuen,** etc. *Imperative :* **Freuen Sie sich !** *Be glad!*

NOTE.—All reflexive verbs in German are conjugated with **haben. Ich habe mich gefreut,** *I have been glad.*

Examples of reflexive verbs : **Sich irren,** *to make a mistake* ; **Sich erinnern,** *to remember.* (But : **erinnern,** *to remind.*)

LESSON 7

Auxiliary, Impersonal, and Separable Verbs

CONTINUING the study of Verbs, in the present Lesson we proceed with a table of simple auxiliaries and the six auxiliaries of mood, going on to deal with the impersonal, separable, and inseparable verbs, and notes on the verbs **WISSEN** and **KENNEN** and conjugations with **SEIN**.

The Six Auxiliaries of Mood. In addition to the three simple auxiliaries **HABEN, SEIN,** and **WERDEN** (see Table in page 513), there are six important verbs called "auxiliaries of mood," because, although alone they may convey no very definite or complete idea, yet when used with other verbs they modify the meanings of these.

The six verbs are: **KÖNNEN, MÖGEN, SOLLEN, DÜRFEN, MÜSSEN, WOLLEN.** As these words recur continually, their uses should be fully grasped.

Können, konnte, gekonnt. Denotes the *power* to do something. It translates all the English words which signify *ability, possibility,* or *likelihood* of doing something. Thus: **Ich kann nicht kommen, schreiben, lesen,** *I cannot come, write, read,* etc.

Mögen, mochte, gemocht: Denotes a *liking* or *preference* to do something, and also a *possibility* (often depending on a wish that it may be so). Thus: **Das mag sein,** *That may be.* **Er mochte fünfzig jahre alt sein,** *He might have been fifty years old.*

Note the following usage of MÖGEN:

ICH MÖCHTE	1. GERN 2. LIEBER 3. AM LIEBSTEN	DEUTSCH SPRECHEN
I should	*1. like to* *2. rather* *3. like best to*	*speak German*

Note the two forms: **MOCHTE** means *might have been* or *seemed to be,* but **MÖCHTE** means *should like.*

And the idiom: **Ich mag ihn (sie) nicht,** *I dislike him (her),* etc.

Sollen, sollte, gesollt. This denotes a *duty* or *obligation.* (It corresponds to the French *devoir.*) Also a *command.* Thus: **Du sollst nicht stehlen,** *Thou shalt not steal*; **Was soll ich tun?** *What shall I do?*

Dürfen, durfte, gedurft: *To be permitted to,* or *to be likely to*; **Darf ich mitkommen?** *May I come with (you)?*; **Sie dürfen,** *You may*; **Sie dürfen es nicht glauben,** *You must not believe it*; **Das dürfte richtig sein,** *That is probably right.*

Müssen, musste, gemusst: Corresponds to the English *must,* denoting *compulsion* or *necessity.* **Er muss geben, kommen,** *he must go, come.* **Sie muss müde sein,** *She must be tired.*

Wollen, wollte, gewolt: A vague word this, so one must be wary. (It corresponds to French *vouloir.*) Denotes *willingness* or *desire.* **Wollen Sie mit mir kommen?** *Will you come with me?* **Wollen Sie die Güte haben . . .** *Would you be so good as to . . .*

Wollen must not be used for simple futures.

NOTE.—A mood auxiliary, in addition to its ordinary past participles given above, also uses its *infinitive* as past participle—the latter when another infinitive depends upon it: **Er hat nicht gehen können** (instead of **gekonnt**), *He was unable to go.* If there is no second infinitive, the ordinary form is used. **Er hat es nicht gekonnt,** *He has not been able to.* **Zu** is never used after a mood auxiliary before an infinitive.

Brauchen, *to use, need,* is also used as an auxiliary of mood, but only in the *negative* form. **Er braucht nicht zu kommen,** *He need not come.* Brauchen in this sense is *always* followed by **zu**: (**"Wenn Sie BRAUCHEN gebrauchen, müssen Sie BRAUCHEN mit ZU gebrauchen, sonst brauchen Sie BRAUCHEN überhaupt nicht zu gebrauchen."** "*If you use* brauchen, *you must use it with* zu, *otherwise you need not use it at all.*")

The following verbs may also command an infinitive: **LASSEN, ließ, gelassen,** *to let, allow*; **HELFEN, half, geholfen,** *to help*; **SEHEN, sah, gesehen,** *to see*; **MACHEN, machte, gemacht,** *to make.* **ICH LIESS IHN KOMMEN,** *I let him come*; **ICH MACHTE IHN LACHEN,** *I made him laugh.*

Irregular Present Tenses

I can	*I may*	*I dare*
ICH KANN	MAG	DARF
DU KANNST	MAGST	DARFST
ER KANN	MAG	DARF
WIR KÖNNEN	MÖGEN	DÜRFEN
IHR KÖNNT	MÖGT	DÜRFT
SIE KÖNNEN	MÖGEN	DÜRFEN
I must	*I am to*	*I want*
MUSS	SOLL	WILL
MUSST	SOLLST	WILLST
MUSS	SOLL	WILL
MÜSSEN	SOLLEN	WOLLEN
MÜSST	SOLLT	WOLLT
MÜSSEN	SOLLEN	WOLLEN

Impersonal Verbs. These are, as in most languages, chiefly concerned with the weather. **Es regnet,** *It rains*; **Es schneit,** *It snows*; **Es regnete, es schneite,** *It rained, snowed.* They are conjugated with **haben.**

There is, are: **Es gibt.** (Also **Es ist, Es sind.**)

There was, were: **Es gab.** (Also **Es war, Es waren.**)

There will be: **Es wird geben.** (Also **Es wird, werden sein.**)

NOTE.—**Es gibt** is vague and indefinite. **Es ist** is more definite.

NOTE ALSO.—**Es tut mir leid,** *I am sorry*; **Es gefällt mir,** *I am pleased*; **Es geht mir gut,** *I am well*; **Es fehlt mir,** *I am short of . . .*; **Es fehlt mir ein Glas,** *I am short of a glass*;

Es (das Essen) schmeckt mir, *I like it (the food).*

Separable and Inseparable Verbs. Most German verbs can change or expand their meaning by the addition of prefixes. These prefixes are either *separable* or *inseparable.* By separable is meant that the prefix may be detached from the "root" verb. Thus: **Zurückkommen,** *to come back* ; **Ich komme zurück,** *I come back.* An inseparable "root" verb *never* parts with its prefix. Thus: **Bekommen,** *to get, obtain* ; **Ich bekomme,** *I obtain* ; **Ich habe bekommen.** And the inseparable prefixes are always *unaccented* : **Bekomm'en,** etc.

1. *Inseparable Prefixes are :* **Be-, ge-, ent-, -emp, er-, ur-, ver-, zer-, miss-, hinter-,** and **wider-.**[1] This list must be learnt and the simple or original meanings of the prefixes on p. 1347 should now be mastered.

An inseparable verb follows exactly the conjugation of its simple "root" verb. **Ich bekomme, du bekommst, er bekommt,** etc.

[1] Distinguish the inseparable **WIDER,** *against,* from the separable WIEDER, *again.*

2. *Separable Prefixes with their original meanings are :*

AB- *off*
AN- *on*
AUF- *up, upon*
AUS- *out*
BEI- *by*
DAR- *there*
EIN- *in, into*
EMPOR- *up*
ENTGEGEN- *against, towards*
ENTZWEI- *in two*
FORT- *away*
GEGEN- *against*
HER- *toward one*
HIN- *from one*
LOS- *loose, off*
MIT- *with*
NACH- *after*
NIEDER- *down*
VOR- *before*
VORAN- *before*
WEG- *away*
WIEDER- *again*
ZU- *to*
ZURÜCK- *back*
ZUSAMMEN- *together*

This list is also very important. A separable prefix is accented.

Examples : **Schneiden,** *to cut* ; **Abschneiden,** *to cut off* ; **Wiederkommen,** *to come again* ; **Zurückkommen,** *to come back* ; **Her-** and **Hin-** can be used either alone or with other prefixes : **Hereinkommen.**

The conjugation of these separable verbs follows certain rules :

(a) In the present and past tense (of both indicative and subjunctive) the prefix is despatched to the *end* of the clause. **Reisen,** *to travel* ; **Abreisen,** *to set out, depart* ; **Ich reise ab,** *I set out.*

(b) In the infinitive and past participle **zu** (when used) and **ge-** are placed between the prefix and the "root" verb. **Er wünscht auszugehen,** *He wishes to go out* ; **Sie sind zusammengekommen,** *They have come together.*

(c) In the imperative the prefix is always sent to the end of the clause. **Gehen Sie sofort weg !** *Go away immediately !*

Otherwise the separable verbs are conjugated like their "root" verbs : **Wieder(zu)kommen,** *to come again* ; **Ich kam wieder,** *I came again* ; **Ich bin wiedergekommen,** *I have come again* ; **Ich werde wiederkommen,** *I will come again,* etc.

3. *Separable or inseparable, according to meaning are* ; **Durch,** *through* ; **Hinter,** *behind* ; **Über,** *over* ; **Um,** *about* (also indicates change) ; **Unter,** *under* ; **Voll,** *full* ; **Wieder,** *again.*

When each part of the verb has its original meaning, the prefix bears the principal accent and is separable : **Wieder'holen (ich hole wieder, wieder'geholt),** *to fetch again* ; **um'stellen,** *to put round, to change.*

TABLE OF AUXILIARIES AND THE WEAK VERB LOBEN

1.	**habEN,** *to have*	**seiN,** *to be*	**werdEN,** *to become*	**lobEN,** *to praise*
2.	**habEND,** *having*	**seiEND,** *being*	**werdEND,** *becoming*	**lobEND,** *praising*
3.	**GEhabT,** *had*	**GEwesen,** *been*	**GEworden,** *become*	**GElobT,** *praised*
4.	**ich habE,** *I have* **du haST** **er, sie, es haT** **wir habEN** **ihr habT** **sie habEN**	**ich bin,** *I am* **du biST** **er isT** **wir sind** **ihr seid** **sie sind**	**ich werde,** *I become* **du wirST** **er wird** **wir werdEN** **ihr werdET** **sie werdEN**	**ich lobE,** *I praise* **du lobST** **er lobT** **wir lobEN** **ihr lob(e)T** **sie lobEN**
5.	**ich hatTE,** *I had* **du hatTEST** **er hatTE** **wir hatTEN** **ihr hatTET** **sie hatTEN**	**ich war,** *I was* **du warST** **er war** **wir warEN** **ihr wart** **sie warEN**	**ich wurdE,** *I became* **du wurdEST** **er wurdE** **wir wurdEN** **ihr wurdET** **sie wurdEN**	**ich lobTE,** *I praised* **du lobTEST** **er lobTE** **wir lobTEN** **ihr lobTET** **sie lobTEN**
6.	**ich habe gehabt,** *I had,* etc.	**ich BIN gewesen,** *I have been,* etc.	**ich bin geworden,** *I have become,* etc.	**ich habe gelobt,** *I have praised,* etc.
7.	**ich hatte gehabt,** *I have had had,* etc.	**ich war gewesen,** *I had been,* etc.	**ich war geworden,** *I had become,* etc.	**ich hatte gelobt,** *I had praised,* etc.
8.	**ich werde haben,** *I shall have,* etc.	**ich werde sein,** *I shall be,* etc.	**ich werde werden,** *I shall become,* etc.	**ich werde loben,** *I shall praise,* etc.
9.	**ich würde haben,** *I would have,* etc.	**ich würde sein,** *I would be,* etc.	**ich würde werden,** *I would become,* etc.	**ich würde loben,** *I would praise,* etc.

In the above Table 1 is the *Infinitive* ; 2, *Present Participle* , 3, *Past Participle* , 4, *Present* Tense ; 5, *Past or Imperfect* Tense ; 6, *Perfect* Tense ; 7, *Pluperfect* Tense ; 8, *Future* Tense ; 9, *Conditional* Tense.

There are two more tenses : Future Perfect, **ICH WERDE GEHABT HABEN,** *I shall have had* . Past Conditional, **ICH WÜRDE GEHABT HABEN,** etc. But these are rare.

But when the compound has a *figurative* meaning, the prefix is unaccented, and inseparable: **Wiederho'len (ich wiederho'le, wiederho'lt),** *to repeat.* **Umste'llen,** *to encircle.*

Wissen and Kennen. Wissen, *to know,* denotes knowledge gained by study or learning (French: *savoir*). **Kennen** (French: *connaitre*) *to know,* indicates an acquaintance with something; **Kennen Sie ihn?** *Do you know him?* **Wissen Sie wie er heisst?** *Do you know his name?* **Ich kenne den Inhalt des Briefes,** *I am acquainted with the contents of the letter;* **Ich weiss nicht von wem der Brief ist,** *I do not know from whom the letter is.*

Verbs Conjugated with Sein. Verbs of motion and their compounds are conjugated with **sein: gehen,** *to go;* **abfahren,** *to depart,* etc. *Also the verbs:* **werden,** *to become;* **sterben,** *to die;* and **sein** itself.

A list of essential verbs is in Lesson 10 (pages 1340–1343).

LESSON 8

Exercises on Verbs: (1) Regular and Irregular Tenses

This Lesson consists of a series of Exercises on the instruction given on regular ("weak") and irregular ("strong") verbs in Lesson 6. These exercises are something more than revision work—they are instructional, and the student should work at them both in this and Lesson 9 in close conjunction with his study of the principles previously stated.

Parts of the Verb which Must be Known

Ich sehe dich essen.
I see you eat(ing).
Ich sitze auf einem Stein.
I am sitting on a stone.
Ich koche in der Küche.
I cook in the kitchen.
Ich lasse dich arbeiten.
I let you work.
Ich lerne schrieben.
I learn (to) write.
Ich werde das Rätsel lösen.
I shall solve the riddle.
Sie haben ihn gefragt, er hat geantwortet.
They have asked him, he has answered.
Sie ist gut gekleidet.
She is well dressed.
Er lobte das Spiel.
He praised the play.
Er erschreckte das Kind.
He frightened the child.

REGULAR AND IRREGULAR ("WEAK" AND "STRONG") VERBS

Sie sind nach Berlin gereist, sie haben einen Zug genommen.
They have travelled to Berlin, they have taken a train.
Ich nehme den Zug.
I take the train.
Sie haben auf der Bank gesessen.
They have been sitting on a bench.
Sie legte sich schlafen und schlief gut.
She lay down to sleep, and she slept well.
Ich glaubte ihm.
I believed him.
Er hat das Bild gesehen.
He has seen the picture.
Er hat viel gewagt.
He has risked much.
Er hat 100 Pfund gewogen.
He weighed 100 pounds.
Er hat das Heer geleitet.
He has led the army.
Er hat schwer gelitten und leidet noch heute.
He has suffered heavily and still suffers to-day.
Er hat darum gebeten.
He has asked for it.
Sie haben gebetet.
They have prayed.
Ich bitte darum.
I ask for it.

The student should ask himself: *Which verbs are strong? Which weak?*

PRESENT PARTICIPLES

Ein Fuchs sah an einer Mauer reife Trauben hangen.
A fox saw ripe grapes hanging over a wall.
Sie waren *auffallend* **schön.**
They were strikingly beautiful.
Er versuchte die *einladenden* **Früchte zu erreichen, aber sie waren zu hoch.**
He tried to reach the inviting fruits, but they were too high.
Mit *brennendem* **Hunger ging er weiter.**
With burning hunger he went on.
Auf einem Baum *sitzend,* **fragte ihn** *lachend* **ein Rabe, der** *den Hungernden* **gesehen hatte: Haben die Trauben gut geschmeckt?**
Sitting on a tree a raven who had seen the hungry fox asked him laughingly: Did the grapes taste good?
Der Fuchs antwortete: Ich will sie nicht.
The fox answered: I do not want them.
Sie sind zu sauer.
They are too sour.

AUXILIARY VERBS

Ich werde das Kleid kaufen.
I shall buy the dress.
Ich habe es im Fenster gesehen.
I have seen it in the window.
Es ist blau und hat weiße Knöpfe.
It is blue and has white buttons.
Das Mädchen im Laden wird es mir schicken.
The girl in the shop will send it to me
Ich werde es morgen haben.
I shall have it to-morrow.
Es wird vielleicht zu schwer sein.
It will perhaps be too heavy.
Denn die Tage sind wärmer geworden.
For the days have become warmer.
Es ist dumm gewesen, es zu nehmen.
It was stupid to take it.
Wir werden es tauschen.
We shall change it.
Das Mädchen hat noch ein andres Kleid gehabt.
The girl had still another dress.
Das war leichter.
It was lighter.
Ich bin entschlossen, es zu kaufen.
I have decided to buy it.

TENSES OF REGULAR VERBS

Golden Rules 1 and 2, page 1333

Er erwartet dich.
He expects you.
Du antwortest ihm.
You answer him.
Was wünsch(e)st du?
What do you wish?
Du trocknest.
You dry.
Er leitet.
He leads.
Ich öffne die Tür.
I open the door.

Er atmet. *He breathes.*
Es regnet. *It rains.*
Wir baden. *We bathe.*
Du schadest. *You harm.*

Golden Rules 3, 4 and 5, page 1333

Ein Fuchs bemerkte einmal einen fetten Hahn im Hof eines Bauernhauses.
Once a fox observed a fat cock in the courtyard of a farmhouse.
Er sagte höflich zu dem Hahn : " Ich hörte einmal deinen Vater singen.
Courteously he said to the cock : " Once I heard your father singing.
Er hatte eine so schöne Stimme.
He had such a beautiful voice.
Ich würde dich sehr bewundern, wenn deine Stimme so schön wäre wie die deines Vaters.
I should admire you very much if your voice were as beautiful as that of your father.
Aber ich glaube es nicht."
But I do not believe it."
Der Hahn antwortete : " Meine Stimme ist viel schöner als die deines Vaters und ich werde es dir zeigen."
The cock answered : " My voice is much more beautiful than that of my father and I shall show it to you."
Dann öffnete er den Mund zum Singen.
Then he opened his mouth (in order) to sing.
In diesem Augenblick faßte ihn der Fuchs und eilte weg.
At this moment the fox seized him and hurried away.
Die Bauern aber bemerkten den Dieb und folgten ihm und sagten : " Der Fuchs tötet unsren Hahn."
The peasants, however, noticed the thief, followed him, and said : " The fox is killing our cock."
Der Hahn hörte das und sagte zum Fuchs : " Ich würde den Bauern sagen, ich hole *meinen* Hahn nicht euren."
The cock heard it and said to the fox : " I should tell the peasants; I am fetching my *cock not yours."*
Der Dieb glaubte, er würde die Bauern damit ärgern, öffnete wirklich den Mund und sagte : " Das ist *mein* Hahn nicht eurer."
The thief believed he would annoy the peasants by it, actually opened his mouth, and said : " This is my *cock, not yours."*
In diesem Augenblick befreite sich der Hahn, setzte sich auf einen Baum und sagte : " Du lügst, Herr Fuchs, du lügst. Ich bin der Bauern Hahn, nicht deiner."
At this moment the cock freed himself, perched on a tree, and said : " You are lying, Mister Fox. I am the peasants' cock, not yours."
Der Fuchs bedauerte seine Dummheit sehr.
The fox very much regretted his stupidity.

Golden Rules 8, 9, and 10, page 1333

Helfen Sie mir ! *Help me !*
Ich bedaure Sie. *I pity you.*
Ich bewundre das Kleid, das Sie kopiert haben.
I admire the dress which you have copied.
Ich sammle illustrierte Zeitungen.
I collect illustrated newspapers.
Ich behandle ihn gut. *I treat him well.*
Behandeln Sie ihn gut ! *Treat him well !*

THE IMPERATIVE, THE NEGATIVE OF VERBS THEIR INTERROGATIVE USE

Ist mein altes Zimmer frei ?
Is my old room free ?
Nein, Sie wohnen nicht in Ihrem alten Zimmer.
No, you are not staying in your old room.
Haben Sie Briefe für mich ?
Have you letters for me ?
Geben Sie sie mir.
Give them to me.
Ist Ihre Tochter mit Ihnen gekommen ?
Has your daughter come with you ?
Nein, ich habe sie nicht mitgebracht.
No, I did not bring her with me.
Schicken Sie die Tasche auf mein Zimmer.
Send the bag to my room.
Sie ist nicht schwer.
It is not heavy.
Lassen Sie uns jetzt in die Stadt gehen !
Let us go to town now.
Sind Sie nicht zu müde ? Nein, also gehen wir !
Are you not too tired ? No, let us go then.

PASSIVE OF VERBS

Der Garten des Nachbars ist verkauft worden.
The neighbour's garden has been sold.
Es wird ein Hotel darin gebaut werden.
A hotel will be built in it.
Man sagt, der Käufer sei sehr reich.
It is said that the buyer is very rich.
Es werden viele Geschichten über ihn erzählt.
Many stories are told about him.
Das Material zum Bauen ist schon gebracht worden.
The building material has already been brought.
Die Pläne des neuen Baus werden von meinen Freunden sehr gelobt.
The plans of the new building are very much praised by my friends.

REFLEXIVES

Wir setzten uns auf eine Bank.
We sat down on a bench.
Erinnern Sie sich daran ?
Do you remember ?
Wir haben uns sehr auf den Frühling gefreut.
We were very much looking forward to spring.
Fürchtet ihr Euch vor dem Winter ?
Are you afraid of winter ?
Ich wasche mich jeden Morgen um sieben.
I wash myself every morning at seven.
Ihr habt euch geirrt.
You have been mistaken.

LESSON 9

Exercises on Verbs: (2) Auxiliaries and Groups of "Strong" and "Weak" Verbs

This Lesson should widen the student's grasp of auxiliary verbs, impersonal and other forms, and phrases and tables arranged in seven groups illustrating the use of "strong" (irregular) verbs and a section of "weak" (regular) verbs. The seven groups refer to the lists of essential verbs given in Lesson 10, pages 1340-1343.

AUXILIARIES

Der König von Persien ging eines Abends, als einfacher Mann gekleidet,
One evening the king of Persia, dressed as an ordinary man, went
durch die Straßen der Stadt, um zu sehen, was die
through the streets of the city in order to see what his
Menschen taten. Sein Minister begleitete ihn. In einer
people were doing. His minister accompanied him. In a
Straße, in der die Ärmsten wohnten, hielten sie vor einer
street, where the poorest lived, they stopped in front of a
Tür. Durch ein Loch in der Tür sah er drei schöne Schwestern,
door. Through a hole in the door he saw three beautiful sisters,
die auf einem Bett saßen und miteinander redeten.
who were sitting on a bed and talking to each other.

Der König *konnte* **hören, was sie sagten. "Ich** *möchte* **am liebsten die Frau**
The king could hear what they said. "I should like best to be the wife
des Bäckers sein," sagte die Älteste, "der für den König und seine
of the baker," said the eldest, "who bakes the bread for the king and
Diener das Brot backt. Als seine Frau werde ich jeden Tag schönes weißes
his servants. As his wife I shall be allowed to eat beautiful white bread every day.
Brot essen dürfen. Ihr beide werdet vor Neid sterben." Die zweite sagte:
You will both die of envy." The second said:
"Ich *will lieber* **die Frau des Kochs werden, der die Speisen für den**
"I would rather become the wife of the cook, who cooks the meals for
König kocht. Das Brot *mag* **gut sein, aber alle Diener** *dürfen* **davon**
the king. The bread may be good, but all the servants are allowed to
essen. Die Speisen des Königs dagegen ißt nur der König und sein
eat from it. The king's food, however, is eaten by the king and his
Koch." Die Dritte der Schwestern aber, die jüngste und schönste
cook alone." The third of the sisters, however, the youngest and
sagte: "O meine Schwestern, mein Herz wünscht sich mehr.
most beautiful one, said: "O my sisters, my heart desires more.

Ich *mag* **kein weißes Brot: gute Speisen sind mir nicht wichtig.**
I dislike white bread; good meals are of no importance to me.
Ich *möchte*, **daß der König mich zur Frau nimmt; ich** *möchte* **die**
I should like the king to take me as his wife; I should like
Mutter eines Sohnes werden, der stolz und mutig ist, und dessen
to become the mother of a son, who is proud and courageous and
Haare wie Gold scheinen." Der König wunderte sich sehr
whose hair shines like gold." The king was very much surprised
und beschloß, die Wünsche zu erfüllen.
and decided to fulfil the wishes.
Am nächsten Morgen ging der Minister zu den Mädchen und sagte: "Ihr
Next morning the minister went to the girls and said: "You
sollt zum König kommen. Ihr *müßt* **ihm noch einmal erzählen, was Ihr**
shall come to the king. You must tell him once more what you
gestern gesprochen habt." Sie *wurden* **vor den König geführt und der König,**
were talking about yesterday." They were taken before the king and the king,
der die Schönheit der Jüngsten sah, begann sofort sie zu lieben und sagte:
who saw the beauty of the youngest one, fell in love with her immediately and said:
"Ihr *braucht* **euch nicht zu fürchten. Ihr** *dürft* **mir ehrlich sagen,**
"You need not be afraid. You may tell me honestly what
was Ihr wünscht." Dann erzählten sie ihm alles. Der König rief
you want." Then they told him everything. The king called
den Koch und den Bäcker. Sie *mußten* **die beiden älteren**
for the cook and the baker. They had to take the elder
Schwestern zur Frau nehmen. Er selbst machte die Jüngste zur
sisters as their wives. He himself made the youngest the
Königin des Landes.
Queen of the land.

IMPERSONAL VERBS

Wie geht es Ihnen? *How are you?*
Danke, es geht mir gut und es gefällt mir hier sehr gut.
Thank you, I am well and I like it here very much.
Es hat gestern geschneit und heute regnet es, aber es fehlt nie an netter Gesellschaft und es gibt gutes Essen im Hotel.
It snowed yesterday, and it is raining to-day, but I am never short of pleasant company and the food is good in the hotel.
Es tut mir nur leid, daß Sie nicht hier sind.
I am only sorry that you are not here.

SEPARABLE AND INSEPARABLE VERBS

Sein Einkommen ist größer geworden, denn er bekommt jetzt mehr Geld für seine Bücher.
His income has grown larger, for he now gets more money for his books.
Er kann abends mit uns ausgehen und er kann häufiger verreisen.
He can go out with us in the evening and he can travel more frequently.

Er geht am Morgen weg und kommt abends zurück.
He goes away in the morning and comes back in the evening.
Er wird seine Adresse hinterlassen : seine Briefe werden ihm nachgeschickt.
He will leave his address ; his letters will be forwarded.
Er weiß nicht, wann er zurückkommt.
He does not know when he will come back.
Tacitus faßt in seinem Buch " Germania " die ersten Nachrichten über die Germanen zusammen.
Tacitus summarises the earliest information about the Teutonic people in his book " Germania."
Er wiederholt manche Tatsachen, die Cäsar vor ihm herausgefunden hat, und widerspricht manchen.
He repeats some facts which Caesar found out before him, and denies some of them.
Er stellt die Deutschen als ein Volk dar, das den Krieg liebt.
He represents the Germans as a people who love war.
Ihre Fürsten führen sie in der Schlacht an.
Their princes lead them in the battle.
Im Frieden spielen und trinken die alten Deutschen viel, oder sie ruhen sich am Feuer aus.
In peace time the ancient Germans gamble and drink a great deal, or rest near the fire.

WISSEN AND KENNEN, AND VERBS CONJUGATED WITH SEIN

Seine Schwester ist gestorben.
His sister has died.
Haben Sie sie gekannt ?
Did you know her ?
Ja, ich bin sehr erschrocken, und bin nicht auf dem Land geblieben, sondern gleich zurückgekommen.
Yes, I was very alarmed and did not stay in the country but came back immediately.
Es ist mir gelungen, ihn zu trösten.
I succeeded in comforting him.
Seine Mutter ist angekommen.
His mother has arrived.
Sie weiß noch nicht, was geschehen ist.
She does not yet know what happened.
Sie ist mit dem Zug gereist.
She travelled by train.
Es ist schwer für sie gewesen.
It has been difficult for her.

STRONG VERBS

GROUP I

Mund und Magen stritten sich mit den Gliedern.
The mouth and the belly quarrelled with the limbs.
Die Beine und Arme klagten, weil der Mund und der Magen alle
The legs and the arms complained because the mouth and the
Speisen bekamen. Sie aber arbeiteten und
belly got all the meals. They, however, worked and
bekamen nichts zu beißen. Also schnitten die Hände kein Brot mehr
got nothing to bite. So the hands no longer cut any bread
und gaben dem Magen und Mund nichts zu essen. Der Mund
and gave the belly and the mouth nothing to eat. The mouth
schrie zuerst, aber dann fühlten sich Magen und Mund schwach
cried at first, but then the belly and the mouth felt weak and
und schwiegen. Nach einigen Tagen begannen die Glieder
they became silent. After some days the limbs
auch zu leiden und fühlten sich schwach. Jetzt wußten sie, daß
began to suffer too and they felt weak. Now they knew that
keiner ohne den anderen leben konnte. Dann gaben sie dem
none of them could live without the other. So they gave their
Mund und dem Magen ihre Speisen. Bald ging es allen wider gut
meals to the belly and the mouth. Soon all of them were well again
und seitdem sind sie Freunde geblieben.
and after that time they remained friends.

GROUP II

Wir genießen den warmen Abend im Garten. Es ist nicht heiß und
We enjoy the warm evening in the garden. It is not hot, so
man muß die Blumen nicht begießen. Er hebt einen
there is no need to water the flowers. He lifts an
Apfel von der Erde auf und bietet ihn mir an. Dann schließt
apple from the ground and offers it to me. Then he shuts
er das Gartentor. Laßt uns den Schlüssel nicht verlieren. Der
the door of the garden. Let us not lose the key. The
Nachbar schießt nach einem Vogel, aber er fliegt davon.
neighbour shoots at a bird but it flies away.

(Now form past tense of the verbs in the preceding sentences.)

GROUPS III and IV

Eines der berühmtesten Gedichte Schillers heißt : " Der Taucher."
One of Schiller's most famous poems is called " The Diver."
Ein König wirft einen goldenen Becher ins Meer und befiehlt
A king throws a golden cup into the sea and requests
seinen Dienern ins Meer zu springen. Er verspricht den
his servants to jump into the sea. He promises the cup
Becher dem, der den Becher wiederfindet und ihm von den
to him who finds the cup again and tells him of the
Wundern des Meeres erzählt. Der Becher versinkt im Meer
wonders of the sea. The cup sinks into the sea
und verschwindet. Aber keiner folgt dem Befehl. Endlich findet
and disappears. But nobody obeys the order. Finally a
sich ein Jüngling : er springt ins Meer. Alle erschrecken und
youth is found—he jumps into the sea. Everybody is frightened and
bewundern seinen Mut. Der Jüngling kommt auf den Grund des Meeres,
admires his courage. The youth gets to the bottom of the sea,
findet den Becher und schwimmt ans Land. Nun beschreibt das Gedicht,
finds the cup, and swims to the shore. Now the poem describes
was der Jüngling unten gesehen hat. Aber der König
what the youth saw below. But the king
ist nicht zufrieden, nimmt einen Ring, wirft ihn ins Wasser und
is not satisfied, takes a ring, throws it into the water, and
verspricht dem Jüngling seine Tochter, wenn es ihm gelingt, den
promises his daughter to the youth if he succeeds in finding
Ring wieder zu finden. Um sie zu gewinnen, springt er noch
the ring again. In order to win her, he dives into the sea
einmal ins Meer und verschwindet für immer.
once more and disappears for ever.

Which of these verbs belong to Group III and which belong to Group IV ? Tell the story in the past tense.

GROUP V

Er gab mir seine Karte und bat mich, ihn zu besuchen. Wir
He gave me his card and asked me to visit him. We
aßen zusammen. Er gab uns guten Wein. Er besaß schöne Bilder.
ate together. He gave us some good wine. He possessed beautiful pictures.
Wir saßen im Haus und lasen die Zeitungen, um zu
We sat in the house reading the newspapers,
wissen, was geschehen war. Wir vergaßen, daß es spät wurde.
in order to know what had happened. We forgot that it was getting late.

GROUPS VI and VII

Die Sportsleute aller Länder sind zu den olympischen Spielen geladen worden.
Sportsmen of all countries were invited to the Olympic games.
Hier hat Norwegen die Schweiz geschlagen. Die Norweger liefen
Here Norway beat Switzerland. The Norwegians were better
besser Ski. Nach ihrem Sieg fuhren sie nach Hause zurück.
at ski-ing. After their victory they went back home.
Er wurde lange gefangen gehalten, dann ließ man ihn laufen.
He was kept a prisoner for a long time, then they let him go.

IRREGULAR VERBS

Rätsel (Aus den Märchen der Brüder Grimm).
Riddle (From the folk-lore collection of the brothers Grimm).
Drei Frauen waren in Blumen verwandelt, die auf dem Felde standen.
Three women had been turned into flowers which stood in the field.
aber eine durfte des Nachts im Hause sein.
One, however, was allowed to be in the house during the night.
Da sagte sie zu ihrem Mann, als der Tag kam und sie wieder aufs Feld gehen und
So she said to her husband, when the day had come and she had to go to
eine Blume werden mußte : " Wenn du heute
the field again to become a flower : " If
morgen kommst und mich brichst, werde ich immer bei dir bleiben."
you will come this morning and pick me, I shall stay with you for ever."
Dies geschah. Nun denkt nach : " Wie hat ihr Mann
So it happened. Now think it over : " How did her
sie erkannt, da eine der Blumen gleich der andern war ? "
husband recognize her, as one flower was like another ? "
Antwort : Weil sie die ganze Nacht in ihrem Haus
Answer : Because she was all the night in her
und nicht auf dem Feld war, war sie nicht naß vom Morgennebel
house and not in the field, she was not wet from the morning mist
wie die andern.
like the others.

LESSON 10

The Essential "Strong" and Irregular Verbs

When the student has become thoroughly familiar with what has gone before, the "strong" and irregular verbs can be attacked. The easiest way to learn them is by the groups into which they fall by the changes of the root vowel in past tense and past participle. Owing to the familiar "sound" of many of these verbs and their changes to English ears, they can be learnt surprisingly quickly.

The lists here given are for reference now—to be learnt little by little as the essential verbs given later in the Lesson are learnt.

Only the principal parts of each verb are given—infinitive, past tense, and past participle—as from them *all* other parts can be formed by means of the " Golden Rules."

Most strong verbs modify root-vowels, **a**, **o**, **au**, in the second and third persons singular of the present indicative ; **ich fahre, du fährst, er fährt.** Nearly all strong verbs with root-vowel **e** in Groups II, IV and V take **i or ie** in the second and third persons singular of the present indicative and the second person singular of the imperative : **ich sterbe, du stirbst, er stirbt.** The second person singular imperative of such verbs does not take the ending **e** : **stirb. Werden** forms **ich werde, du wirst, er wird.**

GROUP I

Infinitive	*Past Tense*	*Past Part.*	*English*
ei	**i, ie**	**i, ie**	
beißen[1]	**biß**	**gebissen**	*bite, bit, bitten*
***bleiben**	**blieb**	**geblieben**	*remain*
gleichen	**glich**	**geglichen**	*be like, to equa*
***gleiten**	**glitt**	**geglitten**	*glide*
greifen	**griff**	**gegriffen**	*seize*
leihen	**lieh**	**geliehen**	*lend*
†reißen	**riß**	**gerissen**	*tear*
scheinen	**schien**	**geschienen**	*shine, seem*
schreien	**schrie**	**geschrieen**	*cry, shout*
schreiben	**schrieb**	**geschrieben**	*write*
schweigen	**schwieg**	**geschwiegen**	*be silent*
***steigen**	**stieg**	**gestiegen**	*mount*
streichen	**strich**	**gestrichen**	*stroke, paint*
streiten	**stritt**	**gestritten**	*quarrel*
†treiben	**trieb**	**getrieben**	*drive, drift*
leiden	**litt**	**gelitten**	*suffer*
schneiden	**schnitt**	**geschnitten**	*cut*
Note :			
heißen	**hieß**	**geheißen**	*call, be called*

[1] All in Group I, take i before ch, f, ß (ss), t ; otherwise ie.
† reißen, when impersonal, and treiben in the sense of *drift*, ake sein.

GROUP II

Infinitive	*Past Tense*	*Past Part.*	*English*
ie, etc.	**o**	**o**	
biegen	**bog**	**gebogen**	*bend*
bieten	**bot**	**geboten**	*bid, offer*
***fliegen**	**flog**	**geflogen**	*fly*
***fliehen**	**floh**	**geflohen**	*flee, escape*

* Verbs marked with an asterisk take SEIN to form their compound tenses.

Infinitive	Past Tense	Past Part.	English
ie, etc.	o	o	
*fließen	floß	geflossen	*flow*
genießen	genoß	genossen	*enjoy*
geißen	goß	gegossen	*pour*
riechen	roch	gerochen	*smell, reek*
schieben	schob	geschoben	*shove, push*
schießen	schoß	geschossen	*shoot*
schließen	schloß	geschlossen	*shut, close*
verlieren	verlor	verloren	*lose*
ziehen	zog	gezogen	*draw, move*
lügen	log	gelogen	*tell a lie*
heben	hob	gehoben	*heave, raise, lift*

GROUP III

Infinitive	Past Tense	Past Part.	English
i	a	u	
binden	band	gebunden	*bind*
finden	fand	gefunden	*find*
*gelingen	gelang	gelungen	*succeed*
klingen	klang	geklungen	*sound (clink)*
*(ver)schwinden	schwand	geschwunden	*disappear*
singen	sang	gesungen	*sing*
*sinken	sank	gesunken	*sink*
*springen	sprang	gesprungen	*spring, jump*
trinken	trank	getrunken	*drink*
zwingen	zwang	gezwungen	*force*

GROUP IV

Infinitive	Past Tense	Past Part.	English
e	a	o	
befehlen(ie)[1]	befahl	befohlen	*command, order*
brechen(i)	brach	gebrochen	*break*
*erschrecken(i)	erschrak	erschrocken	*be frightened*

Erschrecken, *to frighten someone*, is a weak verb.

helfen(i)	half	geholfen	*help*
nehmen (i)	nahm	genommen	*take*
sprechen (i)	sprach	gesprochen	*speak*
stehlen (ie)	stahl	gestohlen	*steal*
*sterben (i)	starb	gestorben	*die*
treffen(i)	traf	getroffen	*hit, meet*
*werden(i)	wurde	geworden	*become*
werfen(i)	warf	geworfen	*throw*
*kommen	kam	gekommen	*come*
beginnen	begann	begonnen	*begin*
gewinnen	gewann	gewonnen	*win*
schwimmen	schwamm	geschwommen	*swim*

[1] The vowels in brackets in this and succeeding Groups after the infinitive represent second and third persons singular of the present.

GROUP V

Infinitive	Past Tense	Past Part.	English
e	a	e	
essen(i)	aß	gegessen	*eat*
geben(i)	gab	gegeben	*give*
*geschehen(ie)	geschah	geschehen	*happen*
lesen(ie)	las	gelesen	*read*
messen(i)	maß	gemessen	*measure*
sehen(ie)	sah	gesehen	*see*
*treten(i)	trat	getreten	*tread, step*

Treten, in the sense of *to kick* takes **haben**; **du trittst, er tritt; tritt.**

vergessen(i)	vergaß	vergessen	*forget*
bitten	bat	gebeten	*ask, beg*
sitzen	saß	gesessen	*sit*
besitzen	besaß	besessen	*possess, enjoy*
liegen	lag	gelegen	*lie*

GROUP VI

Infinitive	Past Tense	Past Part.	English
a	u	a	
*fahren	fuhr	gefahren	*drive (fare)*
graben	grub	gegraben	*dig (grave)*
laden	lud	geladen	*load*
schaffen	schuf	geschaffen	*create (shape)*

Schaffen, *to procure, get, work*, is a weak verb.

schlagen	schlug	geschlagen	*beat, strike (slay)*
tragen	trug	getragen	*carry, wear, bear*
*wachsen	wuchs	gewachsen	*grow*
waschen	wusch	gewaschen	*wash*

GROUP VII

Infinitive	Past Tense	Past Part.	English
a	ie	a	
*fallen	fiel	gefallen	*fall*
fangen	fing	gefangen	*catch, seize*
halten	hielt	gehalten	*hold, keep*
hangen	hing	gehangen	*hang, be hanging*

Hängen, hängte, gehängt: *to hang something.*

lassen	ließ	gelassen	*let, leave*
raten	riet	geraten	*advise, guess*
schlafen	schlief	geschlafen	*sleep*

Infinitive	Past Tense	Past Part.	English
au	ie	au	
*laufen	lief	gelaufen	*run*
o	ie	o	
stossen	stieß	gestossen	*push*
u	ie	u	
rufen	rief	gerufen	*call*

IRREGULAR STRONG VERBS

Infinitive	Past Tense	Past Part.	English
*gehen	ging	gegangen	*go*
stehen	stand	gestanden	*stand*
tun	tat	getan	*do*

IRREGULAR WEAK VERBS

Infinitive	Past Tense	Past Part.	English
brennen	brannte	gebrannt	*burn*
kennen	kannte	gekannt	*know*
nennen	nannte	genannt	*call, tell*
*rennen	rannte	gerannt	*run*
senden	sandte	gesandt	*send*
wenden	wandte	gewandt	*turn*

Wenden, wendete, gewendet, *to turn something.*

bringen	brachte	gebracht	*bring*
denken	dachte	gedacht	*think*

VOCABULARY OF ESSENTIAL VERBS

Many German verbs resemble their English equivalents but for different sound or spelling of the "root" vowel. The similarity becomes obvious when they are read aloud. These similar verbs should be learnt first, and each verb conjugated in accordance with its model—the "weak" verbs follow **Loben** (*see* page 1335); and for the "strong"—see essential "strong" and irregular verbs in the list given for reference above.

* Verbs marked with an asterisk take **SEIN** to form their compound tenses.

Section 1. VERBS SIMILAR IN GERMAN AND ENGLISH

(a) "Weak" Conjugation

backen, to bake
baden, to bathe
[illegible]
bringen, to bring

danken, to thank
denken, to think

enden, to end
erreichen, to reach, attain

folgen, [illegible]
[illegible]
füllen, to fill
fürchten, to fear

haben, [illegible]
[illegible]
hoffen, to hope
hören, to hear

[illegible]

kochen, [illegible] boil
[illegible]
[illegible]

lachen, [illegible]
leben, [illegible]

leiten, to [illegible]
lernen, to learn
lieben, [illegible]
lösen, to loosen, solve

machen, to make
meinen, to mean
merken, to mark, note
mögen, may
müssen, must

öffnen, [illegible]

rechnen, to reckon, [illegible]
reichen, to reach, to [illegible]
*rennen, [illegible]

sagen, to say
senden, to send
setzen, to set, put (sich setzen [illegible])
sollen, [illegible] (to be said to)
sparen, to spare, [illegible]
stecken, [illegible], put
[illegible]
strecken, [illegible]

suchen, to seek, search

tanzen, to dance

wachen, to be awake, to watch
wundern (sich), to wonder

(b) "Strong" Conjugation

beginnen, to begin
beißen, to bite
bieten, to bid, offer
binden, to bind
brechen, to break

essen, to eat

*fallen, to fall
finden, to find
*fliegen, [illegible]
*fliehen, to fly, escape

geben, to give

halten, to hold
hangen, to hang
helfen, to help

*kommen, to come

laden, to load
lassen, to let, leave
liegen, to lie (situation)
lügen, to lie (untruth)

messen, to measure

scheinen, to shine, to [illegible]
schießen, to shoot
schlafen, to sleep
schwimmen, to swim
sehen, to see, look
singen, to sing
*sinken, to sink
sitzen, to sit
*springen, to [illegible]
stehen, to stand
stehlen, to steal
streichen, [illegible] stroke, [illegible] erase, strike out

*treten, to tread, step
trinken, to drink

verbieten, to forbid
vergessen, to forget

waschen, to wash
wiegen, to weigh [also [illegible] rock (a cradle)]

Section 2. VERBS DISSIMILAR IN THE TWO LANGUAGES

(a) "Weak" Conjugation

[illegible]

drucken, to print
drücken, to press
dulden, to tolerate
dürfen, may, must (not)

*eilen, to haste, [illegible]
[illegible]
erfüllen, to [illegible]
[illegible]
erkennen, to [illegible]
erklären, to [illegible]
erlangen, to get, obtain
erlauben, to permit
erschrecken, to frighten [illegible]
erzählen, [illegible]

fassen, [illegible]
[illegible]

gehören, [illegible]
[illegible]

[illegible]

kämpfen, to fight

kaufen, to buy
kennen, to know, be acquainted with
klagen, to complain, lament
kleiden, to dress

legen (something), to put
sich legen, to lie down
lehren, to teach
leisten, to do, afford, achieve
loben, to praise

melden, [illegible]

nähren, [illegible]
[illegible]
nützen, to be of use

ordnen, to arrange

reden, to speak
*reisen, [illegible]
retten, [illegible]
richten, to direct, to judge
ruhen, to rest

sammeln, to collect
schaden, to harm
schaffen, to do, get; [illegible]
[illegible]
schicken, to send
[illegible]
sorgen, to care, trouble
[illegible]
spielen, to play
stammen, to derive
stellen, to put, place, set
stören, to disturb

stützen, to support

teilen, to divide
töten, to kill
trennen, to separate
trocknen, to dry
trösten, to comfort
tun, to do

üben, to practise
überzeugen, to convince
untersuchen, to examine, inquire
urteilen, to judge

verdienen, to deserve, earn
verkaufen, to sell
[illegible]
verlassen, to leave
verletzen, to injure, hurt
versichern, to assure, [illegible]
verstecken, to hide
versuchen, to attempt, [illegible]
[illegible]
[illegible]
verzichten, to renounce

wagen, to risk, dare
wählen, to choose, elect
warten, to wait
wechseln, to change
[illegible]
wissen, to know
wohnen, to dwell, live
[illegible]
wünschen, to wish, desire

zahlen, to pay
zählen, to count
zeigen, to show
*zurückkehren, to return

* [illegible]

Section 2. VERBS DISSIMILAR IN THE TWO LANGUAGES [illegible]

"STRONG" CONJUGATION

abhängen, to depend
[illegible]
angreifen, to attack
[illegible]
*auf[illegible]

befehlen [illegible]

begreifen, to understand
[illegible]

[illegible]
bitten, [illegible]
*bleiben, to remain

[illegible]

*entstehen, to arise
erfahren, [illegible]
erfinden, to invent
erhalten [illegible]
*erscheinen [illegible]
*erschrecken, [illegible]
erwerben, to gain, acquire
erziehen, to educate

*fahren, [illegible]
fangen [illegible]
*fließen, [illegible]

gefallen, to please
*gehen, to go, walk
*gelingen [illegible]
[illegible]
*geschehen, [illegible]
gewinnen [illegible]
gießen, to pour
gleichen, [illegible]
graben [illegible]
greifen, to seize

heben, [illegible]
heißen, [illegible]

klingen, to sound

*laufen, [illegible]
leiden, to suffer
leihen, [illegible]
lesen, [illegible]

nehmen, to take, zu sich nehmen [illegible]

raten [illegible]
[illegible]
riechen [illegible]
rufen [illegible]

schieben [illegible]
schlagen [illegible]
schließen [illegible]
schneiden [illegible]
schreiben, to write
schreien [illegible]
schweigen, [illegible]
*sein, to be
sprechen, to speak
*steigen [illegible]
*sterben, to die
stoßen, [illegible]

streiten, to fight, quarrel

tragen [illegible]
treffen [illegible]
[illegible]
*treiben [illegible]

verbinden [illegible]
vergleichen, to compare
verlieren, [illegible]
*verschwinden [illegible]
versprechen [illegible]
verstehen [illegible]
verzeihen [illegible]
vorschlagen, [illegible]
vorziehen [illegible]

*wachsen [illegible]
*werden [illegible]
werfen [illegible]

ziehen [illegible]
zwingen [illegible]

* Verbs marked with [illegible] SEIN [illegible]

LESSON II

Invariable Words

Invariable words are not inflected. They [illegible] Adverbs, Prepositions and Con[illegible]. The rules for them are given [illegible] appropriate exercises and lists of [illegible] of the three classes.

Adverbs. An adverb is a word used to qualify any part of speech except a noun or pronoun.

[illegible] Most German adjectives [illegible] same as that of adjectives (-er), but the super[illegible] an dem [illegible] Am schönsten, [illegible] aufs schönste, or schönstens)

Irregular Comparison

Bald [illegible]	Eher, sooner	Am ehesten, soonest
[illegible]	[illegible]	Höchst, [illegible]
Viel [illegible]	Mehr [illegible]	Am meisten [illegible]
Wohl [illegible]	Besser [illegible]	Am besten [illegible]
Gern [illegible]	Lieber [illegible]	Am liebsten [illegible]
Wenig [illegible]	Weniger [illegible]	Am wenigsten [illegible]
	Minder [illegible]	Am mindesten [illegible]

[illegible] EHESTENS, HÖCHSTENS, MEISTENS [illegible]

The adverb never comes between subject and [illegible] England.

Noch [illegible] Geben Sie mir noch Wasser, [illegible] Sind Sie noch hier? *Are you still here?*

[illegible] viel for quantity and sehr for intensity: viel Wein, *much wine*; sehr beliebt, *much liked*.

Gern [illegible] Ich spreche gern deutsch, *I like to speak German*. Ich spiele lieber, *I prefer to play*, *etc.*

Prepositions. A preposition is a word placed before a noun [illegible]

The chief [illegible] of German pre[illegible] governed by the preposition.

Prepositions with the Genitive

Anstatt, statt, [illegible] diesseits, [illegible] jenseits, [illegible] längs, [illegible] trotz, *in spite of*; während, [illegible] wegen, [illegible]

All these except während take the genitive [illegible] Trotz des Regens, *in spite of the rain*; während des Tages, *during the day*, etc.

Prepositions with the Dative

Aus, *out of*; ausser, [illegible] bei, [illegible] mit, [illegible] nach, [illegible] seit, [illegible] von, *from*, [illegible]; zu, *to*, *at*

A compound of **gegen** and **über** is made; **gegenüber,** *opposite*. It governs the Dative.

Bei is used for: "*at the house of*" . . . **Er wohnt bei uns,** *He lives with us.*

NOTE.—**Zu Hause,** *at home*; **zu Fuss,** *on foot*; **zu Mittag,** *at noon*; **Wasser zum trinken,** *water for drinking*. Also: **Nach Hause,** *homewards*.

Prepositions with the Accusative

Bis, *up to, as far as, until*; **durch,** *through*; **für,** *for* (in view of); **gegen,** *towards, against*; **ohne,** *without* (not possessing); **um,** *round*; **wider,** *contrary to, against*.

Bis jetzt, *until now*; **bis morgen,** *until tomorrow*; **um den Tisch,** *around the table*; **für mich,** *for me*; **um halb vier,** *at half-past three*.

With Dative or Accusative

The following take the dative to indicate *rest*, the accusative to indicate *motion*:

An, *on, at, by*; **auf,** *on, up, to*; **hinter,** *behind*; **in,** *in, into*; **neben,** *beside*; **über,** *over, beyond*; **unter,** *below, among*; **vor,** *in front of, before*; **zwischen,** *between*.

Thus: **er sitzt an dem (or am) Tische,** *He is sitting at the table*; **Er setzt sich an den Tisch,** *He sits down to the table*.

NOTE.—**Auf deutsch,** *in German*; **auf einige Tage,** *for a few days*; **unter sechs Monaten,** *in less than six months*; **unter allen Menschen,** *among all men*; **vor acht Tagen,** *a week ago*.

Conjunctions. Conjunctions are words used for connecting words or sentences. They can be divided into two classes:

I. Those which do not affect the construction of a sentence.

II. Those which affect it.

I. The following conjunctions do *not* affect construction: **aber,** *but*; **denn,** *for*; **oder,** *or*; **sondern,** *but* (after a negative); **sowohl . . . als auch,** *as well as, both . . . and*; **und,** *and*.

The difference between **aber** (or **allein**) and **sondern** should be noted; **aber** should always be used for *but*, except when there is a contrast to be expressed following a negative statement. Thus: **Es ist nicht die Schwester sondern die Tochter,** *It is not the sister but the daughter*.

II. The following send the verb to the end of the clause:

als, *when*	**nachdem,** *after*
bevor / **ehe** } *before*	**ob,** *if, whether*
bis, *until*	**obgleich** / **obwohl** } *although*
da, *as* (a reason)	**seit,** *since* (time)
damit, *in order that*	**so oft (als),** *as often (as)*
damit nicht, *lest*	**so lange (als),** *as long (as)*
daß, *that*	**während,** *whilst*
falls, *in case*	**weil,** *because*
indem, *whilst, by*	**wenn,** *when, if*

Warten sie *bis* **ich fertig** *bin*, *Wait until I am ready*.

Wann, *when*: used in reference to time only: **Kommen Sie wann Sie wollen,** *Come when you wish*.

Wenn, *when, whenever*: usually denotes action on more than one occasion: **Wenn ich ihn sah,** *When (whenever) I saw him*. But **wenn** also stands for *if*: **Wenn sie wollen,** *If you wish*.

Als, *when*, refers to a definite past occasion: **Als ich den Brief schrieb,** *When I wrote the letter*. Also for *but* after **nichts,** *nothing*; **Nichts als Wein,** *Nothing but wine*.

Note also: **Weder . . . noch,** *neither . . . nor*; **entweder . . . oder,** *either . . . or*.

EXERCISE ON INVARIABLE WORDS

Adverbs

In der Schweiz werden noch zwei Sprachen außer deutsch gesprochen.
In Switzerland two more languages are spoken besides German.
Im Süden, spricht man meistens Italienisch, im Westen
In the South they speak mostly Italian, in the West
nur französisch. Bern ist der Sitz der Regierung.
only French. Berne is the seat of the government.
Hier kann man auf der Straße viel französisch und deutsch hören.
Here one can hear much French and German in the street.
Aber die Schweizer sprechen
But the Swiss people like
am liebsten ein besonderes deutsch—das Schwytzer Dütsch—
best to speak a special German—"Schwytzer-Dütsch"—
sie verstehen es viel besser als das Deutsch eines Berliners.
they understand it much better than the German of a man from Berlin.

Prepositions

Prag, die Hauptstadt der Tschechoslowakei, ist wegen ihrer
Prague, the chief town of Czechoslovakia, is famous for
Schönheit berühmt. Diesseits der Moldau liegt der neue Teil der
its beauty. On this side of the Moldau the new part
Stadt, jenseits des Flusses die alte Stadt.
of the city is situated, on that side of the river the ancient town.
Längs des Flusses stehen schöne alte Häuser. Wenn man
Along the river stand beautiful old houses. If one
zu Fuß über die Karls-Brücke geht, sieht man
goes on foot over the Charles bridge, one notices
das alte Schloß auf einem Hügel. Es ist schwer, mit einem
the old castle on a hill. It is difficult to
Wagen durch die engen Straßen zu kommen.
come by car through the narrow streets.

Conjunctions

Jupiter und das Schaf (nach Äsop)
Jupiter and the Sheep (after Æsop)

Das Schaf mußte von den Tieren viel leiden. Deshalb ging es zu
The sheep had much to suffer from the other animals. Thus it
dem Gott Jupiter und bat ihn, ihm zu helfen. Jupiter sagte:
went to the God Jupiter and asked for help. Jupiter said:
"Ich weiß, daß du zu sanft für diese Welt bist. Ich werde dir
"I know that you are too gentle for this world. I shall give you

kräftige Zähne geben, damit du beißen kannst, wenn du angegrif[...]
strong teeth, so you can bite if you are attacked."
wirst." Aber das Schaf antwortete: " Nein, ich möchte nicht
But the sheep answered: " No. I do not want to become
so böse und wild werden wie Löwen und Tiger." " Wenn du das nicht
as bad and wild as the lions and the tigers are." " If you do not
willst," sagte Jupiter, " kann ich dir scharfe Nägel an den Füßen geben. Oder
want this," Jupiter said, " I can give you sharp nails on your feet. Or I shall
ich fülle deine Zähne mit Gift." " Nein," sagte das Schaf, " ich will
fill your teeth with poison." " No," said the sheep, " I do not want
nicht giftig sein wie die Schlangen, weil niemand die Schlangen
to become poisonous like the snakes, because nobody likes the
mag." " Solange du keinen Schaden tun kannst," sagte Jupiter,
snakes." " As long as you cannot do any harm," Jupiter said,
" fürchtet dich niemand." " Ist das wahr ? " fragte das Schaf.
" nobody will fear you." " Is this true ? " asked the sheep.
" Wer Böses tun kann, lernt bald, das Böse zu wollen. Wenn
" He who is able to do harm will soon want to do it. If
man nur die Wahl hat, entweder Schaden zu tun oder zu leiden,
one has only a choice between either doing harm or suffering,
will ich lieber bleiben, was ich bin." Als Jupiter das hörte,
I had rather remain what I am." When Jupiter heard this,
schwieg er.
he was silent.

VOCABULARY: LIST OF ESSENTIAL INVARIABLE WORDS

Section 1. ADVERBS, CONJUNCTIONS, INTERROGATIVES

ab, *off*
aber, *but*
allerdings, *of course*
als, *as, when, than*
also, *therefore*
auch, *also, too*
aufwärts, *upward (-s)*
außen, *outside*

bald, *soon*
beinahe, *nearly*
bevor, *before*
bisher, *till now*

da ! *there !*
da, *as*
daher, *therefore*
damals, *then, at that time*
damit, *with that, in order that*
dann, *then*
darum, *therefore*
daß, *that*
denn, *for*
deshalb, *therefore*
doch, *however, but*
dort, *there*

ehe, *before*
einander, *each other*
einst, *once*

erst, *first, only*
etwa, *about, perchance*
etwas, *some : -thing, -what*

falls, *in case*
fast, *almost*
ferner, *further*
fort, *away*

ganz, *quite*
genug, *enough*
gestern, *yesterday*

her, *hither, here*
heute, *to-day*
hier, *here*
hin, *there, thither*

immer, *always, ever*
immerhin, *still, yet*
indem, *while*
innen, *within*
inzwischen, *meanwhile*
irgendwo, *somewhere*

jedenfalls, *in any case*
jedoch, *however, yet*
jemals, *ever, at any time*
jetzt, *now*

kaum, *hardly*

kürzlich, *recently*

längst, *long since*
links, *on (to) the left*

morgen, *to-morrow*

nicht, *not*
nie (niemals), *never*
nirgends, nirgendwo, *nowhere*
noch, *still, yet*
nun, *now*
nur, *only*

ob, *whether, if*
oben, *above*
obwohl / **obgleich** } *although*
oder, *or*
oft, *often*

rechts, *on (to) the right*
rückwärts, *back*

schon, *already*
selbst, *even*
so, *so, as*
sofort, *at once*
sogar, *even*

sonst, *else, otherwise*
stets, *always*

überhaupt, *generally, at all especially*
übrigens, *by the way*
umsonst, *in vain*
und, *and*

vergebens, *in vain*
vielleicht, *perhaps*
voran, *in front*
voraus, *ahead, beforehand*
vorher, *previously*
vorn, *in front*
vorüber, *along, past*
vorwärts, *forward*

während, *while*
wann ? *when ?*
warum ? *why ?*
was ? *what ?*
weil, *because, since*
welcher ? *which ?*
wenn, *when, if*
wer ? *who ?*
wie, *how, as, like*
wieder, *again*
wo ? *where ?*

zurück, *back*

Section 2. PREPOSITIONS[1]

an, *on, at, by, to*
anstatt, *instead of*
auf, *on, in, at, by, to, of, up*
aus, *out, from, of, by, for, on*
außer, *except, besides*
außerhalb, *outside*

bei, *by, at, with, about, near, in, on*
bis, *up to, until, to*

durch, *through, by*

für, *for*

gegen, *against, about*

hinter, *behind*

in, *in*

längs, *along*[2]

mit, *with*

nach, *after, past*
neben, *beside, besides*

ohne, *without*

seit, *since*

statt, *instead of*

trotz, *in spite of*

über, *over, above, on*
um, *about, by, in order to*
unter, *under, below, among*
von, *from, of, by*

vor, *before, in front of, ago*

während, *during*
wegen, *because of, for*
wider, *against, contrary to*

zu, *to, at, in, on, too*
zwischen, *between*

[1] The list of prepositions is given here merely for convenience of learning, repetition, etc. They have all been covered in pages 1343 and 1344.

[2] längs shows lack of movement: *Die menge stand längs der Strasse.* The crowd stood along the street. *Entlang* is used for movement (Acc.).

Section 3. INTERJECTIONS, GREETINGS, AND MISCELLANEOUS

Ach! *Ah! Oh!*
Leider! *Unfortunately*
Pfui! *Fie! For shame!*
Hoch! *Hurrah!*
Achtung! [illegible]
Passen Sie auf! [illegible]
So! [illegible]
Bitte. *Please. [illegible] please*
Danke. *Thanks. [illegible]*
Guten Morgen, *Good morning.*
Guten Tag, *Good day, good [illegible]*
Guten Abend, *Good evening [illegible]*
Gute Nacht, *Good night [illegible]*
Entschuldigen Sie, *Excuse me*
Verzeihen Sie, *Pardon me*
Wie geht es Ihnen, *or* Wie gehts? *How are you?*
Gnädige Frau, *Madame*
Gnädiges Fräulein, *Miss.*
Herr, *Mr. (used with a name)*
Frau, *Mrs. (used with a name)*
Auf Wiedersehen, *Good bye*
Haben Sie die Güte . . . *Would you kindly*
Ach so, *I see*
Ja, *Yes*
Nein, *No*
Doch, *Yes (after a negative, or for emphasis).*
Ach wo! *Impossible!*
Da haben wir's! *That's done it!*
Gut! Sehr gut! *Good! Well done!*
Ausgezeichnet! *Excellent*
Na also! *There you are!*
Nanu? *What on earth . . .?*
Natürlich! *Of course!*
Stimmt! *O.K.!*
Umso besser! *So much the better!*
Wirklich? *Really?*
Auf und ab, *up and down.*
Dann und wann, *now and then.*
Hin und her, *to and fro.*
Weit und breit, *far and wide*
Noch einmal, *once again.*
Noch nicht, *not yet.*
Noch etwas? *Anything else?*
Noch ein . . . *Another . . .*
Von oben bis unten, *from top to bottom*
Ein paar, *a few [illegible] than two).*
Zehn bis fünfzehn, *from 10 to 15 (or between 10 and 15).*
Von zehn bis elf (Uhr), *from 10 to 11 o'clock.*
Vor allen Dingen, *first of all*
Darf ich Ihnen Herrn X vorstellen? *May I introduce Mr. X*
Allerdings! *Of course, indeed!*
Es tut mir leid. *I am sorry.*
Was bedeutet das? *What does that mean*

LESSON 12

Four Ways of Word Building

Its truly remarkable capacity for forming derivative and compound words is one of the chief characteristics of German. For the foreign learner it is at the same time a difficulty and an encouragement: a difficulty because of the large number of irregularities which can be mastered only by long experience; and an encouragement because of the wide vocabulary which rapidly comes within the bounds of comprehension.

When the grammar and the "Essential" or "root" vocabulary are known, and the principles of word formation mastered, the meaning of thousands of words can be conjectured with fair certainty, and of many other thousands with absolute confidence.

Take the compound word

Lebensversicherungsgesellschaft

Such a word may at first appear somewhat [illegible]. It need not be so. The first thing [illegible] long words is that [illegible] when [illegible] and an actual [illegible] English [illegible] "Life Insurance Company." Not [illegible]

LEBENS	[illegible] Das Leben [illegible]
VER	A [illegible]
SICHER	A [illegible]
UNGS	A [illegible]
GESELL	A [illegible] meaning *companion*, [illegible] *member*. (GE- is an inseparable prefix denoting a union or putting together.)
SCHAFT	A suffix corresponding to the English [illegible]

Clearly these words and particles strung together could hardly have any other meaning but "Life Insurance Company." And so,

Hamburg-Südamerikanische Dampfschiffahrtsgesellschaft

for "Hamburg South America Steamship Company."

The Four Ways of Word Building

There are, for practical purposes, four ways of making new words by deriving and compounding:

1. By changing the root vowel: **DIE FÄHRE**, *the ferry*, from **FAHREN** *to travel*.
2. By adding a suffix or ending: **KOSTBAR**, *costly, precious*, from **KOSTEN**, *to cost*, and the ending **-BAR**. **KOSTBARKEIT**, *costliness*.
3. By a Prefix: **DAS EINKOMMEN**, *income*, from **KOMMEN**, *to come*, with the prefix **EIN-**, exactly as in English.
4. By [illegible] two or more quite independent words: **DAS NEBELHORN**, *the fog-horn*.

One can see that these principles exist in English:

DIE HÖHE, *height*, from **HOCH**, *high*.
DIE GÜTE, *goodness*, from **GUT**, *good*.
DER BISS, *bit, bite*, from **BEISSEN**, *to bite*.
DAS SCHICKSAL, *fate*, from **SCHICKEN**, *to send*.
DIE FINSTERNIS, *darkness*, from **FINSTER**, *dark* and **-NIS** (*-ness*).

The list of examples could be extended indefinitely. The student should now glance at the sample group of words springing from **kommen**, given on pp. 1347-48.

It will be noticed that nouns can be formed from adjectives, verbs, etc., and vice versa; and that there is indeed great liberty in the formation of one part of speech from another or in compounding.

Prefixes Used in Word Building

All the inseparable prefixes are employed: be-, ent- (emp-), er- (ur-), ge-, hinter-, miss-, un-, ver-, and zer-. It is not necessary here to cover all the shades of meaning which these prefixes denote, but the following will give a good idea of their scope.

BE-	English be-: *to give or apply to, to provide with, to complete.* **BEKLEIDEN**, *to clothe*; **BERATEN**, *to "ply" with advice, to counsel*; **BELACHEN**, *to laugh at*; **BEDECKEN**, *to deck out, cover with.*
ENT-	(EMP-): *removal, or depriving,* also denotes transition, or a change of state. **ENTHAUPTEN**, *to behead*; **ENTSCHULDIGEN**, *to remove blame* (**Schuld**) *from*; **ENTFLAMMEN**, *to burst into flame.*
ER-	(UR-): *out, out from,* completion of a process, also original or primeval. **ERDENKEN**, *to think out, evolve*; **ERFINDEN**, *to find out, invent*; **URMENSCH**, *primeval man*; **URWALD**, *primeval forest*; **URALT**, *extremely ancient*; **ERFÜLLEN**, *to fulfil*; **ERHÖREN**, *to give a hearing to*; **ERKRANKEN**, *to fall ill*; **ERBLEICHEN**, *to grow pale.*
GE-	A collective. **GERINNEN**, *to run together, to curdle*; **GEFOLGE**, *the following, attendance.*
MISS-	English mis-, denotes failure, something wrong, and often, a perversion. **MISSBRAUCHEN**, *to misuse*; **MISSBILDUNG**, *deformity*; **MISSERFOLG**, *failure.*
UN-	English un-. **DER UNDANK**, *thanklessness*; **DAS UNGLÜCK**, *unhappiness*; **UNGLÜCKLICH**, *unhappy.*
VER-	English *for-*, away, negative; and also an intensification of an existing state. **BLÜHEN**, *to bloom*; **VERBLÜHEN**, *to fade*; **BIETEN**, *to bid, command*; **VERBIETEN**, *to forbid*; **LERNEN**, *to learn*; **VERLERNEN**, *to [illegible]*; **VERALTEN**, *to become antiquated, grow old*; **MENSCHLICH**, [illegible]; **VERMENSCHLICHEN**, *to humanise.*
HINTER-	*behind.* **HINTERLASSEN**, *to leave behind*; **HINTERGEHEN**, *to deceive, [illegible]*
WIDER-	[illegible] **WIDERSTEHEN**, [illegible] **WIDERSPRECHEN**, [illegible]
ZER-	*asunder*, with [illegible] **ZERGLIEDERN**, *to dismember*; **ZERREISSEN**, *to tear to pieces*; **ZERTRETEN**, *to crush*

In the same way many thousands of words are made with the separable prefixes: **aufstehen**, to get up; **herkommen**, to come hither; **zusammenkommen**, to come together. And so forth. The meaning of words made with these separable prefixes is much more easily recognized than of those made with the inseparable ones—because the meanings of the separable prefixes are more fixed and less vague.

Suffixes Used in Word Building

A number of suffixes are added to "root" words to make other words. Of these the following are the most important:

Suffix	*Gender*	*Meanings*	*Examples*
-EI	F.	(a) abstract quality (b) place	**KINDEREI**, [illegible]; **BUCHBINDEREI**, *bookbindery*; **BÄCKEREI**, *bakery*
-LEIN, -CHEN	N.	Diminutives	**KINDLEIN**, *little child*; **MÄNNCHEN**, *manikin*
-LING	M.	-ling	**FINDLING**, *foundling*
-NIS	F. or N.	-ness	**FINSTERNIS**, *darkness*
-IN	F.	-ess	**GÖTTIN**, *goddess*
-UNG	F.	-ing	**SCHULUNG**, *schooling*
-ER	M.	-er	**BERLINER**, **LONDONER**
*-HEIT	F.	-hood, -head	**KINDHEIT**, *childhood*
*-KEIT	F.	-ness, etc.	**DANKBARKEIT**, *thankfulness*
*-SCHAFT	F.	-ship, etc.	**FREUNDSCHAFT**, *friendship*
*-TUM	M. or N.	-dom	**KÖNIGTUM**, *kingdom*

*Form abstract nouns.

Note the adjectival endings:

-LICH:	English -ly, -like	**TÄGLICH**, *daily*
-IG:	" -y.	**BLUTIG**, *bloody*
-ISCH:	" -ish.	**KINDISCH**, *childish*
-BAR:	" -able, -ible, -ful.	**ESSBAR**, *edible*

It should be clearly understood that the above statement gives only the basic and essential principles of the German system of word building and not an exhaustive treatment for which a treatise would be required. What is given is intended chiefly as a demonstration of the extremely convenient and flexible methods used in the making of vocabulary. But with what has been given and the Essential Vocabulary of these lessons, the student has a very remarkable instrument at his disposal for reading purposes. To guess at the meaning of a word is often to court disaster. Yet the percentage of words of which the meaning can be correctly worked out when the above principles are known is considerable. Always go for the "root" or "roots" of a compound word. Then take it to pieces and it will generally become clear.

Example of an Essential Word and its "Group" for Reference

To illustrate further the very important principle stated, some of the derivatives and compounds of the "Essential" verb **kommen** are given below. Again, this is not an exhaustive statement, merely an illustrative one.

KOMMEN, [illegible] **KAM**, **GEKOMMEN**.

Nouns: **DAS EINKOMMEN**, *income*; **DER NACHKOMME**, *descendant*; **DIE UNTERKUNFT**, [illegible]; **DIE NACHKOMMENSCHAFT**, [illegible]; **DAS VORKOMMNIS**, [illegible]; **DIE ZUVORKOMMENHEIT**, [illegible]; **DAS WILLKOMMEN**, [illegible]; **DIE ÜBERKUNFT**, [illegible]; **DIE ABKUNFT**, [illegible]; **DIE ZUKUNFT**, [illegible]; **EINKUNFT**, [illegible]; **DER ZUKÜNFTIGE**, [illegible]; **DIE ANKUNFT**, [illegible]; **DAS HERKOMMEN**, *tradition*.

Verbs: ANKOMMEN, arrive ; BEKOMMEN, come by, obtain ; ENTKOMMEN, escape ; ENTGEGENKOMMEN, meet, treat kindly ; ÜBEREINKOMMEN, agree ; UNTERKOMMEN, find shelter, obtain a position ; VERKOMMEN, to go to the bad, to the dogs ; ZUVORKOMMEN, be beforehand, prevent.

Adjectives and Adverbs: WILLKOMMEN, welcome ; ABKÖMMLICH, disengaged ; HERKÖMMLICH, traditional ; ZUKÜNFTIG, future.

Learning a Vocabulary

You would be well advised to keep a record of similar groups of related words as you progress in the language. In your notebook enter new words under the " root " or basic word. Better do it this way than attempt to master in a casual manner the vocabulary of so rich a language as German.

Frisch begonnen, halb gewonnen !

Try to trace through a dictionary the " word families " of **binden, geben, steigen, decken** and other essential verbs.

Order of Words in a German Sentence

The student is advised to be content at first to form only sentences of a simple, straightforward nature, and to avoid grammatical subtleties. Six rules, all of them of vital importance, must be learnt.

I. Infinitives and past participles in compound tenses are placed at the *end*. When there is both an infinitive and past participle, the infinitive stands last :

Ich habe einen Apfel gegessen, *I have eaten an apple* ; **Ich werde den Apfel gegessen haben,** *I shall have eaten the apple.*

II. Dative noun precedes accusative, but accusative pronoun precedes dative.

Der Vater hat dem Kinde ein Buch gegeben, *The father has given the child a book* ; **Er gab es ihm,** *He gave it him.*

III. A dative pronoun precedes an accusative noun.

Der Vater gab ihm ein Buch, *The father gave him a book.*

IV. Adverbs of time follow a simple tense or an auxiliary, but pronouns *precede* them.

Der Vater hat gestern dem Kind ein Buch gegeben, *The father has given a book to the child yesterday* ; **Der Vater gab ihm gestern ein Buch,** *The father gave him a book yesterday.*

V. Adverbs are placed in order of TIME, MANNER, PLACE.

Er kam gestern müde nach Hause, *He came home tired yesterday.*

VI. **NICHT** precedes the word it negatives but follows an accusative, an adverb of TIME, or a simple verb.

Das Kind kann nicht alles machen, *The child cannot do everything* ; **Das Kind sah den Vater nicht,** *The child did not see the father* ; **Ich habe das Kind lange nicht gesehen,** *I have not seen the child for a long time* ; **Ich weiss nicht,** *I do not know.*

Inversion

For emphasis or variety of style, a word which is not the subject may be placed first in a sentence, and then the subject is placed *after* the simple verb or auxiliary. For the remainder of the sentence the rules already stated apply : **Zu Weinachten gab der Vater dem Kinde ein Buch.** *It was at Christmas that the father gave the child a book.*

NOTE.—*See* page 1329 for sentences following a relative pronoun ; and page 1344 for sentences introduced by conjunctions.

For some time—at all events until these lessons have been thoroughly assimilated—the student should not move outside the above rules : they are sufficient to enable him to compose quite advanced German.

LESSON 13

Word Building, Word Order, and Idioms

In the previous Lesson we examined the four ways of word building ; we follow it with exercises in word building and on the proper order of words. Notes on idioms and correspondence are included.

EXERCISE ON WORD BUILDING

1. By Changing the Root Vowel

Es war ein schwerer Verlust (verlieren, *to lose***).**
It was a heavy loss.

Er braucht Trost (trösten, *to comfort***).**
He needs comfort.

Das Schloß (schließen, *to lock***) war erbrochen.**
The lock was broken.

Der Schwamm (schwimmen, *to swim***) lag auf der Wage (wiegen,** *to weigh***).**
The sponge lies on the scales.

Der Dieb nahm die Flucht (fliehen, *to escape***) über das Dach (decken,** *to cover***).**
The thief took to flight over the roof.

Der Entschluß (entschließen, *to decide***) war gefaßt.**
The decision was taken.

Er hat eine schöne Schrift (schreiben, *to write***).**
He has a beautiful handwriting.

Die Lage (liegen, *to lie***) des Flusses (fließen,** *to flow***) war für einen Angriff (angreifen,** *to attack***) günstig.**
The situation of the river was favourable to an attack.

2. By a Prefix

Die großen Erfinder werden durch den Undank der Welt nicht entmutigt.
The great inventors are not discouraged by the thanklessness of the world.

Die Mißerfolge des Erfinders des Zeppelin haben Graf Zeppelin nicht abgehalten, seine Entdeckungen zu verfolgen.
The failures of the inventor of the Zeppelin did not keep Count Zeppelin from following up his discoveries.
Seine Versuche wurden belacht.
His experiments were laughed at.
Aber er wurde nicht ungeduldig oder mißmutig.
But he did not become impatient or ill-humoured.
Selbst als das Luftschiff nach dem ersten Flug verbrannte, verzweifelte er nicht.
Even when after the first flight the airship was burnt he did not despair.
Er bemühte sich die Erfindung zuverbessern.
He tried to improve on the invention.
Schließlich überkam er alle Widerstände, und im Jahre 1906 flog das Luftschiff zum ersten Male die Gebäude von Berlin.
At last he overcame all opposition and in 1906 the airship flew for the first time over the buildings of Berlin.

3. By Adding a Suffix

Die wichtigsten Werke der älteren deutschen Dichtung sind
The most important works of the older German literature have
in Österreich und Bayern niedergeschrieben worden. Es sind
been written in Austria and Bavaria. They are
kriegerische Erzählungen vom Schicksal von Königen und Königinnen,
martial tales of the fate of kings and queens,
von leidenschaftlicher Liebe und Freundschaft.
of passionate love and friendship.
Der Held eines Gedichtes ist Siegfried, dessen Leben
The hero of one poem is Siegfried, whose life
von der Kindheit bis zum Tod erzählt wird. Er ist der Besitzer
is told from childhood to death. He is the owner
kostbarer Waffen. Diese Waffen sind von einem seltsamen
of costly weapons. These weapons are surrounded by
Geheimnis umgeben. Sie bringen Unglück, und nach vielen
a strange mystery. They bring bad luck, and after many
blutigen Kämpfen versinken sie im Rhein.
bloody battles they disappear in the Rhine.

EXERCISE ON ORDER OF WORDS

Der süße Brei (*The sweet pie*).

(From the Grimm Brothers' Folk-lore Collection)

Es war einmal ein armes, frommes Mädchen.
Once upon a time there was a poor, good girl
Es lebte allein mit seiner Mutter, und sie hatten nichts zu essen.
She lived alone with her mother, and they had nothing to eat.
Da ging das Kind in den Wald.
So the child went into the woods.
Dort traf es eine alte Frau, die seine Sorgen schon wußte.
There she met an old woman, who knew about their troubles.
Sie gab ihm ein Töpfchen, zu dem es sagen sollte:
She gave her a little pot, to which she should say:
"Töpfchen koche!" Dann kochte es guten süßen **Brei, und wenn es sagte:**
"Cook, little pot!" Then it cooked nice sweet pie, and when she said:
"Töpfchen stehe!" dann kochte es nicht mehr.
"Stand still, little pot!" then it cooked no more.
Das Mädchen brachte den Topf seiner Mutter, und sie aßen süßen Brei, so oft sie wollten.
The girl brought the pot to her mother, and they ate sweet pie as often as they wanted to.

Einmal war das Mädchen ausgegangen, da sprach die Mutter
Once the girl had gone out, the mother said:
"Töpfchen koche!" Es kochte, und sie aß.
"Cook, little pot!" It cooked, and she ate.
Dann wollte sie, daß das Töpfchen nicht weiter kochen sollte, aber sie wußte das Wort nicht.
Then she wanted the pot not to go on cooking, but she did not know the word.
Deshalb kochte es fort, und der Brei stieg über den Rand
Therefore it went on cooking, and the pie rose over the edge
und kochte immer weiter, füllte die Küche und das Haus und das
and went on cooking, filled the kitchen and the house and the
zweite Haus und dann die Straße, als wollte es die ganze **Welt satt machen.**
next house and then the street, as if it wanted to satisfy the whole world.
Als das Kind endlich nach Hause kam, sagte es nur:
When at last the child came home, she said only:
"Töpfchen stehe!" Da kochte es nicht mehr.
"Stand still, little pot!" Then it cooked no more.
Aber wer wieder in die Stadt gehen wollte, mußte seinen **Weg durch den Brei durchessen.**
But anyone who wanted to go into the town had to eat his way through the pie.

SOME USEFUL IDIOMS

An idiom is an expression peculiar to a language. There are many classes of idioms, and they must be learnt by experience or from a book of idioms if one is to be had. The following are of frequent occurrence, and should be known.

Sie haben recht (unrecht).	*You are right (wrong).*
Mir ist heiß (kalt).	*I am hot (cold).*
Ich habe Durst (Hunger).	*I am thirsty (hungry).*
Bitte (in reply to *Excuse me, Pardon me*).	*Don't mention it.*
Wir haben es eilig.	*We are in a hurry.*
Was heisst das?	*What does that mean?*
Was ist los?	*What's the matter?*
Beeilen Sie sich!	*Hurry up.*
Ich bedaure sehr.	*I'm very sorry.*
Ich glaube ja.	*I believe so.*
Es paßte uns sehr gut.	*It suited us very well.*
Was haben Sie?	*What is the matter with you?*
Danke sehr.	*Thank you very much.*
Wie sagt man das auf deutsch?	*How do you say that in German?*

"**Guten Tag, Herr Schmidt.**"—"**Guten Morgen, gnädige Frau. Wie geht's?**" *"Good morning, Mr. Schmidt." "Good morning, madam.* *How are you?"*

"**Danke, gut. Wie geht es Ihnen? Wie geht es Ihre Frau?**" *Thank you, very well. How are you? How is your wife?"*

"**Sie ist im Krankenhaus. Leider!**" "**Nanu? Was ist los?**" *"She is in hospital, unfortunately." "What on earth is the matter?"*

"**Sie war ein paar Tage im Bett. Aber jetzt darf sie dann und wann aufstehen.**" *She was in bed for a few days. But now she is allowed to be up from time to time."*

CORRESPONDENCE

The date is written thus: Den 3. Juni 1955, Den 25. Dez. 1955. *3rd June 1955, 25th Dec. 1955.*

A business letter begins: Sehr geehrter Herr! Dear Sir. A private letter begins: Sehr geehrter Herr Schmidt. Sehr geehrte gnädige Frau. Dear Madam etc.

A familiar opening: **Mein lieber X.**

A business-like ending: **Hochachtungsvoll.**

A formal one: **Mit vorzüglicher Hochachtung.**

A moderately familiar ending: **Ihr ergebener . . . or Ihr . . .**

With best wishes: **Mit bestem Gruss.**

With kind regards: **Mit freundlichen Grüssen.**

Address envelopes thus:

Herrn (Frau, Fräulein), A. B. C.,

Berlin, W.10.

Grillparzer Str. 32.

Example

Berlin, den 2. August 1955.

Sehr geehrtes Fräulein Schmitt,
Dear Miss Smith.
Ich höre von Ihrer Schwester, daß
I hear from your sister that
Sie in Berlin sind und würde Sie gern sehen. Würden Sie
you are in Berlin and I would like to see you. Would you
die Güte haben, mir zu sagen, ob ich Sie am Sonntag abend
be so kind as to tell me whether I can expect you on Sunday
um 8 Uhr erwarten kann.
evening at 8 o'clock.

Mit besten Grüßen
With kind regards
Ihre
Yours
H. K.

LESSON 14

Extracts to Illustrate the Use and Scope of the Vocabulary and Grammar

A Tale from the Folk-lore Collection of the Brothers Grimm

Der Wolf und der Mensch
The Wolf and the Man

Der Fuchs erzählte einmal dem Wolf von der
The fox once told the wolf about the
Stärke des Menschen, kein Tier könnte ihm
strength of man, (how) no animal could
widerstehen, und sie müßten List gebrauchen,
stand up against him, and (that) they must use cunning
um sich vor ihm zu erhalten. Da antwortete
to support themselves with him about. Then answered
der Wolf: "Wenn ich nur einmal einen
the wolf: "If I could only get to see a
Menschen zu sehen bekäme, ich wollte doch auf
man, I would fly at him
ihn losgehen." "Dazu kann ich dir helfen,"
all the same." "In that I can help you,"
sprach der Fuchs, "komm nur morgen früh
said the fox, "just come to me to-morrow early,
zu mir, so will ich dir einen zeigen." Der
and I will show you one." The
Wolf stellte sich frühzeitig ein, und der Fuchs
wolf presented himself early, and the fox
brachte ihn hinaus auf den Weg, den der Jäger
brought him out on the road which the hunter
alle Tage ging. Zuerst kam ein alter abgedankter
daily walked. First came an old discharged
Soldat. "Ist das ein Mensch?" fragte der
soldier. "Is that a man?" asked the
Wolf. "Nein," antwortete der Fuchs, "das
wolf. "No," answered the fox, "that has
ist einer gewesen." Danach kam ein kleiner
been one." After that came a small
Knabe, der zur Schule wollte. "Ist das ein
boy on the way to school. "Is that a
Mensch?" "Nein, das will erst einer werden."
man?" "No, that will duly become one."
Endlich kam der Jäger, die Doppelflinte auf dem
At last came the hunter, with double-barrelled gun on his
Rücken und den Hirschfänger an der Seite.
back and hunting-knife by his side.
Sprach der Fuchs zum Wolf: "Siehst du,
Said the fox to the wolf: "See,
dort kommt ein Mensch, auf den mußt du
there comes a man, at whom you must
losgehen, ich aber will mich fort in meine
fly, but I will make off to my
Höhle machen." Der Wolf ging nun auf den
lair." The wolf now went for the
Menschen los, der Jäger, als er ihn erblickte,
man, (but) the hunter, when he saw him,
sprach: "Es ist schade, daß ich keine Kugel
said: "It is a pity that I have not loaded
geladen habe," legte an und schoß dem Wolf
with ball," took aim and fired the small-shot at the wolf's
das Schrot ins Gesicht. Der Wolf verzog das
face. The wolf twisted his
Gesicht gewaltig, doch ließ er sich nicht
face violently, though he did not allow himself
schrecken und ging vorwärts: da gab ihm der
to be afraid and went ahead: then the
Jäger die zweite Ladung. Der Wolf verbiss
hunter gave him the second charge. The wolf suppressed
den Schmerz und rückte dem Jäger zu Leibe:
the pain and closed up on the hunter:
da zog dieser seinen blanken Hirschfänger und
whereupon the latter drew his bright hunting-knife and
gab ihm links und rechts ein paar Hiebe, daß
gave him a few cuts to left and right, so that,
er, über und über blutend, mit Geheul zu dem
bleeding all over, he ran howling back to
Fuchs zurücklief. "Nun, Bruder Wolf," sprach
the fox. "Now, brother wolf," said
der Fuchs, "wie bist du mit dem Menschen
the fox, "how did you deal with the man?"
fertig geworden?" "Ach," antwortete der
"Oh," answered the
Wolf, "so hab' ich mir die Stärke des Menschen
wolf, "I hadn't imagined the strength of man
nicht vorgestellt, erst nahm er einen Stock
to be such: first he took a stick
von der Schulter und blies hinein, da flog mir
from his shoulder and blew into it, whereupon
etwas ins Gesicht, das hat mich ganz entsetzlich
something flew into my face that tickled me
gekitzelt; danach pustete er noch einmal in
horribly; after which he blew once again into
den Stock, da zog mir's um die Nase, wie Blitz
the stick, and something flew round my nose like lightning
und Hagelwetter, und wie ich ganz nah war,
and hail and as I was quite near

da zog er eine blanke Rippe aus dem Leib,
he drew a bright rib from his body,
damit hat er so auf mich losgeschlagen, daß
with which he so attacked me that
ich beinah tot wäre liegen geblieben." "Siehst du,"
I was nearly left for dead." "See,"
sprach der Fuchs, "was du für ein Prahlhans
said the fox, "what sort of a braggart you
bist, du wirfst das Beil so weit, daß du's
are: you fling the hatchet so far that you
nicht wieder holen kannst."
can't fetch it back."

Conversations

Der Bahnhof (*The Railway Station*)

Träger: **Träger, mein Herr?**
Porter: *Porter, sir?*
Herr Jedermann: **Ja. Ich möchte den Zug nach Berlin erreichen. Wieviel Zeit habe ich?**
Mr. Anyman: *Yes. I should like to get the train to Berlin. How much time have I?*
Träger: **Zehn Minuten. Der Zug fährt jetzt in den Bahnhof ein. Haben Sie Ihre Fahrkarte?**
Porter: *Ten minutes. The train's coming into the station now. Have you got your ticket?*
Herr J.: **Nein. Wo ist der Fahrkarten-Schalter?**
Mr. A.: *No. Where is the ticket-office?*
Träger: **Gerade dort drüben. Fahren Sie erster oder dritter?**
Porter: *Straight over there. Are you going First or Third?*
Herr J.: **Dritter.**
Mr. A.: *Third.*
Träger: **Das ist auf der anderen Seite.**
Porter: *That is on the other side.*
Herr J.: **Gut. Nehmen Sie meine Sachen zum Zug. Ich treffe Sie dort. Nehmen Sie einen Fensterplatz in einem Raucherwagen—möglichst nach vorwärts.**
Mr. A.: *Right. Take my things to the train. I'll see you there. Get me a window-seat in a smoking-carriage—if possible, facing the engine.*
Träger: **Jawohl, mein Herr. Der Zug ist auf dem dritten Bahnsteig.**
Porter: *Very good, sir. The train is on number 3 platform.*
Herr J. **(zu dem Mann am Fahrkarten-Schalter) Eine Karte dritter Klasse nach Berlin, bitte.**
Mr. A. (to man at the ticket-office): *A third class ticket to Berlin, please.*
Mann am Schalter: **Einfach oder Hin- und Rückfahrt?**
Man at the ticket-office: *Single or return?*
Herr J.: **Ist die Rückfahrkarte billiger?**
Mr. A.: *Is the return ticket cheaper?*
Mann: **Am billigsten ist eine Wochenend-Karte. Aber Sie müssen dann spatestens mit dem letzten Zug am Dienstag abend zurück-kommen.**
Man: *The cheapest is a week-end ticket. But you have to come back not later than the last train on Tuesday night.*
Herr J.: **Gut, geben Sie mir eine Wochenend-Karte. Wieviel kostet sie?**
Mr. A.: *All right, give me a week-end ticket. How much is it?*
Mann: **Zwei und zwanzig Mark fünfzig.**
Man: *Twenty-two marks, fifty (pfennige).*
Herr J.: **Hier ist das Geld.**
Mr. A.: *Here is the money.*
Mann: **Sind das Ihre Handschuhe?**
Man: *Are these your gloves?*
Herr J.: **Ja—meine neuen Handschuhe. Wie dumm! Das war sehr nett von Ihnen.**
Mr. A.: *Yes,—my new gloves. How foolish! That was very kind of you.*
Herr J. **(Geht zum Zug): Sie sind mein Träger, nicht wahr?**
Mr. A. (goes to train): *You're my porter, aren't you?*
Träger: **Jawohl, mein Herr. Ich habe Ihren Platz hier, und alles ist drin. Ihr Koffer ist unter der Bank, und das Paket ist da oben.**
Porter: *Yes, sir. I've got your seat here, and everything's in safely. Your bag is under the seat, and the parcel is up there.*
Herr J.: **Gut. Hier (gift ihm Geld).**
Mr. A.: *Good. Here you are* (gives him money).
Träger: **Danke sehr, mein Herr.**
Porter: *Thank you very much, sir.*
Herr J.: **Kann man im Zuge zu essen bekommen?**
Mr. A.: *Is it possible to get a meal on the train?*
Trager. **Ja, der Mann an der Tür wird Ihnen eine Karte geben. Aber er kommt später.**
Porter: *Yes, the man by the door will give you a ticket. But he'll be coming round later.*
Herr J.: **Dann werden wir sehen, wenn er kommt. (Setzt sich) Darf ich das Fenster zumachen? Es zieht.**
Mr. A.: *Then I'll see about it when he comes.* (Takes his seat.) *May I have the window shut? There's a draught.*
Eine Frau gegenüber: **Oh, bitte nicht. (Atmet den Rauch in tiefen Zügen ein.) Die Zeitungen sagen, Luft hält Erkältungen fern.**
A Woman Opposite: *Oh, please don't.* (Taking in deep breaths of smoke.) *The papers say that air keeps away colds.*
Herr J.: **Was für eine merkwürdige Idee.**
Mr. A.: *What a strange idea!*

Das Hotel (*The Hotel*)

Hotel-Portier: **Haben Sie ein Zimmer bestellt, mein Herr?**
Hotel Porter: *Have you booked a room, sir?*
Herr Jedermann: **Nein. Legen Sie meine Sachen hier hin: ich gehe zum Büro. (Zu der Frau im Büro): Haben Sie ein Einzelzimmer?**
Mr. Anyman: *No. Put my things down here while I go to the office.* (To woman at office) *Have you a room for one?*
Frau im Buro: **Wir sind sehr voll diese Woche. Ich habe jetzt überhaupt keine kleinen Zimmer. Wie lange bleiben Sie?**
Woman at Office: *We're very full up this week. I have no small rooms at all at present. How long are you going to be here?*
Herr J.: **Wahrscheinlich zwei Wochen.**
Mr. A.: *Probably for two weeks.*
Frau: **Ich gebe Ihnen ein Zimmer mit zwei Betten bis Montag, und später können wir Ihnen ein kleines Zimmer geben. Sind Sie damit einverstanden?**
Woman: *I'll let you have a room with two beds until Monday, and after that we shall be able to give you a small room. Will that be all right?*
Herr J.: **Ich bin sehr gegen umziehen, wenn es sich irgend vermeiden läßt. Ich wollte nach der Reise alles aus meinen Koffern auspacken. Haben Sie keinen anderen Vorschlag?**
Mr. A.: *I'm very much against moving if there's any possible way out of it. I was hoping to get everything out of my boxes after the journey. Have you no other suggestion to make?*
Frau: **Nein, das ist das beste, was ich tun kann. Alle andered Hotels sind ebenso besetzt.**
Woman: *No, that's the best I am able to do. Every other hotel is full up in the same way.*
Herr J.: **Na, dann muss ich nehmen, was da ist. Ist das Zimmer ruhig? Das ist die Hauptsache.**
Mr. A.: *Well, I'll have to take what there is. Is this room quiet? That's the great thing.*
Frau: **Ja, sehr ruhig; es geht nach hinten. Das andere auch. Wirklich, es sind die ruhigsten Zimmer im Hotel.**
Woman: *Yes, very quiet; it's at the back. So is the other one. In fact, they're the quietest rooms in the hotel.*
Herr J.: **Hat es ein Badezimmer?**
Mr. A.: *Has it got a bathroom?*

Frau : Das Zimmer, das Sie heute bekommen, hat eins. Das kleinere hat keins, aber es ist nur eine Tür entfernt vom allgemeinen Badezimmer. Und alle Zimmer im Hotel haben Telefon.

Woman : The room you're going into to-day has. The smaller one hasn't, but it's only one door from the public bathroom. And all the rooms in the hotel have telephones.

Herr J. : Gut. Und der Preis?
Mr. A. : Good. And the price?

Frau : Das Zimmer mit Bad kostet zehn Mark. Das andere kostet sieben Mark.
Woman : The room with the bathroom is ten marks. The other will be seven marks.

Herr J. : Das ist ohne Mahlzeiten?
Mr. A. : That's without any meals?

Frau : Ja. Mahlzeiten sind extra.
Woman : Yes. Meals are extra.

Herr J. : Gut, ich nehme das Zimmer.
Mr. A. : All right, I'll take the room.

Frau : Wollen Sie bitte Ihren Namen in das Buch schreiben?
Woman : Will you put your name in the book, please?

Herr J. : Gewiß.
Mr. A. : Certainly.

Frau : Das Zimmer ist im zweiten Stock, nahe am Fahrstuhl. Der Portier wird Sie hinauffahren.
Woman : The room is on the second floor, near the lift. The porter will take you up.

LESSON 15

General Reading

In this extract from H. G. Wells's book *The Shape of Things to Come*, the original text has been put into Basic English. The passage is entitled "The Future of Basic English" (**Die Zukunft von "Basic-Englisch"**).

One unlooked-for development of the hundred years between
Ganz unerwartet war die Entwicklung, die das sogenannte "Basic Englisch"
2000 and 2100 was the way in which Basic English became in that [illegible]
in den hundert Jahren zwischen 2000 und 2100 genommen hat, nämlich daß es
short time the common language for use between nations and the
in so kurzer Zeit zur gemeinsamen Umgangssprache zwischen den Nationen
expansion at an even greater rate—as the outcome of this, and
wurde, und daß infolgedessen und nach vielerlei Änderungen die
[illegible] *itself*
englische Sprache selbst sich noch schneller ausbreitete.
[illegible]
Das Englisch, das die meisten von uns heute sprechen
[illegible]
und schreiben, ist von dem Englisch Shakespeares, Addisons, Bunyans und Shaws
[illegible]
sehr verschieden. Es hat die letzten
[illegible]
Reste aller schwierigen Formen wie den Konjunktiv abgeworfen
[illegible]
das geschriebene Wort entspricht wirklich seinem Klang oder spricht
one gives the same sound to the same word; a number of words and
den gleichen Laut gleich; eine Anzahl von Worten und Wortgruppen
[illegible] *No attempt*
ist aus anderen Sprachen übernommen worden. Es wurde nicht der Versuch
was made at forcing it upon other nations as the one language of the
gemacht, Englisch zwangsweise zur Weltsprache
Earth. In its natural form it was better for the purpose in a
zu machen. In seiner natürlichen Form war es in
number of ways than the chief languages in competition with it,
vieler Hinsicht für diesen Zweck besser geeignet als die
Spanish, French, Russian, German, and Italian. It was simpler,
Sprachen, mit denen es hauptsächlich im Wettbewerb stand, wie spanisch, französisch,
more delicate, more elastic, and even at that time more widely
russisch, deutsch und italienisch. Es war einfacher, feiner, dehnbarer
used, but it was certainly the development of Basic English which
und schon damals weiter verbreitet; dennoch war es sicherlich erst die
in the end gave it the position it now has.
Entwicklung des "Basic English," die ihm schließlich die Stellung verschaffte, die es heute einnimmt.
Basic English was the invention of a man whose quick and
Basic-Englisch war die Erfindung eines Mannes, dessen rascher und
fertile mind was trained at Cambridge in England. This C. K. Ogden,
schöpferischer Verstand seine Ausbildung in Cambridge in England erhalten
through a long and working hard, gave all his [illegible]
hatte. Dieser C. K. Ogden (1889-1990), der lang lebte und hart arbeitete,
the question of getting a simpler relation between language and
verwendete seine ganze Zeit darauf, eine einfachere Beziehung zwischen
thought, and specially to the working-out of this one system. It
Sprache und Gedanke herzustellen, und vor allem darauf, sein System auszuarbeiten.
is an interesting fact that he was living at the same time as
Es ist interessant, daß er zur gleichen Zeit lebte wie James
James Joyce (1882-1941) who, unlike Ogden, was [illegible] *for more*
Joyce (1882-1941), der, im Gegensatz zu Ogden, auf
[illegible] *and greater power of expression. In*
größeren Sprachreichtum und größere Ausdruckskraft hinarbeitete. Schließ-
readers who became less in number every year, were unable to get
lich konnten seine Leser, die von Jahr zu Jahr weniger wurden, seine ver-

t him at all through his knotted and twisted prose, which became
wickelte und gewundene Prosa, die dem sinnlosen Gespräch eines Verrück-
more like the foolish talk of a man who is off his head. He did,
ten immer ähnlicher wurde, überhaupt nicht mehr verstehen. Er brachte
however, get about twenty-five new words into the language which
aber etwa 25 neue Worte in die Sprache, die auch heute noch
are still in use. Ogden, after working hard for a long time in
gebraucht werden. Ogden, der lange Zeit eifrig in der entgegengesetzten
the opposite direction, came through with an English of 850 words
Richtung gearbeitet hatte, kam mit einem Englisch von 850 Worten
and five or six rules for their operation which would make it
und fünf oder sechs Regeln zu ihrer Anwendung aus: das ermöglichte
possible for any person from another country who had a ready
jedem Ausländer, der ein aufnahmefähiges Gedächtnis hatte,
memory to get to the point of talking and writing quite good
so weit zu kommen, daß er in zwei bis drei Wochen
English in two or three weeks. Generally it was harder to get
ganz gutes Englisch sprechen oder schreiben konnte. Im allgemeinen
Englishmen to keep to the words and forms of the Basic selection
war es schwerer, Engländer dazu zu bringen, sich innerhalb der Worte und
than to give outside learners a knowledge of the complete language.
Formen der Basic-Auswahl zu halten, als außenstehenden Lernenden eine vollständige Kenntnis der Sprache zu vermitteln.
It was a teacher of languages, Rudolph Boyle (1910-1959) who first
Ein Sprachlehrer, Rudolph Boyle (1910-1959) hat zuerst ein
put forward a system by which persons using English as their natural
System vorgelegt, mit dem Personen, die Englisch als Mutter-
language were trained to keep inside the Basic limits when necessary.
sprache sprachen, so ausgebildet werden konnten, daß sie sich, wenn nötig, innerhalb der Grenzen des "Basic" hielten.
Basic was taken up in a most surprising way after the
Nach der ersten Konferenz von Basra wurde die Beschäftigung mit
first Conference at Basra. It was made the language for
Basic in ganz erstaunlicher Weise aufgenommen. Es wurde von der Luft- und See-
public and general purposes in every country by the Air and
kontrolle zur Sprache für alle öffentlichen und staatlichen Zwecke in
Sea Control, and by 2020 almost everyone was able to make use
allen Ländern gemacht, und im Jahre 2020 konnte fast jedermann "Basic-
of Basic for talking and writing.
Englisch" sprechen und schreiben.
It is from the starting-point of Basic English, worked
Ausgehend von "Basic-Englisch," aufgebaut auf einem System
with a system in which the form of a word on paper is representative
in dem das geschriebene Wort
of its sound, that the language used by us to-day has come
seinem Klang genau entspricht, hat sich die Sprache, die wir jetzt
into existence, chiefly by putting back, by slow degrees, the
sprechen, entwickelt, hauptsächlich, indem wir nach und nach die
[illegible]
"Verben" und die Spezialausdrücke aus der Mutter-sprache wieder
over [illegible] and word-groups from other languages. To-day our
eingefügt und Worte und Wortgruppen anderer Sprachen hineingenommen haben.
[illegible] 2,000,000 [illegible]
Heute enthält unsere Sprache fast 2 000 000 Worte. Es ist eigentlich eine
language formed from other languages, with roots, words, and
Sprache, die aus anderen Sprachen gebildet ist, mit Sprachwurzeln, Worten
special uses taken from the tongues of all nations. A. H. [illegible]
und Ausdrücken aus den Zungen aller Nationen. Vor kurzem hat K. Wang
made it clear in a paper a short time back that different word
in einem Aufsatz klar gemacht, daß verschiedene Gruppen von Personen
[illegible] On
noch immer eine verschiedene Wortauswahl treffen. Bei der Be-
looking at the work of present-day writers of Italian [illegible]
trachtung der Werke heutiger Schriftsteller italienischen Blutes,
whose names were taken by chance, it was seen that there was a
deren Namen zufällig gegriffen wurden, konnte man sehen, daß eine
[illegible]
ausgesprochene Neigung bestand, Worte mit lateinischen Wurzeln zu ge-
[illegible]
brauchen, während zwanzig ostasiatische Schriftsteller besonderes Gewicht
[illegible] and American." But they all make
auf chinesische und amerikanische Worte legen. Aber sie alle
clear to one another, and the same art, the same learning,
machen sich einander verständlich und die gleiche Kunst, die gleiche
[illegible] them all.
Bildung und der gleiche Standpunkt in der Wissenschaft ist allen eigen.

EXTRACT FROM EINSTEIN'S *RELATIVITY*

With acknowledgement to the author and his [illegible]
Relativitätstheorie. Von Albert Einstein. [illegible] Vieweg). English edition, *RELATIVITY* [illegible] By Albert Einstein [illegible] taken by Dr. R. W. Lawson [illegible] Methuen and Co. [illegible]

Spezielles und allgemeines Relativitätsprinzip	Special and General Theory of Relativity
Nachdem sich die Ein-führung des speziellen Relativitätsprinzips bewährt hat	Since the introduction of [illegible]

muss es jedem nach Verallgemeinerung strebenden Geiste verlockend erscheinen, den Schritt zum allgemeinen Relativitätsprinzip zu wagen. Aber eine einfache, scheinbar ganz zuverlässige Betrachtung lässt einen solchen Versuch zunächst aussichtslos erscheinen.

every intellect which strives after generalization must feel the temptation to venture the step towards the general principle of relativity. But a simple and apparently quite reliable consideration seems to suggest that, for the present at any rate, there is little hope of success in such an attempt.

Der Leser denke sich in den schon so oft betrachteten, gleichförmig fahrenden Eisenbahnwagen versetzt. Solange der Wagen gleichförmig fährt, ist für den Insassen nichts vom Fahren des Wagens zu merken. Daher kommt es auch, dass der Insasse den Tatbestand ohne inneres Widerstreben dahin deuten kann, dass der Wagen ruhe, der Bahndamm aber bewegt sei. Diese Interpretation ist übrigens nach dem speziellen Relativitätsprinzip auch physikalisch ganz berechtigt.

Let us imagine ourselves transferred to our old friend the railway carriage, which is travelling at a uniform rate. As long as it is moving uniformly, the occupant of the carriage is not sensible of its motion, and it is for this reason that he can unreluctantly interpret the facts of the case as indicating that the carriage is at rest, but the embankment in motion. Moreover, according to the special principle of relativity, this interpretation is quite justified also from a physical point of view.

Wird nun aber die Bewegung des Wagens etwa dadurch in eine ungleichförmige verwandelt, dass der Wagen kräftig gebremst wird, so erhält der Insasse einen entsprechend kräftigen Ruck nach vorne. Die beschleunigte Bewegung des Wagens äussert sich in dem mechanischen Verhalten der Körper relativ zu ihm; das mechanische Verhalten ist ein anderes als im vorhin betrachteten Falle, und es erscheint deshalb ausgeschlossen zu sein, dass relativ zum *ungleichformig* bewegten Wagen die gleichen mechanischen Gesetze gelten, wie relativ zum ruhenden beziehungsweise gleichförmig bewegten Wagen.

If the motion of the carriage is now changed into a non-uniform motion, as for instance by a powerful application of the brakes, then the occupant of the carriage experiences a correspondingly powerful jerk forwards. The retarded motion is manifested in the mechanical behaviour of bodies relative to the person in the railway carriage. The mechanical behaviour is different from that of the case previously considered, and for this reason it would appear to be impossible that the same mechanical laws hold relatively to the non-uniformly moving carriage, as hold with reference to the carriage when at rest or in uniform motion.

Jedenfalls ist klar, dass relativ zum ungleichförmig bewegten Wagen der Galileische Grundsatz nicht gilt. Wir fühlen uns daher zunächst genötigt, entgegen dem allgemeinen Relativitätsprinzip der ungleichförmigen Bewegung eine Art absolute physikalische Realität zuzusprechen. Im folgenden werden wir aber bald sehen, dass dieser Schluss nicht stichhaltig ist.

At all events it is clear that the Galilean law does not hold with respect to the non-uniformly moving carriage. Because of this, we feel compelled at the present juncture to grant a kind of absolute physical reality to non-uniform motion, in opposition to the general principle of relativity. But in what follows we shall soon see that this conclusion cannot be maintained.

The word **Beschleunigung** in physics (as here) is applied to change of velocity, whether acceleration or retardation.

HINTS FOR FURTHER STUDY AND READING

Few students who have worked through this course carefully will need to know more grammar. Those who wish to pursue their studies ought not to be content with even the best of the more advanced grammars prepared specially for English students. They should study a good German work, e.g. Schulz and Sundermeyer's *Deutsche Sprachlehre für Ausländer*, published by Berlin University.

Dictionary. A good dictionary is indispensable: either that by Muret-Sandars or, what is perhaps more easily obtainable, Cassell's *German-English and English-German Dictionary*.

Reference. The *Basis and Essentials of German* by Charles Duff and Richard Freund, and the *Basis and Essentials German Reader* by Richard Freund, may be found useful for general revision.

Practical. As soon as the Course has been completed, let the student read, say, J. G. Robertson's *History of German Literature*. A history of the literature will provide the student with an account of the principal authors and works, from which his own taste will select what most appeals to him.

An excellent practical reader for those desiring to acquire conversational German is *Die Abenteuer von Paula und Peter*, by E. Johannsen and A. M. Wagner (University of London Press). For those desiring to study science, *An Introduction to Scientific German*, by J. Henry Wild (Oxford University Press), will be found useful—it contains a wide range of extracts from German works on science.

The German Classics. The student has an immense field from which to select. But he would be well advised to be content to begin with a classic written in simple, straightforward German, e.g. the *Märchen* of the Brothers Grimm, or *Das Soll und Haben*, by Freytag.

From that he may proceed to read any of the classical authors: Goethe and Schiller, Heine and Nietzsche. Many students find it interesting to read Shakespeare in the magnificent translation by Schlegel and Tieck. The works of Heinrich and Thomas Mann can be recommended. And the works of Leon Feuchtwanger, Fritz von Unruh, and Döblin, though not easy to read, provide instruction and entertainment. An outstanding name among modern writers is that of Rainer Maria Rilke, some of whose works have been translated into English.

ART

THIS *Course records the story of man's rich achievement in self-expression through the visual arts painting, sculpture, etc.—from the time of ancient Egypt to the present day. The range is comprehensive, including Lessons on Oriental art of yesterday and to-day, though for obvious reasons the emphasis of the course is on the development of European art.*

For the art of prehistoric man, reference should be made to the Course on ARCHAEOLOGY *elsewhere in this Volume. A practical Course of instruction in* DRAWING AND DESIGN *is included in Volume 5. The history and principles of* ARCHITECTURE *form another Course, which will be found in Volume 1.*

22 LESSONS

LESSON 1

Enjoyment or Appreciation—or Both?

No satisfactory definition of art exists. Philosophers, writers on aesthetics, and art historians have tried times without number to confine its functions within recognizable limits, or to formulate its all-pervading essence in terms of a single phrase. Others—like Tolstoy in his *What is Art?*—have devoted whole volumes to the question without finding a final answer; or if they have found an answer, it differs from a thousand others, being dependent only on their own individual views.

All that can be definitely stated is that art is one of the characteristic instincts and manifestations of mankind. One might go further than this, and say without much fear of contradiction that art in its broadest sense is the expression of spiritual, as opposed to material, consciousness of perception; and that it deals with what is beyond, or outside, the possibility of demonstration by mechanical or scientific testing, being concerned with the expression of imagination and the translation of emotion. Beyond such a statement, it is safest, amid all the conflicting views on the subject, to fall back on some wise words from G. K. Chesterton: "You can dogmatise about such things as Euclid or the making of paper boats; you cannot dogmatise about art."

Self-expression, Beauty, and Taste

But if one cannot dogmatise, one can still recognize. Art can be, and is, recognized throughout a vast range of human creation and expressive activity. It is seen in its crudest forms in, say, the drawings of Bushmen, and in its highest forms in such varied productions as a Gothic cathedral, the symphonies of Beethoven, the paintings of Rembrandt, the sculpture of Pheidias or Michelangelo, the poetry and drama of Shakespeare, the novels of Dostoievsky.

It can inform such evanescent modes of self-expression as speech, gesture, and movement—or, more precisely, oratory, acting, dancing. It may be recognized with equal clearness in the design or decoration of the humblest household objects. It has, indeed, been well said that art may have its place in the way a woman lays a dinner-table or a man brushes his hair.

W. R. Lethaby (1857–1931), who was both inspiring as a leader of art and a fine and devoted practitioner, once defined art as "the well-doing of whatever is worth doing." But this at once begs two questions—What is meant by well-doing? What is worth doing?—and does not get anybody much farther.

There are those who choose to relate art to beauty. But again one is driven to ask a question: What is beauty? And here again nobody has ever been able to provide a comprehensive definition. We know that beauty affords pleasure, but that fails to pin it down. Apart from individual tastes, which are proverbially different, the general human idea of what constitutes beauty has undergone perpetual change almost from one generation to another; also it differs notoriously in different parts of the world. Ibsen wisely said that beauty is no more than "a coin made current by time and place."

Know What You Like

After all, there is something to be said in defence of the man or woman who uses those phrases so scoffed at by the allegedly superior: "I know nothing about art—I only know what I like." So far as the second part of the phrase is concerned, it is as much as anyone can honestly say, however much of an authority he may claim to be. The first part of the phrase is the one that confesses to weakness. There is no good reason for ignorance about art. Everyone can at least be aware, not only of what he himself likes, but also of what many other people like and have liked, even if the reason for their liking it is sometimes beyond his comprehension.

The Lessons in this Course are intended to make the student so aware. In studying what forms of art have been generally accepted and enjoyed throughout the ages and trying to appreciate the reasons, the student may find that the phrase "what I like" becomes stretched, in his case, to cover a great deal more than he expects—to his great benefit. Certainly that is the hope of those who have prepared these Lessons.

But there is one danger. To like or dislike any form of art or any individual work of art, in the sense of enjoying or not enjoying it, is a prerogative of every individual human being and one to be jealously guarded. No one should ever be persuaded into *pretending* to like what he genuinely dislikes, or to dislike what he genuinely likes.

Value of the Historical Approach

Your *enjoyment* of art is entirely your own concern; and if your natural taste leads you to be in a "minority of one," you will still derive far more benefit from indulging that taste than you ever will from trying to go with the fashionable crowd against your own better inclinations. After all, *you may be the one who is right.* Who can say?

But even if you cannot, and need not, learn to enjoy what others enjoy, you can still learn to

understand, appreciate, and even admire. To be wise and well informed about any human activity brings its own obvious benefits—not least among which are the acquisition of a firmer basis for your own judgments and tolerance for the judgments of others. And surely so deep-rooted and universal a human activity as art calls for a perpetually renewed exercise of human judgment. Art is, willy-nilly, everybody's concern.

What has been called the "historical approach to art" has been derided by certain writers on the subject. Probably this is because of their awareness that with the acquisition of factual knowledge may come all too often that false affectation of enjoyment to which we have already alluded. Rightly they feel that people who can experience enjoyment only at second-hand do no good to the cause of true art, and tend to encourage the creation of spurious art. However, we have already stressed this danger, and made our plea to the student. That being so, we know of no more practical method of imparting factual information on art than through a general survey of its history from earliest times to the present day ; and this will be given in the Lessons that follow.

Art and the Arts

For our immediate purposes, one other limitation must be imposed. It is a familiar one. In general usage the word "art" has come to be used for visual art in particular, that is to say, for art as expressed in drawing and painting, sculpture, architecture, and such handicrafts as pottery, metalwork, textiles, woodcarving, stained glass, and embroidery. It is, of course, in this sense that institutions where the practice of any, or all, of these is studied are called art schools ; and this is the sense in which we shall use the word here.

A further distinction that is sometimes loosely made, that between "fine" art and "applied" art, will concern us much less. By "applied" is meant "applied to a utilitarian end," as distinct from "fine," which presumably refers to art that is concerned exclusively with the self-expression of the artist—"art for art's sake," as the hackneyed phrase goes. Yet it may be justly argued that the quality which makes, say, a teapot or a piece of furniture acceptable as an example of art is precisely that which is recognizable in a fine piece of sculpture. Without that quality, each is no more than craftsmanship, and not necessarily good craftsmanship.

A Medium of Delight

Oscar Wilde, who wrote and talked much sense and nonsense about art, made the claim that "all art is quite useless." Like most epigrams, this was an overstatement, though it does convey one aspect of the truth. Certainly art exists in its own right, and can be appraised without reference to any immediate utilitarian purpose. One does not have to be a devotee of the old pagan gods in order to be thrilled by the Parthenon ; and it is a truism that a good painting of a dust-heap may be more relished as art than a bad painting of an angel.

Nevertheless, through all ages man has disciplined art to countless varied uses. It has been made the handmaid of religion, the slave of commerce, a powerful weapon for every kind of argument and propaganda. Nor does it appear to have really suffered by such discipline. On the contrary, some of the most enduring of the world's masterpieces were inspired by some such extraneous purpose as these.

As one of the most powerful of human urges and needs, as a means of expression and communication which both in the imparting and in the receiving raises man's perceptions beyond what is merely material, beyond even what is intellectual, art has ever been, and will remain in its varying forms, the medium of such untold immediate delight and abiding satisfaction that any kind of human existence without it is not to be imagined.

LESSON 2

Art in Ancient Egypt

The study of art history most suitably begins with the art of ancient Egypt, since the valley of the Nile gave birth to the oldest civilization in the world. Research has proved that the civilization of the early historic period of Egypt was preceded by one still older, which takes us back at least another 4,000 years ; and that even in this dim and distant age Egyptians had arrived at a high degree of artistry in the carving of hard stone and ivory. Undoubtedly their art instinct, like that of other primitive races, first expressed itself in personal decoration. From the earliest pictorial representations of human figures on pottery and on the walls of tombs, it would appear that the men and women painted their bodies with red and yellow colour respectively, and tattooed them with figures of animals and other symbolic marks. There is evidence also that when they turned from the decoration of themselves to the decoration of walls, their art was more or less naturalistic—i.e. representational or imitative of natural appearance—as opposed to the symbolism which came later and eventually dominated the medium.

It was not until Egypt's dynastic rulers began to hold sway that Egyptian art assumed its peculiar individuality. To understand the direction it took, one has to remember that the dynastic kings of Egypt were also high priests and almost gods. As temporal rulers, they had to impress their subjects with their power; hence the ideals of size and massiveness that inspired the building of their temples to the dead or living. Although the gigantic in architecture is commonly supposed to be essentially a modern conception, the Great Pyramid of Cheops still dwarfs the largest modern buildings in Cairo. With the idea of size went those of stability and permanence. If the purely human instinct to perpetuate a name and memory was partly responsible for this characteristic, the old Egyptian "cult of the dead" was still more so. Life in ancient Egypt was a conscious preparation for death. The pyramids, the world's last word in permanence, are temple tombs; and although the ruling Egyptian may sometimes have built his house for a lifetime only, he built his tomb for eternity and even called it his "house everlasting."

EGYPTIAN STATUARY. Wooden effigy of a Vth dynasty official ("Sheik el Beled") and gold-plated wooden statuette of Tutankhamen as Horus.
Cairo Museum and Harry Burton: Metropolitan Museum, New York

Art as the Witness of Power

There remains the third element—mystery. Every powerful priesthood in history has for its own purposes appealed not to the reason but to the superstitions of its humble but indispensable votaries; but the Egyptians went beyond all others in impressing their people with the superhuman power of priest-kings and divinities. The mystic element in nature has never found more fitting expression in art, and even to-day ancient Egyptian motifs and ideas are borrowed whenever there are mysteries to be suggested.

The artistic genius of the dynastic Egyptian was architectural and sculptural, rather than pictorial. With the knowledge of the three main characteristics of that architecture—massiveness, permanence, mystery—the student should be prepared to examine some of the most wonderful monuments for himself. He will notice that with the passing of the centuries the main characteristics become intensified. There is a vast difference between the earliest pyramid, the Step Pyramid built by King Zeser of the IIIrd Dynasty (2900 B.C.) with its comparatively modest base of 394 feet by 351 feet, and the Great Pyramid of Cheops, of the next dynasty, which is more than half a mile round, and its companion pyramids of Gizeh. He will see also that a certain lightness of style which characterises the Step Pyramid is not present in the huge unbroken sides of the works over which the colossal Sphinx—by some considered the greatest achievement of Egyptian art—stands guardian.

Of the temples the most wonderful is that of Karnak, near Luxor. Strictly speaking, this is a city of temples, tombs, avenues of sphinxes, and sacred lakes, all within an encircling wall. This house of Amen-ra was already a vast and splendid building when the kings of the XIXth Dynasty cast their eyes upon it and decided to erect in front of the existing building the famous Hypostyle Hall, 338 feet long and 170 feet broad, with roof 80 feet in height, supported by no fewer than 134 columns.

Increase in the size of tomb or temple was accompanied by greater technical perfection.

THE SPHINX AT GIZEH, one of the greatest achievements of Egyptian art. The head is probably a portrait of Khufa (Cheops) or Kafra (Chephren), builders respectively of the Great Pyramid and the Second Pyramid; the latter is directly behind the Sphinx.

Square, unadorned stone pillars, such as were used in the causeway leading to the second pyramid at Gizeh, gradually took the shape of articulated columns. Similar developments can be discerned in the rock tombs of Beni-Hassan.

THE TEMPLE OF AMEN-RA AT KARNAK, near Thebes, begun by a pharaoh of the XIIth dynasty (c. 2000 B.C.) and finished by a Ptolemy (c. 200 B.C.), was especially famous for a vast hall of 134 columns, for wall decorations and inscriptions, and for its grand avenues of sphinxes.

Royal Air Force Official

The Triumph of Impassivity

Famous early Egyptian sculptures include the "Scribe," now in the Louvre, Paris, and the "Sheik El Beled," in the Cairo Museum. Firmly and masterfully modelled, with a marvellous breadth and dignity of treatment, they bear the impress of personal portraiture, of having been done from life. Yet in a human sense there is little or no life in them. Although the muscles of the body are at times indicated, they lack all accent and expression, all suggestion of potential movement. Always the expressionless face looks straight ahead.

The laws governing Egyptian art were extremely rigid. Figures became increasingly conventionalised (i.e. designed in accordance with accepted formal rules), and "static" postures were invariably adopted rather than any that suggested movement.

A later and more vivid aspect of Egyptian art was revealed by the excavation at Karnak in 1922-23 of the tomb of Tutankhamen. Egyptian art is considered by experts to have reached its peak in the Old Kingdom between 2800 and 2500 B.C. This relic of the New Empire (1580 B.C. onwards) was found to contain many objects in metal and wood of a lighter craftsmanship. The story of the tomb must be read elsewhere, but one may mention among its contents thirty statuettes of wood, plated with gold, of which one, representing Tutankhamen himself as Horus the Avenger, has a grace and an animation that mark a definite break with the old tradition of impassiveness. Among the other finds were examples of polychromatic art that make the typical reliefs—following the curious Egyptian convention by which face, arms, hands, legs, and feet are drawn in profile, while shoulders and torso are given a front view—seem dead and rigid. The craftsmen of Tutankhamen's age contributed a most refreshing touch to the tradition before darkness finally descended upon the great Egyptian civilization. Their temporary release from the bondage of convention was a feature of the "heretical" reign of Amenhotep IV, who introduced the worship of Aton (the solar disk), renamed himself Akhnaton, and built a new city, Akhtaton (Tell el-Amarna) as part of his move away from the worship of Amen and the other old gods. The supreme example of the new and spiritualised art which flourished for a few years is the beautiful portrait head of Akhnaton's queen, Nefertiti, carved in limestone and brightly painted. This is now in the Berlin Museum, and two views of it, profile and full face, are reproduced in the colour plate facing page 1360. As a portrayal of essential femininity some judges have ranked this famous work of Egyptian sculpture with Leonardo da Vinci's equally famous painting of the "Mona Lisa" (reproduced in page 1381)—and not without reason!

AMONG THE HUGE COLUMNS of the hall of the temple of Amen-ra at Karnak, many of which retain some of the splendid colouring of their reliefs. To span one of these great central columns, six men must join hands with their arms extended.

LESSON 3

Oriental Art: Assyria, Persia, India

ASSYRIAN WAR-CHARIOTS CROSSING A PONTOON BRIDGE. Assyrian sculptors devoted their art to the glorification of the kings and of their wars and hunting exploits. The bas-reliefs on palace buildings present a degree of realistic statement, particularly in the shapes and attitudes of animals, never surpassed in the history of sculpture.
British Museum

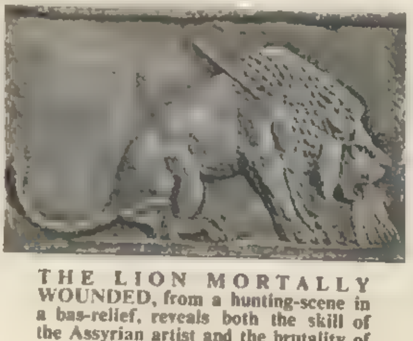
COMPARED with the knowledge we have of the art of ancient Egyptians, our reconstruction of the art of the early Asiatic empires, of Babylonia (now known, with its next-door neighbour Chaldea, as Iraq) and Assyria, is comparatively fragmentary. Their monumental works were not as durable as those of the Nile valley, because of the destructibility of the building material, which consisted mainly of sun-dried bricks clad with coloured and glazed tiles, and alabaster slabs decorated with relief carvings.

A few shapeless sandhills are all that is left of the mighty city of Babylon, but we know from ancient records the colossal extent of the strong walls by which the city was encircled. We know that the pyramidal Temple of Baal was built in eight terraces on a base of about 800 feet, and that the famous Hanging Gardens of Semiramis were laid out in similar fashion on pyramidal terraces. Some Chaldean bas-reliefs and statues of extraordinarily skilled workmanship, unearthed about 1900 at Tello, the site of a royal palace, date from about 3000–2000 B.C.

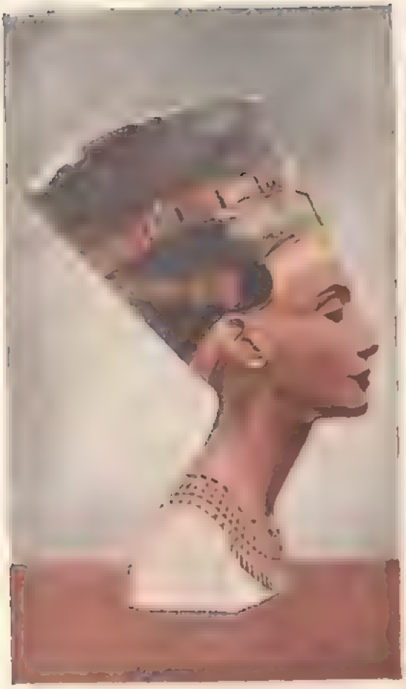
There was a striking difference between Chaldean and Egyptian art. Egyptian sculpture reveals a striving for monumental repose, and a simplification of form which often resulted in smooth surfaces without even a suggestion of bones and muscles. The diorite statues from Tello, notably that now known as "The Architect of Tello," are marked by that sharp characterisation and pronounced realism that are the most striking feature of Chaldean and Assyrian plastic art.

But the Tello discoveries do not represent the whole artistic achievement of Chaldea. In the 1920s the Pennsylvania Joint Expedition to the site of Ur, the city of Abraham, found proof that as far back as 3500 B.C. Sumerian workers in the valley of the Euphrates were turning out veritable masterpieces of design and technique in gold and silver, copper, stone, and shell. Thorough excavation revealed the traces not only of temple and palace but also of domestic architecture showing familiarity with the main principles of modern construction, and dated about 2100 B.C. It is clear that much of the material was imported for the making of artistic treasures, though the houses were built of the native burnt brick.

Assyrian sculptured reliefs are far less naïve in the treatment of the human form than those of Egypt, the attitudes being quite natural. In depicting animals, too, they reached a perfection of realistic statement that has never been surpassed. In the world-famed "Dying Lion," at the British Museum, the suggestion of form and muscular development is the more astounding because the relief is very little raised above the surface of the background. The artistic importance of this work is greater than that of those curious inventions of the Assyrian genius, the winged bull or lion with a bearded



THE LION MORTALLY WOUNDED, from a hunting-scene in a bas-relief, reveals both the skill of the Assyrian artist and the brutality of the militaristic Assyrian mind.
British Museum : photo, Mansell



LINKS WITH AKHNATON: HIS WIFE AND HIS SON-IN-LAW

A triumph for all time, this painted limestone head (left) of Nefertiti, wife to the heretic pharaoh Akhnaton, was discovered in the workshop of the sculptor Thothmes at Akhetaton. Right, the gorgeous anthropoid casket containing the mummy of Tutankhamen.

ART. LESSON 2

To face page 1360, Vol. 3

EARLY CHINESE PAINTING

The earliest known Chinese painting is a silken roll of about A.D. 370, illustrating "The Admonitions of the Preceptress to the Court Ladies." In the portion above, the Lady Feng shows her bravery in the face of a bear. The lower picture is a landscape copied in the Sung period from a work of Wang Wei (699-759) of the T'ang period.

To face page 1361, Vol. 3 *British Museum*

man's head and five legs, or even the winged angel, though the latter has been handed down, somewhat modified, to play an important part in Christian art.

Characteristics of Assyrian Building

In architecture the Assyrians invented the vault and the cupola. Their palace buildings rose terrace-like in several storeys, each storey being crowned by a little gallery with short columns, which provided the interior with light and air. The use of columns played a very subordinate part. Where they occur, they are generally of moderate height, and carry a curious form of capital, consisting of two pairs of volutes placed one on top of the other. Sometimes these short columns rest on the backs of walking lions, a device frequently resorted to by the Italian sculptors of the 13th and 14th centuries.

BULL CAPITAL from the winter palace at Susa : one of the few original Persian contributions to ancient art.
Louvre : photo, Maurice

Of the enormous extent of some of these palace buildings, the excavations at Khorsabad may give some idea. The brick terrace on which the palace was raised has been calculated to have occupied about 40,000,000 cubic feet. About 210 apartments, many of which were decorated with wall-paintings, were arranged around 30 courts. The temple pyramid by the side of the palace had seven steps, four of which —each 20 feet high—are still extant. Each of the seven storeys was resplendent in a different colour, symbolic of one of the seven planets. The porches had round arches of wedge-shaped enamelled bricks.

Derivative Art of Persia

The art of ancient Persia was always eclectic, and never reached any degree of independence. The love of dazzling display evidenced by the powerful dynasty that ruled over Persia after the fall of the Assyrian empire in the middle of the 6th century B.C. was grafted on to the artistic traditions of Assyria, of the Ionian Greeks in Asia Minor, and to a lesser extent even of Egypt. The only special form developed by the Persians is the over-decorated capital, consisting sometimes of a pair of bulls or unicorns back to back, sometimes of two flower-cups—one upright, the other and lower one turned downwards—upon which rest pairs of volutes, rather like those of the Assyrian capitals, but placed upright instead of horizontally. The relief sculpture has something of the priestly dignity of Egyptian art, and something of the realism of Assyrian.

It is in the remains of the royal palaces at Susa and Persepolis that Persian art of the early periods can best be studied (*see* colour frontispiece to this volume). After its conquest by Alexander in the 4th century B.C. the history of Persia becomes fragmentary, and from the standpoint of its art the next great period arose under the Sassanid dynasty, A.D. 226–651. The most famous examples of Sassanian art are to be found chiselled under the rock tombs of the Achaemenians at Persepolis. These bas-reliefs, seven in number, begin with two equestrian figures in a rather archaic style. In a garden (Tak-i-Bustan, the "Garden of the Grotto") near Kermanshah, there are bas-reliefs representing a stag hunt and also a boar hunt. These varied reliefs are the supreme expression of Sassanian art.

In this period fine goldsmith's work—cameos and coins—was also produced. The best extant specimen of Sassanian metalwork is the "Cup of Chosroes." The frame is composed of a network of hammered gold in which are inserted medallions of rock crystal, cut in relief to represent Chosroes II (Parvis). According to ancient tradition, this cup, which for many centuries was known as the Cup of Solomon, was presented to Charlemagne by Haroun-al-Raschid.

Buddhist and Hindu Art in India

The earliest remains of Indian art belong to the 3rd century B.C. The emperor Asoka (272–232) was a convert to Buddhism, and his peaceful and prosperous reign resulted in high artistic development. He issued a series of edicts, which were incised upon the highly polished surfaces of rocks and monolithic pillars. Among the fragmentary remains of the pillars is one found in the ruins of Sarneth, of which some 17 feet of the original 50 feet remain. It was surmounted by a lion capital, 7 feet high, which is the finest piece of ancient Indian sculpture surviving from a golden age that lasted until the Muslim conquest of 1001.

The magnificent series of Buddhist wall-decorations, painted and sculptured, in the cave temples at Ellora, Ajanta, and Karli, which date from the 2nd century B.C. to the 8th century A.D., constitute one of the greatest achievements of Asiatic art.

Indian art is not concerned with the real or the temporal, but with the spiritual and eternal, not with physical beauty as such, but with the soul. It is idealistic and symbolic.

Influence of Islam

The Muslim prohibitions in the Koran against images and pictures not only repressed the pursuit of naturalistic art, but also fanatically encouraged the mutilation of existing works, and a great many fine examples of Indian painting were destroyed in the 11th century A.D.

Indian painters wrought chiefly in fresco, a method still practised, since it is peculiarly adapted to interior decoration in dry, tropical climates, and, with care, is more durable than oil painting. While few examples of early Indian painting remain, a number of small Nepalese paintings, recovered from Tun-huang, on the western borders of China, reveal the artistry of their creators.

The sculptors were averse to portraiture, partly because individualism was repugnant to the spirit of their faith. Their aim was to produce an impersonal type of face. In the sculpture of animals there is nothing to surpass the gigantic war horse at the Black Pagoda at Kanarak, in Orissa.

Art Under the Moguls

Indian art raised its head again after the Mongols had overthrown Arab rule (at the beginning of the 16th century), and under the Mogul emperors a school of miniaturists reached its apogee.

At the Mogul court at Delhi, Akbar (1542 1605), the grandson of Babur (founder of the dynasty), greatly encouraged the arts, and under his patronage painting received a fresh stimulus. Many of his artists were Hindu. Others came from Persia, and during the first phase of his rule Persian miniatures were imitated. Basawan is the most famous of Akbar's artists.

By the 17th century the Persian element had been completely absorbed, but Indian painting received a setback during the reign of Aurungzebe towards the end of the century, because that emperor was a strict Muslim. During the 18th century Hindu themes became popular, but thereafter European influence began to make itself felt.

THE EMPEROR AKBAR HUNTING, by a Delhi court painter of the 16th century
British Museum

The present century has seen a revival of Indian art based on traditional methods. Lyrical in mood, allied to folk song and dance, these works express the idealistic nature of the Indian viewpoint. This modern school has received great encouragement from such artists as Abanindro Nath Tagore, Nanda Lal Bose, and Surendra Nath Ganguly.

LESSON 4

Oriental Art: China and Japan

THE Chinese claim that their civilization is at least 5,000 years old. It seems certain that the beginning of this culture had its roots north of the Yangtse valley in the third millennium before Christ. In China the past is a living tradition, and Chinese culture is a synthesis of ancient and modern.

A wonderful sense of rhythm and form is displayed in the early pottery and bronzes, in the Buddhist sculptures, and in great examples of the T'ang and Sung periods. Painting was considered by the Chinese to be a branch of calligraphy. The importance of technique lay in the quality of the brush-stroke

expressed on paper or silk. Painting was therefore related to writing, and writing to painting, and this view is still strongly held at the present day. So far as painting is concerned, naturalistic representations are alien to the Chinese mind. One thought, one mood, is conveyed in a picture or a piece of sculpture, and this convention has imposed a strict discipline, a single-mindedness, not easily appreciated by the uninitiated Westerner.

Shang, Chou, and Han Periods

Chinese culture has survived several barbaric invasions, but has absorbed many alien influences.

In spite of the diversity of styles over a period of three thousand years, Chinese art is homogeneous. The talents of the Shang dynasty (*c.* 1500–1100 B.C.) are shown in the casting of bronze, an art which was brought to perfection. These early bronzes express rhythm, balance, space, and sensitive treatment of bird, animal, and flower motifs. The Shang craftsmen realized, as few artists realized before or since, the limitations of their medium. Bronzes also characterise the Chou period (*c.* 1100–481 B.C.), but a greater perfection of technique had been achieved.

The paintings of the Han era (*c.* A.D. 206–221) have long since disappeared, though references to them occur in the literature of the time, and copies in stone engraving have been discovered in tombs dating from the 3rd century A.D. Movement is emphasised, and forms are expressed in thin lines. Han sculpture is mainly represented by small objects of bronze, jade, or clay. Later the process of dry lacquer was invented, combining hardness with lightness and durability. But plastic art had outgrown the stage of infancy long before it was transformed by Buddhist influence, as is attested by the fine bas-reliefs found in the tombs of the Later Han. In these reliefs the delineation of the figures is simple and masterly.

The earliest known specimen of Chinese painting dates from about A.D. 360, and is in the form of a silken roll containing exquisitely drawn figure subjects by Ku K'ai-chih for a tract called *The Admonition of the Preceptress to the Court Ladies* (*see* colour plate facing p. 1361). Of the painting of the 5th and 6th centuries no authentic specimen survives.

T'ANG STATUETTES. Left, probably a priest, in painted glazed pottery from the tomb of T'ing-hsun (d. A.D. 728). Right, a court lady in unglazed ware. Both figures show high technical skill.

T'ang Period

The T'ang age, A.D. 618–907, was the begetter of China's grandest and most vigorous art, but no original painting has survived of the most distinguished artist of that period, Wu Tao-tze. He was the creator of stupendous designs and initiator of an expressive, calligraphic use of outline. But to-day all our notions of the excellence of his work has to be deduced from a number of medieval copies.

T'ang painters excelled in portraying action and movement, which is not characteristic of later Chinese art. A new school of landscape was founded by Wang Wei (699–759), and specimens of his work were copied by a later master, the great Chao Meng-fu (late Sung).

Tombs dating from the T'ang dynasty have furnished many beautiful examples of both glazed and unglazed pottery, including some vigorously modelled animal and human figures, as well as some painted vases with incised ornamentation of flowers.

The 11th century saw the birth of a special academy of painting at the Chinese court and the development of the suave, accomplished style of landscape painting of which Ma Yuan is the typical exponent. At the court of Hui Tsung (himself a painter of importance) there arose a reaction against stereotyped and traditional modes of representation, which synchronised with an intellectual movement against Confucianism. The emperor and his artists discarded all the ancient models, and proceeded to devote themselves to the study of birds, flowers, and animals, which they drew vividly from life.

Sung Period

This impressionistic style was exemplified by the Sung school (960–1279). Ink was used, rather than colour. A fashion for ugliness, distortion, and abnormality in the religious representations of saints and Buddhas characterised certain aspects of Sung art. At the same time, the painting of birds and flowers had reached an astonishing perfection. It

EXQUISITE EXAMPLE OF SUNG ART. This study of a bird on a bough by an anonymous Chinese painter exemplifies the superb treatment of natural subjects found in works of this period (A.D. 960–1279).
Eumorfopoulos Collection

was the great age of landscape painting, and the mysteries of nature were expressed in visible form; indeed, the greatness of Chinese art is to be found not in its reverence for nature alone, but in the unity of man with nature. These painters of the 10th to 12th centuries showed that art was no mere imitation, but reality experienced at first-hand. A certain sect of Buddhism, the Dyana or Zen sect of Contemplation, strongly influenced Sung painting.

In 1264 the Sung dynasty fell before the invading Mongols, but Chinese civilization absorbed the conquerors. The resultant period (1280–1368) ended with the establishment of the Ming dynasty.

Ming Period, and After

The great period of the Ming era extends from 1368 to 1644. There is a noticeable change of style: more intense colouring, and an increasing trend towards the eternal aspect of objects. In time this influence led to artificiality and over-stylisation, and it was examples of the Later Ming period that propagated many false notions regarding Chinese art that were current in the West during the 18th century. Yet there are many admirable examples of Ming art, and decorative motifs reached a high degree of colour and harmony.

China's isolation from the outside world from the 16th century to the middle of the 19th had an inhibiting effect on many works of art. With the establishment of the Manchus (Ch'ing dynasty, 1644–1912) tradition fettered imagination and artistic styles tended to be reduced to a formula. Nevertheless Chinese art was still vigorous during the 18th century. Calligraphic rhythm was increasingly discarded, and this led to a traditional academism.

At the present day there is no great school of painting in China. Traditional art forms have sometimes been allied to Western ideas, and Chinese painters maintain that innate fidelity to experience so vividly expressed by the majority of their countrymen.

Derivative Art of Japan

With the adoption of Chinese civilization and the introduction of Buddhism into Japan in the 6th and 7th centuries, painting became the major art of that country. Few works of this period have survived, yet it is evident that Japanese artists of the 7th century derived their ideas from the Chinese Y'ang period and even made copies of their masterpieces.

Three hundred years passed before a native painter of outstanding ability and original genius appeared in Kanaoka, the founder of the Kose school. He might well be termed the precursor of Japanese pictorial art. Religion restricted artistic genius, yet it was a priest who finally broke the long spell of mediocrity. This was Meicho, or Cho Densu of Kyoto (1351–1427), a painter of originality and vigour. His work is impressionistic, and is distinguished by light and silvery tones. Cho Densu rescued Japanese art from insularity, but at the same time there was danger that the cult of Chinese

CHINESE CERAMIC ART. Left, wine-jar with three-colour glaze and relief depicting fairy draughts-players (15th cent.). Upper right, stand for artists' colours, Chia Ching period: lower right, porcelain wine vessel ("Chang Te," 1506–21).
Eumorfopoulos Collection and British Museum

models might become pedantic. Landscapes in the Chinese style were painted by Japanese artists who had never been to China.

The inevitable reaction produced the Kano school, of which Masanobu was the founder. Many of the painters belonging to this school were Buddhist priests, and they sought to restore colour, for which the earlier Japanese artists had been renowned.

The second phase of the Kano school is known as the Momoyama period. Magnificent screens painted on a gold ground were among the fine productions of this time. The fourth phase (Tokugawa) is remarkable for its light colouring and simpler style.

The Popular school of Japan in the 16th century concerned itself with everyday affairs and caricature. Hishigawa Moronobu (*c.* 1646–*c.* 1716) carried on this tradition, introducing sketches of contemporary manners and customs. These provide valuable examples of Japanese life at that period, and also led to the establishment of a school of wood-engravers. Hanabusa Itcho (1651–1724) freely introduced wit and humour into his drawings of open-air life. The revolutionary assault upon the classical tradition, however, was initiated by Maruyamo Okio (1733–95), who established a school in Kyoto expressly to practise naturalistic art.

The 18th century saw the flowering of the art of the colour-print—that aspect of Japanese art with which Europeans are most familiar. The greatest genius of this school was Hokusai (1760–1849). He found endless subjects in men and nature, and his views of Fuji are world-famous. He had a unique gift for composition, and his drawings are highly imaginative. With him may be grouped Hiroshige (1797–1858), whose colour-blocks have also acquired a well-merited and universal reputation.

LESSON 5

The Minoans and Mycenae

The civilization of Crete and Mycenae is of great antiquity and remarkable interest. Crete, indeed, was the first European land to attain any high achievement in art between 2200 and 1600 B.C. The chief cities of this pre-Hellenic culture were Troy in Asia Minor, Mycenae and Tiryns in Greece, Knossos and Phaestus in Crete itself.

The name Minoan has been given to phases of Bronze Age civilization in Crete. Minoan art is divided into Early (*c.* 3600–2100 B.C.), Middle (*c.* 2100–1600 B.C.), and Late (*c.* 1600–1200 B.C.) periods. The golden age of Crete—in the Late Minoan period—lasted about a century (1500–1400 B.C.), towards the end of which the Mediterranean island was invaded by the more robust Mycenaeans and the palace of Knossos burnt. The remains of this intricately-built palace, with its ancient Labyrinth, exhibit architectural and decorative talent of an unusual order.

Some excellently preserved wall-paintings (fresco) show processions, bull-ring scenes, warriors and their women seated in their courts or looking out from balconies, landscapes and seascapes, and other scenes of national life. The human figures are narrow-waisted and long-limbed. From the fresco paintings which decorated the public rooms and main corridors of the palaces, from the delicate work of the gem engravers and, above all, from statuettes and plaques in glazed earthenware, one becomes aware of a complex civilization and a sophisticated art.

LIBATION VESSEL from Knossos: a bull's head of black steatite (c. 1350 B.C.).
Photo, G. Maraghianni

Furniture and household goods, costume, and weapons show little change throughout the three periods, but during the Late Minoan age the polychrome vase-painting, characteristic of Middle Minoan pottery, gave place to black silhouette-drawing on a light ground. Vases were usually wheel-made, of dignified but not very varied shapes. The floral designs ceased to be naturalistic and became formal combinations of a few popular forms—lily, iris, and rosette. Spiral ornament, so easily reproduced and adapted, became customary. Magnificent work of a more realistic kind was still done in fresco painting, ivory carving, stone work, and gem-cutting, and there was presumably fine metal-work copied in commoner materials, though the originals have perished.

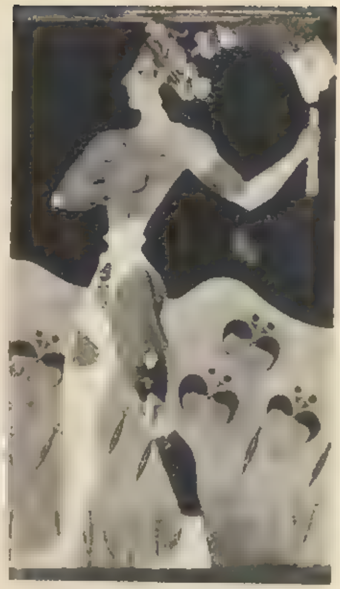
A number of smaller objects executed by the artist-craftsmen of the time prove that great skill and efficiency had been attained at this remote period in goldsmith's work, enamelling, cameo-cutting, and pottery. A table of gold-plated ivory, set with crystal plaques, backed with silver and blue enamel, was one of the Knossos discoveries.

During the course of the Late Minoan age in Crete, a similar

culture was being developed in the southern regions of the Greek mainland. There were native communities in Aegina and on the Corinthian Isthmus, settlements in Attica and Euboea. The Aegean Islands took the lead in the decoration of pottery by painted instead of incised ornament. Whence they learned this art is not certain; possibly it was from the Greek mainland, where it had been practised for a long time in Thessaly.

Golden Mycenae

Guarding the pass through the hills to Corinth and the north, Mycenae was the fortress of Agamemnon in the days of the Trojan War. Because of its geographical position the city could hardly have avoided the fame and prosperity which befell it as soon as the Greek mainland began to share the dawning civilization of Crete and the Aegean Islands some 3,000 years B.C. Schiemann's excavations on the site of Mycenae, which were begun in 1876, revealed an astonishing array of gold rings and necklaces, and embossed and intricately decorated gold fittings and platings for furniture and clothing, of which the perishable parts are dust or splinters. Schiemann's discoveries brought to light not only an elaborate craftsmanship and a wealth of complicated design, but a whole style of art, with traditions and ideals of its own, not merely pre-Hellenic, but, at the time of these excavations, unrelated to any comparable discoveries.



FRAGMENT OF FRESCO from the royal palace at Knossos, which probably represents a Minoan prince taking part in a dance or other ceremony.
Courtesy of Hellenic Society

Early shaft graves, or repositories for grave contents, reveal as nothing else has done the great wealth of the Mycenaean civilization. Much of the funerary equipment is of gold: drinking cups, personal ornaments, embossed plaques for the decoration of robes, hangings, or wooden chests now perished. These were found in great number and infinite variety of design, with spiral patterns commonly, but also with figures of animals and plants and occasional human scenes, such as that of the famous lion-hunt inlaid in a dagger blade with alloys of several colours.

Silver is rare, bronze abundant and worked into gold designs, which serve to explain some peculiarities of the pottery belonging to this period and also to identify as Mycenaean work some of the vessels brought from Mycenae in tribute to the Egyptian Thothmes III (16th century B.C.). Mycenaean ornamentation corresponds partly to that of ancient Egypt, partly to that of the Bronze Age in northern Europe.

The Mycenaean civilization came to an abrupt end about 1000 B.C., when the Dorian invasion from the north drove the Mycenaeans (or Achaeans) to the Aegean Islands and to Asia Minor, where their influence made itself felt among the Lydians and Phrygians. Greece itself relapsed into a state of semi-barbarism. Not until the 7th century B.Ç. did the current of culture return, and Greece enjoy a period of artistic development and achievement unsurpassed in the history of the world.

LESSON 6

Art of Ancient Greece

THE ancient Greek conceived his gods and goddesses in human form, and made his statues of them accordingly. He strove to achieve by means of balance and proportion the utmost perfection of which human form is capable, eliminating all accidental and personal elements. His human gods or heroes are generalised; they are persons, yet they are impersonal.

His art arose naturally from his daily life. A natural instinct for beauty was constantly fed by the sight of the graceful movement of beautifully developed bodies, draped or naked, engaged in dance or physical exercise. The last was almost as much a part of his religion as was his physically perfect god.

The earliest Greek works of plastic art—Greek, as differing from Minoan and Mycenaean—clearly show Egyptian and Assyrian influences, though already in these archaic works there is an undeniable striving after truth, a searching for the essential beauty of the human form. The Greeks imbued their statuary with a general feeling of flight. For the first time in the

history of art man had learned to soar away from his surroundings. With the dawn of the 6th century B.C. the emancipation from the archaic was well on its way. French excavations at Delphi led to the unearthing of a wonderful bronze statue of a charioteer, life size, in which the strongly realistic treatment of the feet contrasts with the conventionalised drapery. Skilled reconstruction has thrown fresh light on some of the work of this period. A restoration of the pediment of a temple at Aegina, representing nude athletes fighting across the bodies of their stricken comrades, reveals an unexpected freedom from stiffness.

PART OF THE GREAT FRIEZE OF THE PARTHENON AT ATHENS showing magistrates and maidens in the Panathenaic procession held every five years in honour of the goddess Athene. The frieze is carried above the architrave of the inner row of columns at either end of the cella or temple itself, and along the side walls of the cella. It was thus protected by the surrounding colonnade.

The Great Greek Sculptors

But the great period of Greek sculpture begins about 480 B.C., with the Argive school of Polycleitus, Myron, and Pheidias. The "Doryphorus" of Polycleitus was one of the earliest statues in which the weight of the body, instead of resting on both feet, is thrown on one foot, while the other leg is "free standing" with the heel raised from the ground. A wonderfully easy pose results. According to the Greek writers, Polycleitus interested himself especially in the study of human proportions and produced the "Doryphorus" as a model of the human form at rest.

Myron was one of the first to discard the rigid uprightness of chest and head, and to show the full flexibility of the body in action. His statues of athletes, among them the well-known Discobolos (discus thrower), are notable instances of his art. This school reached its culmination point in Pheidias, who was undoubtedly the leading spirit in the sculptural decoration of the Acropolis and was responsible for the colossal statue of Athene, 40 feet high, by which this group of buildings was once dominated. It is considered likely that the sculptures of the Parthenon, if not actually the work of Pheidias, were made under his direction.

The 5th century, especially that part of it which is known as the age of Pericles, witnessed the highest point reached by Greek art, but it remained at a high level for at least another

ANOTHER PART OF THE PANATHENAIC PROCESSION on the Parthenon frieze, showing Athenian knights or cavalrymen on their magnificently-sculptured horses. The climax of the procession is depicted on the east side; there a new robe is presented to Athene in the presence of the principal gods and the other principal goddesses of Greece. Of the 524 feet of the frieze, about 247 are now in the British Museum; these sculptures were brought to London in 1806 as part of the "Elgin Marbles" rescued by Lord Elgin from imminent destruction on the Acropolis.

MASTERPIECES OF ANCIENT GREEK SCULPTURE. Upper left : Hermes with the infant Dionysus, ascribed to Praxiteles, now in the museum at Olympia in the Peloponnese. Upper right : the Nike (Victory) of Samothrace, now in the Louvre, Paris. Lower left : the Aphrodite of Melos (or " Venus de Milo "), by an unknown master, also in the Louvre. Lower right : the Doryphorus (spear-bearer) of Polycleitus, in the Vatican, Rome.

hundred years. Then in the 4th century, the sculptor's ideal became modified ; instead of aiming merely at the interpretation of a robust physical life and spiritual serenity, he sought the expression of human emotion and passion. The three great masters of this period are Scopas, Praxiteles, and Lysippus. To Scopas has been ascribed a head from the temple at Tegea (in the Athens National Museum), and masterpieces such as the Niobe group and the Nike of Samothrace are ascribed at least to his influence.

The most famous of the ascriptions to Praxiteles are the Venus of Cnidus, at the Vatican in Rome, and the Hermes with the infant Dionysus at the Olympia museum—works that show less passion and more dreamy tenderness than is seen in the art of Scopas. There is much human charm in the figure of the Hermes, but this hardly extends to the child, whose man-like expression is curiously suggestive of those representations of the child Christ in early Christian art. Lysippus, who is said to have executed a vast number of statues, including many of Alexander the Great, delighted in the rendering of physical vigour, as in the " Apoxyomenus " of the Vatican.

" Hellenic " and " Hellenistic "

Historians of artistic development draw a sharp line of division between the art that followed the death of Alexander the Great and that of the preceding period. The change in political conditions caused by the formation of independent states under various princes is held to have been responsible for a certain debasement of the artistic ideal. The works of the Hellenistic schools of Rhodes and Pergamum—notable among which are the Laocoön group (Vatican), the Farnese Bull (Naples), the Apollo Belvedere, and the Dying Gaul—are considered to mark a decline. The first and last of those named have been accused of " playing to the gallery." But there are works of genius and real beauty belonging to the same age.

Marble and Terra-cotta

In order to arrive at a just estimate of Greek sculpture, the student should consider not merely the works ascribed to the major sculptors, but also the far more homely aspect of life represented by the workers in terra-cotta. In the 4th and 3rd centuries B.C. Greek sculptors produced thousands of terra-cotta figures illustrative of Greek daily life and charmingly free from the classic conventions. In this phase of the art they would seem to have had much in common with the Etruscan realists, and to have expressed their own sense of humour.

A very attractive form of this lighter Greek sculpture is seen in the Tanagra figurines, thousands of which have been found in graves. In the art of cutting cameos and striking coins the Greeks also achieved perfection.

There is no extant Greek painting on which to form a judgment, apart from the Greek vases and the Pompeian wall-paintings executed by Greek artists during the late Hellenistic period. But there is no lack of literary records, and many names of Greek painters have come down to us. We know that they used the fresco technique for wall paintings, and the tempera (white of egg) for panels, and—in the best period—the encaustic method, i.e. painting with dry wax-sticks and burning the colours into the carefully prepared surface. Later came the mosaic work which was to become vulgarised in the decoration of Roman houses.

The decorative paintings of Polygnotus and Micon were, it is safe to assume, coloured outline drawings, without modelling, shadow, or perspective. According to the records, Agatharchus, at the end of the 5th century, was one of the first artists to study the problems of perspective. Then came Apollodorus, a pioneer in the rendering of light and shade; and those reputed masters of realism, Zeuxis, Parrhasius, and Apelles.

Although no means exist of comparing a single example of Greek painting with medieval or modern work in that medium, we have at any rate the evidence of the red and black figure pottery to guide us in our estimate of the painters' ability in general. This ware, produced in great quantities from the 6th to the 4th centuries and distributed through the Mediterranean countries, represents in many cases the loveliest combination of ceramic design and pictorial embellishment ever devised.

LESSON 7

Outline of Roman Art

The statement of Horace that "conquered Greece led her conqueror captive" in the arts has frequently been taken to mean that the Romans were mere plagiarists from the older civilization. This is true only in so far as they were content for a time to sit, metaphorically, at the feet of their artistic masters and learn. The Romans were soldiers and practical men of affairs; the Greeks were philosophers, poets, artists, with the wisdom in such matters that centuries of peaceful endeavour can give. They were scientists whose pre-eminence lay in the science of beauty.

At the beginning Roman art was avowedly Hellenistic and derivative. Later came certain developments to fit the needs of its patrons. The Roman's pride in his own home was at variance with that of the Greek. It is in the decoration of the Roman villa that the social history of the empire is reflected.

Etruscan Art the Parent of Roman

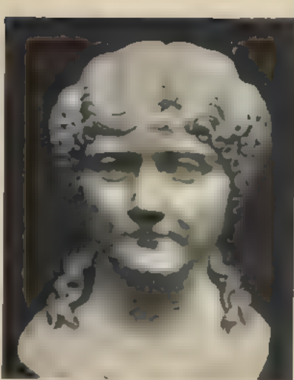
Researches have brought into prominence a truth which the 19th-century enthusiasm for Greek art tended to obscure, namely that there is such a thing as Roman art, pursuing its own lines independently of Greek influences with which it never completely coalesced. The origin of this art we must seek in the neighbouring region of Etruria. The latter state, although originally owing its culture mainly to Greek sources, embarked on an artistic course peculiarly its own from the beginning of the 6th century B.C. Roman art from the time of the kings down to the middle of the republican period was in reality Etruscan.

The Etruscans handed on their gift for realistic portraiture to their Roman successors. Two salient features of Etruscan civilization were belief in a future life and a reverence for ancestors, and both were displayed in their

ETRUSCAN TOMB FRESCO showing a husband and wife at table. The man holds a large wine-vessel and appears to be inviting the woman to drink; their servant brings two ladles and a wine-strainer. The unclothed male figures contrast with the elaborate costume of the woman.

From Weege, "Etruskische Malerei"

funerary urns and sarcophagi, each surmounted by a bust or full-size model of the deceased. Many Roman tombs were Etruscan in style, especially the effigies on cinerary urns.



AGRIPPINA, called "The Younger," (c. A.D. 15-59), mother by her first husband of a son who became the emperor Nero. Later she married the emperor Claudius, whom she poisoned in A.D. 54.

Development of Portraiture

Sculpture became mainly a vehicle for personal portraiture. The Greeks had made statues of their gods, of mythological heroes, and of athletes, whereas statuary in Rome was towards a perpetuation of the forms and features of living personalities. Where the Roman gods were depicted in sculpture, considerable scope was allowed. The Romans thus found full expression for their realistic tendencies, and it was maintained down to the days of the later empire. Many busts in museums represent unknown individuals and show "with matter-of-fact, even pitiless realism the Roman gentleman as he was."

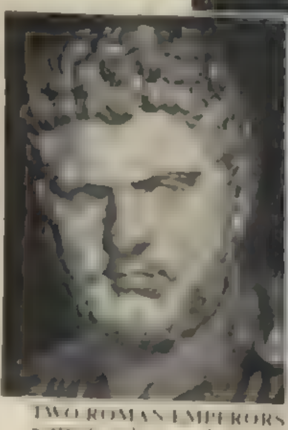
The radical difference between Greek and Roman portraiture is worth noting. The realism of Roman portraits is well exemplified at the beginning of the imperial age by portraits of Julius Caesar and Augustus (about 60 and 20 B.C.). In the succeeding period, from Augustus to Vespasian, it is not in the portrayal of imperial personages that best specimens of the art are found, but in the portraits of unknown persons. In Roman sculpture the anonymity of the artist forbids identification of his work, and there are no schools of sculptors, as in Greek art, in which the creation of a type often enables us to recognize the artist.

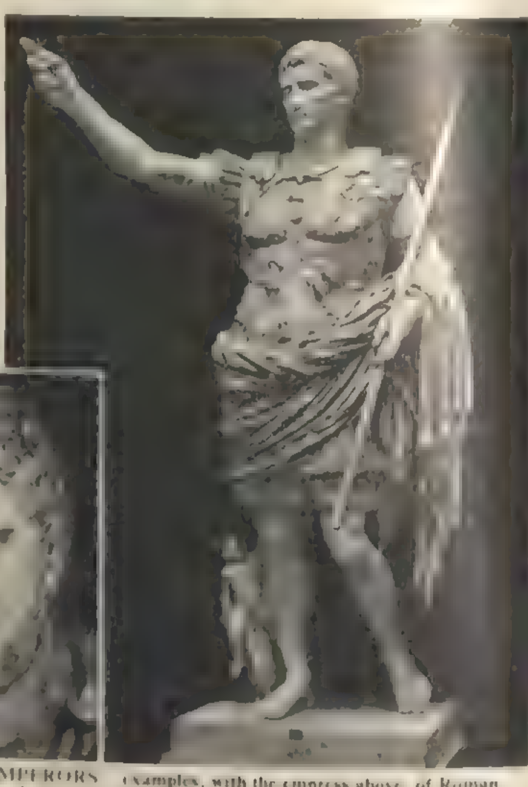
Under the Flavian emperors sculptured portraiture reaches its highest point, and extraordinary skill is shown in the combination of the Hellenic and Roman elements. Good examples are of the two portraits of Titus, in the British Museum and the Vatican.

The great impetus given [illegible] art under the Flavian emperors, w[illegible] is exemplified in the portraits [illegible] the time (A.D. 80 onwards), was [illegible]e to the increased skill of the [illegible] Roman artist, who broke free [illegible] the conventions of the Augus[illegible]ge and produced a new nationa[illegible] full of vigour and realism. Duri[illegible] century from Vespasian to Ma[illegible] Aurelius, Roman art was at its be[illegible]

The Frescoes of Pompeii

Owing to the preserv[illegible]ction of the volcanic ash, many [illegible] mural paintings in the villas at [illegible]peii and Herculaneum (buried A.D. [illegible]) were discovered in a remarkabl[illegible]h condition. It should be em[illegible]ed that the easel picture (i.e. [illegible] painted separately on wood or [illegible]as and therefore movable to any p[illegible]on) was unknown to the Roma[illegible]t then mural pictures frequently serve[illegible]e same purpose as an easel picture, that of [illegible]ating a window-like illusion.





TWO ROMAN EMPERORS examples, with the empress above, of Roman portrait sculpture. Left, Caracalla, ruled A.D. 211-17; right, Augustus, adopted son of Julius Caesar, and the first emperor (ruled 27 B.C.-A.D. 14).

One [illegible] chief illusions aimed at and ach[illegible] was that of extra space to a room [illegible]nings imitated in the panels of a w[illegible] made to frame landscapes [illegible]he open country outside. By represe[illegible] dexterou[illegible]position of painted columns and sce[illegible]cts, a completely enclosed room c[illegible] made to resemble a clois-tered c[illegible] One of the frescoes of the villa o[illegible]a at Pompeii occupies an entire [illegible] with the representation of a garden [illegible] achieves both realism and consid[illegible] decorative charm

The [illegible] unusually brilliant, were flat. A [illegible] or a figure was thrown into relief b[illegible]ck or by that Pompeian red still us[illegible] some contemporary schemes of de[illegible]n. Landscape, allegory, and myth[illegible]lly demanded light and ten-der c[illegible] Modern authorities have called [illegible]ion to the curiously free hand-ling o[illegible] of these frescoes. Certainly the P[illegible]n painters outstripped the Greek [illegible] other conventions in mural paint[illegible] their efforts to satisfy their essent[illegible] practical and realistic-minded patro[illegible]

WALL DECORATION AT POMPEII: a striking example of "trompe l'œil" (deceive-the-eye) painting, giving an illusion of framed openings on to other rooms and outside spaces, in the House of the Vettii.
Photo Brogi

Painti[illegible] Replaced by Mosaic

[illegible]rious fact that the destruction of P[illegible] seems to have synchronised with [illegible]lmost complete disappearance of the [illegible] of painting in the Roman world Its p[illegible]vas taken by a new method of deco[illegible] that of mosaic, which, despite its [illegible]s limitations, yet proved an effect[illegible] form of graphic art. Originally practis[illegible] in the East, it was first introduced into [illegible]about the middle of the 2nd century [illegible] was used for floors and for wall dec[illegible]n. Subsequently it spread to the pro[illegible] often with a very skilful and happy pict[illegible] result, and some of its finest achieve-men[illegible] during the period of the empire are to b[illegible] found in the pavements of houses in France, Germany and Britain

The Roman potter inherited technical skill from his Etruscan prototype, profited consider-ably from Greek designers, and achieved a striking success in the well-known Arretine ware, in which the decoration is stamped in relief by moulds. The pottery itself was made of red clay. The Portland vase in the British Museum is a triumphant expression of Roman genius in glass work, which indeed, following the invention of the blow pipe, was frequently made to take the place of pottery in everyday use. From glass was devel-oped the so-called mosaic glass used by the Romans for producing ware unequalled in richness of colour. Gold and silver plate and bronze articles for household use or orna-ment have been found at Pompeii and the former port of Ostia

Pompeii, it must be remembered, was an ordinary Italian seaside town of the early empire. The standard of comfort and decoration in the houses of its wealthier citizens hints at the glittering extravagance of life among their social superiors in Rome itself

SILVER CUPS once used by a wealthy Roman, found at Boscoreale. Their decoration consists of garlands of flowers, hung as for a feast, and below them the figures of skeletons with masks, torches, and other equipment for the occasion

LESSON 8

Byzantine Art

While Christian art may be said to have begun in the obscurity of the catacombs, there is not very much in what remains of the frescoes and carvings of these underground refuges that denotes a definite artistic impulse distinct from what had preceded it in pagan Rome. The Christians of the catacombs took over many of the motifs and subjects that the pagan artists had used, and gave them, so far as their craftsmanship allowed, a new symbolic meaning. Floral patterns such as the vine were equally applicable to the New Testament and to the worship of Bacchus, and many a pagan myth lent itself to the illustration of a Christian precept.

But there was one important difference. The early Christians were strongly averse to rendering the divine story in terms of the human form, and employed unpretentious symbols, such as the fish, the alpha and omega, the cross, the palm branch, and the lamb, wherever they could. When the human form is represented, it is an abstraction, a symbol, not a naturalistic interpretation.

Yet this art of the catacombs was extraordinarily widespread. The catacombs of Rome were but a fragment of the whole. There are other catacombs at Naples, where the Christians used old quarries ; at the once-magnificent city of Syracuse in Sicily ; at Ziza, near Palermo, in the same island ; and in northern Africa, at Alexandria and other places where the Roman colonists had penetrated. It is important to remember the existence of these outlying catacombs, if only as evidence of the extensive growth of the Christian community, whose members, though separated by vast distances, were united in a common ideal, which was later to find expression in Byzantine art

Growth of Byzantine Art

In A.D. 330 the emperor Constantine, having formally adopted Christianity as the state religion, moved his capital from Rome to Byzantium (Constantinople). With Rome relegated to the position of a provincial city, her importance as an artistic centre rapidly dwindled and her craftsmen forsook her for the new capital. At the same time Christian artists and worshippers in Greece, in Egypt, and in Syria, who had all along developed their faith and the art connected with it on eastern rather than on Roman lines, gravitated to the new centre. It was almost inevitable then that Byzantium should become a rallying ground for world-Christian art.

BYZANTINE TAPESTRY was developed as a fine art after the introduction of the silkworm to the Empire in the 6th century. This early specimen, with a doubled design of Samson slaying the lion, was probably woven at Alexandria.
Victoria and Albert Museum

In the shaping of Byzantine art this eastern influence proved a good deal more potent than the western. To the eastern artists Christianity presented itself in terms of colour rather than of form, and it was colour that assumed the predominance in the eastern capital and later spread its influence through the west. The naturalistic predilections of Rome disappeared ; sculpture in the round almost entirely perished, except in the art of the ivory workers, who developed the very highest skill. Decoration became flat once more, relying on colour to give it a sensuous appeal.

Mosaic Replaces Sculpture

Mosaic work was the outstanding characteristic of Byzantine art—very much as, in the centuries to follow, stained glass was to become the crowning glory of medieval Gothic art. The walls of Byzantine churches were often resplendent with light

and colour, formed into images which, though conventionalised, had an impressive nobility of design. In Roman times coloured glass and gold had been used for mosaic work, but by the time of Constantine these materials had been replaced by fragments of inlaid marble and other hard stones, enriched with brilliant enamels and glass-work and gold. On a stucco foundation the innumerable tiny pieces, or tesserae, were fitted together with amazing skill, and polished smooth. Originally applied to pavements, in imitation of carpet, mosaic art produced rich geometric designs ; later it was used for huge mural decorations, confined at first to the interiors of churches, but by the 12th century appearing on the facades also. The conventions of this medium were as austere a discipline as the conventions later imposed by the nature of stained glass.

BYZANTINE MOSAIC was adapted from Oriental sources and attained a frequently sombre magnificence. This dignified conventional portrait of Christ is a portable example, composed of stone tesserae.

National Museum, Florence

Architecture alone continued to develop along Roman lines. Descriptions of the early Christian basilica, in which the Byzantine builders introduced their new principle of dome construction, can be found in our Course on Architecture. The Byzantine style of church building lent itself admirably to the use of colour. Craftsmen in stone, wood, ivory, metal, enamel, and glass produced superb carvings and mouldings, plaques, candelabra, and altar furniture, including chalices of outstanding beauty. The textile workers of the East were kept busy designing and executing the richest of ecclesiastical vestments. Nor must one forget the illuminators of manuscripts, whose exquisite work has never been surpassed. Except for architecture, the major arts languished throughout this period, but all the richness and glowing radiance of the East was brought into the service of Byzantine decoration.

LESSON 9

Medieval Art in Europe

MEDIEVAL art found its highest expression in church architecture, culminating in the majestic cathedrals of the so-called Gothic period. These are dealt with at length in our Course on Architecture. But virtually all other forms of art were equally inspired by religion and directed all over Europe to its requirements.

The main function of medieval sculpture, for example, was to ornament the cathedrals and churches, especially the façades ; and to embellish with effigies the tombs of kings, prelates, and other dignitaries. All over Europe the beautiful work of thousands of anonymous sculptors and carvers in stone, wood, and ivory remains a delight and an inspiration. One needs only to mention the exquisite statues of the kings of Judah on the west portal of Chartres cathedral, all severely and properly disciplined to their function as an essential part of the architecture and therefore deliberately structural, rigid, and compact in form, yet each possessing (as is soon found on closer inspection) an individual character. To quote Helen Gardner in her book, *Art Through the Ages :* "The modelling in these figures, which are carved from building stone, has not the subtle surface modelling of the Greek ; but such a surprising amount of feeling has been secured by concentrating upon such a detail as the smile about the mouth or the expression of the eyes. This again is in marked contrast to the Greek, who disregarded physiognomy, making his figures in their entirety expressive of his thought."

To take an example nearer home, the west front of Wells cathedral, in Somerset, has no fewer than 300 statues, arranged in a formal cohesive design ; and there are displays of the same kind scarcely less impressive at Lichfield cathedral and Westminster Abbey. But this figure sculpture was never developed in England to the same extent as in France and Italy ; Milan cathedral has the largest collection of such statuary, the exterior being embellished with about 2,000 marble figures.

The great play of fancy in the carving of grotesques—odd combinations of man and beast, originating from the MS. "bestiaries"—is also characteristic of Gothic ecclesiastical sculpture. They represent a freedom and light-heartedness which appears to be at variance with the solemn purpose of the buildings in which they find a place; but it is a licence mostly used with discretion, tucked away in odd places high up on the walls and columns and pinnacles, suggesting some jolly medieval friar cracking jokes between his penances.

The recumbent figures on the tombs followed certain conventions in the matter of drapery and the disposition of limbs, and are almost invariably of truly monumental dignity, communicating in their very lines a wonderful sense of profound repose; but the portraiture is finely naturalistic. The names of some of the greatest English sculptors of such effigies are known: they include one John of St. Albans, and William Torel, whose effigy of Henry III in Westminster Abbey is one of the world's most beautiful bronze figures.

The Glory of Stained Glass

Within the cathedral there was little room for paintings or mosaics on the walls. As Gothic architecture developed, windows became larger and larger until they represented the greater portion of the interior surface of the walls, the thrust of the roof being taken by huge exterior buttresses. Nevertheless the interiors were gloriously enriched with colour more radiant than could have emanated from any painting, when these large windows were filled with stained glass, the principal and most striking decorative feature of Gothic interiors.

As stained glass is so essentially an art of the Middle Ages, this is a convenient point to consider this branch of pictorial and decorative art in some detail.

A stained-glass window is a design in terms of coloured light. The colour of the glass is for the most part produced chemically (e.g. from metals) while the glass is in a molten state. Such glass is called "pot metal." Small pieces of glass of different colours are held together by strips of lead, which also supply outlines for the design; but in a large window, the sheet of glass and lead is reinforced by iron bars. The iron bars and the lead strips are therefore taken into account by the designer, as well as the various colours; indeed they help to determine the general design. The separate pieces of coloured glass are cut to the required shape by a diamond and fitted into the pliable lead, and the various pieces of lead are soldered where they join.

Further refinements, e.g. human features, drapery folds, shading, are obtained from the use of a brown enamel (consisting chiefly of iron oxide mixed with fusible glass), applied with a brush and then fixed into the main glass so that it becomes part of it. In the 14th century it was discovered that silver salts fired on the glass have pleasing yellow and orange stains on white glass, and these in combination with the brown enamel gave a wider range for pictorial representation as well as increasing the resplendent effect.

Grisaille: The "Five Sisters"

Beautiful effects were also obtained from a limited use of colour. Indeed as early as 1134 the Cistercians prohibited the use of coloured glass, and this helped to develop the art of "grisaille," i.e. windows of white glass decorated only with the brown enamel, either in solid lines and masses or as a thin wash, relieved only here and there by small and judiciously placed pieces of coloured glass. Early "white" glass had a faint greenish tint and a silvery opalescence, and with the skilful use of lead strips and stain in the design a grisaille window could be one of the most exquisite products of the medieval period. An outstanding example is the wonderful "Five Sisters" window of York Minster, made up of nearly 100,000 separate pieces of glass. York also contains one of the world's richest examples of stained glass in full colour, in its great east window (78 feet by 33 feet), made in 1405. It is rivalled only by the east window at Gloucester (72 feet by 38 feet), made in *c.* 1350, and by the glass in the great French cathedrals, especially that of Chartres.

Later History of Stained Glass

As technical resources improved, the art degenerated. Enamels were used as colouring materials, and the general effect became increasingly that of painted glass rather than stained glass, and the qualities inherent in the nature of the materials—coloured light and lead strips—were sacrificed to naturalistic picture-making, complete with backgrounds and distances. By so much as windows became increasingly pictorial were they the less harmonious as architectural decoration.

The Renaissance and Reformation ended the long reign of the Church as almost the sole patron of art, and stained glass suffered a long decline. Reformers were more interested in destroying stained glass than in creating it. Although developments in domestic and other secular architecture widened the range of artists in stained glass, most of the designs they produced would have suited an opaque background equally well.

The 19th century brought a revival, especially in England, following the Oxford Movement in the Church of England and the accompanying restoration of colour and beauty to church ceremonial; and it is interesting to note that

the revivers of the art—men like Winston, Clayton, Burne-Jones, and Morris—went back to the principles of the medieval designers, consciously seeking to reproduce what had long been thought of as defects. In other words, they sought always to subordinate pictorialism to the simple beauty of pure coloured light and the fine decorative and structural effect of the lead strips.

Medieval painting as such found its most exquisite and most characteristic expression in the illumination of MSS. Here again the main concern was with decorative effect rather than with naturalism, but for all the conventional design and application of gold leaf, these craftsmen, like the sculptors, contrived increasingly to seize all kinds of opportunities not only for individual fancy, but for miniature portraits and landscapes of the utmost delicacy and freshness.

LESSON 10

Basic Materials of Painting

This historical survey now reaches the period of the European Renaissance, and several subsequent Lessons will be concerned chiefly with the emergence during that period of the great historical "schools" of painting—Florentine, Venetian, Flemish, Dutch, French, Spanish, and so on. The period of the historical schools may be taken as coinciding with the conception of painting as the art of representing visual truth, sometimes for its own sake, sometimes subserviently to other ends, e.g. religion, and the gradual perfecting of the painter's craft to the achievement of this purpose in combination with harmonious and pleasing design

Before the varied achievement of the historical schools is studied in any detail, it is advisable that some general information should be given about the technique of the painter and his materials.

The pigments used by the artist are various mineral, vegetable, and (occasionally) animal substances, prepared chemically. They are applied to a given surface, such as wood, canvas, or paper, usually but not invariably by a brush, after they have been mixed with a suitable medium, e.g. linseed oil, tempera (white of egg), or plain water. When an oil medium is used, the brushes are commonly of hog-bristle, but other media demand softer and more pliable brushes, such as sable.

Modern Materials

In earlier days it was the artist's practice to grind and mix his own pigments, but "artists' colours" are now prepared by manufacturers, ready mixed to a stiff paste with linseed oil or poppy-seed oil, and packed in metal-foil tubes. Those intended for water-colour work are sometimes sold in small hard flat cakes, prepared with size or gum arabic ; but usually to-day they are softened with glycerine and sold either in tubes or in china pans. The artist using an oil medium may choose to dilute the pigments, and will certainly re-mix them in small quantities on his palette according to his immediate requirements. The range of pigments is continually extending, but the artist's choice of them is governed by consideration, not only of their colour, but also of their durability, their relative transparency or opaqueness, and the possible chemical action of one upon another. The ready solubility in water of certain pigments (e.g. prussian blue) is an obvious recommendation for their use in water-colour painting.

Other media used include fresco (explained later in this Lesson), distemper (gum), encaustic (wax, worked with hot tools), pastel or crayon, and body colour, also known as gouache, which is water colour made opaque by admixture with Chinese white (zinc oxide).

If the artist is using oil colours or an oil medium, the surface on which he paints, whether it be canvas or wood, requires to be primed with a coating of size, usually mixed with white lead. Canvases are to-day obtainable already primed, and stretched on a frame to give a firm, flat surface. Paper for painting in water colours is also usually stretched to prevent it from "cockling" through saturation during the process of painting.

Fresco and Tempera

The craft of painting as we know it to-day began in Italy with the dawn of the Renaissance. Early Italian masters painted in fresco on the walls of buildings, especially churches, and in tempera on panels of wood over which canvas had been pasted, with a plaster ground over the canvas.

Fresco is water-colour pigment applied to freshly laid plaster with no glue to hold it. The pigment is applied to the wet plaster, to become the surface of the plaster as it dries. Thus, once applied, it cannot be retouched or altered. Any plaster which dries before the colour has been applied must be chipped away. The severe discipline of this medium helped to develop sure draughtsmanship.

The early Italians drew their outlines first, then laid on their colour in flat coats, concentrating on simple statement in line, colour, and

pictorial composition. But the Renaissance brought first an urge to give whatever objects were depicted a more solid and "rounded" appearance by the help of light and shade, and to place them realistically among their surroundings. Then came discovery of the laws of linear perspective and experiments in foreshortening to give an appearance of depth not only to individual objects depicted but to the whole picture. Another achievement was the representation of "aerial perspective," i.e. the effect of atmosphere, in giving the appearance of depth to the picture, and achieving greater pictorial unity. By all these means the later Italian painters, culminating in Michelangelo and Leonardo da Vinci, strove and learnt to present ever more skilfully the effect of three dimensions on their flat two-dimensional surface.

Development of Oil Painting

For a long time the use of oil in Italian painting was confined to an addition of coloured glazes over tempera painting. But in northern Europe the oil medium, which had been known as early as the 10th century, came to perfection in the hands of the Flemish painters, of the 14th and early 15th centuries, especially the two Van Eyck brothers, the secret of whose method is popularly supposed to have died with them. It is possible that the oil medium developed the more readily in these northern latitudes because the moister air was deleterious to fresco. The Van Eycks painted in oil on a white gesso ground. The method adopted by later Flemish artists was that of a monochrome underpainting upon which was imposed a smooth impasto, with solid paint for the lighter portions of the picture and transparent paint for the darker.

Oil painting was soon adopted by the Italians, if only because the medium allowed more scope in alteration and retouching. In other words, the discipline exacted by the medium was infinitely less strict. Michelangelo is said to have scorned it for that very reason, as "fit only for women," and there are certain critics to-day who date a general decline in the quality of painting from the time of its introduction—from which moment, they say, paintings inevitably suffered from lack of precision in statement and in design. This, of course, is true only if precision of statement is accepted as the sole criterion by which a painting may be judged—and, of course, not everybody would agree that this is so.

Italian painters who adopted the oil medium include Perugino, Verrochio, and Leonardo da Vinci. They followed the Flemish method. On the other hand, the painters in oils of the Venetian school prepared their underpainting in full solid impasto, then added transparent oil glazes. Titian, the greatest of the Venetians, used underpaintings of tempera in massed lights and darks, then superimposed his superb colour in oil glazes.

Technique in Oils

Finally came the method of oil painting that gradually came to predominate and is the one most in use to-day, the "direct" method. In this the solid colour is applied directly to the canvas without either underpainting or subsequent glaze. The first to experiment with this method were Caravaggio and his Neapolitan school of Tenebrists at the end of the 16th century. The method was introduced into Spain and Holland early in the 17th century. Velazquez in Spain and Rembrandt and Hals in Holland were all masters of the "direct" method.

The chief effect of the method is of greater freshness and vigour. Historically it induced for the first time admiration of a picture not only in respect of its fidelity to natural appearance but also in respect of its brushwork. An artist could now take legitimate pride in displaying the sweep and mark of the brush, rather than concealing it. Indeed he could make such things almost as expressive and significant and as subservient to his purpose as good draughtsmanship or composition. Thereafter the pictorial representation of nature was no longer confined to an aping of forms; it became, rather, a frank translation of nature in terms of individual brush-strokes, and technique could be enjoyed for its own sake.

The technique of painting in water colour will be separately considered in a later Lesson.

LESSON 11

Precursors of the Italian Renaissance

THE great movement in art and letters known as the Renaissance had its origin in Italy at the end of the 14th century. Early in the 15th it was becoming international in scope. Then it was that under the rule of the Medici family in Florence, humanism became a cult. Scientists and men of letters, architects, painters, and sculptors devoted themselves to the study of classic literature and antique art. The writings of Greek and Roman philosophers were popularised, fragments of ancient sculpture unearthed, ruins of

classic buildings investigated, and the lessons learnt from them applied to the creation of new monumental buildings. In a short time what remained of the Gothic taste was swept away. The gradual flowering of the Renaissance was the outcome of five hundred years of darkness, of a period that had lost the flexibility and aesthetic discrimination of the best Greek and Roman epochs.

Art Becomes Secular

Humanism stood for much more than the mere study of the antique. It was identified also with the liberation of thought and ideas. Even as early as the 13th century the Gothic architects and sculptors of the North were reaching out towards a more individual conception of art than had previously been permitted by the Church. In the North the new ideas of the 15th century coincided with the Reformation. In Italy there was no Reformation in the Church ; but in Florence arose the new religion of science. It arose with such force that it spread over Europe. The numerous city states into which Italy was divided were presided over by merchant princes who challenged the lay authority of the Pope while themselves remaining within the body of the Church. Under their patronage and that of the wealthy citizens who supported them, art became to a great extent secularised, and artists were encouraged to work under less restricted conditions. The building and decoration of palace and civic building became at least as important as the building and decoration of church and cathedral.

The theory that Renaissance art merely repeated antique classical art has long been exploded. Even in architecture it was very far from being a mere reproduction of rediscovered Greek and Roman forms. The great designers put so much genius into scientific interpretation of realism that the classic forms became a new means of scientific expression and were used to solve constructive problems arising from quite new types of building.

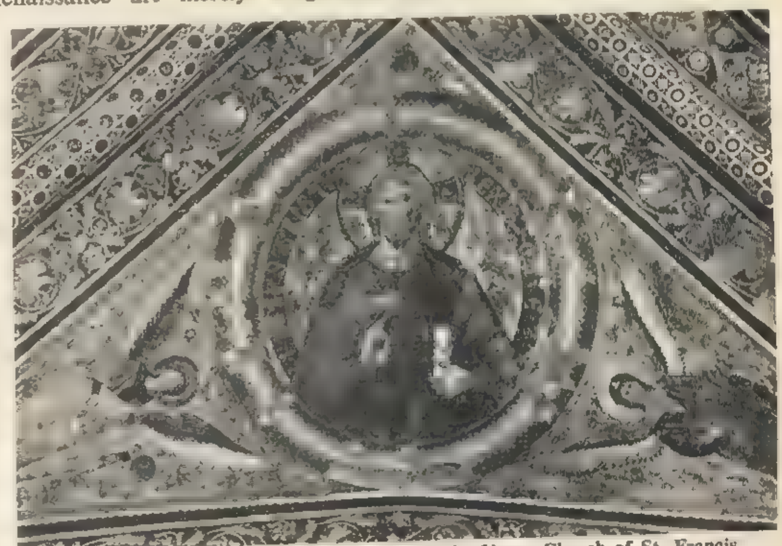
The Italian Pioneers

Historically the Renaissance flourished in the 15th century. More than a hundred years earlier than this, Cimabue (1240–1302) showed himself to be no longer fully controlled by the Byzantine models ; his painting shows a form of " impressionism." He executed several important frescoes in both the churches of St. Francis at Assisi. His last work is said to have been the mosaic of Christ in Glory in the apse of the cathedral at Pisa. His work in this medium was without equal at the period.

Niccola Pisano (*c.* 1206–78), the creator of the sculptures (1260) on a famous pulpit at Pisa and of those on the great pulpit in Siena cathedral, revived the noble grandeur of classic form ; he was also a skilled engineer and great architect. Giovanni Pisano, his son, executed the pulpit of St. Andrea at Pistoia, which gives an early glimpse of that naturalism and emotional expressiveness which were to become so marked a feature of later Renaissance painting. These artists represent already the twin currents of Renaissance thought—humanism and the urge to realism—at their source.

Giotto : Andrea Pisano

The first man of the pre-Renaissance period to paint objects in a manner that enables the spectator to realize their likeness to actuality without mental effort was Giotto (1266–1337). He preserved traditional ideals, but created reality. His form is not merely plastic, but shows the constructive lines of composition. There is a vital suggestion of drama, a striving for more than mere beauty of line and colour. There is more form in his work but less vision, less grandeur, than in that of his predecessors. A great series of frescoes at Assisi and a famous



CHRIST WITH ANGELS on a ceiling in the Upper Church of St. Francis at Assisi : one of the few paintings now ascribed to Cimabue, the early Florentine artist, also famous for mosaic work and as the teacher of Giotto.

altarpiece in St. Peter's at Rome bear witness to his naturalistic genius and sincerity. He had several followers in painting, but their work—mostly uninspired—did little towards the development of the master's ideals. It was a sculptor, Andrea Pisano, who did more than any contemporary painter to further that movement which Giotto had begun, away from the old conventions. Andrea and his pupils infused vigorous life into the art of relief sculpture, and there are few things finer in that art than this master's bronze gate to the Florentine baptistry.

The Italian Schools Contrasted

Side by side with the school of Florence a school of painters arose at Siena, of which Duccio di Buoninsegna was perhaps the greatest member. But in the early paintings of the Sienese school there is only a slight advance from the immobility of the Byzantine models. The style never quite freed itself from the hierarchic tradition. It had not the vitality of the Florentine school, and was soon left behind by the latter in the race for pre-eminence. Outside Tuscany the influence of Giotto's art was shown in the work of painters of Rimini, but the Venetians for a long time remained more faithful to the Byzantine tradition.

PULPIT, PISA CATHEDRAL, dating from 1268 : one of the famous works of the sculptor and architect Niccola Pisano, who revived the imaginative use of classical forn

THE VIRGIN MARY WITH ANGELS, a fresco by Giotto in the Chapel of the Arena, Padua. Giotto as painter, architect, and sculptor, led the Italian break-away from the conventions of Byzantine art.

Versatility of Italian Masters

One characteristic of these precursors remains to be mentioned—their many-sidedness. Giotto was not only painter but also architect and sculptor. The great campanile of the cathedral at Florence is striking testimony to his achievement in both these other arts. Orcagna, Andrea Pisano's most celebrated pupil, was equally famed as painter, sculptor, goldsmith, and architect. Even in these early days to know one art was to be conversant with many, if not actually distinguished in more than one; and as the Renaissance matured, this habit of versatility became more marked, until the culmination was reached in those two giants of artistic endeavour, Michelangelo and Leonardo da Vinci. The restless intellect of Leonardo, indeed, could not be kept within the confines of art, but ventured courageously and effectively into military engineering and the then unknown science of aeronautics; while Michelangelo demonstrated his superb powers equally in sculpture, painting, and architecture, in addition to being no mean poet. In the Renaissance the barriers between the various arts were to a great extent broken down in a common effort for the enrichment of life.

LESSON 12

Masters of the Italian Renaissance

In the early days of the Italian Renaissance the Florentine school took the lead in the art of painting, Siena occupying the second place. But the subsequent history of the movement in the arts reveals the rise and development of several other local schools, each of them being identified with the name of at least one great master; in fact, there was hardly a town of any importance that did not have its school of painters or sculptors. Thus the schools of Rome and Venice developed when the influence of Florence began to wane; Milan, Bologna and Genoa, Parma, Naples and Modena, Ferrara, Pisa, and Lucca—all had their periods of artistic activity and fame. Most art historians do not adopt a severely local classification, but range the painters under more general headings, such as Tuscan or Umbrian or Lombard, finding in those within the category some common bond in style or technique. Many of the great artists travelled, and were influenced by other schools or imparted their own knowledge to them.

Florence and Venice

It is not always easy to distinguish among these schools, especially those of central Italy, where the influence of Florence was dominant. But so far as pictorial art is concerned, a difference was to develop in the early part of the 16th century between the Florentine school and that of Venice. Whereas the Florentines thought mainly in terms of form and line, the Venetians were first and foremost colorists. Venice was always less concerned with the revival of the ancient learning than with the colourful amenities of her own life—the pomp and ceremony of government, the pageants of her canals, the domestic luxuries—anything that gave pleasure and relaxation from the daily cares of this sternly mercantile republic. These wealthy traders demanded an art that should be neither academic nor didactic, an art that should not reflect classic knowledge or otherwise exact an effort of the intellect, but that should reproduce the splendour of their surroundings and appeal directly to the senses through harmony of colour. Venice had never lost touch with those aesthetic motifs that had come from Byzantium; and Byzantine art was an art of colour.

Of the painters working in the 15th century, Fra Angelico, Giovanni Angelico da Fiesole (1387-1455), a Dominican monk, who is associated with the Florentine school, stands on the threshold of the Renaissance, yet nevertheless apart. His was the triumph of the Gothic style. His frescoes in the museum of San Marco in Florence, and those in the chapel of Nicholas V in the Vatican, at Cortona and elsewhere, are considered examples of purest religious art. In the National Gallery, London, is one of his easel pictures: a "Glory" of Christ with 265 saints. The deeply sincere yet joyous nature of his art has often been stressed.

Emergence of Humanistic Feeling

Fra Filippo Lippi (*c.* 1409-69) expressed emotions of a different character. A Carmelite, he studied under Masaccio (1401-*c.* 1428), a master who improved perspective in art and the relation of figures and objects to the background. Fra Filippo found beauty in Italian peasants, painting religious subjects from the human rather than from the idealistic viewpoint. His art reveals emancipation from the exaggerated religious fervours of some of his contemporaries. Lippi's frescoes in the choir of Prato cathedral, depicting events in the lives of SS. John the Baptist and Stephen, are remarkable for their grouping and colour. His son, Filippo Lippi

THE ANNUNCIATION, by Fra Angelico: one of his frescoes in the Museum of San Marco at Florence. Perhaps the supreme exponent of religious sincerity in art, this Dominican friar lit the fine austerity of the Gothic style with the first hues of the Renaissance.

THE ANNUNCIATION, by Fra Filippo Lippi. Gothic angularity has disappeared : the Renaissance spirit humanises the Holy Personages.

(1457–1504), is generally considered to be Raphael's truest precursor. His work displays meekness and piety, whereas the elder Lippi's is robust and turbulent.

Masters of Differing Styles

Sandro Botticelli (*c.* 1444–1510), Filippo Lippi's most brilliant pupil, was frankly Greek in spirit. With unequalled beauty of decorative line and rhythm, Botticelli stood apart from the scientific aims introduced into the art of the Renaissance by such masters as Donatello, Masaccio, and Pollaiuolo. Though Botticelli could express blithe and lovely pagan feeling—for example, in his famous "Primavera" in the Florence Academy—he also possessed the capacity for deep and sincere feeling. From 1482 to 1492 his style, from the brilliant and exquisitely fanciful, became more austere. Examples of this period are his religious pictures of the Annunciation and the lovely circulars of the Madonna ; also the magnificent pagan decorative painting of "The Birth of Venus" (National Gall.). His frescoes in the Sistine Chapel of the Vatican are representative of his greatest work.

Piero della Francesca (*c.* 1416–92) gave to his paintings a crystalline purity of light and colour. He was one of the first masters of "aerial perspective," and evenness of colour. His backgrounds, e.g. in the Baptism (National Gallery), are singularly pleasing in their blithe delicacy. A pupil of his, Luca Signorelli (*c.* 1441–1523), painted a number of frescoes, and of his easel pictures there are several examples in the National Gallery, London. He shows himself in these, and in his frescoes in the Cathedral at Orvieto, as excelling in the painting of the human body in terms of muscular activity. Pietro Perugino (1446–1523) is perhaps chiefly noted as the teacher and forerunner of Raphael ; his paintings, though charming in colour and graceful in form, possess an almost cloying sweetness.

Correggio (1494–1534), unrivalled as a master of chiaroscuro, possessed excellent command of technique and of his medium, whether oils or fresco. There are oil paintings by him in the National Gallery, London, but his greatest work was the fresco of the Assumption of the Virgin in the dome of Parma Cathedral.

The Three Titans

So we come to the three great names which represent between them the culmination of Italian Renaissance art—those of Raphael, Leonardo da Vinci, and Michelangelo.

Critical opinion in the 20th century has denied to Raphael (1483–1520) that supreme place in this hierarchy which he held undisputed through

LA PRIMAVERA (Spring), by Sandro Botticelli, is a masterpiece of his earlier period, during which he preferred classical subjects. Later, he turned often to Christian religious motives, though still retaining his superb linear, "open air" style of composition.

THE HOLY FAMILY "with the Holm-oak," one of more than fifty paintings by Raphael of the Holy Family or the Virgin and Child. This group, dated 1518, is in the Prado Gallery at Madrid.

earlier centuries ; but critical opinion is often only a matter of fashion, and fashion will surely change once more. Technically Raphael's paintings are as near perfection as has been reached by any man. He has been described as the symbol of the Italian spirit of sublime grace. His colour is luminous and lovely, his draughtsmanship and sense of lovely harmonious design are impeccable. Never were pictures more expressive of easy graceful movement. It is this very perfection of smoothness and sweetness that makes his pictures less acceptable to the present age with its preference for uncompromising harshness. Raphael's earlier work closely resembles Perugino's—understandable, as he assisted Perugino in the painting of that artist's pictures, after the custom of the time for apprentices in the studio of a master. Later, having acquired all the graceful sense of form that Florence could teach him, he was influenced by both Michelangelo and Leonardo da Vinci. In common with them, Raphael passed easily from religious to pagan themes ; but in the popular mind he remains the pre-eminent painter of Madonnas (i.e. pictures of the Virgin and Child).

Leonardo da Vinci

Leonardo da Vinci (1452–1519) is best known to the world by reproductions of his pictures "The Last Supper" on the wall of the church of St. Maria delle Grazie at Milan, and the "Mona Lisa" at the Louvre, Paris, the only existing example of Leonardo's work in portraiture. A pupil of Verrocchio, Leonardo advanced towards a new synthesis of the scientific aims of the precursors of the Renaissance. Aerial perspective is now taken for granted. The figures in any work by Leonardo are subordinated in detail to the whole and conform to effects of light. An outstanding

TWO MASTERPIECES OF RENAISSANCE ART. Left : "Mona Lisa," by Leonardo da Vinci. The subject is sometimes called "La Gioconda" not because *gioconda* is Italian for "joyful," but because the lady, Lisa Gherardini, was the wife of Zanobi del Giocondo. The painting was done over a period of four years, 1502–06, and is now in the Louvre. Right : central portion of "The Last Judgment," by Michelangelo, a colossal fresco in the Sistine Chapel of the Vatican, perhaps the most famous single painting in the world. It was completed in 1541, the master having worked on it during the six preceding years.

example of the magnificent arrangement of his lighting effects is seen in the Louvre version of "The Virgin of the Rocks." Leonardo is the great structural painter, investigating everything and putting his knowledge clearly into his drawing of forms.

Michelangelo

Painter, sculptor, engineer, and architect, Michelangelo Buonarotti (1475–1564) astounds in all four branches with the grandeur and vastness of his artistic conception. It seems almost incredible that the man who executed the fresco of "The Last Judgment" on the east wall of the Sistine Chapel, with its titanic tortured figures, its overwhelming sense of violence, should also have carved the classically beautiful and infinitely solemn figures of the Medici tombs (in San Lorenzo, Florence), and yet found energy and time to carry out the duties of architect-in-chief of St. Peter's, Rome, and design a dome for that mighty church. Michelangelo carries to its limits the modelling of the human form. His output of sculpture alone is enormous. He could strike the classical note impressively, as in the Medicean figures, or more lightly, as in his "David" (Florence Academy). He could also express the most tender and sincere religious feeling, as seen in the Pietà in St. Peter's, Rome. In none of his work does he consciously aim at beauty, but at grandeur and sublimity

A lesser contemporary of these three supreme artists was Andrea del Sarto (1486-1531), nicknamed by his contemporaries "the faultless painter." He is well represented in the National Gallery, London, by two very fine works, a self-portrait and a Holy Family. These, with other paintings still in Florence, including a series of frescoes in the Church of the Annunciation, reveal him as possessor of a strong sense of structure, and of colour, and a homely, unaffected style. The conflict of his art and his amours is the subject of a discerning poem by Browning.

SUPERB SCULPTURE by Michelangelo: upper, the Pietà in St. Peter's, Rome, showing the Virgin with the dead Christ on her knees; directly above, the figure of Night from the tomb of Giulio de' Medici at Florence.

The Venetians

The Venetian school of painting may be said to have been founded with Giovanni Bellini (*c.* 1430–1516) and Gentile Bellini (*c.* 1426–1507) in the 15th century, in so far as both these artists were in search of that rich colour that was to become a characteristic

AN ALFRESCO CONCERT, by Giorgione, the first great master truly representative of the Venetian school. Now in the Louvre, this celebrated work illustrates the entirely secular trend of much of Venetian painting, together with the growing importance of landscape in composition.

TITIAN, the supreme Venetian master, painted religious subjects, allegories, portraits, and magnificent mythological scenes such as the "Bacchus and Ariadne," now in the National Gallery, London.

feature of the later Venetians. But maturity of the school was not reached till the turn of the century, the age of Giorgione (1477–1510) and Titian (*c.* 1480–1576), and, in its latter half, of Tintoretto and Paolo Veronese. Giorgione is the real creator of the Venetian school; he liberated Venetian art from servitude to Florentine form and established a new relationship between colour and light and shade. Titian, his greatest pupil, took the art of Giorgione as a starting-point, but in later life developed and created new tonal harmonies splendid in their strength.

As a portrait painter, Titian ranks with Velazquez and Rembrandt. The great men of Venice, their characters, their pomp and trappings, live again vividly in the pictures of this aristocratic artist and citizen. The religious side of the most sumptuous colorist of any age is revealed in his gorgeous yet solemn "Assumption" in the Venice Academy. Tintoretto (1518–94) and Veronese (1528–88) upheld and developed the traditions of Venetian painting when the Renaissance throughout the rest of Italy had lost its fire.

TINTORETTO, a great Venetian, was inspired by Michelangelo's work, and without imitating it combined a similar fiery approach to celestial themes with an unexampled contrasting of light and shade, as in this "Paradise," in the Palace of the Doges.

With Tiepolo (1696–1770) and Canaletto (1697–1768) the glory of the Venetian school reached its close. Tiepolo was a prolific painter, and the amount of work accomplished by him in fresco, on vast ceilings and wall spaces of Venetian villas and palaces, was enormous. His compositions are grandiose, and his colour and draughtsmanship display an astonishing vigour and power. Canaletto was essentially a painter of detailed architectural landscape, especially of the Venetian scene, though he also visited England and painted remarkable panoramic views of London.

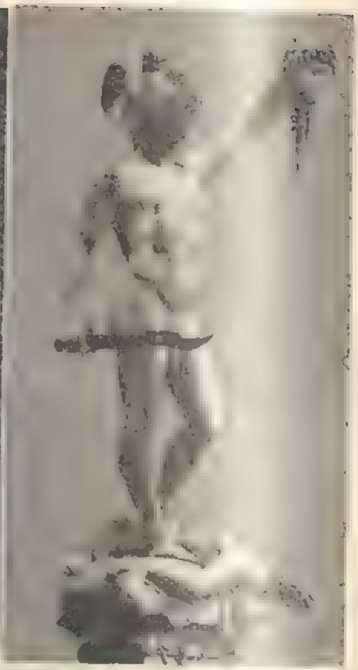
Venice and the Tenebrists

In the 15th century Venetian art had been strongly influenced by the neighbouring school of Padua, represented by Schiavone (*c.* 1435–74) and Mantegna (1431–1506); and these were essentially classical painters, exponents of the new humanism. Mantegna added his enthusiasm for archaelogical construction and the asceticism of the engraver's line. He stands above his contemporaries as a classical artist with a Roman rather than a Greek interpretation.

As the creative spirit of the Italian Renaissance departed and original genius gave place to mere imitation, one lesser painter stood out

and still stands out as relatively important. Caravaggio (1569–1609), whose real name was Michelangelo Amerighi but who became famous under the name of his Lombardy birthplace, sought his own interpretation direct from nature, and was the first to make use of the heightened power to be gained by strong contrasts of light and dark. He founded (in Naples) the school of painters called the Tenebrists, an influential but turbulent group. Caravaggio's own works are usually turbulent, over-dramatised in lighting and gesture, but inescapably forceful, far more so than those of any of his contemporaries.

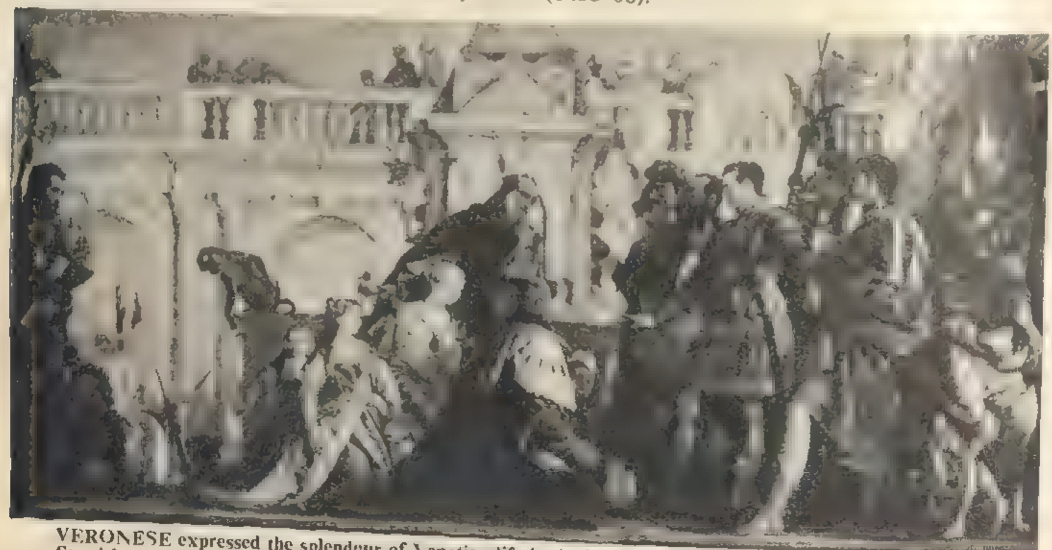
Caravaggio and the Tenebrists are also important in art history as first to use the "direct" method of painting in oils, i.e. painting directly on the canvas in full opaque colour instead of applying glazes of transparent colour to an underpainting in monochrome.



EARLY AND LATE SCULPTORS of the High Renaissance were Donatello, whose marble "St. George" (left) shows Gothic spirit expressed with Florentine energy and naturalism, and Benvenuto Cellini, whose bronze "Perseus and Medusa" (right) reveals his masterly sixteenth-century interpretation of classical forms and subjects.

Other Great Sculptors

The earliest of the great sculptors of the Italian Renaissance was Donatello (1386–1466). His work created an epoch in the art. Although influenced and inspired by Greco-Roman sculpture, he expressed all the spirit of the Renaissance in that his work reflects a vigorous, inquiring, strongly individual mind. His bronze equestrian statue at Padua, the "Gattamelatta" challenges in its impressive dignity the more famous Colleoni monument in Venice, the work of Andrea del Verrocchio (1435–88).



VERONESE expressed the splendour of Venetian life in the later sixteenth century, bringing something of Spanish gravity into his great compositions, such as this classical scene in sixteenth-century costume, "The Surrender of the Family of Darius to Alexander," which is in the National Gallery, London.

MASTERPIECE OF RENAISSANCE ART

Such paintings as this superb portrait by Giovanni Bellini, of Loredano, Doge of Venice in 1501, is not only a superb example of a great painter's art, but also a direct illustration of one of the most brilliant and fascinating chapters in the human story.

National Gallery, London

Luca della Robbia (*c.* 1400–82) was a Florentine sculptor in bronze and marble, and the founder of a complete dynasty of workers in majolica or glazed and enamelled terra-cotta. He, like a true son of the Renaissance, broke fresh ground in his beautiful treatment of the homelier human emotions: the mother's love, the happiness of children. Among his works are coloured medallions and enamelled reliefs, and he is also specially famed for the two panels of children carved in marble for the singing gallery of the cathedral at Florence.

A notable figure of the later Renaissance period was Benvenuto Cellini (1500–71), the greatest goldsmith of all time, who also sculptured in bronze. His "Perseus and Medusa," which stands in Florence, is a superb creation in bronze; but he displayed the same passion for perfection in designing and making so small a piece as a gold salt-cellar.

LESSON 13

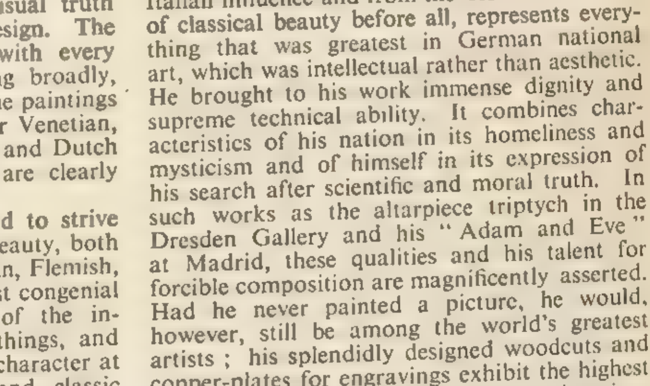
Old Masters of Northern Europe

In Lesson 10 it was stated that from the time of the Renaissance the painters of Europe aimed at a combination of visual truth with harmonious and pleasing design. The balance between the two varies with every individual artist, but also, speaking broadly, with particular schools. Between the paintings of the Italian masters, Florentine or Venetian, and those of the German, Flemish, and Dutch masters certain broad differences are clearly recognizable.

On the whole the Italians tended to strive after types and forms of idealised beauty, both in subject and design; while German, Flemish, and Dutch painters found their most congenial expression in the representation of the individual character of people and things, and preferred to stress that individual character at the expense of idealised forms and classic design. In other words, the Northern Europeans were more *naturalistic.* They excelled in portraiture and landscape, and found beauty—and much inspiration and exhilaration—in the things of every day.

Little is known of the personalities and lives of the early artists in Germany. In the 14th and 15th centuries there were fairly vigorous schools of art at Prague, Cologne, Augsburg, and some of the cities on the Rhine, but the "masters" preferred to remain anonymous. The asceticism of the medieval spirit was translated in Germanic art into terms of the stark horror of realism. This is especially noticeable in the treatment of religious subjects, such as martyrdoms or the Crucifixion. At best, these painters desired to be truthful at all costs, their work possessing a tortured passion of sincerity; at worst, they twisted the truth into unendurable ugliness. The work of Mathias Grünewald shows both aspects.

Towards the end of the 15th century Albrecht Dürer (1471–1528), with his freedom from Italian influence and from the Renaissance cult of classical beauty before all, represents everything that was greatest in German national art, which was intellectual rather than aesthetic. He brought to his work immense dignity and supreme technical ability. It combines characteristics of his nation in its homeliness and mysticism and of himself in its expression of his search after scientific and moral truth. In such works as the altarpiece triptych in the Dresden Gallery and his "Adam and Eve" at Madrid, these qualities and his talent for forcible composition are magnificently asserted. Had he never painted a picture, he would, however, still be among the world's greatest artists; his splendidly designed woodcuts and copper-plates for engravings exhibit the highest degree of finish with boldness of line, imagination, and grasp of the technical requirements of his materials. His drawings, of which there are many examples in the British Museum



DÜRER'S ALTARPIECE in the form of a triptych (painting on three panels, hinged to open and shut) is now in the Dresden Gallery. One of the master's earlier works, it is painted in tempera on linen; the centre panel shows the Madonna and Child, the flanking panels show St. Anthony and St. Sebastian. Dürer, who shared with the younger Holbein pre-eminence among the northern European painters of his age, was also one of the greatest engravers of all time.

show his supreme skill as a draughtsman. The "Praying Hands" have latterly become famous to television viewers.

During his later years the ideas of this profound artist were influenced by Luther—himself fundamentally a German patriot. With zeal for the doctrines of the Reformation, Dürer inscribed admonitory texts from the Epistles—taken from Luther's translation—on his pair of vigorously painted panels of the Evangelists, given to the council of Nuremberg, his native town, and later transferred to Munich.

Hans Holbein the Younger

The next great master produced by Germany was Hans Holbein the Younger (c. 1497 1543). He was born at Augsburg and there instructed by his father, the elder Hans, himself at the time an honoured painter, though later to be eclipsed by his famous son. For a while the younger Holbein worked at Basle, Zürich, and Lucerne, and important paintings and drawings by him are in the Museum at Basle. Here he was employed in making designs for wood-engravers, in which art, like Durer, he was a supreme master. About 1527 he visited England, and subsequently became court painter to

A MASTERPIECE OF PORTR... by Holbein the Younger, dated 1533. ... shape in front of the carpet is a "trompe l'oeil"; seen foreshortened from below, it appears as a skull peeping out of the picture

Henry VIII. During this period he executed remarkably fine portraits in oil of contemporary personages and also a number of wonderful portrait drawings in chalk and pen preserved in the royal collection at Windsor. His painting of "The Ambassadors" and the exquisite masterpiece "Queen Christina of Denmark" are in the National Gallery. Holbein's famous woodcuts of "The Dance of Death" are distributed between London, Paris, and Munich. He learned something of his pictorial composition from the Italians, notably Mantegna and Leonardo da Vinci, but his art is characteristically German in its intellectual veracity and attention to the minutest details. His keen observation of human character, together with his unerring technical ability, places him amongst the world's greatest portrait painters.

JAN ARNOLFINI AND HIS WIFE, by Jan van Eyck, who is sometimes called the greatest portraitist in the history of art. This picture is in the National Gallery, London.

The Van Eycks, and the Flemish School

The great commercial activity of Flanders in the 14th and 15th centuries provided a powerful stimulant to the development of the notable Flemish school of painting, centred on Bruges, Ghent, and Antwerp. As already stated (Lesson 10), Hubert and Jan Van Eyck (the former born about 1366, some twenty years before the latter) perfected the use of the oil medium, so that the brilliance and finish of their work remain unique to-day. The brothers painted in partnership at Ghent for some years; then Jan was employed at Bruges in the service of Philip the Good, of Burgundy. Hubert's great work, "The Adoration of the Lamb," an altarpiece with folding doors, for the cathedral

of St. Bavon, in Ghent, was completed after his death by Jan. The National Gallery, London possesses in the portrait of the Arnolfini and his wife a masterpiece by Jan Van Eyck in which the homely character and setting reveal the perfection of his technique down to the smallest detail, as well as the wonderful state of preservation characteristic of the Van Eycks' work. The Van Eycks painted their sacred subjects also, literally and faithfully from the models they saw in their daily lives, the landscapes and architecture they looked upon, the familiar objects of their homes.

PIETER BRUEGHEL the Elder, most distinguished of his family of Flemish painters, excelled in robust treatments of peasant life, such as this scene, called "Robbing the Orchard."

Other notable names among the earlier Flemish painters are those of Roger van der Weyden, Dierick Bouts, Hans Memlinc, Quinten Matsys, and Gerard David—all religious painters. Unlike the Italians, they painted altarpieces rather than walls, and their works are therefore smaller in scale and more intimate. In them the Gothic spirit, rather than that of the Renaissance, found intense expression. Their painting reflects the richness and elaboration of detail inseparable from the Gothic ideal. Indeed their style was directly derived from the illumination of missals.

No Italian could rival these early Flemings in brilliance and delicacy of technique, even after oil painting had spread from Flanders to Italy. On the other hand, there came a time when Italian influence began to prevail in Flemish religious painting, introducing an alien note which served to confuse the native vision without bringing any compensatory advantage. However, the decline of religious inspiration in painting in the early 16th century allowed a new type of picture to be fostered. This was the "*genre*" picture, wholly indigenous to the Low Countries, a satirical or humorous transcript from common life, which was to be fully developed by the Dutch painters of the 17th century.

Brueghel and Teniers

In this new type of picture Pieter Brueghel (1526–69) excelled, with his delightful crowded pictures of peasant merrymaking and other village scenes, forcefully composed in rich colour. Brueghel's work, for all its fresh jollity and animation, has a satisfying "bigness" about it that attracts much appreciation to-day.

A later Flemish painter of this type who has been greatly admired

RUBENS, the principal master of Catholic Flanders, added to his successful career as a painter the reputation of a diplomat. This magnificent "Descent from the Cross," in Antwerp Cathedral, displays his unrivalled facility in composition; it was painted in 1611, immediately after his return from Italy.

was David Teniers (1610–90). Because of his period he is often ranked with the Dutch painters of the 17th century. It is more correct to see him as a link between Flemish and Dutch schools. He was a painter of little masterpieces, his works being as humdrum in theme as they are small in scale and extraordinarily satisfying in pictorial treatment. Never were tiny brush-strokes applied with more dexterity to reveal the mastery of a man over his chosen medium of expression.

Rubens and Van Dyck

The greatest master of the Flemish school is Peter Paul Rubens (1577–1640). Like that of all the great masters, his work transcends national or regional characteristics. He came near achieving the perfect blend of naturalism and classicism. He took from Italy all that was necessary for the full development of his own robust Flemish art. He was a brilliant colorist, possessed of a quick and fertile imagination. He liked huge canvases and the grand manner, which sometimes could become almost a swaggering manner. The superabundant vigour and vitality of his style expressed itself occasionally in the opulence of his nude figures and in excess of movement, which takes the eye of the spectator out of the picture. He does not equal the gracefulness and dignity of his pupil Van Dyck in aristocratic portraiture, but his range was considerably wider. He was one of the most prolific of painters, credited with over 1,250 authenticated works. Over 30 are in the National Gallery, London. Though not primarily a religious painter (for indeed he painted anything that came his way with equal verve, including some majestic landscapes), Rubens rises to magnificence in some of his religious compositions, notably the Antwerp " Descent from the Cross."

A PORTRAIT OF CHARLES I, by Van Dyck. **Although this masterpiece is in the Louvre, it recalls the fact that the artist spent much of his working life in England, where many of his portraits remain as splendid historical documents.**

Second only to Rubens among the Flemings is Anthony Van Dyck (1599–1641). He has a special appeal to the people of Great Britain by reason of his many portraits of the court of Charles I (of whom he is reputed to have painted 36 portraits between 1635 and 1640). He was knighted by Charles I, died in London, and was buried in Old St. Paul's. His influence on English portraiture as it developed in the following century is inestimable. Reynolds and Gainsborough were his devoted disciples.

Van Dyck, like Rubens, learnt his art in Italy, studying the masterpieces of Titian and Veronese, and if he was less courageous as a colorist than Rubens, he was more subtle. In addition to his fine portraiture, he painted religious subjects with distinction and great beauty of form ; he was also a master etcher and executed a series of line portraits of contemporaries later published as engravings.

THE DUTCH LANDSCAPE, in such a form as Hobbema's " The Avenue, Middelharnis," was in demand by wealthy townsmen whose Protestant religion precluded pictures of saints or classical subjects.

So we come to the famous Dutch school of 17th-century painters. It arose directly out of the Puritanism of the Reformation. The Dutch Republic had generally accepted the strictest Protestant doctrines. In the unpretentious new churches there was no room or desire for religious pictures. But the Dutch burghers had grown well-to-do, and demanded relatively small paintings for the decoration of their private houses, not of the sacred or pagan classical subjects of the Italian Renaissance, but landscapes of the country they lived in, and scenes of the daily life of burgher and peasant at work or recreation. The demand was met by painters of landscapes like Ruysdael, Hobbema, and Cuyp, by seascape painters like Van der Velde, and by a host of "little masters," among whom stand out the names of Jan Steen, Terborch, Pieter de Hooch, and Gerard Dou.

SPLENDID DUTCH INTERIORS of the seventeenth century: left, "Lady and Gentleman at the Virginals," by Jan Vermeer; above, "Interior with Woman and Boy," by Pieter de Hooch. The work of these two painters reveals all that is at once most dignified and most endearing in the achievement of the Dutch masters of "genre."

The Dutch Masters

Technically these men are superb. The greatest of them was Jan Vermeer of Delft, whose paintings, though small and dealing with humble subjects, possess a monumental quality and a serenity in composition and colour that those of the others sometimes lack. Never have cabinet pictures been produced with a keener appreciation of the handling of pigment. All show high skill in the representation of textures (such as velvet) and the handling of lights and darks. Their detail is

VIEW OF DELFT from across the river Schie, by Vermeer, who was a native of the town. For more than two centuries after his death Vermeer (the name is a shortened form of Van der Meer) was forgotten; his works were sold under the names of other Dutch masters of landscape and genre. This famous painting is in the museum of The Hague.

THE MAN WITH A GLOVE is typical of the work of Frans Hals, the robust portrait painter who is sometimes said to have favoured black and white because they were the cheapest colours for an impoverished artist to buy.

often amazing in the fidelity and precision of its representation.

Rising from among these excellent "little masters," Hals assumes a great, and Rembrandt a gigantic, stature. Frans Hals (*c.* 1580-1666) achieved his greatest triumphs in portraiture, especially in group portraiture, such as the fine series of Burgomasters at Haarlem. He is especially notable for the spontaneity and liveliness of his rendering of the passing expressions on the faces of his subjects, as seen in his famous works "The Laughing Cavalier" (popularly so called, though the man depicted is not a cavalier, nor is he even smiling) and "The Lute Player."

It has been said that Hals as a portrait painter excelled in the depiction of the outward and momentary man, the casual glance, the superficial expression ; but that Rembrandt depicted a man's eternal soul. Rembrandt (*c.* 1607-69), one of the supreme painters of all time, was a master of chiaroscuro, e.g. light and shade, which he often employed with haunting dramatic effect. But the dominant characteristic of his work is a profound understanding of, and sympathy with, his fellow human beings. His subjects were often enough commonplace by

PORTRAIT OF A RABBI by Rembrandt. Six hundred paintings, three hundred engravings, remain as the gift to art and civilization of his many-sided and magnificent genius.

choice, or at least would seem so to ordinary eyes. Humble people, beggars, soldiers, cripples, old women—Rembrandt looked upon them with the compassionate eyes of one who loved all humanity and could recognize in the meanest of God's creatures the image of the Creator. To look at a portrait by Rembrandt is to see the finest side of humanity, its courage, its frailty, its fears and hopes—above all, its dignity.

LESSON 14

Great Painters of France and Spain

THE influence of the Italian Renaissance was felt much earlier in France than elsewhere. This was due to the almost continuous political connexion between Italy and France during the 15th century. Social manners in France became Italianised. French scientists and men of letters visited the lecture rooms of Tuscany and Lombardy. Italian painters, notably Leonardo da Vinci and Andrea del Sarto, were welcomed at the French court.

Early in the 16th century French architects began to plan along the lines of the classic Renaissance builders. Indeed, then, as later, it was the striking achievement of French artistic

genius to borrow and re-create. Thus when the real Renaissance in French architecture matured in the 17th century, it had evolved a synthesis of northern warmth of feeling with classical abstraction.

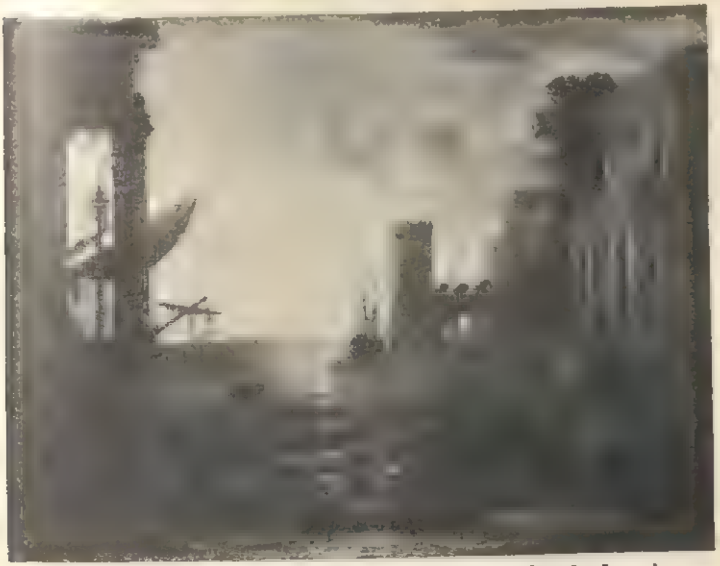
French painting was similarly eclectic, and the emergence of a truly national style was not seen until even later. Its earliest manifestation derived, like those of the German and Flemish school, from the art of illumination and miniature portraiture. Jean and Francis Clouet, who worked in Paris in the 16th century, strengthened the influence of the Netherlands in skilled portraiture. Contemporaneously, on the other hand, a second-rate Italian painter, summoned to Fontainebleau by Francis I, founded there a school of Italian mannerists.

BACCHUS AND ARIADNE, by Poussin, the leading painter of the Neo-Classicist French school, who spent much of his life in Italy, although appointed King's Painter to Louis XIII in 1640. His celebrated scriptural, classical, and allegorical subjects have been described as "painted bas-reliefs."

The "Grand Manner in France"

These were followed early in the 17th century by the so-called Neo-Classicists, at the head of whom was a great painter, Nicholas Poussin (1594-1665). He may be regarded as the founder of a dynasty of painters in the grand, or classical, manner. In the latter half of the 19th century Poussin's work was discredited, but to-day the same degree of admiration expressed for it by Sir Joshua Reynolds and Gainsborough is accorded by most critics. Poussin used geometrical and thoroughly architectural shapes on his pictorial compositions. The finest collection of his works is at the Louvre, but he is represented at the National Gallery and at Dulwich.

Of the same period as Nicholas Poussin is Claude Gelée, often called Claude Lorrain, or simply Claude (1600-82). Concerned—as were the Impressionists after him—with light as it affected colour, his achievement may be described briefly as the discovery of sunlight as a governing factor in landscape. Claude never painted a landscape without paying homage to the classic ideal by including classic architecture, but the real subject of his pictures is the varying effect of sunlight.



THE EMBARKATION OF ST. URSULA, by Claude Lorrain. This 17th-century French artist "never painted a landscape without paying homage to the classic ideal by including classic architecture, but the real subject of his pictures is the varying effect of sunlight."

At this period, when, under the influence of Poussin and Claude, painting in France was mainly devoted to classical subjects and landscapes, or to portraits of great persons, the influence of the Netherlands was equally clear in the works of the three brothers Le Nain, renowned (like Brueghel and Teniers and the "little masters" of the Dutch school) for their paintings of humble domestic life.

By the end of the 17th century French painting had become frigid and statuesque.

A FÊTE CHAMPÊTRE, by Watteau, who led the "Romantic" revolt against the Neo-Classicism of the age of Louis XIV. The influence of Rubens, and a theatrical sense of values, combined with Watteau's exquisite use of colour to produce his masterpieces of elegance and charm.

Rigaud and Mignard were the high priests of academic artificiality. Rigaud's portraits of the famous people of his time, which are exhibited in the chief European art galleries, have a value that is almost exclusively historical.

The School of Watteau

It was only with the dawn of the 18th century that French painting at last acquired a characteristic virtue and flavour of its own. The artist who pointed the way was Antoine Watteau (1684–1721). His "*fêtes champêtres*" and "*fêtes galantes*" mark a reaction against the stiffly set styles of Rigaud and Mignard. Gaiety, airiness, and sparkle characterise his elegant groups arranged under trees, thus breaking the tradition of stuffy interiors and cold classical vistas of corridors. He made them alive with an atmosphere of charm and humanity. His arcadian scenes—French courtiers and their ladies masquerading as harlequins and shepherdesses in appropriate gardens—held up a faithful mirror to the artificial life of the 18th-century society. But his subjects throughout remained free from the indecencies introduced by some of his followers. Grace and poetry are manifest in all his work—etchings as well as paintings. His delicate, brilliant colour is inimitable in the latter, and so, in both branches of his art, is his fluent drawing.

Following in the Watteau tradition, J. B. J. Pater (1695–1736) excelled in "*fêtes galantes*." Watteau's pupil, Lancret (1660–1743), was a charming painter, but lacking the outstanding genius of his master. Nattier (1685–1766) was a successful portraitist of the Rococo period. Boucher (1703–70) and Fragonard (1732–1806) were much influenced by Watteau in both subject and treatment. The former was a fine etcher and decorator, and his portraits of Madame de Pompadour are well known. Fragonard, who studied in Rome and outlived the Revolution, was also a court painter and an etcher. He decorated the apartments of Madame du Barry with exquisite skill.

An artist whose paintings, especially those of girls on the threshold of womanhood, possess delicacy of colour, and a certain superficial charm, was Jean Baptiste Greuze (1725–1805), but his work is sentimental, mawkishly indecent, and somewhat weak in draughtsmanship.

"LA FONTAINE," by Chardin, the Frenchman who "painted Dutch" with a wonderful sense of the beauty and dignity of simple household scenes. Born soon after Watteau, he died soon after Boucher, having kept alive a tradition in the strongest possible contrast with theirs.

A little apart from these is that most lovable of painters, Jean Baptiste Simeon Chardin (1699-1778). His paintings of still life, flower pieces, and "household" scenes represent to some the fulfilment in the 18th century of all that the Dutch *genre* painters of the previous century had attempted. Chardin found and revealed in his bottles and loaves of bread and bunches of grapes and kitchenmaids, the monumental beauty that Velazquez might have seen, and in his pictures of these things simplicity is touched with a rare profundity of vision, which makes many of them truly great works of art. He painted humble things as reverently as Rembrandt painted his tramps and beggars and aged people. Chardin was also an exquisite portraitist.

Velazquez

In Spain any direct impression that the rise of humanism brought about by the Renaissance might have had was checked by the Counter-Reformation. The Church held life and art alike in a firm grip. In spite of this there were unmistakable signs there of the intense mental and physical activity which was informing the whole Western world at this period of history. They are evident in the writings of Cervantes and in the exploration of the oceans by Columbus, in the conquest of Mexico and Peru ; and Spain contributed at least one predominant name to the roll of truly great masters of the brush, that of Velazquez (1599–1660).

The insight of Velazquez into human character can be studied in the long series of court portraits of the Bourbons and of the men who stood behind them, now scattered among the great European galleries. One marvels the more at his achievement in that the court held etiquette hardly second to religion and every one of its members assumed a mask of cold dignity to conceal his inner self from the world. Velazquez penetrated the masks. He was, of course, more than a portrait painter. The National Gallery contains excellent examples of his historical scenes and religious paintings, as well as the celebrated "Rokeby" Venus.

In addition he perfected the technique of portraying visual or optical truth. He is justly famous for his rendering of what artists call "tone values." In other words he was concerned less about the detailed and known facts or even the inner significance of a subject than with just so much of them as was revealed to his eyes by the way in which the light fell upon its surface. Furthermore, as the range of the human eye, like the photographic camera, is limited by a particular focus, so that to bring that focus to bear on one particular object is to render neighbouring objects relatively indistinct, so Velazquez reproduced the effect of optical focus in his paintings. His pictures are thereby unsurpassed in their effect of unity. In contemplating one of his pictures the eye is never tempted to wander off into interesting corners of the background, for his background—and the less important figures in his groups—are consistently subordinated to the main subject on which he required the eye to remain fixed. Rembrandt before him had frequently attempted the same effect, and later this fidelity to optical truth was the inspiration of the 19th-century Impressionist painters. With all this Velazquez also possessed and developed a magnificent sense and extensive range of colour, very different from the prevalent fashion among Spanish painters, who were over-fond of warm reds, yellows, and browns, painting all their shadows in heavy brown.

A SPANISH LADY, by Velazquez, who more than any other great master captured on canvas the whole life of a country at a particular period. His career displayed the unbroken success of a profoundly personal genius.

Zurbaran and Murillo

There are few other names of any significance. Francisco Zurbaran (1598–1661) is the best and most characteristic representative of the national character. He was a painter of monks and of the dramatic side of the Counter-Reformation. His is emotional art, with the emotion pitched in one key ; but he had a fine sense of form and handled his limited palette with skill.

Murillo (1617–80), painter of religious subjects and beggar-boys, has always had a popular

appeal, but he was little more than a journeyman artist, whose work is characterised by over-sweetness and strained sentiment. Ribera (1588–1656) is classed with the Spaniards, but he spent most of his working life in Naples and his large historical pictures reveal the direct influence of Caravaggio (*see* Lesson 12).

El Greco

One other and earlier painter among the Spaniards stands completely isolated not only in Spanish painting but in all the Europe of his time. This is the Cretan-born Theotocopuli, better known as El Greco (1545–1614). Though the force and originality of his work have always been found inescapable, he was, until the present century, customarily looked upon as something of a freak. So long as the representation of visual truth was looked upon as the highest aim of the painter, it was inevitable that the work of a painter who strove to portray something more should be considered incomprehensible. It meant nothing to 19th-century Impressionists. The kindest explanation they could offer of his alleged shortcomings was that he suffered from an astigmatism or some other such optical defect. But just as Velazquez was the idol of the Impressionists, so in his turn did El Greco become in the 20th century the idol of the Post-Impressionists and the exponent of contemporary "modern" theories of art.

THE AGONY IN THE GARDEN, by El Greco. This mystical Cretan, whose real name was Theotocopuli, was a pupil of Titian; his brilliant colour and daring treatment of form, his search for inward rather than visual truth, set him apart from all other painters of his time.

Certainly El Greco, like the English painter William Blake, was born out of time. His attempt to express inner truth rather than visual truth anticipated "modern" art by three hundred years. Again as with Blake, his search for inner truth was inspired by mysticism. His inner vision was essentially religious, for he was in fact deeply influenced by Ignatius Loyola, founder of the Society of Jesus; and such a masterpiece as "Christ's Agony in the Garden," now in the National Gallery, reveals an imaginative force and creative originality of which Velazquez, for all his superior technique, could never have been capable.

Goya

The only other Spanish painter to make any effective impact upon the rest of the world came after a whole century of undistinguished quiescence. This was Goya (1746–1828), an artist of great versatility and a rebel by nature, who delighted in offending conventional susceptibilities. Much of his work was done in a spirit of bitter satire against an effete and corrupt church and state. His vitriolic pungency is seen at its most effective in three famous series of etchings; but his paintings—portraits and subject pictures—are hardly less characteristic in their uncompromising realism.

LESSON 15

The English School of Painters—1

The English school of post-Renaissance painting was a late arrival. It came to its greatest flowering in the second half of the 18th century and the early decades of the 19th.

The earliest English painting, that of the illuminators, bears comparison with anything of the kind produced elsewhere in Europe; but the invention of printing dealt a death-blow to the illuminator, and at the same moment of history the Reformation removed from art the patronage of the Church. English painters were not ready for the great developments that accompanied elsewhere the rapid change from an art that was almost exclusively ecclesiastical to one that was almost wholly secular; and they learnt only slowly—from foreign visitors.

Portraiture was the first fashion in English secular art, and portrait-painters were imported

to set that fashion, such as (in the Tudor period) the Dutch Sir Anthony More (Antonis Mor), the German Hans Holbein, and the Italian Federigo Zucchero, who painted portraits of Elizabeth I. Under the Stuarts the position was little better. Sir Anthony Van Dyck (1599–1641), court painter to Charles I, was Flemish; Sir Peter Lely (1618–80), court painter to Charles II, was Dutch, his real name being van der Faes; Sir Godfrey Kneller (1646–1723), who succeeded him in that office, was a German, whose real name was Gottfried Kniller. These were the virtual founders of the great English tradition in portraiture.

Van Dyck's work has already been noticed (*see* Lesson 13). Lely's portraits can be seen at their best at Hampton Court. Kneller has left long-familiar portraits of James II, William III, Queen Anne, George I, Marlborough, Newton, Wren, Dryden, Cowper, and many other notable people.

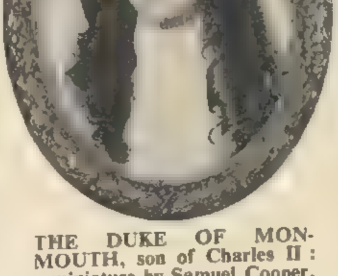
English Miniaturists

English painters did their best to emulate these foreign visitors, but only in one direction did the native genius excel, and that was in the painting of miniatures. Nicholas Hilliard (1547–1619) was a superb miniaturist working in the reign of Elizabeth I, who was one of his sitters. The tradition was maintained during the centuries that followed by such outstanding miniaturists as Isaac Oliver (d. 1617) and his son Peter Oliver (d. 1647), John Hoskins (d. 1664), Samuel Cooper (1603–72), Richard Cosway (1742–1821), George Englehart (1750–1829), John Smart (1741–1811), and Andrew Plimer (1763–1837). In the early days of the Royal Academy's exhibitions the walls were always crowded with miniatures, and the art survives to this day—though, as a critic has pointed out, the costumes and hair-styles of the late 19th and early 20th centuries have not been marked by that elegance which inspired the delightful work so characteristic of the earlier periods. The work of all the miniaturists named above is highly esteemed, especially that of Samuel Cooper.

The Demand for Portraiture

Kneller was governor of the first art school in London. He therefore had ample opportunity to influence the work of the rising generation of English painters. By the end of the 17th century a demand had been created among the wealthy not only for large portraits but also for huge paintings of classical subjects suitable for palatial buildings and stately mansions. But soon there were also the prosperous members of the professional class and the wealthy merchants who required smaller pictures to adorn the walls of those Georgian houses which arose terrace by terrace and square by square in city and town; and sporting country squires wished for paintings of hunting scenes and of their horses and dogs. The immediate demand was therefore for something more after the Dutch type of *genre* picture.

THE SHRIMP GIRL, by Hogarth, the first great painter of the English school. A portraitist of great distinction, he was also a harsh satirist of his times, and an early exponent of the art of the "story picture."



THE DUKE OF MONMOUTH, son of Charles II: a miniature by Samuel Cooper, most famous of the 17th-century English miniaturists.

Three Great Englishmen

The first great native painter of the English school was William Hogarth (1697–1764), who supplied a great part of this "wealthy middle-class" demand with a remarkable output of narrative pictures, stage scenes, "moralities," and small conversation pieces, i.e. portrait groups in which members of a family or household are united in some occupation or interest.

In his portraits Hogarth was no flattering painter

"IMPRISONED FOR DEBT," the fourth episode in Hogarth's famous sequence "The Rake's Progress," exemplifies the exactitude of observation, the powerful use of action, light, and shade, and the relentless force with which he attacked social evils of his time.

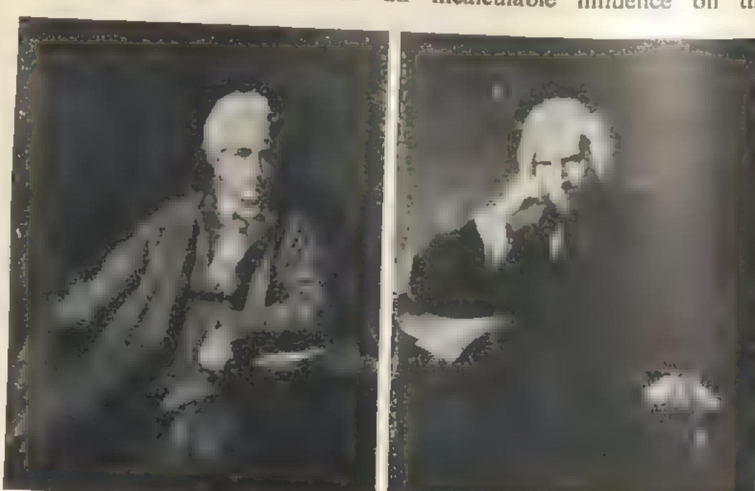
of monarchs and court ladies, demanding to be painted as beautiful in accordance with an established formula. One of his most admirable paintings is a group of six heads on one canvas, portraits of his own servants, each an extraordinarily interesting, amusing, and lively character study. It is now in the National Gallery.

Hogarth professed contempt for foreigners, but owed much to study of Dutch painters. His method of composition by means of light and shadow owed much to Rembrandt. He can also be compared to Hals in his gift for catching to perfection a momentary facial expression—as in his famous "Shrimp Girl" (National Gallery).

Hogarth was also a satirist, who used his brush to expose the hypocrisies of his time. Yet the fame of his pictorial moralities—the three series called "Marriage à la Mode," "The Rake's Progress," and "The Harlot's Progress" are the best remembered—is not based only on their narrative interest or even their moral interest. The six pictures in the first-named series, now in the National Gallery, belong to his most brilliant period as a painter. Charles Johnson, in his book *English Painting* (Bell, 1932), says of him :

> His wide sympathy inspired in him a curiosity to investigate every aspect of human life, so that he proved that anything can be interesting and even picturesque. His excitement in noting movement and life naturally led to a [illegible] enjoyable method of painting pictures ; just as his perception of the beauty of lighting led to a transparent painting of its gradations. The same wide sympathy led him to moralise. Whether this was also good for his art has been doubted. Certainly it often caused him to overstress his meaning or overcrowd his canvas, and sometimes to become obscure or even to forget the requirements of a picture altogether. It has been said that he was at his best as a cool, detached observer. But sympathy makes complete detachment impossible ; and without such sympathy with the inner workings of men's minds he could never have recorded their outward actions with such vivacity.

The two greatest names in the history of English portraiture are those of Sir Joshua Reynolds (1723–92) and his acknowledged rival Thomas Gainsborough (1727–88). Reynolds exercised an incalculable influence on the



EIGHTEENTH-CENTURY PORTRAITURE in England and Scotland flourished at the hands of master-colourists who painted the social and intellectual leaders of the period. Left, "Professor Robison," by Raeburn ; right, "The Rev. Laurence Sterne," by Reynolds.

course of English painting, not only by the example of his own excellence but also as first president of the Royal Academy, who imparted much sound advice to his students, both individually and in his lectures, which can still be read with profit. He possessed an unsurpassed intellectual appreciation of the characters of his sitters. He painted always like the cultured, scholarly gentleman that he was. Perhaps he was most successful in the painting of men, and especially men of some intellectual capacity, like the many literary celebrities of the day whom he painted and with whom he was so much at home—Johnson, Goldsmith, Sterne, Gibbon, and others. But he also painted cherubic, lovable children.

Gainsborough was not only the more versatile painter—he excelled in landscape as well as in portraits, as will be noted later—but he was also the more imaginative. There is a vibrant, delicately poetic quality about his portraits such as Reynolds never achieved, but they rarely reveal Reynolds's profound appreciation of human personality. One can almost recognize Gainsborough's great love for music in his work, but it is like the music of violins, whereas Reynolds may be more readily compared to an organ.

Gainsborough painted over 200 portraits, including eight of George III, seven of Pitt, and five of Garrick. His women are painted with more sympathy and warmth than those of Reynolds. He derived additional dexterity from close study of the work of Van Dyck.

THE COUNTESS OF SUSSEX AND HER DAUGHTER, by Gainsborough, who shared with Reynolds the leadership of the English school of portrait painters and was also a great master of landscape.

Romney, Raeburn, Lawrence

Second only in esteem to-day to Reynolds and Gainsborough, as portrait-painters of the same magnificent period, are George Romney (1734–1802) and Sir Henry Raeburn (1756–1823). Both used a simpler, more " direct " technique than either of the others, and if their portraits never reach the same depth of feeling as those of the greater men, their more vigorous statement is altogether admirable. Romney's direct brush-strokes, like those of Hals, give life to his work ; those of Raeburn give strength. Romney is especially renowned for his many portraits of Lady Hamilton, but there are many other charming portraits of other women and children. Raeburn, on the other hand, was, like Reynolds, primarily a painter of men. He was a Scot, who, although he became a Royal Academician, lived and worked almost entirely in Scotland. He had a robust and sure touch and a fine appreciation of character that was well suited to the depicting of strong and straightforward Scots lairds and old ladies. His finest portraits were painted in the first decade of the 19th century.

The front rank of the Reynolds tradition in portraiture embraces one more name, that of Sir Thomas Lawrence (1769–1830), a painter of extraordinary facility. His portraits of children have a place of their own in English art, and those which he painted of the fashionable women of his time are always sweet and vivacious, even if they lack the sincerity which distinguished the work of his predecessors.

Mention of Raeburn, the Scotsman, makes it advisable to state that the term " English school " is used here because it is the accepted historical term both in Great Britain and abroad. It should be remembered by any who think it a misnomer that not all painters of the Florentine school were natives of Florence, that one of the greatest artists of the Spanish school was a Greek, and that at least three great painters usually included in the English school—Benjamin West, Whistler, and Sargent—were American. Throughout the period of the school its activities and development were centred firmly on London, and especially on the Royal Academy. For more than a century every British painter of any note—Englishman, Irishman, Scotsman, or Welshman—learned his craft in the schools of the Royal Academy and eventually became a member.

LESSON 16

The English School of Painters—2

THE list of great English portrait painters is paralleled by another list, that of the great English landscape painters. Some might say that the second list, containing as it does the resounding names of Turner and Constable as well as that of Gainsborough, is even more distinguished. Be that as it may, it can assuredly be said that the English school resembles the Dutch school in this : that its members never approached the sublime nobility of the Italian masters, nor the delicate lyrical note struck by such French painters as Watteau, but were most at home in portraying, as faithfully as they could, the world of down-to-earth reality which lay about them ; and for that reason—again like the Dutch—they excelled in both portraiture and landscape painting.

In addition, the English—again like the Dutch—have always shown a liking for anecdotal pictures, i.e. pictures that portray some recognizable incident or tell some sort of recognizable story. Portraits, landscapes, and anecdotal pictures of one kind or another have therefore made up the great bulk of good English painting down to our own time ; and in the practice of these arts they have more than once achieved their own kinds of poetry and nobility.

Wilson and Gainsborough

The earliest name to stand out in the history of English landscape painting is that of Richard Wilson (1714–82). His style was based on the Neo-Classical tradition of the French masters Poussin and Claude. Wilson's pictures of Welsh mountains and lakes were not so much realistic transcriptions of what he saw as they were idealised scenic compositions—based on what he saw, indeed, but presented with a peculiar dignity that was more pictorial than naturalistic, composed within the terms of certain rules of composition. Nevertheless, like Claude, he portrayed a serenity of atmosphere that belongs essentially to the world of outdoors.

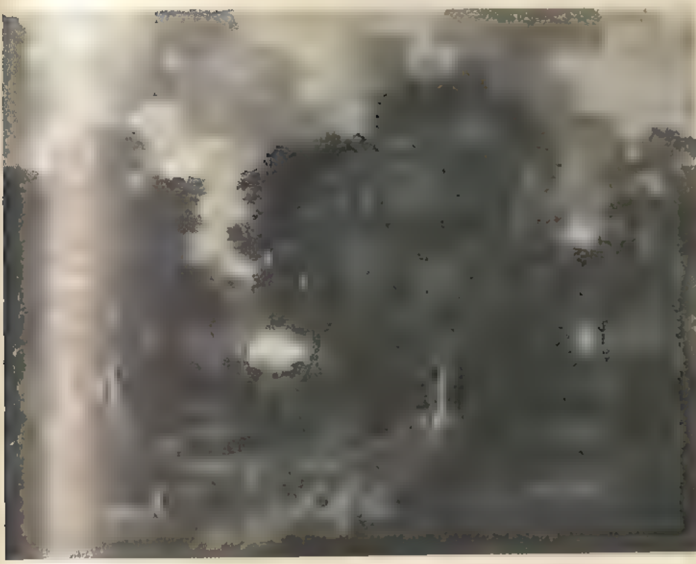
Meanwhile Gainsborough was discovering the beauty and majesty of the English rural scene. Many critics have preferred his landscapes above his portraits. The artist himself would have agreed, for he painted his landscapes only to please himself, not to satisfy sitters. Though it is clear from them that he deeply appreciated the work of Poussin and Claude, it is more important that he evolved his own strongly individual vision. Many of his landscapes are painted thinly, with rare beauty of execution and luminous colour. Ruskin deemed him the greatest colorist after Rubens.

ON THE WYE, by Richard Wilson, who treated landscape in the idealised but masterly manner of Claude Lorrain, and stood apart from the "natural" painters of the characteristic English school.

Climax of English Landscape Art

The first English landscape painter—indeed, the first British artist of any kind—to achieve renown and the highest esteem on the Continent was John Constable (1776–1837). He devoted his whole time, energy, and skill to the representation of nature as he saw (and loved) it, and his pictures are faithful interpretations, not only of the familiar, homely English countryside, but also of the familiar English weather in all its variety—sunshine, showers, storms, lowering skies, gladdening rainbows, fresh breezes—and the effect of changing light on trees and meadows and buildings. At his hands the strange 18th-century convention of painting trees as brown in colour received its *coup de grâce*. Constable, it might be said, turned them green !

Constable, who was slow in achieving full maturity, learnt to handle his paint superbly. In later life he made lavish use of his palette knife to dab solid highlights on his picture in an attempt to reproduce the vibrant sparkle of natural light. The technique was derisively called "Constable's snow," but no one before him had even thought of making such an attempt.



WOODLAND SCENE NEAR CORNARD, SUFFOLK, by Gainsborough, one of the "inventors of natural landscape," who painted portraits to please sitters and landscapes to please himself. His appreciation of the work of Poussin and Claude did not hamper the development of his own individual style.

In addition to his large canvases he left many sketch-books as well as numerous small sketches in oil painted rapidly out-of-doors as he made notes of the passing moods of wind and weather. Some admirers rate these more highly than his finished pictures.

Mention should also be made of Richard Parkes Bonington (1801–28), who has been called the Keats of English painting. Like the poet, he loved beauty, influenced successors in his art by his exquisite workmanship, and died young of consumption. Many of his water-colours, noted for bright colouring and delicacy of draughtsmanship, were acquired for the Wallace Collection, London.

Towards the end of the 18th century a school of landscape painters flourished at Norwich. Its leading spirit, John Crome (1768–1821), a self-taught artist, never travelled far from that city. From Dutch masters of landscape he learned to observe and accord natural beauty at first hand. By mid-19th century the fame of "Old Crome" was so forgotten that his superb "Mousehold Heath," now in the National Gallery, changed hands for a few shillings.

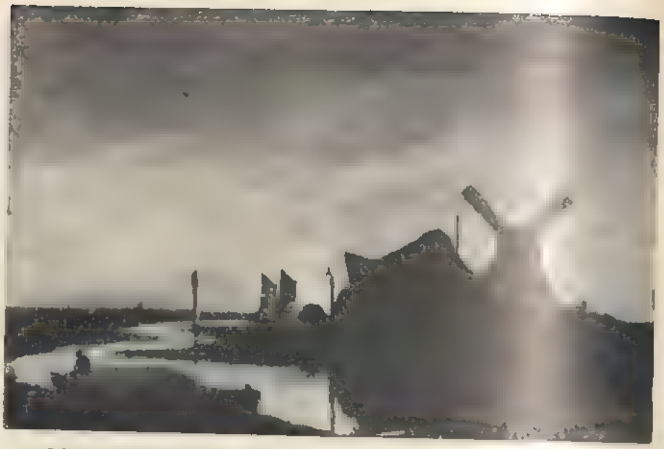
The artist who set a glorious golden crown on English landscape painting was Joseph Mallord William Turner (1775–1851), often considered the supreme genius of the English school, and certainly one of the most remarkable painters of all time. His earliest pictures—many of them "seascapes," for he could paint the sea perhaps better than anyone else who ever lived—gave more than a hint of his exceptional power and his sure observation of natural effects. He studied nature profoundly, then used his observations as a basis for pictures that eventually far transcended nature. He became increasingly interested in effects of bright sunlight. It could be fairly said that he attempted to paint the sunlight itself rather than the objects on which that sunlight happened to fall.

So his work became "impressionistic," full of wonderful resplendent colour, intense, brilliant, dramatic, astonishing in its range. He painted glowing sunrises, vivid sunsets, sun-drenched mists, snowstorms, the dazzling light that can

A LOCK ON THE STOUR, by John Constable, most famous interpreter of English landscape in cloudy English weather, whose masterly paintings of his homeland on the borders of Suffolk and Essex have given it the name of "the Constable country." He was the first British artist whose reputation spread widely on the continent of Europe.

suffuse a drawing-room on a summer afternoon, and the bleak, relentless light that accompanies a black frost—all with equal breadth of vision and mastery of touch.

In his youth Turner was apprenticed to an architectural draughtsman, and the training must have been one reason for the structural soundness of drawing which enabled him to combine as nobody else had done the solidity of natural forms with the atmospheric softness of light and air. There was a precision about his draughtsmanship that was almost mathematical. Only the human figure defeated him. On the other hand, as a draughtsman of trees he remains unsurpassed.



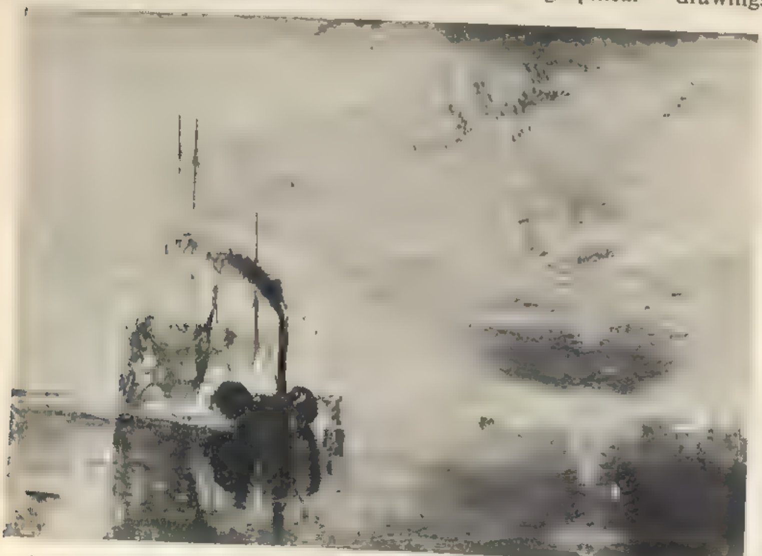
MOONLIGHT ON THE MARSHES OF THE YARE, by John Crome, called " Old Crome " to distinguish him from his artist son John Bernay Crome. John Crome was the founder of the Norwich Society of landscape painters, and for many years after his death his work was barely known outside his own county.

To quote Charles Johnson once again, in a passage comparing Turner and Constable :

> They have both been called Impressionists, and were both in many ways pioneers of the movement of that name. For they were both students of light. But while Constable loved to record a fleeting instant of time when light was passing, Turner's peculiar gift lay in representing the gradual increase or decrease of light upon an abiding spacious area. Turner was ultimately always concerned with form. Even when most vague in his drawing he aimed at making the horizon appear immeasurably far away. Turner's accuracy of perception of tone, combined with the system of colour he used in his later works, makes him the forerunner of the scientific Impressionists. Constable's indifference in his sketches to fact and absorption in the momentary effect make him the forerunner of the emotional Impressionists.

As Turner was also the supreme exponent of the art of landscape painting in water-colours, his name serves conveniently to introduce another unique expression of the genius of the English school.

The beginnings of English water-colour painting are to be discerned in the " topographical " drawings popular in England in mid-18th century, when artists like Paul Sandby (1725 1809) found a ready market for studious delineations of the streets and buildings of London, Bath, and other cities, as well as of many of the mansions of aristocracy. To give a pleasing finish to such drawings they began to add simple washes of transparent water-colour—cool greys and warm browns—to suggest shadows and give extra solidity to their buildings. From this it was an easy step to the washing-in of a blue-grey tint to represent the sky, possibly with some simple unobtrusive cloud effect.



THE FIGHTING TÉMÉRAIRE, by Turner, who has frequently been considered the greatest of all English painters. Study of his large output, which included magnificent water-colour work, leaves unchallenged his sense of the poetic value of landscape and seascape—and, above all, sunlight.

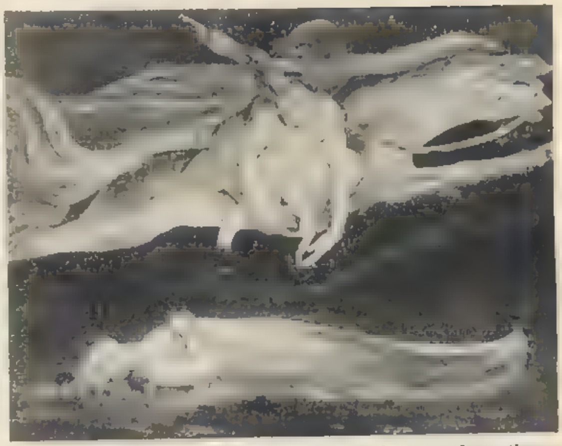
Other early water-colorists were Alexander Cozens (*c.* 1698 1786) and his son John Robert

Cozens (1752–1799), who used the medium to depict Alpine landscapes, and introduced deeper tones of colour and more suggestion of atmosphere, besides discovering a method of representing trees that was particularly suitable for the soft-pointed, pliable, water-colour brush.

Of Thomas Girtin (1775–1802), a fine draughtsman and a master of the "cool grey and warm brown" technique in water colour, Turner is reputed to have said: "Had Tom Girtin lived, I should have starved." The breadth, sweep, and certainty of Girtin's water-colour landscapes, such as those depicting Bolton and Kirkstall Abbeys, rank them among the most satisfying pictures ever painted. But equally attractive to most judges is the work of John Sell Cotman (1782–1842), another Norwich artist. He used a much wider range of colour and tone than Girtin. He was also the first to exploit the purely decorative quality of water-colour, using it to emphasise, and even to extract, sheer pattern in landscape. His pictures are delightful patchworks of limpid colour washes —delicate blues and greens, rich golden browns. David Cox (1783–1859) used water-colour, rather dry in its application, to record, like Constable, the moods of wind and weather, so that all his pictures seem to be full of fresh air. Peter de Wint (1784–1849) usually preferred to present the tranquillity of the English scene by a free use of "wet" washes.

Turner used in his water-colours every technique and trick to obtain his effects—washes of transparent colour, fine stipple with the point of the brush, spots of opaque body-colour, scratches with a penknife, wipes with a sponge, blacking out with ink or lamp-black. Such devices broke through all accepted rules, and might have proved disastrous in any other hands. They make Turner's small water-colours as forceful as many of his larger works in oils.

These were the men who first discovered that the limpid, atmospheric qualities of the water-colour medium were peculiarly suited to the English scene and, even more, to the English climate. Since then the medium has been more popular and more skilfully used by English painters than by any others with the exception of the Japanese.



PITY, by William Blake, the poet, painter, and mystic who made his own mythology. This design expresses a vision of the images in Shakespeare's Macbeth, Act I, Scene 7: "And pity, like a naked new-born babe, striding the blast, or heaven's cherubim hors'd upon the sightless couriers of the air."

Contemporary with this highly creative period in English painting, but as remote from its general trend as was El Greco from the trend of the Spanish school, was William Blake (1757–1827), the great mystic who was both painter and poet. He was influenced by the ideas of Reynolds to the extent that he produced "historical compositions," a term that included literary or classical subjects, which Reynolds had always claimed to be a superior form of art to portraiture and landscape. But Blake owed little else to anybody or anything beyond his own highly imaginative genius. His figures and his scenes are not of this world. Rather are they a transcript from ethereal and apocalyptic visions, set down with such clarity and force that they compel attention and seem for a moment to be more real than reality. Some of his most impressive allegorical and illustrative designs are in the form of colour-printed designs. Others are water-colour drawings. The Tate Gallery, London, has an excellent collection of his works, but possibly his finest engravings are the 21 illustrations to the Book of Job, now in the British Museum.

LESSON 17

Painting in the 19th Century

With the 19th century, European painting emerged from the period of the great schools, and became less national in its expression. Not that it thereby became international, because for the greater part of the period it was dominated by the output of one nation—France. Paris became the centre of all that was vital and progressive in European painting, and led the way in courageous experiment, with London sometimes taking a

MADAME RÉCAMIER, a famous example of the portrait painting of J. L. David, greatest exponent of the Classical style which succeeded the "frivolous" work of the age of Louis XV.

notable second place—and the rest of Europe producing virtually nothing that was not imitative of, or derivative from, France or, to a lesser extent, England.

By the end of the century it had become almost as inevitable that a professional painter of standing should have studied in Paris as that a professional musician should have studied in Leipzig, and the romantic tradition of the *ateliers*, as depicted in George Du Maurier's *Trilby*, dies hard.

The "Classical" Revival

Art historians still differentiate by the term "school" the various groups and movements which emerged at different times during the century—the Classical school, the Romantic school, the Barbizon school, the Realist school, the Impressionist school, and so forth, in France; and in England, the Pre-Raphaelite school. But these were never schools of painting in the former, wider sense, so much as groups of artists who shared, or were influenced by, particular and often completely opposing doctrines about both the purpose and the practice of painting.

After the turmoil of the French Revolution there was, under the influence of Napoleon, a harking back to classical, or at least Roman, ideas. France had become, like ancient Rome, an empire, with a senate and tribunes and an emperor who liked to ride under triumphal arches. It was possibly the natural reflection of the emergence of order from chaos.

In sculpture, and as details in contemporary portraits as well as in historical paintings, laurel wreaths, acanthus leaves, columns, eagles, the toga, and classical draperies and sandals were introduced. The so-called "empire style" was reflected even in England in the architecture, decoration, and feminine costume of the Regency period.

Painting in this style found its most satisfying expression in the work of Jacques Louis David (1748–1825), court painter to Napoleon I. Well-known pictures by this artist are his early painting of "Marat Assassinated" and his dignified portrait of Mme. Récamier, reclining on her classical sofa, also a large group, "The Rape of the Sabines." His work has a sculpturesque quality by reason of his clear-cut accuracy in the depiction of the human form, though it suffers from coldness and monotony of colour and a certain hardness of texture.

A greater influence on the younger artists of his day was David's most famous pupil, Jean Auguste Dominique Ingres (1780–1867), who also evinced a fine restraint of style as well as draughtsmanship disciplined with admirable severity. The National Gallery, London, has one or two excellent examples of his work, including the portrait of M. de Norvins; but possibly his most famous picture is "La Source," an exquisite study in the nude.

The "Romantic" Movement

By 1830 the Classical movement was being rapidly outpaced and displaced by the urge of the Romantic movement, which was in complete opposition to it. The great Romantic revival of the early 19th century was international, and affected all arts alike. It inspired the revival of the Gothic style in architecture, the novels of Sir Walter Scott, the poëtry of Byron, and the music of Schumann. French artists were the first to give it expression in painting, most conspicuously Eugène Delacroix (1798–1863). He was considered a revolutionary, so little did his paintings conform to the classical tradition in theme and treatment. But his mastery received official recognition when he was employed to decorate a ceiling in the Louvre. A famous picture of his is "Liberty Leading the People," also now in the Louvre. Until recently his art was quite out of fashion, but he retained his place in the history of art as the first leader in the revolt against official and academic tradition. Since his day French painters—and often English painters—have been ready to take a certain pride in having their work rejected by the Academy and in affecting to paint only "to please themselves."

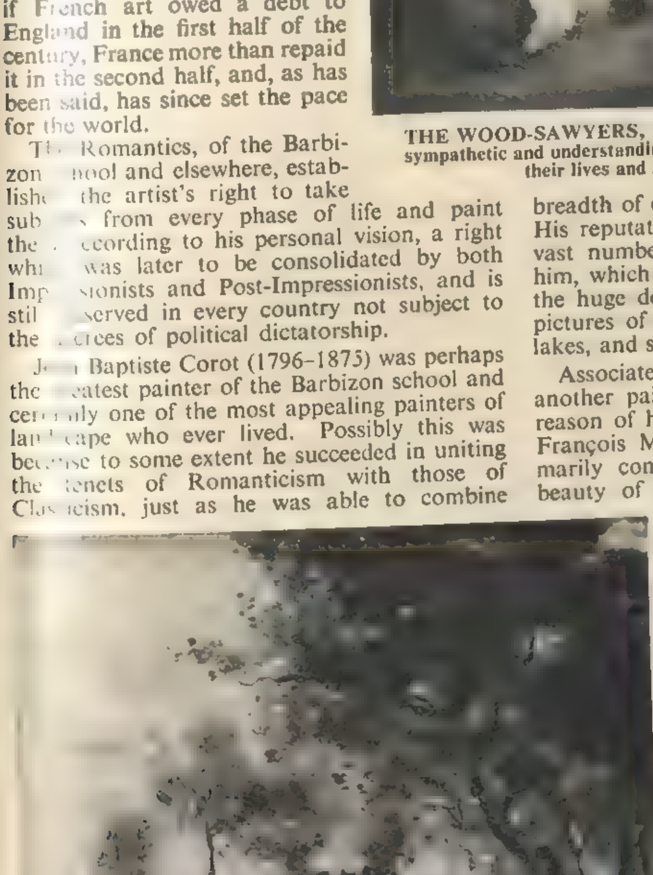
A similarly defiant artist was Théodore Rousseau (1812–67), whose early paintings were

THE WOOD-SAWYERS, by Millet, the "idyllic realist," whose sympathetic and understanding paintings of French peasants invested their lives and labours with new dignity.

so regularly rejected for exhibition that he was given the nickname of "Le Grand Refusé." Settling in the Forest of Barbizon, he became the founder of the so-called Barbizon school, a group of Romantic landscape painters, including Diaz, Daubigny, and Corot. The Barbizon painters were directly influenced by the paintings of the English John Constable. It can also be claimed that Constable and Turner showed the way to the later French Impressionists. But if French art owed a debt to England in the first half of the century, France more than repaid it in the second half, and, as has been said, has since set the pace for the world.

The Romantics, of the Barbizon school and elsewhere, established the artist's right to take subjects from every phase of life and paint them according to his personal vision, a right which was later to be consolidated by both Impressionists and Post-Impressionists, and is still preserved in every country not subject to the decrees of political dictatorship.

Jean Baptiste Corot (1796–1875) was perhaps the greatest painter of the Barbizon school and certainly one of the most appealing painters of landscape who ever lived. Possibly this was because to some extent he succeeded in uniting the tenets of Romanticism with those of Classicism, just as he was able to combine breadth of composition with exquisite delicacy. His reputation suffered somewhat through the vast number of spurious works attributed to him, which were produced by copyists to meet the huge demand for his poetically composed pictures of shimmering trees, misty rivers and lakes, and serene moonlight.



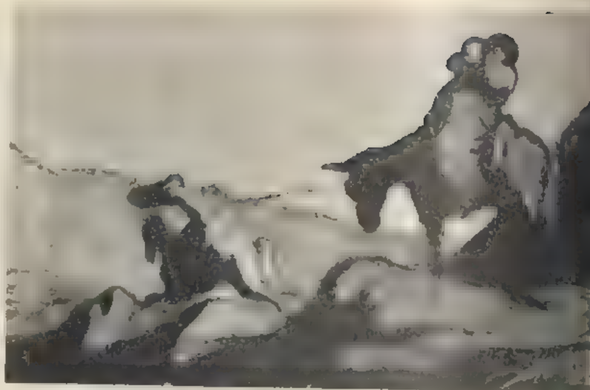
MORNING LANDSCAPE, by Corot, who passed during his long career from Classicism to the beginnings of Impressionism, and whose pictures of shimmering trees and serene moonlight in the Barbizon woods combine breadth of composition with a truly exquisite delicacy.

Associated with the Barbizon school was another painter who stands out powerfully by reason of his simple directness of vision, Jean François Millet (1814–75). He was not primarily concerned with landscape, though the beauty of spring was never more effectively presented than in his picture "Le Printemps." His chief works expressed one prevailing motive: "Man goeth forth to his labour until the evening." The son of a peasant, who had spent his boyhood in his father's fields, he visualised peasant life with sympathetic understanding and invested it with massive dignity in such well-known pictures as "The Gleaners," "The Angelus," "The Wood-Sawyer," and "The Man with the Hoe."

The Impressionists

The next great French movement, which dominated the whole of European painting for the rest of the century and beyond, was the Impressionist movement. It is sometimes claimed that Edouard Manet (1832–83) was the first Impressionist, but this is true only in so far as Impressionism arose from the ideas of the *plein air* group,

of which Manet was the principal exponent. He and his immediate disciples of this group aimed at the realistic rendering of objects in full daylight rather than in the modified and controlled light of a studio—not a very revolutionary aim, it would seem, until one realizes the prevalence at that time of large easel pictures "concocted" in the studio. Even Constable's large works for exhibition were worked up in his studio from his wonderful direct sketches made out of doors. The *plein air* schools completed their work "on location," as the makers of cinema-films would say, and rated the faithful, direct transcription of vision a quality far superior to technical finish.

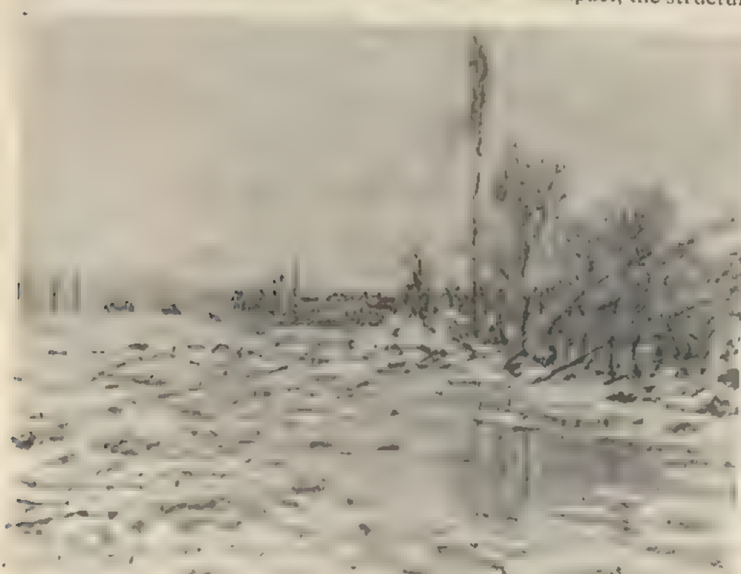


DON QUIXOTE AND SANCHO PANZA, by Daumier (1808 79), an example of the imaginative work of this painter, poster artist, caricaturist, and sculptor, who has been called the Charles Dickens of French art, but whose more serious fame was almost entirely posthumous. He excelled in social and political satire, and was once imprisoned for a caricature of King Louis Philippe.

Manet, then, was a realist. When he painted his "Olympia," the seemingly deliberate ugliness of the subject he had chosen to transcribe so faithfully was a flat challenge to all previous ideas of beauty in art. Rembrandt before him had done something of the sort when he painted his magnificent picture of a flayed ox in a slaughter-house. But Manet owed much to the influence of Rembrandt as also to that of Titian, Hals, and Velazquez. He was a fine and sincere artist, and if "Olympia" startled Paris and shocked the critics, many of his other works are delightful in their sympathetic portrayal of character, e.g. "Le Bon Bock" and "The Bar at the Folies Bergère" (National Gallery, London). His "Dejeuner sur l'herbe," a kind of 19th-century rendering of one of Watteau's "*Fêtes Champêtres*," was rejected by the Paris Salon in 1863, but now hangs in the Louvre. All his work has a peculiarly penetrative quality, and evinces a grand perception of colour and form and even weight.

In Manet's earlier works the design is more compact, the structures are more clearly defined, than in his later pictures, painted when his supreme interest was the relation of colour to light.

It is this semi-scientific study of light that ranks him with the true Impressionists, whose central principle was that the hand should record exactly as the eye sees at a given moment. The eye, in fact, was made to serve as a snapshot camera, which records just as much, no more and no less, as the light of a moment allows it to observe. The Impressionists were therefore less concerned with known facts than with how these facts appeared when revealed in a particular light and particular atmospheric conditions—in other words, with optical, as



WINTER SCENE, by Manet, the great mid-nineteenth-century realist who broke with classical traditions of beauty, form, and design, and led the practice of completing a subject in the open air rather than working it up in the studio. Among his most famous later works were character-studies in Parisian settings.

distinct from intellectual, truth. It was the leader of the movement, Claude Monet (1840–1926), who declared that light itself was the most important subject in any picture.

Th Monet's most characteristic paintings are pictures of light and atmosphere with only the vaguest suggestion of form. It was after a close study of Turner's work in England that he evolved both his theory and a suitable method for its expression. He began to paint with broken strokes of the brush, laid upon the canvas in touches of pure primary colour—for pure light, as the spectroscope proved long ago, is composed of all the primary colours in juxtaposition. The effect produced was brilliant and luminous, as can be seen in his "Vetheuil: Sunshine and Snow," now in the Tate Gallery, London. Incidentally Rubens, Watteau, Gainsborough, and especially Turner had all made occasional and partial use of this method to obtain the effect of sunshine in their landscapes.

When Monet and his associates in the movement exhibited their works in Paris in 1874, one of Monet's pictures was labelled "Impression: Soleil Levant." The name was seized on by the periodical *Charivari* as a peg on which to hang a string of vitriolic abuse of the exhibition and all it stood for. The artists retaliated by accepting the label of "Impressionists" with pride. They continued to be reviled by the more academic critics for close on a decade; but they maintained their principles, and gradually—as has happened so often in the history of art—their views prevailed; and as younger painters increasingly adopted those views, the critics and even the public eventually followed—for the time being!

Monet's chief associates and disciples were Alfred Sisley (1840–99), Pierre Auguste Renoir (1841–1919), Camille Pissarro (1850–1903), and Manet's pupil Berthe Morisot (1840–95). Edgar Degas (1834–1917), popular in England to-day for his paintings of ballet dancers, is usually classed with the group, but was really allied to it only by his revolt against Romanticism and his extraordinary feeling for movement. Sisley's landscapes possess a spontaneous charm; while the sensitive and brilliant painting of Renoir is seen to perfection in his portraiture and scenes of everyday life. Renoir painted with long strokes of the brush, using a preponderance of red and green. A characteristic work is "Les Parapluies," in the National Gallery, but possibly his masterpiece is "La Loge," in which a man and woman in black and white evening dress are seated in a box at the theatre, both figures being superbly realized.

A later development, sometimes called Pointillism, was introduced by Georges Seurat (1859–91) and his friend Paul Signac (1863–1935), who carried the absolute painting of

LES PARAPLUIES, by Renoir, one of the chief Impressionist painters, whose 6,000 paintings and over 150 lithographs reveal him as a master of portraiture and of the contemporary scene in street, house, and theatre. Emphasising colour without sacrificing light, his art remained always spontaneous and impetuous throughout.

light even farther by the use of spots (*points*) or small rectangles of primary colour.

As already suggested, Impressionism, contemporaneous as the movement was with the growth of interest in photography, was parallel to photography in its attempt to turn the artist's eye and the artist's hand into a camera, recording everything in terms of light. It is equally fair to suggest that the revealed limitations of photography served to reveal the limitations of the Impressionist theory, especially as expressed in the pseudo-scientific devices of the Pointillists; and inevitably artists were soon searching for escape from those limitations.

Nevertheless the movement, like all vital art movements, however extreme, did serve to "clear the air" of conventions that had become outworn. The greatest art searches continually for new ways of fulfilling its age-old function. When it ceases to do so, it will be dead indeed.

LESSON 18

English 19th-Century Painting

English 19th-century painting experienced its own minor revolution, that deriving from the formation in 1848 of the Pre-Raphaelite Brotherhood. It was a minor revolution because its influence was restricted entirely to English painting, and even so, though it had results that endured, they were by no means the results at which the original revolutionaries had aimed.

It was stated in Lesson 16 that although the genius of the English painters found its most natural and successful expression in portraiture and landscape, academic tradition, following the lead of Reynolds, rated "historical painting," after classical models, a superior pursuit. It was called "painting in the grand manner," and sometimes a certain level of competence was reached in this direction—notably by Benjamin West (1738–1820), who came from what was then the British colony of Pennsylvania and was appointed "historical painter" to George III and succeeded Reynolds as president of the Royal Academy. West produced numerous dignified *tableaux* depicting great events of his time, such as the death of General Wolfe. Later came Daniel Maclise (1806–70), whose two huge panels in the House of Lords, one of the death of Nelson, the other of the meeting of Wellington and Blücher on the field of Waterloo, are well known from countless engravings. They are fair examples of the style of "historical painting" once popular.

But the "grand manner" could also serve in the representation of literary, classical, or Biblical subjects. William Etty (1787–1849) followed it in his classical and allegorical paintings, such as "Youth at the Prow and Pleasure at the Helm," in the Tate Gallery, and is reckoned the most competent English painter of nude figures. More commonplace painters, like Charles Robert Leslie (1794–1859) and William Powell Frith (1819–1900), found their themes in scenes from the works of Shakespeare and other writers, though Frith in addition is famous for his large crowded canvases, full of incidents or "stories," depicting Derby Day, Ramsgate sands, and Paddington railway terminus—and, uninspired as these may be in conception in comparison with the sublime creations of a Michelangelo or a Titian, they are nevertheless distinguished by much deft painting and skilful composition.

The Anecdotal Picture

As already stated, the English genius, as studied from Hogarth onwards, is more suited, like that of the Dutch, to humbler themes and less dignified treatment than are demanded by the "grand manner." Moreover, English painters have always favoured a strong anecdotal or illustrative element in the subjects they chose. Hence the popularity of works by George Morland (1763–1804), who painted with great charm scenes in country inns, stables, and smithies; of Francis Wheatley (1747–1801), who did the series called "Cries of London," engravings of which are so eagerly sought by collectors; of Sir David Wilkie (1785–1841), whose scenes of homely Scottish life have all the verve of Teniers; of Sir Edwin Landseer (1802–73), a favourite artist of Queen Victoria and the most popular painter of his day, who painted animals that were almost human in their behaviour and gave his pictures titles like "Dignity and Impudence" and "The Monarch of the Glen."

OPHELIA, painted by Millais during his Pre-Raphaelite period, exemplifies the exact representation of nature — which was a guiding principle of the artist and his associates in their short-lived but successful revolt against the conventions of contemporary grand manner.

All these, in addition to being more than competent craftsmen, sought to make their pictures tell or illustrate an anecdote, perpetuate some amusing episode, even to point some moral. Even great landscape painters like Constable and Turner did not consider a picture to be seriously complete unless their glorious trees and skies and sunsets were made the setting for some particular incident like a leaping horse, or a

hay-waggon crossing a ford, or a boy drinking from a brook; or, in Turner's case, until they were labelled as "Dido Returning to Carthage," or "Ulysses Deriding Polyphemus." To the majority of people the real point of his most famous picture, "The Fighting Téméraire," is not the gorgeous sunset, but the fact that the old battleship is being towed to the breaker's yard; the sunset takes its place as a mere symbol of the incident.

Nevertheless the Royal Academy still stood by the requirements of the "grand manner," though it became even less of an inspiration. By the 1840s "historical painting" was very much in the doldrums, following a hidebound set of conventions, essentially imitative, devoid of all spontaneity and vitality. At the Academy schools all natural genius was being stifled in attempts to copy the manner of Raphael—and very hideous many of those attempts were.

PICTURES BY PRE-RAPHAELITE LEADERS. Left, "The Annunciation," by D. G. Rossetti, savagely attacked for its unconventionality; right, "Claudio and Isabella," by Holman Hunt. Such works were condemned as inferior to the almost lifeless academical paintings to which the taste of the age had become accustomed.

The Pre-Raphaelites

The Pre-Raphaelite Brotherhood was formed in 1848 by a little group of very young and very earnest men, the eldest of them only 21. Their purpose was to paint scenes inspired by great literature, especially the Bible, in what they conceived to be the exact manner of the Italian painters *before* Raphael—even trying to copy their technique of painting thinly on a wet gesso ground—and reconstructing their chosen scenes with the most literal fidelity to nature down to the smallest detail, as the Italian painters had done, and certainly in full opposition to what was demanded by academic convention.

The leading spirits in the venture were John Everett Millais (1829–96), William Holman Hunt (1827–1910), and Dante Gabriel Rossetti (1828–82). In 1849 they exhibited their first pictures to be conscientiously painted according to these principles, signing their canvases, in all the hopeful arrogance of youth, with a mysterious "P.R.B." If they hoped their work would make a stir, they were disappointed for the time being. But by the following year the two literary members of the little group had boldly publicised the P.R.B. principles in a periodical called *The Germ*, and academism was ready to teach the Brotherhood a lesson. Again the three young men exhibited paintings, two of them in the Royal Academy, and this time they received all the publicity they could desire. So bitterly was Rossetti's hauntingly beautiful picture "The Annunciation" received that never again in his lifetime would he consent to exhibit his work publicly. The picture is now in the National Gallery. Millais' picture was the well-known "Christ in the House of His Parents," sometimes called "The Carpenter's Shop," which was eventually (in the 1920s) purchased for the Tate Gallery by public subscription. In 1850 it was fiercely attacked, abused, and derided, by critics and public alike.

Thereafter the Brotherhood rapidly disintegrated. The little adventure was over, and disastrously so. Yet not entirely; for, quite unexpectedly, John Ruskin, who was already making his mark as a discerning art critic and a persuasive writer, defended the Pre-Raphaelites stoutly in a series of letters to *The Times* pointing out the virtues of such a revival of the spirit of the primitive Italians, and thereafter he made the cause of the Pre-Raphaelites his own. Millais and Hunt at least were encouraged by his support to persist in their Pre-Raphaelite practice, Millais for some years, Hunt for the rest of his life. Moreover, other artists, counting on the support of Ruskin, began to paint in the same style, and colour and life and intelligence began to replace pretentiousness on the walls of the Royal Academy.

The glowing colour and wealth of accurate detail which characterised the original Pre-Raphaelite technique are seen at their best in paintings like "The Hireling Shepherd" (Manchester City Art Gallery), "The Triumph of the Innocents" (Tate Gallery), and "The Light of the World" (Keble College, Oxford), all by Hunt, and in Millais' "Ophelia" (National Gallery) and "The Blind Girl" (Birmingham Art Gallery). Other works by the same artists reveal some of the weaknesses of the style, such as its tendency towards pseudo-medievalism in the painting of human figures, its insistence on stiff, awkward, or even ugly attitudes, in deliberate defiance of the grace of Raphael.

The most notable of the painters outside the Brotherhood who adopted the style with success was Ford Madox Brown (1821–93), whose little picture "The Last of England" (Tate Gallery) is an anecdotal picture, beautifully composed and painted and deservedly popular.

The "Literary" Pitfall

Short-lived as the P.R.B. had been, its influence was out of all proportion to its importance. Its greatest achievement was to doom the "grand manner" to early and, one may hope, permanent extinction. Its other more accidental achievements were less happy. For one thing it swept Raphael from the high pinnacle he had up to then occupied, and this superb artist has since been sadly underrated.

Instead of the "grand manner" there arose an over-emphasis on literary inspiration in painting, which, through the English love of a "story," soon degenerated at the hands of artists with commonplace minds into the commonplace "story picture" so popular on the walls of the Royal Academy exhibition rooms for the rest of the century. For this Millais himself must bear part of the responsibility. When the Brotherhood broke up, he was the one who most missed its stimulus. He was by far the most accomplished craftsman of the three, but he gradually fell away from his early ideals to expend his superb technical gifts on increasingly commonplace and sentimental "anecdotes." For a time he was interested in drawing book-illustrations, and very satisfying illustrations they were. Unfortunately the paintings he exhibited at the Royal Academy year after year became little more than large-scale book-illustrations in colour. A host of less competent artists followed his lead, and it is only by comparing their work with his that one realizes his superior talent. Even in such a picture as the notorious "Bubbles" there are a pleasing freedom and breadth of treatment and a soundness of drawing and composition that are missing from pictures in a similar vein from other painters.

The tradition of English "anecdotal" painting, therefore, which at its best had given scope to Hogarth, Wilkie, and the Pre-Raphaelites, became most prominently represented by the many enormous canvases of painters whose names are not worth recording in this brief survey, which filled most of the wall-space at the Royal Academy exhibitions towards the end of the 19th century and in the early years of the 20th. Looking like huge coloured photographs of people in fancy dress, or grossly inflated illustrations to the stories in a magazine, they presented, as realistically as the painters could contrive, dramatic *tableaux* from history and literature ; or they were stories in themselves, with such titles as "Saved !", "The Fallen Idol," "When the Heart is Young," "Between Two Fires," "A Hopeless Dawn," "And When Did You Last See Your Father ?". In their day they were exceedingly popular. Cheap reproductions of them were found in homes all over the country. Gradually the British public became surfeited with them, and by the 1920s the "story picture" was almost everywhere derided. This is to be regretted. There is nothing wrong with the "story picture" as such, and Millais remains an artist who offers far more to admire and enjoy than to condemn. What is regrettable is that so many of these pictures were poor in execution, produced by men with commonplace minds devoid of poetry or any sense of beauty, pictures with nothing but the story to commend them, and that story too often as trivial as the painting was pretentious.

Burne-Jones and Watts

But as the 19th century wore on, the influence of the Royal Academy waned. Other and livelier influences were at work. There was a further indirect stemming from the Pre-Raphaelite experiment, through the influence of Rossetti, another of the original Brotherhood. He developed his own increasingly sensuous style, pursuing and exploiting a poetic vision in which his ideal of feminine feature and form played a prominent part. He, too, influenced a number of painters who adopted a new kind of sentimental classicism, sometimes tinged with religiosity. The outstanding exponent of this style was Sir Edward Burne-Jones (1833–98), notable for many over-refined pictures of languid female figures, often allegorical, clad in classical draperies and all with the same expressionless features. Rossetti and Burne-Jones led the painting fraternity in the "aesthetic" movement, which affected all the arts in the England of the 1870s and 1880s. (*See* Course on English Literature, Lesson 23). Through a confusion of ideas due to Rossetti's part in it, this movement was also popularly called Pre-Raphaelite.

GAINSBOROUGH. "The Blue Boy," a masterpiece now in the U.S.A., is the portrait of the son of an ironmonger of Soho. It was painted by Gainsborough about 1770 in refutation of Reynolds's dictum that a cool colour such as blue could never be successfully used as the main colour of a painting

ENGLISH WATER-COLOUR LANDSCAPE IN ITS PERFECTION

Many people think this picture of Greta Bridge, Yorkshire, by John Sell Cotman (1782-1842), the loveliest work in water-colours ever painted. Apart from its crisp pattern and serene colour, it shows how admirably suited is this delicate medium to the beauty of the English scene.

Courtesy of the Trustees of the British Museum

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Allied to this spurious Pre-Raphaelitism, but far more robustly masculine in approach and more profound in conception and feeling, are the allegorical paintings of G. F. Watts, whose "Hope," "Love and Death," "The Dweller in the Innermost," and many others, are too readily underrated to-day. Watts possessed—though he used them unequally—a natural sense of grand design and a masterly technique, and his allegories are the supreme example of didactic art. His colour, which owes something to the study of Titian, is subtle and expressive. He was also a great portrait painter who probed with the eyes of a Rembrandt deep into the characters of such inspiring contemporaries as J. S. Mill, Carlyle, Newman, Tennyson, Gladstone, and Matthew Arnold.

Whistler and Impressionism

Finally, the closing decades of the century brought from across the Channel the impact of Impressionism. The persuasive personality, no less than the artistic integrity, of the American-born James Abbot McNeill Whistler (1834–1903) made the impact a forceful one. Like the Pre-Raphaelite Brotherhood in England and Manet and the Impressionists in France, he received more than his share of storm and scorn and abuse, but he faced it with an air of imperturbable defiance. This time Ruskin, champion of the Pre-Raphaelites, was not on the side of the revolution. On the contrary, he called Whistler a coxcomb and accused him of "flinging a pot of paint in the public's face." There followed a notorious action for libel, in which Whistler was awarded damages of one farthing; but it was wonderful publicity for Impressionism.

Like Manet, Whistler was not strictly an Impressionist; but again like Manet, he had in common with the movement the fact that he rebelled against the prevalent conceptions of the purpose and practice of art. He held that the beauty and interest of a picture, like that of a musical composition, resided primarily, if not solely, in the artist's treatment, and least of all in the associations of its subject matter. He therefore adopted the habit of labelling many of his canvases after the manner of musical composers, e.g. "Symphony in White No. 2," "Nocturne—Blue and Gold" (Whistler's title for the familiar picture of old Battersea Bridge now in the Tate Gallery). Even what has become his most popular picture, the portrait of his mother, painted in 1872 and now in the Louvre, Paris, was exhibited in the Royal Academy as "Arrangement in Grey and Black," and he said of it: "To me it is interesting as a portrait of my mother, but why ought the public know or care about that?"

Whistler settled in London after studying and working in Paris, where the Salon had rejected paintings submitted by him in 1859 and 1863. The second of these rejected pictures, "The White Girl," is now in the National Gallery.

In his earlier paintings he was much influenced by the art of the Japanese, then newly revealed by the opening-up of the East to the West. He appreciated, as the Japanese painters did, the decorative value of empty spaces. His subordination of detail to effects of serene colour-harmony, especially in his so-called "Nocturnes," where the detail is vague, implies a great deal in common with Impressionists, though his ultimate purpose was unlike theirs in being more concerned with beauty than with truth. Nevertheless the close alliance with French painting so marked in England at the close of the century was due as much to Whistler's influence as to that of any other one man; and that alliance meant a spread of Impressionism among English painters, and a general revival of English landscape painting, but this time in Impressionist terms.

The English Impressionists

Among leading English Impressionists were Walter Sickert (1860–1942), a pupil of Whistler, Wilson Steer (1860–1942), and Lucien Pissarro

OLD BATTERSEA BRIDGE, originally called "Nocturne—Blue and Gold," is the most celebrated of Whistler's series of night studies of the Thames. Whistler's influence drew together elements of English and French art towards the close of the nineteenth century.

(1860–1944). Sickert and Steer were also foremost in adapting portraiture to the demands of Impressionist theory. In landscape Steer at his best seemed to have short-circuited the Impressionists by deriving his perception, if not his treatment, directly from Constable—the Constable of the rough sketches rather than of the finished Academy pictures. Steer's own water-colour sketches reveal the same ardent devotion to nature in all her moods. Sickert had a gift of expressing atmosphere that was not only physical but emotional. His favourite method of developing a picture was to prepare a series of careful drawings so that the subject was learnt by heart, then to paint from them in a systematic scale of tones, with a given colour schedule.

Raphael and the principles of classical composition had been thrust from their pinnacle, and at the end of the century English painters seeking inspiration from among the old masters found it in the profound shadows of Rembrandt, the dexterous brushwork of Hals, Constable's storms and Turner's sunlight, and the luminous tone-values of Velazquez. Such is fashion in art! The turn of El Greco was yet to come.

LESSON 19

20th-Century Painting

The upheavals of world order, the revolutions in human thought and in human values, and the violence of human action, all those unprecedented changes which have marked the first half of the 20th century—changes in which Epstein and Freud have played no less formidable a part than Lenin, Hitler, and the atomic bomb—have been clearly reflected in the art of the 20th century. No one need search far for examples of architecture, poetry, music, and painting, popularly and vaguely called "modern," which are to the lay mind as bewildering as, say, Riemannian geometry, or some aspects of "modern" finance.

It has been a period of extreme experiment in the arts, each experiment apparently different from the last, and having in common with the rest only its startling opposition to the old standards and apparently to any rules or formulas whatever. In other words, the old standards have been thrust into the melting-pot, and no reliable new ones have yet emerged to take their place. We have had Post-Impressionists, Fauvists, Cubists, Vorticists, Futurists, Purists, Dadaists, Surrealists, Expressionists, and mixtures of all of them, all stridently forcing themselves into public notice in rapid succession, and to many people meaning less than nothing.

TE RERIOA, a study of native life in Tahiti, by Gauguin, the almost self-taught French master who renounced civilization except for its painting materials, and produced magnificently designed and coloured scenes from the South Sea Islands of his adoption.

What does it all mean?

The bewilderment, even of the younger generation, is no doubt in large measure the result of our nearness to contemporary art. It is difficult to see the genuine wood for the bogus trees. It is also instructive to remember that our forbears, in their way, felt as bewildered and as outraged in the face of the Pre-Raphaelites or the Impressionists as are many people to-day by any form of "modern" art. It is human nature to mistrust the unfamiliar. Let it be remembered also that the British public has long recovered from the violent shock occasioned by the first exhibition in London of the work of Cézanne, Gaugin, and Van Gogh, and that their paintings, hanging on the walls of public galleries or reproduced by the thousand on the walls of schools and homes, have become old and valued friends, giving real delight to almost everybody, probably much more delight than the works of many strict Impressionists, which to-day seem so tame and dead by comparison.

The reaction to Impressionism was inevitable. Structure had been sacrificed to optical effect. The Pointillists had taken theories of light as

far as they could go in complexity. If the ultimate and only goal of all painting was to be fidelity to optical vision, the photographic camera could always reach it more quickly, more economically.

It was time to indicate once more the fact that art is not nature, but a pattern or rhythm of design imposed on nature by the individual vision of the artist.

All reactions tend towards exaggeration, towards the extreme. If it were not so, they could never gain impetus enough to attract attention. New movements arise out of the exasperation suffered by individual genius in the face of any attempt to standardise or dogmatise, and that exasperation causes a heavy swing over to the opposing side.

Cézanne, Van Gogh, Gauguin

Again the reaction began in France, where Paul Cézanne (1839–1906), striving towards perfection with unceasing individual experiments, finally hit upon the method which revolutionised painting. He was the leader of the movement which was eventually called, in Britain, Post-Impressionism. The name was invented by two British critics, Roger Fry and (Sir) Desmond MacCarthy, when they organized in 1910 a London exhibition of the works of Cézanne and others, including Vincent Van Gogh (1853–90) and Paul Gaugin (1848–1903). It may be noted, as evidence of the long lead of France in the world of painting, that by the time this exhibition shocked the London public (as it unmistakably did) the chief painters represented were dead.

In fact, the painting of the three men had little in common apart from being revolutionary in their repudiation of the tenets of Impressionism. Each was revolutionary in his own individual way. They were not a "school," as such, not even a group like the Pre-Raphaelites. Gaugin, for example, while repudiating naturalistic representation and indulging in a certain defiance of perspective, developed as a supremely decorative artist on somewhat exotic lines—he was partly Peruvian on his mother's side. Bold, almost violent in colour and design, his work has chiefly been the means of influencing design for decorative industries. He did not mind in one and the same picture painting some of his figures "flat" and others in relief. He used a heavy outline to unify his design.

Gaugin led a restless and unhappy life until he emigrated to Tahiti in 1891, specialising thereafter in portraying the rich colour and spirit of the South Sea Islands scene, and leaving many impressive pictures of native women, charged with a feeling that is always mysterious, often tragic.

Van Gogh was a Dutchman by birth who began to paint in Paris in 1886. His early work was influenced by Pissarro, Renoir, and Seurat. Like Renoir, he used much green and red, with resultant masses of brilliant and vibrant colour. His paint was applied with heavy strokes, sacrificing any delicate qualities of texture to the demands of form and structure and, especially, pattern. This did not impair his power of characterisation in his portraits, while it gave strange clear-cut beauty to his landscapes and flower-pieces (*see* colour plate facing page 1416).

Of the three, Cézanne was not only the greatest but also the most immediately influential. He inspired a host of followers; his very failures were to serve as stepping-stones for other artists, who could see in his many uncompleted efforts what it was he had been striving for.

Cézanne was always searching beyond visual impressions for some poetic or spiritual quality in all he saw. Turning to painting rather late in life, he was never able to acquire facility or a satisfactory figure memory, and he disliked working from models. He set himself a difficult ideal—to combine the best qualities of Impressionism with the structural qualities of Poussin. He has been called a poor draughtsman, but at least he was an original one. He conceived that everything could be resolved into geometrical forms such as cylinders, cones, and spheres. His very brush-strokes were mostly angular. This was draughtsmanship according to a theory. It was as though he had first mentally obliterated whatever subject he intended to depict, then reconstructed it afresh from nothing—and had left something out! Yet what he did introduce had its own effect of solidity and weight, as well as all the powerful appeal of individual creation. His many still-life studies are typical. He made them like patterns, just for the pleasure a pattern can give. But the pattern was structural, three-dimensional, as well as original and inventive.

Moreover, some of his pictorial conceptions possess a colour value as sensitive as that in a painting by Turner. For an example of his subtle colour modulation in landscape, *see* the colour plate facing page 1416.

Matisse

Of the other artists associated with the term Post-Impressionist, the most distinguished was Henri Matisse (1869–1954), who carried even farther the search for abstract pattern, so far that some of his subjects are almost wantonly deprived of any resemblance to natural appearance, though they are recognizable as landscapes, interiors, figures, flowers, and so on. His appeal lies in his direct simplicity. Like a primitive painter, he worked freely with colour and line; like the early Italians, he submitted to the "discipline of rhythm" rather than the discipline of nature.

For a time Matisse led a group called the Fauves or Fauvists, also derisively called the "incoherents," and Matisse-like painting found some favour for a time in other countries, especially Germany. But copyists of Matisse's style were, and are, doomed to failure because no rules can be made for their guidance. Matisse painted each picture with the treatment he felt it required, ignoring shadow, modelling, and perspective as he pleased, but ever inventing new qualities in the sensitive relations of colour, and of colour with line.

VASE OF FLOWERS, by Matisse, follower of the Post-Impressionists, whose search for abstract pattern set him for a period at the head of a group called the "Fauves," meaning the wild men. Picasso said that Matisse "could make a canvas laugh and sing."

Picasso

The Cubists came to the fore in 1908, with Pablo Picasso (b. 1881) as their leader and prophet. In its purest form Cubism dealt solely with abstract patterns. Roger Fry defined its aim as the "abstract language of form—a visual music." Its practitioners, developing Cézanne's geometrical simplification, declared that the primitive abstract form is the cube, and that circles are cubes with the edges rounded off. Flat surfaces rather than curved ones were therefore emphasised—and, as is the way with extremists, over-emphasised—in their compositions. Six cubes were made to form a primitive human figure, four for the limbs, two for the head and trunk. The cubes were thereafter split up into facets as required.

Cubist pictures were therefore hard and very clearly defined. Atmosphere and texture were discarded. Light was valued only as a means of defining structural (Cubist) form. The contribution of the Cubist theory to "modern" painting has been its insistence on resolving objects to the simplest forms and grasping their essential shapes, and on preferring even a distortion to an outline that is weak and blurred. But anyone who looks at "modern" design in rugs, tapestry, wallpaper, dress fabrics, or pottery will soon discover in what direction both Cubism and Fauvism (with its insistence on simple direct patterns in line and colour) have exerted the greatest influence.

Picasso, who was born at Malaga and settled in Paris in 1903, outlived and outgrew the Cubists, and his work has gone through extraordinarily different phases, all equally arresting. His earliest paintings were both realistic and poetic, evincing a particular interest in the backstage of the world of entertainment, pierrots, the circus, the ballet. He was then for a time interested in Negroes and Negro art. The paintings of his Cubist period affect one with a sense of disintegration that is expressive of the analytical thought and disruptive tendency of the years immediately preceding the First World War. His portraits were sometimes put together in fragments from different points of view, perhaps showing the eye full-face in a profile, the shoulders square to the spectators, and the legs again viewed sideways. Such experiments were always directed to the clearing away of the artist's former concepts and the creation of fresh ones. To Picasso the subject of a picture was important only in so far as it might be suggested in abstract terms.

About 1915, in company with a French painter, Georges Braque (b. 1881), Picasso began to produce studies in abstract pattern, in which violins, jars, pipes, tools, etc., were taken as objects on account of their intrinsically decorative shapes, and made the basis of new patterns a practice which has also had direct influence on modern industrial decorative design. In his later years Picasso's painting, still experimental, has become highly individual,

PICASSO AND CUBISM: Left, "Pierrot" by Picasso before he led the way into the Cubist mode, which is illustrated (right) by the portrait of Picasso himself painted by his fellow-Cubist Juan Gris. Picasso passed beyond Cubism to experiment in many other directions (see picture at foot of opposite page).

more difficult to understand; but he has been accepted as a master of organized form and constructive design, and his work has influenced architecture and sculpture as well as industrial design.

ABSTRACT PATTERN, by Braque, who in 1915 joined Picasso in experimenting with such patterns in this creation of new forms. The still life, which distinguished Dutchmen laboured to produce so that the drop of water seemed to tremble on the skin of the peach, became for these modern masters the threshold of adventure into a new world of light, colour, shape and pattern.

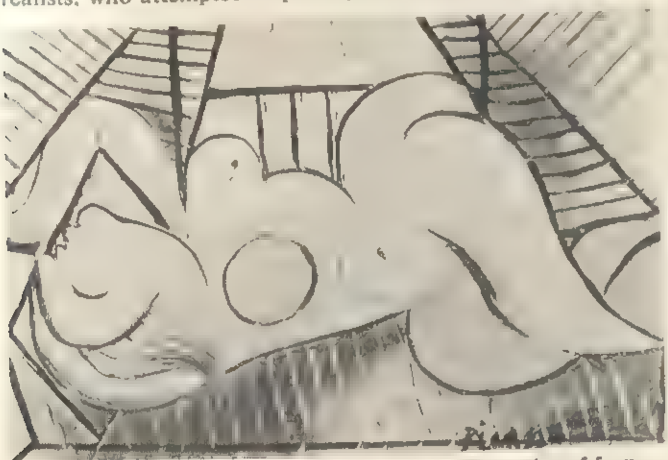
Futurists and Vorticists

Futurism and Vorticism were short-lived movements of the years preceding the First World War, the former originating in Italy in 1910 under the leadership of the Italian writer Marinetti, the latter in England, heralded by Wyndham Lewis. Futurism used arbitrary symbols—"force lines and rhythms"—together with quasi-geometric patterns, in an attempt to make each painting a purely subjective expression in an absolute, personal "language." A phase of Futurism which caught the popular imagination was the device of drawing, say, ten arms to represent a single arm in motion, and thus to express movement or activity. Vorticism found its most appropriate subjects in the mechanism of modern weapons and in vast armies of robot-like creatures at drill or on the march. Wyndham Lewis composed his pictures in planes and wedges, giving cast-iron countenances to his inhuman tubular figures, to symbolise mass production and machines.

The Value of Extremists

Later influential developments in France included Dadaism, which took machinery as the basis of its designs; the Purists, who were in opposition to the Cubists; and the Surrealists, who attempted to portray, often with a remarkable finesse of detail that would have done credit to the Pre-Raphaelites, the rhythms, fantasies, freaks, beauties, horrors, obsessions, and distortions of the subconscious. There were, and are, and possibly will be for a long time to come, many other such movements.

The value of extremists is that they break down the barriers, leaving the new paths free for more moderate and orthodox people to explore in their own ways. There are few artists in, for example, Great Britain to-day whose art remains unaffected by the impact of Post-Impressionism and the rest. Even the Royal Academy has become, year by year, ever more Expressionist. The term Expressionism was coined as the opposite of Impressionism, to cover *all* the lesser "isms" which have contributed to the "modern" trend. It suggests that the artist is not content only to *see* something and share what he sees with you, the spectator; he must *feel* something and express his feelings in such a way that (he hopes) you may share them. So art has once again become emotional, as well as representational.



NUDE, by Picasso, in which simplification and distortion of form leave the observer with the responsibility of his own interpretation of the pattern of curves imposed upon a grid formation.

The finest and most vital representative painters of the 20th century, e.g. Augustus John (b. 1878), Duncan Grant (b. 1885), Paul Nash (1889–1946) and his brother John Nash (b. 1893),

C. R. W. Nevinson (1889–1946), Henry Lamb (b. 1885), Stanley Spencer (b. 1892), Graham Sutherland (b. 1903), Edward Bawden (b. 1903), Vivian Pitchforth (b. 1895), Stephen Bone (b. 1904), and John Piper (b. 1903)—this list does not pretend to be exhaustive—all in different ways owe much in their development to the iconoclastic idiom of "modern" movements, yet none can be called unintelligible by anyone with unprejudiced eyes and an open receptive mind. Moreover the kinship of their art with that of great artists of any age or clime can be clearly discerned. The living spirit is dressed in the fashion of the time; but peer behind the fashions of two periods and you will find no basic difference in a portrait by Reynolds and one by Graham Sutherland. John's vigorous and unforgettable "Smiling Woman" and "Madame Suggia" (both in the Tate Gallery) are stepsisters to Leonardo's "Mona Lisa"; a water colour by Paul Nash or Pitchforth can link the message of Turner with that of Cézanne; and the deeply sincere religious pictures and beautifully detailed pattern-compositions of Stanley Spencer combine contemporary Expressionism with Victorian Pre-Raphaelitism and the deep-rooted English partiality for anecdotal painting.

POWERFUL PORTRAITS by modern painters are (left) "Somerset Maugham," by Graham Sutherland and (right) "The Smiling Woman," by Augustus John.
Courtesy of the Trustees of the Tate Gallery

As for all the Expressionism of any kind that is less immediately intelligible—well, there never was an easy road to the appreciation and understanding and enjoyment of art. No doubt much is derivative and imitative, not to say insincere; and in art that refuses to conform to canons and conventions it is hard to be sure of what is worthless and what is commendable. Moreover, in considering the effect of any painting there is always the possibility that it may have been achieved by pure accident. There is nothing new in this. An old and possibly unreliable tradition records that Ruskin once took a party to the Royal Academy exhibition to lecture on "a square-inch of Turner," demonstrating the superb mastery of the artist over every part of his canvas. Turner, hearing of this, asked to be shown which particular square-inch had been so honoured by the great critic's approbation. When the spot was indicated, he said, "Oh, yes, I remember—I dropped my paint-brush against that, and the mark looked rather good, so I left it."

SUNFLOWER WORSHIPPERS, by Stanley Spencer, a modern mystical and religious artist whose work combines arresting spiritual power with frequent deliberate distortion of the image, a realism sometimes affectionate and comical and sometimes sinister, and a detailed execution in the most skilful Pre-Raphaelite tradition.

Nevertheless it seems sound advice to say this: if one of these pictures, however unintelligible or unusual, strikes within you, the observer, a responsive chord of any particular kind, if mood appears to speak directly to mood and concept

to concept, as happens so universally in listening to music, it is only fair to give the artist credit for intending that this should be so, and that in satisfying himself he has contrived to communicate his satisfaction to you, and thus art is in this case fulfilling its purpose.

Meanwhile the student is urged to visit all notable picture galleries and loan collections and exhibitions of paintings, old and contemporary, within his reach, also to attend lectures on painting whenever opportunity arises. Only art itself can teach you to enjoy art.

LESSON 20

The Art of the Etcher

In reviewing the history of painting it has been mentioned once or twice that certain great painters were also skilful etchers. This Lesson gives a brief description of the art of etching. It involves a process whereby a number of prints may be taken from an artist's original drawing. But each print from an etched plate may be in itself a work of art. Many people collect etchings, often paying quite high prices for a single print by a master-etcher.

The process of etching is somewhat complicated. The plate from which the prints are taken is usually of copper. This is covered with a thin coating of wax, called the etching-ground. The artist draws on the wax with a fine steel needle. Wherever the needle moves, a thin line of copper is left exposed. The plate is then soaked for a time in nitric acid (or hydrochloric acid), which attacks all the lines of exposed copper, eating them into the plate. Those lines which are intended to be the finest and lightest in the print are then " stopped out " with varnish, so that they become impervious to any further action of the acid ; and the process is repeated *ad lib.*, the remaining lines being ever more deeply and broadly etched. Those lines representing heavy shadows and blacks are exposed longest to the action of the acid.

The process completed, the acid is cleaned off, the varnish removed, and the plate evenly inked and wiped over. All is then ready for the printing. Plate and paper rest on a flat bed over which a roller passes with the turning of a wheel. The pressure forces the plate into the paper, leaving a " plate mark," an indentation of the paper characteristic of all etchings.

In printing, the artist may choose to leave more or less ink on some part of the surface of the plate, according to the effect desired ; or he may deliberately wipe one part of the plate cleaner than the rest. Such devices give a print its peculiar personal quality. There is nothing mechanical-looking about a good etching. The lines can be even more sensitive and expressive than in a fine pen-and-ink drawing. Each individual print from the same plate may wel. give the effect of having received the artist's special care and attention. It would almost look like his own direct handwork but for the " plate mark."

Soft-ground etching is a special form, by which the lines, when printed, resemble those made by a lead pencil or crayon. The usual wax etching-ground is mixed with about half its weight in tallow or some other fat. Thin paper is then stretched tightly over the surface of the plate, and upon this the drawing is made with a pencil. Wherever the pencil-point travels, the back of the paper sticks to the ground, so that when the paper is removed, that part of the ground comes away with it, leaving lines exposed on the copper plate with a grain corresponding to the grain of the paper. The rest of the process is the same as for ordinary etching.

Drypoint and Aquatint

An allied art, which is really less etching than engraving, is " drypoint." In this the artist uses his needle directly on to the plate, scratching his lines into the surface. This raises a " burr " on either side of the incised line, which gives an attractive " fuzzy " quality to the print. The drypoint method is often effectively combined with that of ordinary etching, but fewer effective prints can be taken from drypoint plates because the burr naturally becomes flattened very soon.

Aquatint is a process of etching in which the prints resemble water-colour drawings. A resinous ground is used. On exposure to the acid, this coagulates into microscopic blobs. The blobs are impervious to the acid, which eats into the plate in all the spaces between and around them. The effect of this in the print is a level mass of grey. By stopping out portions of the plate as required and exposing other portions for successively further periods, a wide range of light and dark tones can be obtained in the prints.

Etching was probably invented early in the 15th century. Great painters like Van Dyck and Rembrandt were practitioners of the art ; indeed Rembrandt remains the outstanding figure in its history, his etchings being often rated as highly as his paintings as great works of art and the expressions of a great mind and personality. It was such men as these who discovered the chief appeal of etching, which is

the wide range it offers between extreme delicacy and extreme force, and to dramatic contrast between the two.

Another dominating master of the medium was the Spanish painter Goya, and the later decades of the 19th century saw its revival in England, due in great measure to the work of Whistler, Sir Francis Seymour Haden (1818–1903), from whom Whistler learnt the art, and Alphonse Legros (1837–1911). Since then the tradition has been maintained by many brilliant etchers, including Sir Muirhead Bone (1876–1953), Sir Frank Brangwyn (1867–1956), and Sir David Young Cameron (1865–1945).

LESSON 21

Sculpture since the Renaissance

Although many references have been made to sculpture in earlier Lessons of this Course, dealing with, for example, the art of the ancient Egyptians, Greeks, and Romans, the medieval Masons, and the Italian Renaissance, the general aspect of the art has not yet been considered.

It is, of course, expression by means of carved or moulded form, differing from pictorial art essentially in that it introduces a third dimension. Not always is sculpture carved or moulded "in the round," i.e. fully three-dimensional so that you can walk round it and examine it from every angle. It may be "in relief," i.e. so carved or moulded that the form stands out from a flat surface, either boldly (high relief) or slightly (low relief). Yet even in the flattest relief the three-dimensional element is still present ; there is all the difference between the Queen's image on a coin and the same head on a postage-stamp.

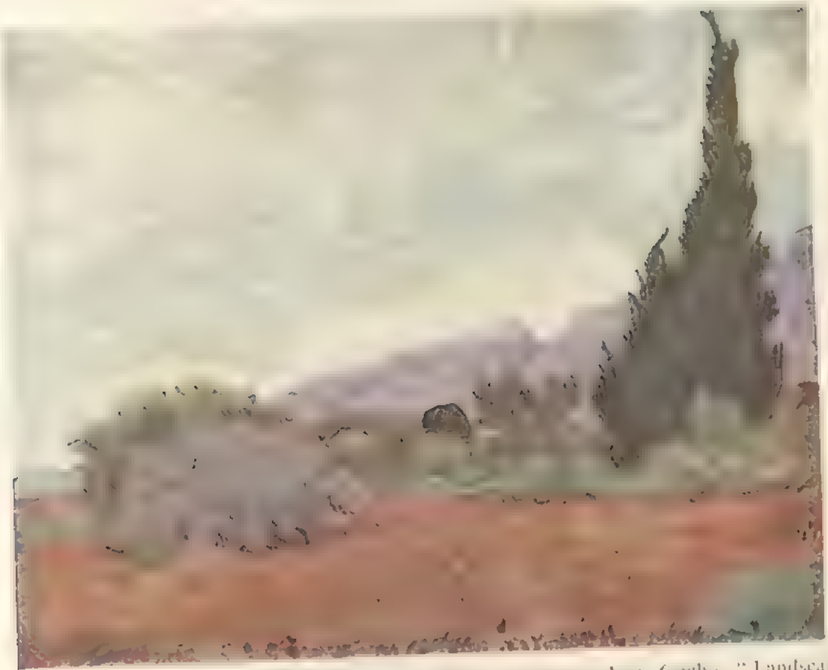
The Chisel and the Clay

The alternative, "carved or moulded," marks the difference between *glyptic* sculpture and *plastic* sculpture. The former is produced by chiselling down from a block of solid material until the desired shape is achieved. The material most commonly used is marble or stone (e.g. granite), though wood and ivory have not been overlooked by the sculptor. Marble was made famous by the Greeks. The Egyptians favoured granite, basalt, sandstones, and limestones. When wood was used, it was usually covered with gesso and coloured. Indeed, all sculpture was coloured until about 350 years ago.

Plastic sculpture is the result of adding piece to piece of some plastic material, e.g. clay or wax, until the design is complete. Sculpture produced by this method usually requires to be cast in some more durable material, so that clay or wax becomes plaster, bronze, or concrete. The process of casting from a mould of the original work has not varied through the centuries. Bronze is probably the most satisfactory alloy for such sculpture, by reason of its ductile quality, its strength, and its colour. With age, bronze acquires an appealing "patina"—and this is sometimes also acquired artificially by chemical treatment, rather in the way that new paintings were at one time treated with dark varnish to make them resemble "old masters."

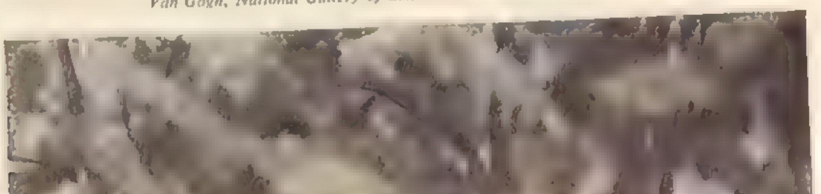
Broadly speaking, carved sculpture tends to be fuller, glyptic or modelled work to be thinner. Obviously the plastic method is the more suitable for the subtleties of naturalistic representation. Direct carving demands greater

SENTIMENTALISED CLASSICISM AND DIGNIFIED REALISM are contrasted in (left) "Apollo and Daphne," by Bernini, the gifted sixteenth-century sculptor and architect whose work displays "all the intellectual vices of his time," and (right) "George Washington," by Houdon, the eighteenth-century precursor of Rodin.



POST-IMPRESSIONISTS. The upper picture, Van Gogh's "Landscape with Cypress Trees," exemplifies his pre-occupation with pattern and clear-cut beauty of colour. The Cezanne landscape below shows the artist's subtle power of colour modulation.

Van Gogh, National Gallery of British Art



PAUL NASH. The modernist work of Paul Nash (1889–1946) is the antithesis of the Impressionists both in the striking patterns which he seeks and in his powerfully structural methods. This painting "Battle of Britain" clearly displays these characteristics.

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accuracy and clarity of concept—and greater labour. There are no other ways of producing sculpture, except by a combination of both methods, as when cast concrete finished by chiselling.

Michelangelo carved directly in marble, usually without the aid of a preliminary model beyond a small sketch in wax or clay. He regarded plastic sculpture with the same disfavour that he regarded painting in oils. The greater the self-imposed discipline, the greater was his inspiration. He believed (to quote R. H. Wilenski, in his book *The Meaning of Modern Sculpture*) that "the finest artist has no concept which the marble alone does not contain within itself" and that sculpture is essentially "collaboration between the sculptor and the block of resistant substance beneath his hand."

TRUTH AND FALSEHOOD, an allegorical group designed for the monument to the Duke of Wellington in St. Paul's Cathedral: an example of the work of Alfred Stevens.

Sculpture, the most durable of the arts, can be of all scales, from the colossal to the diminutive, from the huge statues cut by the Egyptians out of solid rock to the miniature jade carvings of the Chinese. Nine-tenths of all the sculpture ever made deals with the human form and draperies, portraying gods, heroes, personified virtues, and famous men and women. Animals are also popular subjects, whether they be the noble horses on Greek reliefs, the strange beasts copied from the medieval bestiaries, or Landseer's lions in Trafalgar Square. Recent years, as will be told in the next and final Lesson, have seen exercises in the carving and modelling of forms that are purely abstract.

Decline of Sculpture

From the time of Michelangelo, sculpture languished throughout Europe for the greater part of two centuries, only galvanised from torpidity by flamboyant design and the "movement" which characterises decorative art of the Rococo period. This decadence may be broadly described as the aftermath of the revival of interest in everything Greek by scholars and pedants—purveyors of the dead letter of the Renaissance as opposed to its living spirit of scientific inquiry and adventure. It was due in some measure to the antique-dealers who arose in the Renaissance era and flourished thereafter on the sales of spurious and restored Greco-Roman statues, to the propaganda of such dealers and collectors, and to the popular taste created thereby for everything pseudo-classical, which came to be recognized as the standard of beauty irrespective of the sculptural merit of any particular piece.

Bernini

The work of that elegant craftsman Lorenzo Bernini (1598–1680) affords a striking example of the decadence. Successful in his day as sculptor, painter, and architect (in architecture his most notable achievement is the colonnade of St. Peter's, Rome), on contemporary sculpture he exercised a baleful influence with his pseudo-classicalism, his extravagance of design, and his emotional theatricality. Some of his work, after the type of "Apollo Pursuing Daphne," possesses a pretty grace which, when dwindled to the proportions of chimney-piece ornaments, could serve as an inspiration of mythological figure groups in the chinaware factories. His portrait busts of royal and noble personages show his delight in the long hair or huge wigs and in the intricate patterns of the lace collars in vogue at the time.

Houdon: the Neo-Classicists

Jean Antoine Houdon (1741–1828), French sculptor, working in the latter half of the 18th and early 19th centuries, forestalled Rodin in his interest in emotional character, realistic detail, and expression. He certainly put in the blemishes. His portrait bust of Gluck, the composer, shows on the animated face traces of smallpox. Ideal groups were sculptured by him, but it is as a portrait sculptor that he is famous; busts and statues include J. J. Rousseau, Voltaire, Mirabeau, Lafayette, Franklin, Washington, and Napoleon.

Contemporary with Houdon were the Italian sculptor Antonio Canova (1757–1822), who produced a vast amount of highly finished work on academic pseudo-classical lines, and John Flaxman (1755–1826) in England. Flaxman was emptily academic in style. His claim to admiration lies in his designs for engravings to illustrate Dante and Homer rather than in his sculpture.

Bertel Thorwaldsen (1770–1844) was a Danish sculptor who worked in Rome for forty years and executed many allegorical and "pagan" statues. He followed Canova's example of smooth grace and was equally hidebound by "Greek" convention as understood in his day. The principal collection of his works is at Copenhagen. On the other hand, his pupil-assistant Alfred Stevens (1817–75) possessed in common with other great artists—Poussin, David, Ingres, and Turner, for instance—the faculty of abstracting knowledge for his own purpose from his profound study of the work of Michelangelo and other masters. Stevens returned to England in 1842 the most thoroughly educated artist of the 19th century. His was the inherently classical temperament—using the word classical in its meaning of austerity grand and dignified, not of Greco-Roman. A representative collection of his work is in the Tate Gallery, and plaster models for the Wellington monument in St. Paul's Cathedral, and of the fireplace for the former Dorchester House are at the Victoria and Albert Museum.

THE THINKER, by Rodin: the heroic and enigmatic figure designed to brood above the sculptor's "Gate of Hell," the bronze door commissioned for the Musée des Arts Décoratifs, Paris, which was left unfinished at his death.

Auguste Rodin

The greatest sculptor of the 19th century was Auguste Rodin (1840–1917). Seeking and experiment are the marks of an original artist, but they are not commonly welcomed by the public. Rodin's contemporary was the French painter Manet, and the two had much in common artistically. Both were essentially Romantics; both have been loosely ranked with the Impressionists, the one in paint, the other in sculpture. As with Manet, Rodin's first recognition by the critics was effected by shock. In 1864 he exhibited the statue of a "Man with a broken nose." By 1877 he had "arrived." His "Age of Bronze" in the Salon was acquired by the French government and placed in the Luxembourg Gardens. Between 1882 and 1885 he sent to the Salon busts of Victor Hugo and of the sculptors Dalou and Carrier-Belleuse.

A bronze replica of his "Burghers of Calais"—which outraged all the conventions of his day—was set up in the Victoria Tower Gardens, London, while his magnificent gift of representative sculptures to the British nation during the First World War is at the Victoria and Albert Museum, where the nature of his contribution to sculpture can provide an absorbing study. His famous "Thinker," his "Eve," which excited quite as much protest and initial disgust as any statue by Epstein, and his greatest work, "The Gate of Hell," illustrating Dante's *Inferno*, and containing 186 figures—which took up almost all his lifetime from 1880, and was left unfinished—all these show his dramatic expression and power of conveying emotion by gesture and pose. His habit of leaving a figure emerging from an unfinished block is intentional and characteristic of his Romantic tendency, not only to force contrasts but also to focus the spectator's attention on the particular point of interest which the sculptor wishes to stress by completing that portion and leaving all else in the rough.

Rodin never carved; clay modelling was the only possible medium for his work. The varied surfaces when cast in bronze aided the impressions he desired to create.

LESSON 22

Universal Forms in Sculpture

To label the art of our own period the end of the Romantic-Impressionist movement and the return to Classicism (using the last word in its austere meaning) is partial truth. Form is certainly set above feeling, structure before impression; there is appeal to the intelligence rather than to sentiment, reliance on mathematical axiom and fundamental shapes rather than on association of ideas; but it is here that we branch away from the Classic revivals of the past.

The French artist-architect Le Corbusier has

said: "Art is no longer anecdotal, it is a source of meditation; after the day's work it is good to meditate." Painters and sculptors have been experimenting and concentrating on patterns and shapes of abstract beauty, or of universal character, rather than on literal representation, on individual or story interest; where their work possesses such interest, it is subordinated to the formal design. Everywhere art is simplified, liberated of excess and sham. There has been a return to the elemental in the endeavour to state the problem afresh.

Experiments continue. An immense clearing and productive work has been accomplished. There is harmony between the new architecture, sculpture, and painting, and between these great arts and all their decorative dependants—pottery, metal, glass, furniture, mural design, and textile design.

Contemporary original sculptors have endeavoured to return to the starting-point and re-educate themselves. They began with the idea of essential sculpture as an activity consisting in the fashioning of form; a work which has sculptural meaning, i.e. the meaning of its form, need have no anecdotal or sentimental meaning.

Ruskin, in *The Seven Lamps of Architecture*, writes:

> I have said that all art is abstract in its beginnings; that is to say, it expresses only a small number of the qualities of the thing represented. Curved and complex lines are represented by straight and simple ones; interior markings of form are few and much is symbolical and conventional.

Some 20th-century sculptors have advanced towards greater completion of their work; others have continued of their own free will along the line of what Ruskin terms "noble abstraction," gathering out of objects "those arrangements of form which shall be pleasing to the eye in their intended places"; or they have experimented with the essential characters of symbolic life.

Ruskin defined sculpture as "the reduction of any shapeless mass of solid matter into an intended shape," and he displayed a crystal sphere as the essential type of sculptured form in the round (i.e. not relief carving); thus—as R. H. Wilenski points out—foreshadowing the modern sculptors' initial creed and their initial concept of form:

> They thought of themselves as architects of sculptures in the round, and their first concern was to discover the simplest type of three-dimensional meaning. That type they found in the sphere, the cube, and the cylinder; and they sought to fashion statues which would be apprehended in the way that the sphere, the cube, and the cylinder are apprehended . . . And in the early stages they rigorously restricted their studies to this field

Greek philosophy has been placed above Greek art. According to Plato's *Philebus*, Socrates affirmed that geometric forms are not only relatively beautiful, "but they are eternally and absolutely beautiful," while that which is commonly termed art is mere guesswork *plus* skilful craftsmanship.

The sculptors Brancusi and Gaudier-Brzeska (1891–1915) made pioneer experiments in geometrisation early in the 20th century. The former, in accordance with the Socratic idea of beauty, made sculptures which have meaning only as universal or decorative shapes; the latter resolved figures into geometrical forms and also formalised natural structures. Gaudier-Brzeska was killed in the First World War. Three small pieces of his sculpture are at the Victoria and Albert Museum: "The Fallen Workman," a bronze cast from an early study; a marble torso of a girl; and a statuette in plaster, "The Dancer."

Revival of Renaissance Curiosity

With regard to formalisation of natural structures, the modern sculptor shares with the sculptors of the Renaissance the scientific spirit of inquiry. Just as the latter were deeply interested in anatomy and death-masks for the purpose of realism, so the former are interested in results of photo-micrography. They may go back to nature, but from a new and scientific viewpoint not the guesswork which Socrates condemned which has revealed the almost unbelievable perfection of detail in the geometrical construction of organisms. Fossilised skeletons of minute sea-plants diatoms, for instance—when magnified hundreds of times

GEOMETRICAL FORMS IN NATURE are illustrated by (left) the opened seed capsule of the nettle and (right) the young fruit of the common rue with calyx. These magnified examples from nature dispel the conception of plant life as a mass of haphazard forms. Like minute animal organisms, they reveal shapes eloquent of "intentional" order and logical development, from which the artist may choose basic forms to be manipulated for his own constructive purposes.

Courtesy of Messrs. A. Zwemmer

PAX, by Frank Dobson, whose calm and dignified sculpture combines a purity of line and a grace of treatment with the building up of massive and monumental form full of vigour in repose.

show octagonal, hexagonal, and pentagonal varieties in absolute symmetry. The spiral curve, which is the plan on which the ammonite shell was developed millions of years ago, has been used throughout the history of art and again appears in modern geometric sculptured compositions.

Art Forms in Nature

In two volumes of series of remarkable photographs, entitled *Art Forms in Nature*, Professor Karl Blossfeldt shows that an almost inexhaustible variety of lovely geometrical designs exists in plant organisms. When these are magnified, all idea of their haphazard growth is dispelled. In his foreword to the second series Professor Blossfeldt says :

> Every sound expansion in the nature of art needs stimulation. New strength and stimulus for its healthy development can be derived only from nature . . . The plant may be described as an architectural structure, shaped and designed ornamentally and objectively. Compelled in its fight for existence to build in a purposeful manner, it . . . combines practicability and expediency in the highest form of art. Not only then in the world of art, but equally in the realm of science, nature is our best teacher.

By such studies sculptors have arrived at what Mr. Wilenski calls " the concept of the universal analogy of form, the concept of all human, and animal and vegetable forms as different manifestations of common principles of architecture, of which the geometric forms in their infinity of relations are all symbols " ; thus modern sculpture may be seen as an effort towards truth to life, not merely truth to nature in the old individualistic sense. The next generation of sculptors may very probably achieve balance by proceeding along more individualistic lines, but they will have learnt much from these experiments, just as all painters have learnt much from Cézanne (*see* Lesson 19).

LA MÉDITERRANÉE, by Maillol, of whose work the English sculptor Dobson has said that it has " the rounded beauty of the Downs." This example is in the courtyard of the Hôtel de Ville, Perpignan.

MAN WITH BIRD, by Maurice Lambert, a stylised but realistic work ; the stone is alive with nervous and muscular tension about to be resolved.

Greek Echoes

Although Greek statues are viewed by them without sentimental bias, various contemporary sculptors have taken a backward glance at Greek art without becoming enslaved in past academic tradition. Aristide Maillol (1861–1944), who, with Sir Jacob Epstein, may be termed a traditional master between Rodin and the contemporary sculptors, went in 1909 to Olympia, there to study the Greek temple fragments, and afterwards to Athens and to Naples. A pupil of Rodin, Maillol reacted against the emotive quality of the great Romantic, but did not become a convert to the antique ; his work is modern in his creation of generic and not

NIGHT, by Epstein, is one of his pair of symbolic sculptures on the Underground Building, Westminster, London. Like others of this master's stone carvings, these figures at first aroused hostility among sections of the public.

particular forms. The most famous of his sculptures are the monument of Cézanne and the war memorial at Elne ; and various important works are in collections in Germany. His French followers are many, and in England Frank Dobson (b. 1888) has developed Maillol's work along somewhat academic lines. One of Dobson's best works, the bronze figure of "Truth" (1930) was acquired for the nation.

A GRECIAN ECHO in the sculpture of the many-sided, modern master Pablo Picasso.

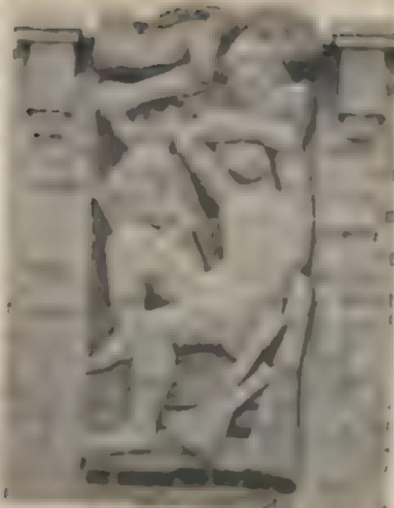
Zadkine and Archipenko, who are not French but exhibited in Paris before the Second World War, are other sculptors whose work shows traces of Greek affinity as differentiated from Asiatic. They find formal meaning in some of the archaic Greek statues in the round, though the essential cubic or cylindrical shapes of these were borrowed from the Egyptian academic tradition, which was already centuries old when the Greek sculptors used its formula.

Pablo Picasso has also made many interesting experiments in sculpture He too seems to have looked with his eager and critical gaze at the Greek fragments, as well as at all other plastic forms of the past—to have constructed astounding models for himself from wood and from wire in order to wrench from these unusual materials new patterns and forms Sculptors of less ability have copied his methods, but with very limited success.

Epstein and Other Masters

Violent controversy has raged round the sculpture of Sir Jacob Epstein (b. 1880) since he first gained notoriety in 1907 by his figures for the buildings of the British Medical Association in London. (When these buildings were reconstructed for the Rhodesian government, the figures were either broken up or left in a sadly depleted state.) That Epstein's work is the expression of genius of a very high productive order is now generally conceded. His clay-modelled busts cast in bronze have found many admirers, because they are his own development of the Romantic style of Rodin and, farther back, of Donatello ; therefore they have shocked less than his direct carvings in stone, such as "Rima," "Day" and "Night," "Genesis," "Ecce Homo," "Consummatum

FAMILY GROUP, by Henry Moore. This commissioned work, set in a large open space in the centre of Harlow New Town, Essex, symbolises the new humanitarian civilization of which the whole town itself is an expression. The group includes the artist's first stone carving of a male figure.



Shakespeare Memorial Theatre

us realise that all forms in Natu

BOOK LIST

Introduction to Italian Painting.

Flemish Art.

Dutch Art. R. H. Wilenski

Johnson (Bell).

(Harper). The Mea…

R. H. Wilenski (Faber).

ACCOUNTANCY

[illegible] punched card accounting are dealt with.

[illegible]

22 LESSONS

LESSON 1

Basic Principles of Double-entry Book-keeping

A reliable and accurate system of accounting is essential to the modern business. The executive of to-day demands of the accountant more detailed information and more intelligent arrangement of figures than in the past, and it is the accountant's duty so to collect and arrange information about the operations and position of the business as to enable the management to make its plans for the future in the knowledge that nothing whatever has been overlooked.

The first and most important task is to grasp clearly the meaning of double-entry book-keeping. All business transactions involve exchange, which necessarily implies two parties.

If I part with anything having value, someone has received it; if I acquire anything of value, somebody has parted with it; if I receive money, somebody has parted with that money. A proper record of the transaction will show how all the parties have been affected.

Debit and Credit

"Debit" and "credit" correspond respectively to the conceptions of receiving and imparting. If I lend a sum of money to John Smith, I will *debit* John Smith in my books, because he has *received* the money. If W. Jones lends me a sum of money, I will *credit* W. Jones, because he has *parted* with the money.

Every transaction is recorded in what are called accounts, and under the double-entry system every transaction is recorded in two accounts. Each account is divided by a line running down the middle of the page; the left-hand side of the account is the debit side, the right-hand side is the credit side.

When I lend John Smith £100, I debit his account—I write £100 on the left-hand side of an account headed John Smith. When I borrow £100 from W. Jones, I credit his account —I write £100 on the credit side of an account headed W. Jones.

Absolutely Unvarying Rule

Every transaction must be recorded in two accounts. One of the recording entries must be on the debit side, the other on the credit side. This is an absolutely unvarying rule, to which there are no exceptions. For every amount that appears on the debit side of an account, a similar amount appears on the credit side of another account. Thus every transaction has a twofold aspect. For every debit there is an equal and opposite credit, and vice versa. The debit entry is the record of the receiving. The credit entry is the record of the imparting.

When I lent £100 to John Smith, I debited his account, because he received the money. In so far as transactions affect other individuals, the course is clear. In so far as the transactions affect the proprietor of the business in whose books the records are being made, he does not debit himself, but debits an account which indicates the *nature* of the benefit which he receives. When the business parts with a benefit, the credit is made to an account which indicates the nature of the benefit imparted.

It may be convenient for the beginner to regard these accounts as the accounts of imaginary persons inside the business. When £100 was paid to John Smith, the business parted with cash. The credit entry will be made in an account called the cash account, which is the account of the cashier

If £20 is spent on the purchase of goods, the procedure is to credit the cash account, because we have parted with cash, and to debit a goods account, thus indicating the nature of the benefit received. This goods account can also be regarded as the account of the storekeeper whose duty it is to look after the goods. We look upon him as having received something worth £20, and we debit him with the amount accordingly.

No Credit Given or Received

Now to consider the transactions of a man who conducts all his business on a cash basis, no credit being given or received. Every transaction will involve an entry in the cash account. The cash account is debited with all sums received, and credited with all payments made.

For every payment an entry must be made on the credit side of the cash account. The debit entry will in every instance indicate the nature of the payment made. £2 is spent on a railway fare; the debit will be made in an account called travelling expenses. It would, of course, be possible to record the transaction in greater detail. The debit could be made to the account of the "person" who received the money, i.e. the railway company.

We should then, however, be compelled to credit the railway company with £2, since we have received *from* the railway company the benefit of travelling; and, to complete the double-entry, travelling expenses account must be debited.

The account of the railway company would show £2 on each side, and we should be left as we were before, with a credit to cash, and a debit to travelling expenses. No useful purpose, therefore, is to be served by the more roundabout method.

Further payments are made. £20 is paid out in wages ; £50 is spent in buying furniture ; £4 on rent ; £100 on machinery. In every instance cash account will be credited, since we have parted with cash. In every instance a debit will be made to an account which indicates the nature of the benefit received in exchange for the money. Wages account will be debited with £20 ; furniture account with £50 ; rent account with £4 ; machinery account with £100.

At this stage we are in a position to state the principle that all debit balances are either assets or losses. At first sight this appears contradictory. One is inclined to regard an asset as being opposed to a loss, rather than as being in any way similar. A little reflection on the transactions mentioned will make its truth apparent. If money is spent, it is spent for some purpose ; no one pays cash for nothing.

Benefit of a Transitory Nature

The benefit received in exchange for cash can be represented by a physical object having some degree of permanency, such as a table or a building. The debit on furniture or buildings account represents an asset. The benefit may be of a transitory nature, and not represented by any physical object. If I pay £3 for a railway ticket, I receive the benefit of travelling in a train. This benefit is exhausted when the journey terminates. No object of lasting utility remains. The debit on travelling expenses account is said to be a loss.

LESSON 2

How Accounts Are Classified

Every item of cash received will be debited to the cash account, and will be posted to the credit of an account which indicates the source from which it has come. The corollary of the axiom that all debit balances are either assets or losses is that all credit balances are either liabilities or gains.

Let us examine this statement in relation to the business which conducts all its transactions on a cash basis. If I receive cash, it is under one of two conditions ; either I am under an obligation to repay it, or I am not. There is no third choice. If I borrow £100 from W. Jones, I debit cash and credit W. Jones. This credit to W. Jones's account indicates that I received the money from Jones, and that I owe this amount to him. That is to say, the credit item on Jones's account is a liability.

If I perform a service for Robinson and he pays me £30 by way of commission, I will debit cash and credit an account headed commission. I am under no obligation to repay the £30 to Robinson ; it has become mine without any obligations attaching to it. The credit on commission account represents my gain, or profit.

Where Payment is Deferred

Now to consider transactions where payment is deferred, or, in ordinary language, on a credit basis. " Credit " in this sense is not to be confused with the special term used in accountancy in opposition to debit.

It is possible to acquire benefits without paying cash for them immediately. If I obtain the benefit of using a building and I pay for that benefit in cash, the double-entry necessary to record the transaction is a credit to cash and a debit to rent account. If, instead, I owe the money to my landlord, the double-entry is to debit rent as before, and to credit not cash, but the account of my landlord. The credit on my landlord's account is a liability, representing my obligation to pay him in the future.

Credit on Commission

Conversely, I may impart a benefit without receiving cash immediately. If I perform a service for Robinson for which he agrees to pay me a commission, the double-entry is a credit to commission account and a debit to Robinson's account. The credit on commission account is a profit, and the debit on Robinson's account is an asset representing my right to receive the money from Robinson at a later date.

Personal Accounts

Accounts can be classified into certain well-defined groups. First, there are personal accounts. These are the accounts of individuals or firms with whom business transactions have been conducted. Personal accounts are debited with all benefits received by the persons concerned. If we part with cash, and pay it to a person with whom business is being transacted, that person's account is debited, cash being credited.

If we sell goods to a person without receiving immediate payment, that person's account is debited and goods account credited. A debit balance on a personal account (that is, an

excess of debits over credits) is an asset representing our right to receive a certain sum of money at some time in the future.

Personal accounts are credited with the benefits imparted by those persons, and therefore received by us. If we buy goods without paying for them immediately, we credit the personal account and debit a goods account. If we receive cash, we debit cash and credit the person from whom we received that cash. Of course, where there is no obligation to return the money, as in the sale of goods for cash, the credit is made direct to the goods account.

But where there is an obligation to return the money, the personal account is credited; and where we receive cash in discharge of a debt due to us (a debt represented by a previous debit to a personal account) the personal account of the debtor is credited. The effect of this is to leave no balance on the personal account, the debit and credit sides being of equal amount, denoting that the debt is no longer due to us.

Real Accounts

Secondly, there are real accounts. Real accounts are accounts of things. If we buy £100 worth of furniture or spend £1,000 on machinery, cash will be credited, and furniture or machinery account will be debited correspondingly. It is impossible for a credit balance (an excess of credits over debits) to exist on a real account.

Nominal Accounts

Thirdly, there are nominal accounts. Nominal accounts are either profits or losses. A debit balance on a nominal account, such as rent, wages, or travelling expenses, is a loss or an expense. The existence of the debit balance does not necessarily imply that we have parted with cash.

It only signifies that an expense has been incurred. If payment has not been made, the credit goes to a personal account. When the cash is subsequently paid, cash is credited, and the debit then goes, not to the expense account, but to the personal account.

The two examples can be illustrated as follows.

1. Stationery worth £20 is purchased, and cash is paid immediately. The entries will be a credit to cash, and a debit to stationery, account, viz. :

Dr.	*Cash*	Cr.
		By stationery £20

	Stationery	
To cash £20		

2. Stationery worth £20 is purchased from A.B., Ltd., on January 1, but payment is not made until February 3. The entries will be a debit to stationery account, and a credit to A.B., Ltd., on January 1, followed on February 3 by a credit to cash and a debit to A.B., Ltd., viz. :

Dr.	*Stationery*	
Jan 1. To A.B., Ltd. £20		

Dr.	*A.B., Ltd.*	Cr.
Feb. 3 To cash £20		Jan. 1. By stationery £20

Dr.	*Cash*	Cr.
		Feb. 3. By A.B., Ltd. £20

The ultimate effect of the second method is the same as that of the first. We are left with a debit to stationery and a credit to cash. A.B.'s account is "closed," both sides being equal and there being no balance.

The point is that we could not make a direct entry from cash to stationery before February 3, whereas we incurred the expense on January 1. This fact must be recorded. If an inquirer were to examine the books on, say, January 21, he would expect to find a record of the state of affairs at that point of time. The state of affairs on January 21 is that an expense of £20 has been incurred, and that there is an undischarged liability to A.B., Ltd.

Where the payment of cash is coincident with the incurring of the expense, no personal account is necessary. An entry can be made direct from cash to the expense account. This is an everyday occurrence. Consider cash paid for travelling expenses, goods purchased for cash, the payment of wages, etc.

Similarly, a credit balance on a nominal account, which is a profit or a gain, does not necessarily imply that the money has been received, but only that it has been earned. If there is a lapse of time between the earning of the profit and its receipt in cash, then a personal account intervenes.

Reflection of Facts

To sum up. Double-entry book-keeping is not a mere mechanical device but a reflection of facts. When a profit is earned, that fact is recorded by a credit in a nominal account. It is also reflected by a debit to an asset account, thus increasing the assets of the business. The asset account can be real, i.e. cash, furniture, etc., or personal—a debt due.

In any event the profit, which is a credit balance, is represented by an increase of assets. This is common sense. If a man says: "I have made a profit of £100," one is entitled to ask: "Where is it?" The answer is that it is represented by assets, i.e. debits, whether in cash or in something else.

In the same way, if an expense or a loss is incurred, it is shown as a debit balance on a nominal account. But that debit balance

on the nominal account is reflected in a credit to cash, which diminishes assets, or else in a credit to a personal account, which increases liabilities. Thus the incurring of expenses is necessarily accompanied by a decrease of assets or an increase of liabilities.

Twofold Aspect of All Transactions

The basis of double-entry book-keeping is that every transaction has a twofold aspect, that of receiving and that of imparting. To present a complete picture of any transaction, both these aspects must be recorded. If I acquire assets, I must show how I have acquired them; if I have parted with cash in exchange, my cash account is credited; if I have not yet paid for them, I have incurred a liability and a personal account is credited. If I incur expenses in the conduct of my businesses, I either part with cash or increase my liabilities. Both aspects must be recorded; the appropriate expense account is debited, and cash account or a personal account is credited.

Every debit to a nominal account (expense) involves a credit to another account, which has the effect of decreasing the net assets. Conversely, every credit to a nominal account (profit or gain) involves a debit to another account, which has the effect of increasing the net assets. In other words, profits necessarily imply an increase in net assets, and losses or expenses imply a decrease in net assets.

Principle of the Trial Balance

In as much as every transaction requires two entries, one to the debit and one to the credit, it follows that the total of debit entries in any set of books must be equal to the total of credit entries. If, therefore, at any point of time a list of all debit entries is set down on paper, together with a list of all credit entries, the totals of the two lists must necessarily be equal. This is the principle of the trial balance.

It is not customary to draw up a trial balance showing total debits and total credits. It is sufficient if the balance of each account is extracted. For instance, John Smith's account may appear in the book as follows:

John Smith

	£		£
To goods	100	By cash	95
,, goods	250	,, discount	5

In a list of total debits and total credits, John Smith's account would appear as a debit of £350 and a credit of £100. The usual practice is to offset the credit against the debit items, and to show a debit balance only of £250. The two columns (debit and credit) of the trial balance must still be equal. All that has been done is simply to deduct £100 from each column.

It is most important to recognize that the trial balance is not an integral part of the double-entry system; the figures on the trial balance are not entries that require a complementary credit or debit. All that has been done is to set down on paper a list of the balances existing in the books at a given point of time. This list includes all the balances on all accounts, both assets and liabilities, and losses and gains—real, personal, and also nominal accounts.

Summary of Expenses and Gains

The next step is the preparation of the trading and profit and loss account. This account is a summary of all the balances on the nominal accounts, that is to say, of expenses and gains. If one were to take a trial balance, collect all the nominal accounts together, cancel out compensating debit and credit items, and thus leave only the net balance of all the nominal accounts, the two sides of the trial balance would still agree.

The principle of cancelling compensating debits and credits in each individual account has simply been carried a stage further. By cancelling out debit balances against credit balances on nominal accounts, equal amounts have been deducted from each side of the trial balance. We are left with real and personal account balances, together with the final balance of all nominal items. That is to say, we are left with a list of assets and liabilities, together with the net profit or loss.

This collection of all nominal account balances is called the profit and loss account. The list of asset and liability balances, together with the net balance on the profit and loss account, is called the balance sheet.

The balance sheet is a statement of the financial position of a business at a given point of time. It is a summary of possessions, or assets, on the one side, and a summary of obligations, or liabilities, on the other. The possessions are debit balances; the obligations are credit balances; and all are classified by the recognized methods.

Capital Account

It becomes necessary at this point to explain an account of a special character, viz. the capital account. This can be defined as the personal account of the proprietor. The business must be regarded as having a separate existence, distinct from the proprietor. If an outside individual lends a sum of money to a business, the double entry is a debit to cash account and a credit to the personal account of the lender. This credit balance represents the liability to repay the lender the sum he has advanced.

If the proprietor advances a sum of money to his own business, his personal account,

which is called the capital account, is credited. In everyday language, any sums put into a business by its proprietor are his capital. The debit indicates the nature of the asset the business receives; the credit indicates where the asset has come from—in this instance, from the proprietor himself.

Net Credit Balance

If business is being successfully carried on, the assets will exceed the liabilities to external persons. The excess of the assets over the liabilities is what the business owes to the proprietor; it is the measure of what he is actually worth.

To return to the profit and loss account. If the credit balances on all the nominal accounts exceed the debit balances, the net credit balance is the measure of the profit earned for the period under review. This is the net profit, as disclosed by the profit and loss account.

As pointed out, the credit items on the nominal accounts, which have been transferred to the profit and loss account, were reflected by debits to asset accounts of one kind or another; and the debit items on nominal accounts, which appear on the expenses side of the profit and loss account, were reflected by credit entries to asset or liability accounts.

Net assets have been increased to the extent of the credit items in the profit and loss, and have been diminished to the extent of the debit items in the profit and loss.

If no additional sums have been introduced by the proprietor during the trading period, and if he has withdrawn no sums for his personal use, a credit balance on profit and loss account must be accompanied by a corresponding increase in the assets, which, it will be remembered, are debit balances.

Balance Sheet

When the nominal accounts appearing in the trial balance are collected together to form the profit and loss account, we are left with asset and liability balances, including the proprietor's capital account. A list of these asset and liability balances, together with the net balance on nominal accounts (the net profit), must agree. This list is called the balance sheet.

If we compare the balance sheet at the end of a trading period with that at the beginning, and if a profit has been earned, and the proprietor has neither introduced nor withdrawn anything, we shall find that both sides of the balance sheet have been increased by an equal amount. Among the credit items we find the figure for net profit—the credit balance on the profit and loss account. Corresponding to, and representing this net profit, we find that the assets have increased by a similar amount.

LESSON 3

Double-entry in Practice

We have traced the steps by which, under double-entry book-keeping, the final results of all the transactions of a business appear in the profit and loss account and in the balance sheet. This process is made clear by an example. Consider the following set of facts.

Arthur Harris opens his business on January 1 with a capital of £1,000 in cash. During January the following transactions take place. Harris buys £100 worth of furniture, and pays in cash; buys £500 worth of goods on credit from W. Robinson; buys £400 worth of goods for cash; pays Robinson £200 on account; sells part of the goods to R. Jameson for £800, on credit; sells the remainder for cash, £400; pays wages £50, and sundry expenses £25 in cash.

The first entry in Harris's books will be to debit cash £1,000 and to credit his capital account. The ledger accounts will be as shown in the next column.

Cash

To—		By—	
Capital ..	£1,000	Furniture ..	£100
Goods	400	Goods	400
		W. Robinson	200
		Wages	50
		Sundry Expenses	25
		Balance *c/d*	625
	£1,400		£1,400
Balance *b/d* ..	625		

Goods

To—		By—	
W. Robinson	£500	R. Jameson ..	£800
Cash	400	Cash	400
Balance *c/d* ..	300		
	£1,200		£1,200
		Balance *b/d* ..	300

W. Robinson

To—		By—	
Cash	£200	Goods	£500
Balance *c/d*	300		
	£500		£500
		Balance *b/d* ..	300

Wages

To— Cash	£50		

Capital

		By— Cash	£1,000

Furniture

To— Cash	£100		

R. Jameson

To— Goods ..	£800		

Sundry Expenses

To— Cash	£25		

It will be noticed that those accounts containing more than one entry have been "balanced." Take the cash account, for instance; the debit entries exceed the credit entries by £625. By making a balancing entry of £625 on the credit side, the two sides of the account agree and can be ruled off.

The double-entry is preserved by "carrying down" the balance, i.e. making a debit entry of £625 below the ruled line; the effect is that the balance can be seen at a glance. The abbreviations c/d and b/d mean "carried down" and "brought down"; it is important to remember to "bring down" the balance before ruling off the account.

We are now in a position to extract a trial balance. The trial balance is simply a *list* of the balances appearing in the books; and it is no part of the double-entry.

Trial Balance

	Debit Balances	Credit Balances
Cash	£625 0 0	£
Capital		1,000 0 0
Goods		300 0 0
Furniture	100 0 0	
W. Robinson		300 0 0
R. Jameson	800 0 0	
Wages	50 0 0	
Sundry Expenses .	25 0 0	
	£1,600 0 0	£1,600 0 0

The balances can be classified as follows:

(1) real—cash and furniture.
(2) personal—capital; W. Robinson and R. Jameson.
(3) nominal—goods, wages, and sundry expenses.

The goods account in this form is not a real account. The goods, when purchased, were physical and tangible; at the end of the period they have gone—no physical object remains. In practice there is always some stock of goods on hand, and their value (at cost, or under) is a balance sheet item, like cash and furniture.

A second classification is as follows:

(A) debit balances: Assets (cash, furniture, and R. Jameson) and losses or expenses (wages and sundry expenses).
(B) credit balances: Liabilities (capital, W. Robinson) and gains or profits (goods).

The next step is to prepare the profit and loss account. All the nominal accounts must be collected together. Unlike the trial balance, the profit and loss account *is* an integral part of the double-entry.

Every entry in the profit and loss account has its equal and opposite debit or credit. In order that this principle should be clear, the balances as they appear in the books are again reproduced.

Cash

To— Balance ..	£625		

Goods

To— Profit and Loss a/c ..	£300	By— Balance	£300

W. Robinson

		By— Balance	£300

Wages

To— Cash	£50	By— Profit and Loss a/c	£50

Capital

		By— Cash	£1,000
		Net Profit ..	225

Furniture

To— Cash	£100		

R. Jameson

To— Goods ..	£800		

Sundry Expenses

To— Cash	£25	By— Profit and Loss a/c	£25

Profit and Loss Account

To— Wages	£50 0 0	By— Goods ..	£300 0 0
Sundry expenses ..	25 0 0		
Balance, being net profit transferred to capital account	225 0 0		
	£300 0 0		£300 0 0

In the trial balance it was seen that there was a credit balance on goods account of £300. This has been "transferred" to the profit and loss account, by debiting the goods account and crediting the profit and loss account with £300. The goods account is now said to be "closed"; the debit side equals the credit side; there is no balance either way. This account has now served its purpose, and its place is taken by the entry in the profit and loss account.

Similarly, wages account and sundry expenses account have been "closed off" to profit and loss account. They have fulfilled their function and are no longer required; their places are taken by the entries on the debit side of the profit and loss account.

The profit and loss account presents a picture of the operations of the period ; a net profit of £225 has been earned. Harris sold the goods for £300 more than he paid for them ; out of this he had to pay £75 in expenses, leaving a net profit of £225.

If the books are examined *after* the nominal accounts have been closed off to profit and loss account, it will be found that the following balances remain.

	Debit Balances	Credit Balances
Cash	£625 0 0	£
Capital		1,000 0 0
Profit and Loss		225 0 0
Furniture	100 0 0	
W. Robinson		300 0 0
R. Jameson	800 0 0	
	£1,525 0 0	£1,525 0 0

It will be noticed that with the exception of the capital account and the profit and loss account, only assets and liabilities remain.

When Harris opened his business on January 1, his assets totalled £1,000 (all cash) and he had no liabilities. If the list of balances remaining on January 31 is examined we find that his assets are :

Cash	£625
Furniture	100
R. Jameson	800
	£1,525

He has also a liability of £300 to Robinson. His net assets are therefore £1,225. If he were to pay W. Robinson, he would be left with assets totalling £1,225. His net assets have increased from £1,000 to £1,225 during the month, i.e. by £225 ; and £225 is the amount of the net profit.

The meaning of the statement that double-entry book-keeping is not a mere mechanical device but a reflection of an actual state of affairs now becomes clear. A profit is not something abstract. A profit is represented by something really solid—either money or money's worth.

The nominal accounts summarised in the profit and loss account show how the profit has been earned ; the asset accounts, minus the liability accounts, show *where* the profit is.

The remaining balances in the trial balance are normally summarised in a document called the *balance sheet*. This can be defined as a classified summary of the balances remaining open in a set of books after the collection of the nominal balances have been summarised into one account, and including the balance of that account, so arranged as to show the assets and debit balances upon the right-hand side, liabilities and credit balances upon the left.

That the items appear on the reverse sides must be accepted as an historical survival, and should not be allowed to obscure the true nature of the balance sheet.

The balance sheet of Arthur Harris on January 31 will be as follows :

Balance Sheet

(Credit Balances)		*(Debit Balances)*	
Capital (including net profit) ..	£ 1,225 0 0	Cash ..	£625 0 0
Creditors (W. Robinson..	300 0 0	Debtors (R. Jameson) .	800 0 0
		Furniture ..	100 0 0
	£1,525 0 0		1,525 0 0

The balance of the profit and loss account (the net profit) is transferred to the proprietor's capital account. The reason for this is that the business is deemed to be accountable to the proprietor for profits, just as much as for the cash he originally invested in the business.

In everyday terms—the capital account is the measure of what the business is worth. When Harris started with a capital in cash of £1,000, he was worth £1,000. During the month of January he has increased his net worth by £225, by earning a profit of that amount. His net worth is always the amount by which his assets exceed his liabilities, and a profit in the business is always accompanied by an increase in net assets.

The balance sheet is a statement of the " real " position at a point of time, while the profit and loss account is a record of transactions *over a period of time.* Each, in a sense, confirms the other. The profit and loss shows what has been happening ; the balance sheet shows the *result* of these events.

LESSON 4

Principal Books of First Entry

We have sketched the theoretical working of the double-entry system and examined the final results, as shown by the profit and loss account and the balance sheet. The next three Lessons deal with the working of the system in practice and, in particular, the purpose of the various books.

At the outset, an important distinction has to be made between books of original or prime entry, as they are called, and the financial books. Strictly speaking, only the financial books are an integral part of the double-entry system. Every transaction has its place in the financial books ; all the debits and credits

and all the ledger accounts that have been given by way of illustration are of the kind entered in the financial books.

The financial books are divisions or parts of the theoretical ledger. The ledger is, in theory, the book containing all debits and credits and all the accounts, personal, nominal, and real. In practice, transactions are too numerous to be recorded conveniently in one book. Such a book would be excessively bulky. The ledger is therefore split up, for the sake of convenience, into a number of ledgers ; these ledgers, combined, constitute the essential double-entry system.

Divisions of the Ledger

The usual divisions of the ledger are as follows. 1. The bought (or purchases) ledger, which contains all the personal accounts of suppliers. 2. The sales (or sold) ledger, which contains all the personal accounts of customers or debtors. Personal accounts only are found in the bought and sales ledgers. 3. The private or general ledger, which contains real and nominal accounts and the proprietor's capital account. 4. The cash book, which contains the cash account, the bank account, and details of the discount account.

In most businesses daily routine transactions are very numerous. It would be difficult for the book-keepers to make a double-entry in the ledger every moment a transaction took place. Many people might require the book at the same time.

Journal, or Day Book

The books of original entry, or books of first entry, are designed to meet this practical difficulty. Just as in earlier days one ledger was sufficient to contain all the accounts, etc., at one time, so one book of first entry was sufficient to record all transactions. This book was called the *journal*, or day book.

The journal was a kind of notebook, in which all transactions were jotted down in chronological order and at the time they occurred. At the end of the day, or at some time when he had leisure, the book-keeper would enter the transactions in the ledger, making a double-entry for every transaction.

Methods of Entry

The entry in the journal was not a part of the double-entry. Two entries were made in the ledger in respect of every transaction, in addition to the record in the journal. At accounting periods the trial balance was extracted in its entirety from the ledger, without reference to the journal. The latter was simply used as a matter of convenience. In the rush of business it would not always be possible for the book-keeper to find time to make the full ledger entries.

In the busy hours of the day the book-keeper made notes in the journal to serve as a reminder that certain transactions had occurred and that he must, at a more appropriate time, record these transactions in the ledger. To avoid duplications, it became the rule that *all* transactions must be entered in the journal before they were entered in the ledger. This is still the rule in some countries.

Divisions of the Journal

In modern practice, the journal, like the ledger, is divided into specialised books. There is one exception to the rule that *all* transactions must first be recorded in a book of first entry : cash transactions are entered direct in the cash book, which is itself a part of the ledger.

All cash received is entered on the left-hand or debit side of the cash book ; the corresponding credits are made to the appropriate accounts in the other divisions of the ledger. Similarly, cash payments are entered on the right-hand (or credit) side of the cash book, the corresponding debits being made to the appropriate ledger accounts. No book of first entry is used in respect of cash items.

For convenience, the cash book is often ruled so as to combine the cash account, the bank account, and the discount account, that is, with three money columns on each side, thus :

1957		L.F.	Discount	Cash	Bank	1957		L.F.	Discount	Cash	Bank
	To—						By—				
Jan. 1	Balance	b/f		46 0 0	573 0 0	Jan. 2	R. Jones	BL 17	1 15 0		68 5 0
4	S. Smith	SL 64	1 0 0	39 0 0		5	W. Richards	BL 103	3 0 0		97 0 0
7	T. Taylor	SL 37	2 10 0	97 10 0		7	Bank	—		136 10 0	
7	Cash	—			136 10 0	7	Wages	GL 49		15 0 0	
	Discount % Dr.	GL 19	3 10 0				Discount % Cr.	GL 19	4 15 0		
							Balance	c/d		31 0 0	544 5 0
			£	182 10 0	709 10 0				£	182 10 0	709 10 0
Jan. 8	To Balance	b/d		31 0 0	544 5 0						

The entries mean that on Jan. 1 there was £46 in the office cash and a balance of £573 at the bank. On Jan. 4, S. Smith, whose account is on sales ledger folio 64, paid £39 in satisfaction of an account for £40. On Jan. 7, T. Taylor, S.L. folio 37, paid £97 10s., and was allowed £2 10s. discount. On Jan. 7, the £136 10s. that had been received was paid into the bank. On Jan. 2, R. Jones, whose account is on bought ledger folio 17, was sent a cheque for £68 5s. in payment of his account for £70. On Jan. 5, W. Richards, B.L. folio 103, was sent a cheque for £97 in payment of his account for £100. On Jan. 7, £136 10s. was paid into the bank. On Jan. 7, wages were paid out of the office cash.

Discount, Cash, and Bank

This cash book consists of three ledger accounts telescoped into the same section: discount, cash, and bank. No other cash or bank account will normally be kept in the ledger.

The discount columns give the details that are transferred in total either to a combined discount account or to two separate accounts, discount allowed (a "loss" account), and discount received (a "profit" account). Note that the discount columns are not balanced. The discount columns are the details of a nominal account. The cash columns are the cash account, a real account. The bank columns are a personal account, that of the firm's bank.

The first entries of the transactions will be made in the cash book. The double entries will be completed by "posting" the cash book to the ledger. To post the debit side, credit S. Smith with £40 and credit T. Taylor with £100; debit the discount account with the discount allowed, £3 10s. To post the credit side, debit R. Jones with £70 and debit W. Richards with £100 and debit wages account with £15; credit the discount account with the discount received, £4 15s. The double-entry for the payment into the bank has already been completed.

In large businesses the whole cash book may be split into a debit side, cash receipts book, and a credit side, cash payments book. Various other arrangements of the cash book are used to meet the special needs of individual businesses; but they do not differ fundamentally from the three-column type exemplified.

The principal books of first entry are the purchases day book; the sales day book; and the journal proper.

Purchases Day Book

All purchases on credit are recorded in this book in chronological order. Purchases for cash are entered in the cash book, not in the purchases day book. Spaces are provided for the date of the transaction, for the name of the supplier, and for the money value of the goods. A narrow column is provided for a note of the page in the bought ledger containing the personal account of the supplier. Each individual item is "posted" to the credit of the appropriate personal account in the bought ledger. At the end of a definite period, usually a month, the items in the day book are added up and debited in total to the purchases account in the private ledger.

By debiting totals only to the purchases account the private ledger does not become overcrowded. The details appear in the day book. The totals, which give a compact view of a series of transactions, appear in the private ledger. It is very desirable to keep the private ledger as compact as possible. The double-entry is preserved. The total debit to purchases equals the sum of all credits in the bought ledger:

Purchases Day Book

		Credited to Bought Ledger Folio	Amount		
1957			£	s.	d.
Jan. 1	R. Jones	17	73	10	1
„ 15	W. Richards	103	165	0	0
„ 25	J. Wilkins	88	40	8	3
			£278	18	4
		Debited to Purchases a/c in private ledger			

Sales Day Book

The purpose of this book is analogous to that of the purchases day book. In this book all sales on credit are entered in chronological order. The individual amounts are debited to the appropriate personal accounts in the sales ledger, and the total, which is usually made up monthly, is credited to the sales or goods account in the private ledger.

Cash sales do not appear in the day book; the double-entry is made direct from the debit side of the cash book to the credit of goods account in the private ledger. It has already been pointed out that cash transactions are an exception to the rule that all transactions must first be recorded in a book of original entry. Other specialised books of first entry are dealt with later.

The journal proper is the survivor of the old journal in which at one time all transactions were recorded. Its use is now limited to transactions of a peculiar or unusual character, which are not sufficiently numerous to make a specialised day book necessary. Ordinary day-to-day routine transactions, such as the purchase and sale of goods, are recorded in the special day books just referred to. Receipts and payments of cash are entered direct in the cash book. The journal is thus strictly limited to entries that concern only two ledger accounts.

Closing entries, that is to say, the entries by which nominal account balances are transferred to the profit and loss account, are recorded in the journal. Other matters, such as provision for depreciation, bad debts, and outstanding expenses, are the subject of journal entries. By way of illustration, consider the following transaction.

I perform on March 1 a special service for Arthur Jackson, for which he agrees to pay me £20 commission on April 1. I must record this transaction on March 1 thus :

Journal

		Page	Debit Column	Credit Column
Mar. 1	Arthur Jackson Dr.	40	£20 0 0	
	To commission ..	78		£20 0 0
	being amount due from A. J. for services rendered			

I will then proceed to make a double entry in my private ledger. I will open an account for Jackson on page 40, which I will debit with £20 ; I will then credit commission account, which is on page 78, with £20. The credit balance on commission account is a profit, which will eventually be transferred to the profit and loss account.

Narration

The journal entry is nothing more than a memorandum that a double-entry is *to be* made, and indicating which account is to be debited, and which account is to be credited, and with how much. The entry of the figures 40 and 78 in the narrow column is an indication that the double-entry has been made in the appropriate ledger. It will also be noted that the journal entry is followed by an explanation of the nature of the transaction, which is called a "narration."

LESSON 5

The Trading Account

In Lesson 3 appears a specimen set of transactions, with trial balance, profit and loss account, and balance sheet. On one point—the goods account—a very simplified assumption was made. It was assumed that when goods were purchased, the goods account was debited, and that when goods were sold, the goods account was credited with the proceeds ; in this illustration *all* the goods were sold during the period under review.

In practice, the purchase and the sale of goods are dealt with in two separate accounts. Furthermore, only in very exceptional cases are all the goods that have been purchased in a given period sold within the same period ; nearly all traders find it necessary to keep a stock of goods continuously in hand. This being so, the next thing to be done is to trace the development of the goods account.

Suppose a trader buys goods for £1,000, and sells them all for £1,500, in both instances for cash. In its simple form the goods account would appear thus :

Goods

To—		By—	
Cash ..	£1,000	Cash ..	£1,500

The credit balance of £500 on the goods account represents the profit on the transaction.

Now suppose that the trader sells only half of the goods for £750 :

Goods

To—		By—	
Cash ..	£1,000	Cash ..	£750

There is a debit balance of £250 on the goods account. What does this balance represent ? It is a composite of two things. The trader has sold half the goods only, the other half remaining in his possession ; the unsold half of the goods constitutes an asset, value £500, which is half the total cost price. The half that has been sold for £750 also cost £500. Therefore there is a profit of £250 on the sale.

Assets are debit balances, and profits are credit balances. This debit balance of £250 on the goods account is made up of a debit balance of £500, being the value of the stock still on hand, and a credit balance of £250, being the profit on the sale of half the goods. The goods account can be balanced in this way :

Goods

To Cash ..	£1,000	By Cash ..	£750
Profit ..	250	Stock .. (balance) c/d	500
	£1,250		£1,250
To Balance (stock) b/d ..	£500	By Balance (profit) b/d ..	£250

Purchases and Sales

In practice, the goods account is split up in the following manner. When goods are purchased, the debit is made to a "purchases" account. When goods are sold, the credit is made to a separate account, called the "sales" account. At the end of an accounting period the total of the purchases account is transferred to the debit of a trading account, and the total of the sales account is transferred to the credit of this account.

The value of stock on hand at the end of the period is credited to the trading account, and debited to a stock account. Given the same figures as above, the ledger accounts would be :

Purchases

To Cash ..	£1,000	By Trading account ..	£1,000
	£,1000		£1,000

Sales

To Trading account ..	£750	By Cash ..	£750
	£750		£750

Trading Account

To Purchases ..	£1,000	By Sales ..	£750
Gross profit, carried to profit and loss account	250	Stock ..	500
	£1,250		£1,250

Stock Account

To Trading account ..	£500		

The trading account here shown is identical with the old goods account, with the exception that the stock is not brought down as a balance but is debited to a new account, which remains in the books and is shown in the balance sheet as an asset.

Summarised Transactions

The profit on the sale (which is called the *gross profit*) is transferred to the profit and loss account. Against this profit are debited the various loss or expense accounts. In other words, the profit and loss account is a continuation of the trading account. The distinction between trading account and profit and loss account is made for the sake of clarity.

It is desirable to be able to see at a glance what is the extent of the margin between cost price of goods and selling price, and then to compare this gross profit with the expenses, rather than to lump all nominal debits and credits indiscriminately in one account. The balance on profit and loss, which is the gross profit minus expenses, is called the net profit.

It might be asked, If the trading account is so similar to the goods account, why go to the trouble of opening separate purchases, and sales accounts ? The answer is that in the course of a year the entries in the purchases and sales accounts are fairly numerous. If the purchase and the sale of goods were recorded direct in the trading account, this account would become unwieldy. It is the purpose of the trading account to present a summarised bird's-eye view of the whole transactions for a given period.

Just as one of the purposes of the purchases day book is to avoid too great a multiplicity of detail in the purchases account, so the purpose of the purchases account is to avoid unnecessary details in the trading account. Every individual transaction is recorded in detail in the purchases day book. The trading account shows in a concise form the *final* effect of the transactions of a period.

It has been shown that the trading account is credited with the value of the closing stock, and that a new account is opened, to which the same figure is debited. This account appears in the balance sheet as an asset. This balance will remain in the ledger unchanged throughout the whole of the following accounting period.

As fresh goods are bought, the cost is debited, not to the stock account but to the purchases account ; as goods are sold, sales account is credited. What is to be done with the stock account ?

The procedure is this. At the end of the subsequent accounting period the balance on the stock account, which has remained in the ledger since the close of the preceding period, is transferred to the *debit* of the trading account. The value of the stock at the end of the subsequent period is found by a physical stocktaking, and this figure is credited to this new trading account, and debited to a new stock account.

The old stock account has been closed by transfer to the debit of the trading account. Consider the following illustration. A man commences business on Jan. 1. 1956. without any stock. During 1956 his purchases are £4,000, his sales £7,000, and his stock on hand at Dec. 31, 1956, is £1,000.

The ledger accounts and trading account for the year will appear as follows :

Purchases

To—		By—	
Sundries ..	£4,000	Trading account	£4,000

Sales

To—		By—	
Trading account ..	£7,000	Sundries ..	£7,000

Trading Account (Dec. 31, 1956)

To—		By—	
Purchases ..	£4,000	Sales ..	£7,000
Gross profit, to profit and loss	4,000	Stock Dec 31 ..	1,000
	£8,000		£8,000

Stock Account

1956 Dec. 31. To— Trading account ..	£1,000	1957 Dec. 31. By— Trading account ..	£1,000

The balance of £1,000 on the stock account on Dec. 31, 1956, appears as an asset in the balance sheet at that date. During 1957 the purchases are £7,000, sales £10,000, and the stock on Dec. 31, 1957, is £3,000. The stock on hand at the commencement of 1957 is transferred to the debit of the trading account and a new stock account is opened, being debited with £3,000, while the trading account is credited.

Trading Account (Dec. 31, 1957)

To—		By—	
Stock, Jan. 1	£1,000	Sales ..	£10,000
Purchases ..	7,000	Stock, Dec. 31 ..	3,000
Gross profit	5,000		
	£13,000		£13,000

Stock Account

1957	
Dec. 31. To— Trading account ..	£3,000

The purpose of the trading account is to compare the cost of the goods sold with the selling price of those goods. The stock in hand on Jan. 1 plus purchases during the year is the total of goods to be accounted for. But not all of these goods have been sold. If there is deducted from the sum of opening stock and purchases the cost price of the stock in hand at the close of the year, the difference is the cost price of the goods sold.

There is on the debit side of the trading account opening stock plus purchases, with closing stock on the credit side. An addition to the credit side is the same thing as a subtraction from the debit side. The trading account thus gives a true picture.

An alternative arrangement is the following:

Trading Account (Dec. 31, 1957)

To Stock *b/f* ..	£1,000	By Sales ..	£10,000
Add Purchases	7,000		
	8,000		
Less Stock *c/f*	3,000		
Cost of goods sold ..	5,000		
Gross profit ..	5,000		
	£10,000		£10,000

LESSON 6

Corrections for the Profit and Loss Account

In Lesson 5 it was shown that certain adjustments had to be made in respect of stock-in-trade and the trading account. This Lesson deals with some other elementary adjustments that normally have to be made at the end of every trading period.

For the profit and loss account to be an accurate record, corrections must be made for the following: outstanding expenses; expenses paid in advance; bad debts; depreciation. At the end of the accounting year, some expenses will usually have been incurred which will not be paid until the following period, and of which there is no record in the financial books. For instance, it may happen, in the example of a concern whose accounting year ends on December 31, that the rent for the three months October to December will not be paid until the following January.

If it is assumed that the purchases day book does not include any expenses (some day books contain analysis columns for certain of the more important expense items), then at December 31 there will be no entry in the books to record the fact that the rent is due. The rent account in the general ledger will have been debited only with the payments in respect of the first nine months of the year.

If this total only were transferred to the debit of the profit and loss account, the profit for the year would be overstated by the amount of three months' rent. The rent shown would not be the true rent for the year.

Assuming that the rent of the premises occupied by the business is £100 per annum, payable in four quarterly instalments of £25, on March 31, June 30, September 30, and December 31, in respect of the preceding three months in each instance, the rent account in the general ledger would appear on December 31, as shown at A in the Table on page 1436. It is assumed that the rent due on December 31, in respect of the last three months, had not been paid on that date.

In order that the true rent for the year (£100) may be transferred to the profit and loss account, another £25 must be debited to the rent account. The corresponding credit cannot, however, be made to cash, since no cash has been disbursed. This difficulty is overcome by debiting the rent account with the amount due and crediting the new rent account as shown in example B overleaf.

In this way the profit and loss account is debited with £100, the true expense for the year. The credit balance of £25 in the rent account is left as a balance in the books, and it must appear in the balance sheet, which

is a list of all balances remaining after the nominal accounts have been closed off to the profit and loss account. It will, of course, appear on the liabilities side. It is not shown in the balance sheet because there is nothing else to do with it. On the contrary, it *must* appear, if the balance sheet is to be a complete statement of assets and liabilities, since a very clear and definite liability to pay this £25 to the landlord exists on December 31.

Assuming the rent due on December 31 is paid during January, cash will be credited and the rent account will be debited. If all the subsequent payments in this second year are all made on the due dates (including the last payment, on December 31), there will be five debits of £25 each, totalling £125.

Just as it would have been incorrect to charge the profit and loss account with £75, so also it would be incorrect to transfer £125. The true rent is neither of these figures, but £100. In this instance, however, the credit of £25 on January 1 reduces the *balance* of the rent account to £100, which may be correctly transferred to the profit and loss account.

NOMINAL LEDGER ACCOUNTS

A — *Rent Account*

1957					
Mar. 31	To Cash ..	£25			
June 30	" " ..	25			
Sept. 30	" " ..	25			

B — *Rent Account*

1957			1957		
Mar. 31	To Cash ..	£25	Dec. 31	By Profit & Loss account	£100
June 30	" " ..	25			
Sept. 30	" " ..	25			
Dec. 31	.. Rent accrued c/d	25			
		£100			£100
			1958		
			Jan. 1	By Balance b/d ..	£25

C — *Insurances*

1957					
July 1	To Cash ..	£20			

D — *Insurances*

1957			1957		
July 1	To Cash ..	£20	Dec. 31	By Amount unexpired c/d	£10
			Dec. 31	By Profit & Loss account	10
		£20			£20
1958					
Jan. 1	To Balance b/d ..	£10			

It may also happen that certain expenses are paid before they are due, the payments having been debited to the appropriate expense account. In such an event, the total debit to the expense account in question will be greater than the true expense for the year.

Take the example of a man commencing business on January 1, 1957. For the first six months he pays no insurance, but on July 1 he takes out a policy and pays a premium of £20, which covers him for the succeeding twelve months. He makes up his accounts on December 31, on which date there is a balance of £20 on the insurance account, as shown at C in the Table in this page.

This premium of £20 covers the period July 1, 1957, to June 30, 1958. Therefore only £10, or half, relates to the accounting year 1957, and this amount only should be charged against the profits of that year. To enable this to be done, the balance on the insurance account at Dec. 31 must be reduced to £10. This is done by crediting insurances with £10, and debiting the new insurance account, as shown in the example, with a similar sum. The debit balance on insurances account is now £10, and this is transferred to the profit and loss account, as seen at D.

The profit and loss account has now been debited with the proportion of the premium that relates to the year under review. The debit balance on the insurances account appears as an asset in the balance sheet. This balance represents a very real benefit; the business possesses on December 31 the advantage of being insured for six months ahead. For if the premium lapsed on December 31, a further premium would have to be paid on January 1. The business is relieved of the necessity of making any fresh disbursement until the following July 1.

LESSON 7

Writing Off

A DEBIT balance on a personal account is an asset. It represents the obligation of the debtor to pay at a future date the sum of money indicated. It is the right to receive cash, combined with the expectation of payment.

It may, however, happen that the debtor becomes insolvent and is unable to pay. In such an event, the debt due ceases to be an asset. The mere right to receive cash, if it cannot be enforced, is worthless. Debit balances are either assets or losses. The debit

balance on the personal account of an insolvent debtor ceases to be an asset and becomes a loss. An account headed "bad debts" is opened, and the balance on the personal account is transferred thereto. This is called "writing off."

It is likely that in the course of the year a number of bad debts will be sustained. All bad accounts are written off to the bad debts account, and the total of this account is ultimately transferred to the profit and loss account. For example :

John Smith

To Goods	£300	By Bad Debts ..	£300

Bad Debts

To J. Smith ..	£300	By Profit and Loss..	£600
" W. Robinson ..	250		
" H. Jones ..	50		
	£600		£600

W. Robinson

To Goods	£250	By Bad Debts ..	£250

H. Jones

To Goods	£50	By Bad Debts ..	£50

The three amounts in this illustration, totalling £600, have been previously included in the credit to sales account, and will therefore be included on the credit side of the profit and loss account. But because no cash will ever be received in payment for the goods, this must be offset by the debit of "bad debts."

There may be some doubt as to the capacity of the debtor to pay. There is no *certainty* that he will be unable to pay, but there is a definite possibility. If that is true of a number of personal accounts, it is almost certain that in a proportion of the cases the expectation of non-payment will be correct.

Thus at the time the profit and loss account is being prepared there is a moral certainty of some loss, but no definite knowledge of its precise amount, or of which personal accounts will be involved.

It is therefore impossible to write off any specific accounts as definitely bad. But because some loss is certain, some provision must be made, otherwise the net profit will be overstated. This provision is made by the creation of a provision for bad and doubtful debts. After examination of balances revealed by the books an estimate of total probable loss is made ; this sum is debited to the profit and loss account, and is credited to a provision account.

No entry is made in any of the personal accounts. The provision is shown in the balance sheet as a deduction from "sundry debtors," "sundry debtors" being the total of the sales ledger balances. Thus the *net* figure in the balance sheet is the same as it would have been had specific accounts, equal in total value to the amount of the reserve, been written off. The provision remains as a balance in the ledger until the following accounting period.

When this point has been reached, a number of the personal accounts which were previously doubtful will now be known to be definitely bad, and will be written off to the bad debts account. Other accounts may prove to be good. The total of the bad debts account can be transferred to the debit of the provision account.

But the story does not finish there. It now becomes necessary to make a fresh estimate of doubtful accounts at the end of this second period. The amount of this new provision is debited to the provision account, and brought down to credit. The balance of the account is written off to the profit and loss account.

Consider the following example. A business commences on January 1, 1957. At the end of the first year's trading it is estimated that the probable loss from doubtful debts will be £500, and a provision of this amount is created. During 1957 personal accounts totalling £400 are written off as bad. On December 31, 1957, the total of all sales ledger balances, *excluding* those that have been written off as bad, is £10,000. It is estimated that 10 per cent. of these debts will eventually prove to be bad, and it is determined to bring the provision up to this figure.

It will be noted that :

1. The debit to profit and loss for 1956 is £500.

2. The provision deducted from sundry debtors in the balance sheet of December 31, 1956, is £500.

Provision for Bad and Doubtful Debts

1956			1956		
Dec. 31.	To Balance *c/d*	£500	Dec. 31.	By Profit and Loss Account	£500
		£500			£500
1957			1957		
Dec. 31.	To Bad Debts %	£400	Jan. 1.	By Balance *b/d*	£500
" 31.	To Balance *c/d*	1,000	Dec. 31.	By Profit and Loss Account	900
		£1,400			£1,400
			1958		
			Jan. 1.	By Balance *b/d*	£1,000

3. Bad debts actually written off in 1957 were only £400, which is £100 less than the estimate.

4. The debit to profit and loss for 1957 is £900. This amount is made up in this way. It was estimated that £1,000 of the debtors' accounts at December 31, 1957, would be bad; but the debit to profit and loss at December 31, 1957, turns out to have been excessive to the extent of £100. By charging to P. and L. at December 31, 1957, £100 *less* than the amount of the estimated loss, the total debit to P. and L. for the two years taken together is correct, always assuming, of course, that the final estimate is correct:

Debit to P. and L. 1956	£500
" " " " 1957	900
	£1,400
Debts known to be Bad and written off.. ..	£400
Balances estimated Bad, Dec. 31, 1957.. ..	1,000
	£1,400

5. The provision deducted from sundry debtors (not including the £400 written off) is £1,000, which is the proportion of the debts then due, which it is not expected to realize.

Wasting Assets

Assets can be divided into current and fixed assets. Current assets consist either of cash or of assets which will eventually be converted into cash, such as debtors or stock-in-trade. With these we are not at the moment concerned.

Fixed assets are those which are acquired with a view to permanent retention, and not with a view to re-sale and conversion into cash. They are held in order to enable the business to earn profits. Some fixed assets are called wasting assets—they gradually wear out and shrink in value as they are used. Examples of wasting assets are office furniture, plant and machinery, leaseholds, and motor vans.

A wasting asset cannot be allowed to remain in the books at its original cost price. This original value must be gradually reduced as the real value shrinks. The double-entry by which this reduction in value is recorded is a credit to the asset account, thus reducing the balance remaining, and a debit to the profit and loss account.

The question at once arises: By what amount are we to reduce a given wasting asset in a given year? The market price or the selling value of the asset has no bearing whatever on this question. Fixed assets are not held with a view to re-sale, therefore the market price is entirely irrelevant.

The true nature of wasting assets is that they represent revenue expenditure (i.e. business expense) paid in advance. The difference between the cost of a machine which lasts for five years and the cost of a man's labour for a week is time. Both are expenditure necessarily incurred in earning profits; both must therefore be charged against profits. At the end of five years our machine is worn out, and useless; the money we paid for it is now represented by nothing of any value.

Because accounts are usually prepared once every year, and not once in five years, the object will be to spread the cost of the machine over the profit and loss accounts of the five years during which it is used. This is done by writing off one-fifth of the original value of the machine at the end of each year. The depreciation account is, of course, transferred to the profit and loss account.

LESSON 8

The Balance Sheet of a Small Business

On page 1439 is the trial balance of a sole trader on a small scale, as taken out at December 31, 1956.

This gives a picture of all balances in the ledger of J. Harris at December 31, 1956. The following adjustments are to be made.

3 months' rent is outstanding	£80
" " gas " "	4
" " electricity "	3
" " telephone "	3
Insurance paid in advance	4

The stock on December 31 is valued at £3,220. The motor vans are to be depreciated at 20 per cent. p.a. of the original cost, which was £1,000. The fixtures and fittings are to be depreciated at 5 per cent. on the balance which is outstanding.

£325 of the book-debts are to be written off as bad; in addition, the provision for bad and doubtful debts is to be increased to 5 per cent. of the total debts outstanding.

The trading and profit and loss account is prepared by transferring thereto the balances of the nominal accounts. It should be remembered that the trading and profit and loss account is a ledger account, and that all entries therein require an equal and opposite debit or credit in some other account.

The appropriate entries to give effect to the foregoing adjustments must also be made. The trading and profit and loss account is shown overleaf.

Having prepared the trading and profit

Trial Balance

	Dr.	Cr.
Capital account—J. Harris		2,000
Loan account A. Harris		500
Motor vans	840	
Fixtures and fittings	500	
Stock in trade, Jan. 1, 1956	2,850	
Sundry debtors	4,525	
Sundry creditors		5,785
Purchases	28,320	
Sales		33,830
Returns inwards	500	
Returns outwards		400
Wages	1,840	
Rent	240	
Telephone	12	
Insurances	20	
Gas	12	
Electricity	8	
Sundry expenses	88	
Rates	75	
Bank charges	10	
Audit fee	42	
Carriage	170	
Discounts	955	730
Interest on deposit		5
Drawings	900	
Income tax	63	
Bad debts provision		180
Cash at bank	460	
Deposit account	1,000	
	£43,430	£43,430

and loss account, the balance sheet can be drawn up, as shown above. This consists of a list of all balances remaining after the closing of the nominal accounts.

It should be noted that the balance sheet is *not* a ledger account. It is merely a list of balances. The preparation of the balance sheet does not require any entries in the ledger.

(1) *Stock at December* 31. The trading account is credited with £3,220, and a new stock account is debited. This item remains as a balance in the ledger, and appears therefore in the balance sheet.

The purpose of the trading account is to compare the total value of all sales with the cost of the goods sold. The stock of goods at the beginning of the accounting period (at cost price), with purchases (also at cost price), together equal the cost of all goods available for sale ; but not all these goods are sold. In order, therefore, to arrive at the cost of the goods which have been sold, the cost price of the closing stock must be deducted.

This is done by crediting closing stock to the trading account. Another method of presentation would be to show the value of closing stock as a deduction from the debits to the trading account ; in this way the cost of goods sold would be thrown up as a single figure.

(2) *Expenses accrued due*. The appropriate expense accounts are debited with the amounts outstanding and each new expense account is credited. The expense accounts are then closed off to the profit and loss account, while the balance brought down in each expense account appears in the balance sheet.

(3) *Insurance* paid in advance. The insurance account is credited with the amount to be carried forward, thus reducing the balance to be transferred to the profit and loss account, and a new insurance account is debited. The balance of this account (debit) appears in the balance sheet as " insurance paid in advance."

(4) *Depreciation*. The asset accounts (motor

Trading and Profit and Loss Account for the year ended 31.12.1956

To—			By—		
Stock		£2,850	Sales	£33,830	
Purchases	£28,320		*Less* returns	500	
Less returns	400				33,330
		27,920	Stock		3,220
Gross Profit carried down		5,780			
		£36,550			£36,550

To—			By—	
Wages		£1,840	Gross profit, brought down	£5,780
Rent		320	Discounts received	730
Rates		75	Interest on deposit	5
Lighting and heating		27		
Telephone		15		
Insurances		16		
Carriage		170		
Discounts allowed		955		
Sundry expenses		88		
Bank charges		10		
Audit fee		42		
Bad debts		355		
Depreciation—				
Motor van	£200			
Fixtures and Fittings	25			
		225		
Net profit transferred to Capital account		2,377		
		£6,515		£6,515

vans, and fixtures and fittings) are credited with the amounts to be written off, and a depreciation account is debited. The depreciation account is closed by transfer to the profit and loss account, while the balances of the asset accounts (reduced by the amount of the depreciation) appear in the balance sheet.

(5) *Bad debts* and provision for bad debts. If it is definitely known that certain debtors are unable to pay, the personal accounts in the sales ledger are credited, and " bad debts " account is debited.

Anticipated Loss

If, in addition, a proportion of the remaining book-debts are expected to be bad, profit and loss account will be debited with the amount of the anticipated loss, and a provision account will be credited.

In the foregoing example, the balance on the provision account at January 1 was £180. On December 31 it was found necessary to increase the provision by £30 to £210. The personal accounts actually written off totalled £325, thus making a total of £355 to debit to profit and loss. The bad debts provision account would appear as follows :

Bad Debts Provision Account

To—		By—	
Bad debts . .	£325	Balance . .	£180
Balance . .	210	Profit and Loss . .	355
	£535		£535
		By balance . .	£210

The total anticipated loss due to the inability of debtors to pay amounts to £535. But £180 of this has already been provided for by debiting the profit and loss account of the previous year, thus leaving only £355 to be charged in 1956. The following are important items:

Discounts and Returns

(1) *Discounts.* Personal accounts of debtors are credited, and " discounts " accounts debited; personal accounts of creditors are debited, and " discounts " credited. The amount of cash necessary to settle the personal account is thus diminished.

(2) *Returns.* The entries are reversals of the entries necessary to record purchases and sales on credit. For returns outwards, debit the personal account and credit " returns outwards." For returns inwards, credit the personal account of the customer, and debit " returns inwards."

Value of Proprietor's Possessions

(3) *The Capital Account.* The balance of the profit and loss account is credited to the proprietor's capital account. A profit is necessarily represented by an increase in net assets, which belong to the proprietor. His capital account shows the value of his possessions—the excess of his assets over his liabilities, which is increased by a profit.

This excess is diminished if the proprietor withdraws cash from the business for his own private use. The amount of the proprietor's " drawings," as such withdrawals of cash are called, is debited to his capital account. Such an entry is necessary if the capital account is to record the amount by which assets exceed liabilities. Assets (i.e. in this instance, cash) are diminished by drawings ; the capital account must be diminished to an equal extent.

Debited to Capital Account

(4) *Income Tax.* This has been debited to the capital account, not to the profit and loss account. Income tax has been added to drawings, or the proprietor's private expenditure, and not to business expenses. A person earning a salary does not regard his income tax payments as being a reduction of salary ; he regards the tax as a payment *out* of his salary or income.

In the same way, the profit and loss account of a business is designed to show the income of the proprietor. Income tax is paid out of that income ; it is not a payment which reduces the amount of income.

Balance Sheet *As at December* 31, 1956

Capital Account				Cash at Bank			
As at Jan. 1 . .		£2,000		Current account	£460		
Add Net profit		2,377		Deposit account	1,000		
		4,377				£1,460	
Less Drawings	£900			Sundry Debtors	4,200		
Income tax	63			*Less* provision	210		
		963				3,990	
			£3,414	Stock in trade		3,220	
Loan account			500	Motor vans			
Sundry creditors				As at Jan. 1	840		
Trade		5,785		*Less* depreciation	200		
Expenses accrued		90				640	
			5,875	Fixtures and fittings			
				As at Jan. 1	500		
				Less depreciation	25		
						475	
				Insurance paid in advance		4	
			£9,789			£9,789	

LESSON 9

Loose-leaves and Machine Accounting

THE development of machines to do the detailed entries and calculations in keeping a set of accounts has not introduced any new principles of book-keeping : the rules of double-entry still have to be observed.

It has, however, increased the need for vigilance, to ensure that the double-entry is actually completed ; for the whole work is so sub-divided that it is easy to overlook a section. The accounting has to be well organized and proper control accounts maintained.

In even moderately sized businesses loose-leaves or cards are generally used instead of books for the bought ledgers (containing the accounts of suppliers) and the sales ledgers (containing the accounts of customers).

The loose-leaves can be kept in binders or arranged in vertical files. The cards can be in suitable drawers or in visible indexes (i.e., arranged so that successive cards overlap and the names are visible), or upright in trays. There will be one sheet or card for each customer. Debit and credit entries can be made on the sheets or cards either by hand in ink or indelible pencil or by machine in typewriting.

Accounting machines can be used for " posting " to the ledger, i.e. debiting the customer's account with the total of the invoice of goods supplied or crediting his account with the amount of cash and discount or a remittance. Generally, at the same time as the customer's account is debited or credited, a copy of the entry is made on his statement of account, which will be sent to him monthly.

Most businesses pay amounts received into the bank as soon as possible. The amounts received each day can be quickly machine-listed—they can be typed on an add-listing machine, which automatically provides the correct total. This is the control total, by which subsequent work can be checked.

The bank pay-in sheets can be typed in triplicate, one sheet to be handed in to the bank, one to be initialled by the bank cashier, the third to remain as the proof copy. The initialled sheets can be filed together to form the debit side of the bank column of the cash book.

Simultaneous Preparation

Alternatively, using an accounting machine (somewhat similar to a typewriter), one can put into the machine the following pieces of paper. (a) A receipts cash sheet ; (b) two copies of a bank pay-in slip, interleaved with carbon paper ; (c) receipt forms in duplicate with interleaved carbon, feeding from the front of the machine, so as not to disturb the other papers already on the platen of the machine.

In this way there can be prepared simultaneously (a) the receipt to be sent to the payer, (b) a duplicate receipt, which is a posting slip to be used when crediting the payer's account, (c) the receipts cash sheet, which is the debit side of the cash book, and the pay-in slips, one to be initialled by the bank cashier, one to be left with the cheques paid in.

The receipts cash book for machine operation might be ruled thus, in order to facilitate the combined operation :

RECEIPTS CASH BOOK

Ref.	Receipt No.	Name	Date	Amount £ s. d.	Discount £ s. d.

For each sales ledger (or in a big business, for each section of a sales ledger) a control account will be kept, giving the totals of all the amounts debited and credited to the individual accounts during the period (say, a month), so that a separate trial balance of each ledger or section of the ledger can be prepared as a check.

In most loose-leaf and card ledgers the debit and the credit columns are adjacent, and there is a third column for the balance, thus :

J. Smith,
64 High Street, Exton. 63.

1956		Dr.	Cr.	Bal.
June 1	Bal.			64 10 0
15	Gds.	37 5 0		101 15 0
17	Csh.		64 10 0	37 5 0
20	C.N.		2 5 0	35 0 0

The pay-in slips are narrower than the cash book, and would show only the date and the amount. Suitable stationery can be bought to permit such carbon copying methods to be used for handwritten work.

The successful application of these labour-saving methods depends upon the ability of the one in charge to ensure that each debit has a corresponding credit.

Copies of each day's invoices, accompanied by a summary sheet, can be stapled together and filed in order of date. In this form they constitute a sales day book. Similar arrangements can be made with invoices received for purchases. The journal proper is used only for exceptional items involving two ledger accounts. It still retains the bound book form.

LESSON 10

Special Features of Partnership Accounts

So far it has been assumed that the ownership of a business is vested in a single person. But ownership of most undertakings is shared by a number of individuals. These undertakings are of two types—partnerships (general or limited) and limited companies.

Partnership is defined as " the relation which subsists between persons carrying on a business in common with a view of profit." But it is sometimes difficult to determine exactly what constitutes a partnership. An employee or servant may be remunerated by a share of the profits of a business, but that does not necessarily make him a partner. In general terms, however, the right to a share of the profits of a business is *prima facie* evidence that the recipient is a partner.

Each partner is liable for the debts and obligations of the firm to the full extent of all his personal property, and not merely to the extent of his share of the partnership property. Should the assets of a partnership be insufficient to pay the firm's debts, the partners must make good the deficiency out of their private resources.

All partners must contribute to such a deficiency. If one of two partners is insolvent the other is bound to make good the whole of the deficiency, even if it entails the sacrifice of all his personal effects.

Provisions of the Partnership Act

Normally, partners will draw up a document called a partnership agreement, which will define their rights and duties as between themselves ; but such a written agreement is not essential—agreement may be verbal or understood. If there is no agreement, either written or understood, then certain rules laid down by the Partnership Act apply. The provisions which affect the accountant are :

(1) All partners are entitled to share equally in capital and profits and losses.

(2) Partners are not entitled to interest on capital.

(3) Partners are not entitled to salaries for their services.

(4) Partners are, however, entitled to interest at 5 per cent. per annum on loans.

It must not be assumed, if one partner contributes a larger share of the original capital of the firm than the other partners, that the total capital must be credited in equal proportions to each partner. Even if there were no written agreement, the mere fact of unequal contributions would imply a definite understanding to share capital in those proportions.

In the absence of specific agreement, profits must be shared equally, even if capitals are unequal. Similarly, losses would be borne equally. In practice there will usually be a specific agreement on all these points.

Interest on Partner's Capital

The proportions in which partners contribute the capital of the firm are not necessarily, or even usually, the proportions in which they share profits. If partners, for instance, agree to share profits equally, while they have contributed capital in unequal proportions, it is possible to compensate those partners who have contributed larger shares of capital by an agreement to credit all partners with interest on capital.

Such interest would be debited to the profit and loss account of the firm, and credited in the due proportions to the capital accounts of the partners. The effect of this entry is to diminish the balance of profit available for division.

The interest on partners' capitals is not a business expense in the ordinary sense. The entry debiting profit and loss and crediting partners' capitals amounts to a transfer of a portion of the net profit.

Division of Profits

The final effect of the transaction is that the true net profit of the concern is divided unequally. The circumstances make it impossible to arrange an equitable division of profits by an agreement to share in fixed proportions, and so a more complex method is adopted : the partner who has made the largest contribution to capital receives, in effect, the largest share of the profits. But the amount by which his share of the profits exceeds the shares of the other partners is limited to a fixed sum, which is a different thing from a fixed proportion.

If all the partners, by their work, contribute equally to the prosperity of the business, it is reasonable that the extra compensation for a larger contribution of capital should be limited to interest on the excess.

Salaried Partners

It frequently happens that one or more partners devote more time to the business than others. The usual practice then is for the partners who devote most time to the business to receive an agreed salary, irrespective of what their shares of the net profit may be.

The effect of all these circumstances on the books of account can now be briefly

considered. In the first place, there will be not one capital account, as in the books of a sole trader, but one capital account for each partner. To these accounts will be credited, first, the amount of each partner's original contribution; secondly, if so agreed, interest thereon annually; thirdly, in some cases, a salary; fourthly, a share of the final net profit.

Shares in Net Assets

Should a loss be incurred, a share of the loss will be debited to each capital account. Any sums withdrawn by a partner, whether in respect of interest, salary, or profit, are debited to his capital account. The balances on all the capital accounts combined represent the net worth of the business, that is, the excess of assets over liabilities.

The balance on each partner's account shows the share of each in the ownership of the net assets, and is the amount which each would receive in cash if the business were wound up and all assets sold at a price equal to their book value.

The usual practice is to divide each partner's capital account into two sections, one of which includes only the partner's original contribution, while the second section is reserved for interest, salary, profits and losses, and drawings. This second section is called the current account.

Typical Accounts

The following example shows typical accounts. A, B, and C are partners, sharing profits and losses in the proportions 3 : 2 : 1. Under the partnership agreement, interest on capital is charged at 5 per cent. p.a. C receives a salary of £200 p.a. On Dec. 31, 1956, the following balances appear in the partnership books:

A	Capital account	£10,000
B	" "	8,000
C	" "	2,000
A	Current account	1,000
B	" "	500
C	" "	100
B	Loan account	1,000

The net profit for the year, before charging interest on capital, interest on B's loan, or C's salary, amounts to £3,050.

Drawings for the year amount to:

A	£1,500
B	900
C	750

The profit and loss account is usually balanced before charging any internal items of this nature, and the balance is brought down to an account called the profit and loss appropriation account. In this account, which appears in the ledger as a continuation of the profit and loss account proper, the division of profits between the partners is dealt with.

Profit and Loss Appropriation Account

To—			By—	
Interest on Capital:	£		Balance b/d	£3,050
A	500			
B	400			
C	100			
		1,000		
Interest on Loan, B		50		
Salary, C		200		
Net Profit transferred to Current accounts:				
A	900			
B	600			
C	300			
		1,800		
		£3,050		£3,050

Current Account. A

To—		By—	
Drawings	£1,500	Balance	£1,000
Balance c/d	900	Interest	500
		Net Profit	900
	£2,400		£2,400
		By— Balance b/d	£900

Current Account. B

To—		By—	
Drawings	£900	Balance	£500
Balance c/d	650	Interest	400
		"	50
		Net profit	600
	£1,550		£1,550
		By— Balance b/d	£650

Current Account. C

To—		By—	
Drawings	£750	Balance	£100
		Interest	100
		Salary	200
		Net profit	300
		Balance c/d	50
	£750		£750
To— Balance b/d	£50		

Partners' Loans

Note that a partner's loan account must not be confused with his capital account. A loan is not a contribution to capital. If the business were to be wound up and the proceeds of the assets were insufficient to meet the partners' capitals in full, then any partners' loans would take priority of capital and would be paid before any division was made on account of capital.

LESSON 11

Goodwill and Capital in Partnerships

The ordinary trading records of a partnership concern are in no way different from those of a sole trader. It is, however, necessary to keep accounts in which the rights of the partners (as between themselves) to capital and to profits are recorded.

The adjustment of these rights requires special treatment upon a change in the constitution of the firm, that is, upon the admission of a new partner, or upon the death or retirement of an existing partner, and also in the event of dissolution of the partnership.

The terms upon which a new partner will be admitted into an existing business will be a matter for individual agreement in every instance, but normally he will be required to introduce a certain amount of fresh capital. Whatever sum he contributes will be credited to his capital account.

A partner, by definition, is entitled to a share of the profits of the business in which he participates ; it follows that the existing owners, if they decide to admit a new partner, will be surrendering their right to a part of the future profits of the undertaking. In return for this surrender they will usually require some compensation.

This compensation can take the form of a money payment, called a premium. The new partner is, in effect, purchasing a part of the goodwill, since goodwill can be defined as the right to future profits. The premium usually paid for a share of the goodwill must not be confused with the capital usually brought in by a new partner.

Any sum introduced as capital does not directly benefit the existing partners. This amount is credited to the capital account of the new partner, and it represents his share in the ownership of the assets of the business ; if the undertaking were subsequently dissolved, the sum would be returned to him.

The premium for goodwill need not appear in the books of the firm at all ; it can be paid direct to the existing partners, who can treat it as a completely private possession. Should the premium be paid into the firm, it will be credited to the capital accounts of the existing partners in the proportions in which they share profits. Because goodwill is simply the capitalised value of the right to future profits, the proportion in which partners are entitled to receive future profits is the same as that in which they share the ownership of the goodwill.

Consider the following example. A and B are in partnership, sharing profits in the proportions of 3 to 2. On January 1, 1957, they decide to admit C as a partner ; C is to bring in £5,000 as capital, and is to pay a premium of £1,000 for a one-fifth share of the goodwill. The balance sheet of A and B, before the admission of C, is as follows :

A and B

Capitals : A	..	£6,000	Sundry Assets ..	£12,000
B	..	4,000		
Creditors..	..	2,000		
		£12,000		£12,000

After the admission of C it will be :

A, B, and C

Capitals : A	..	£6,600	Sundry Assets ..	£12,000
B	..	4,400	Cash from C ..	6,000
C	..	5,000		
Creditors..	..	2,000		
		£18,000		£18,000

Had A and B received the £1,000 privately from C, and subsequently decided to invest it in the firm as additional capital, the position would still be as above.

In some instances a new partner is not required to pay a premium. In such an event the existing partners will usually compensate themselves by raising a goodwill account in the books of the firm. If we take the same set of circumstances as in the foregoing example, and assume that C can raise only £5,000, then, since the premium for which C had been asked was £1,000 for a one-fifth share of the goodwill, it follows that the whole goodwill is worth £5,000. A double-entry would be passed, debiting a new account, headed *goodwill*, with £5,000, and crediting £3,000 to A's capital account and £2,000 to B's capital account.

The balance sheet is then as follows :

A, B, and C

Capitals : A	..	£9,000	Sundry Assets ..	£12,000
B	..	6,000	Cash from C ..	5,000
C	..	5,000	Goodwill.. ..	5,000
Creditors..	..	2,000		
		£22,000		£22,000

It will be noted that goodwill is not credited to C's capital account. Normally the goodwill account will not be allowed to remain in the books. It will be written off—a double-entry will be passed crediting goodwill account with £5,000 and debiting the capital accounts of the *three* partners, in the proportions in which they share profits and losses, i.e. A $\frac{12}{25}$, B $\frac{8}{25}$, and C $\frac{5}{25}$. Because C is now a partner, he must

bear his share of the nominal loss involved in eliminating goodwill from the books. Thus A will be debited with £2,400, B with £1,600, C with £1,000.

The balance sheet will then appear as follows :

A, B, and C

Capitals : A	.. £6,600	Sundry Assets	.. £12,000
B	.. 4,400	Cash from C	.. 5,000
C	.. 4,000		
Creditors ..	.. 2,000		
	£17,000		£17,000

Although C has contributed £5,000 as capital, only £4,000 remains to his credit, whereas A and B have benefited to the extent of £600 and £400 respectively. In effect, the position is the same as it would have been had C contributed only £4,000 and paid £1,000 as a premium. A and B benefit to the extent of £1,000 in any event.

Upon the retirement or death of a partner, he or his executors are entitled to repayment of his capital. Legally in such an event the old partnership ceases to exist, and if the survivors continue in business, a new partnership comes into being. Thus, in theory, the obligation to repay the deceased or retired partner's capital is a private obligation of the surviving individual partners, and not a liability of the new firm.

In practice, however, the sum which is due to the outgoing partner is treated as a partnership liability, and it appears as such in the books of the firm. It is important to note that in the event of the firm's becoming insolvent all other creditors would take priority of a retired partner or a deceased partner's executors, because the assets of a firm must be first applied in discharge of the liabilities of the firm.

Ascertainment of the amount due to an outgoing partner is not always simple. It does not follow that the amounts standing to the credit of his capital and current accounts constitute the amount due to him. If the death or retirement takes place at a date during an accounting period, the accounts must be made up to that date, and an appropriate share of the accrued profits must be credited to the account of the outgoing partner.

Furthermore, it is possible that the assets may be shown in the books of the firm at less than their true value. The partners may, for various reasons, have agreed to this course, but the surviving partners cannot thereby claim to limit the amount due to the outgoing partner to a corresponding extent.

The assets of the firm must in these circumstances be valued upon some fair and equitable basis. Any excess of the true value of the assets over the book value must be credited to the partners in the proportion in which they share profits. In particular, the goodwill of the firm, which seldom appears in the books at all, must be valued, in order that the outgoing partner may receive compensation for the surrender of his part ownership of this asset.

The amount standing to the credit of the outgoing partner's capital account, after crediting his share of accrued profits, his share of the goodwill, and his share of any difference between the true value and the book value of the assets, is the amount due to him or his executors. The following illustrates these principles.

A, B, and C are three partners. C retires on June 30, 1957, and the balance sheet of the firm on that date is as follows :

A, B, and C

Capitals : A	.. £5,000	Plant and Machinery	.. £8,000
B	.. 4,000	Debtors ..	.. 4,000
C	.. 6,000	Stock in Trade	.. 12,000
Creditors ..	.. 10,000	Investments	.. 1,000
	£25,000		£25,000

It is decided that a fair value for the plant is £10,000, for the stock £12,500, for the investments £1,500. The goodwill is valued at £4,000. The partners share profits in the proportions 3, 2, 2.

Entries will be passed, increasing the assets by £7,000 in total, and crediting £3,000 to A, £2,000 to B, £2,000 to C. Thus the total due to C will be £8,000 and not £6,000.

A and B can, if they so desire, subsequently reduce the book value of the assets to their original figures, but the capital accounts of A and B only, and not of C, must be debited.

LESSON 12

Dissolution of Partnership

WHEN a partnership is dissolved, it becomes necessary to distribute the assets among the partners, and to determine the amount due to each. The first step will be to prepare a profit and loss account up to the date of dissolution, and to credit each partner with his share of the profit accrued.

As previously pointed out, it does not follow that the value of the assets as shown in the balance sheet is always their true value. For purposes of dissolution the true value must be ascertained. The normal practice is to realize, or sell, the assets, and to distribute the proceeds. The business can be sold as a going concern,

for a lump sum, or, alternatively, the assets can be realized piecemeal. In some instances a proportion or all of the assets may be taken over by one or more of the partners at a mutually agreed valuation.

Realization Account

The circumstances in which the assets are realized may vary in several ways, but the principle is the same: the assets of the partnership are sold, and the proceeds are used, first to pay the liabilities of the firm, secondly to repay the partners' capitals.

The proceeds of the realization may be greater or less than the book value of the assets. Any surplus or deficiency is a profit or a loss, and it must be shared by the partners in the same proportions as any normal profit or loss upon trading. The accounting procedure for these transactions is as follows:

(1) The assets must be transferred to a realization account, which is opened for the purpose.

(2) As the assets are sold, the proceeds are credited to the realization account.

The realization account now becomes a kind of profit and loss account. On the debit side appears the book value of the assets to be sold; on the credit side appears the selling price.

The difference between the two sides of the account is the profit or loss on realization, and it is transferred to the capital accounts of the partners, in the proportions in which they share profits.

(3) The proceeds of the realization account are debited to cash account. The liabilities are paid off, and the balance of the cash account should now equal the total amount standing to the credit of the partners' capital accounts. An entry crediting cash and debiting the partners' capital accounts will now close the books of the firm.

The following example illustrates this procedure. A, B, and C are in partnership, sharing profits in the proportions 5, 3, 2. On June 30, 1957, they decide to dissolve partnership, and on that date the balance sheet of the firm is as follows:

A, B, and C

Capital Accounts:			Cash at Bank		£ 700
A ..	£8,000		Sundry Debtors		6,000
B ..	3,500		Stock in Trade		8,600
C ..	500		Leasehold		
		12,000	Premises	..	2,400
Loan Account,			Plant and		
B ..	..	1,000	Machinery		5,300
Sundry					
Creditors	..	10,000			
		£23,000			£23,000

The assets are at once realized, at the following figures: debtors £5,400; stock £5,000; leasehold premises £3,200; while the plant and machinery are taken over by A, at an agreed valuation of £2,700. All the asset accounts, with the exception of cash, will be closed by transfer to the debit of realization account, which will appear as shown below.

The realization account has been credited with the cash proceeds, the corresponding debit being, of course, to the cash account. Strictly speaking, A should pay £2,700 in cash for the plant and machinery, but as the final balance on A's capital account will exceed this figure, and as A will receive a larger sum in cash when the final distribution is made, it is more convenient to debit this amount of £2,700 to his capital account, thus reducing the amount he will receive. Whichever method is followed, the result is the same. The method here adopted reduces the transferring of cash to a minimum.

Realization Account

To—		By Cash—		
Sundry Debtors ..	£6,000	Debtors	£5,400	
Stock in Trade ..	8,600	Stock	5,000	
Leasehold Premises ..	2,400	Premises	3,200	
Plant and Machinery ..	5,300			13,600
		A—Plant and Machinery ..		2,700
				16,300
		Loss on Realization—		
		A, $\frac{5}{10}$ ..	[illegible]	
		B, $\frac{3}{10}$ ..	[illegible]	
		C, $\frac{2}{10}$ ..	[illegible]	
				6,000
	£22,300			£22,300

The cash received will be used, first to pay the creditors, secondly to repay B's loan account, and finally to repay the partners' capitals.

Cash Account

To—		By—	
Balance % ..	£ 700	Sundry Creditors ..	£10,000
Realization %	13,600	B. Loan % ..	1,000
		Balance ..	3,300
	£14,300		£14,300
Balance	£3,300		

The partners' capital accounts appear as follows:

A—Capital Account

To—		By—	
Realization %, Plant and Machinery	£2,700	Balance ..	£8,000
Loss on Realization ..	3,000		
Balance ..	2,300		
	£8,000		£8,000
		Balance	£2,300

B—Capital Account

To—		By—	
Loss on Realization	£1,800	Balance ..	£3,500
Balance ..	1,700		
	£3,500		£3,500
		Balance ..	£1,700

C—Capital Account

To—		By—	
Loss on Realization	£1,200	Balance ..	£500
		Balance ..	700
	£1,200		£1,200
Balance ..	£700		

The credit balances on the accounts of A and B total £4,000. If the debit balance of £700 on C's account is deducted, the net credit balance is £3,300, which equals the balance of cash available for distribution. As C's capital account is in debit, he must make good the deficiency, and pay in £700 in cash. This will allow A and B to be paid in full. The closing entries will appear as follows :

A—Capital Account

To Cash ..	£2,300	By Balance ..	£2,300

B—Capital Account

To Cash ..	£1,700	By Balance ..	£1,700

C—Capital Account

To Balance ..	£700	By Cash ..	£700

Cash Account

To—		By—	
Balance ..	£3,300	A—Capital % ..	£2,300
Capital % ..	700	B—Capital % ..	1,700
	£4,000		£4,000

Final Distribution

The books of the firm are now closed. One special point must be noted. Had C been insolvent and unable to make good his deficiency, A and B could not be paid in full. The question arises : in what proportions would A and B share this additional loss of £700 ? C's account must be closed, and the capital accounts of A and B must be debited. The next question is : in what proportions ?

The answer is *not*, as one would expect, in the proportions in which they share trading profits and losses, that is, 5 to 3. This ratio is applied for profits and losses upon realization, but not for losses due to the insolvency of a partner. Such losses must be borne in the ratio of the solvent partners' last agreed capital accounts.

It will be observed that A's and B's capital accounts are in the proportions of 16 to 7. It might be necessary to ascertain the ratio from the last normal balance sheet before the dissolution. In any event this rule might have a very important bearing on the final distribution of cash between the solvent partners.

Interim Distribution

In practice, the process of realization may be fairly prolonged, and it is not improbable that in such cases the partners may desire an interim payment on account of the capital due to them. Any such interim distribution should be made very cautiously.

No payment should be made which might, in possible circumstances, have to be returned ; if the partner to whom such a payment was made was in financial difficulties, it might be impossible to recover the money. The only way to prevent this is to assume that the assets remaining unrealized will be found to be worthless, i.e. the greatest *possible* loss on realization must be provided for.

Consider the following case of the balance sheet of A, B, and C :

Capitals, A ..	£10,000	Assets ..	£20,000
B ..	8,000		
C ..	2,000		
	£20,000		£20,000

Three months after the cessation of business the total proceeds amount to £10,000. Assuming that nothing further will be received, the loss on realization would amount to £10,000 (£20,000—£10,000).

If the partners share profits and losses in the proportions A $\frac{2}{5}$, B $\frac{2}{5}$, and C $\frac{1}{5}$, the loss on realization would be borne as to A £4,000, as to B £4,000, as to C £2,000. In this instance C's loss equals his capital, and he would receive nothing. Therefore in distributing the £10,000 that is available, nothing should be paid to C.

This possible loss on realization would reduce A's capital to £6,000 and B's to £4,000. The £10,000 cash should be divided between A and B on this basis, A receiving £6,000, B receiving £4,000.

The capital accounts of the partners will now be :

A, £4,000	(£10,000, less £6,000 cash).
B, £4,000	(£8,000, less £4,000 cash).
C, £2,000	
£10,000	

The capital accounts are in the proportions of 2, 2, 1, which is the profit-sharing ratio. If any further sums are realized, any further interim distributions should be made in the proportions of 2, 2, 1. In this way no partner will receive a penny which in any circumstances he can be called upon to return.

LESSON 13

Joint Stock Company Accounts

The distinctive features of company accountancy are due to the special character of the ownership of joint stock undertakings and to the law governing their constitution. The law of joint stock companies has been consolidated by the Companies Act of 1948. It will be necessary to outline briefly the provisions of this Act as they affect the accounts.

The capital of companies, unlike the capital of sole traders or partnerships, is divided into shares of fixed amounts. The value of each share is generally small and the most usual denomination is £1. The vast majority of joint stock companies are limited liability companies—the liability of members is limited to the amount they have agreed to pay on their shares. In contrast, sole traders and partners are liable for the debts of their undertakings to the full extent of their private possessions.

The security and relative freedom from anxiety which the limited liability type of ownership affords accounts for its popularity. Furthermore, the division of capital into shares facilitates changes in the ownership of the undertaking, for shares are transferable by private contract from one person to another. This is a great convenience to persons who wish to dispose of either the whole or part of their interest in an undertaking without being involved in cumbersome legal procedure.

Joint stock companies are bound to comply with the numerous regulations of the Companies Act, the provisions of which are designed to prevent abuses which the principle of limited liability would otherwise make possible. There are two kinds of limited liability company—public and private. Private companies are exempt from some of the regulations which bind public companies.

Memorandum and Articles

The first step in the formation of a company is to draw up a document called the Memorandum of Association, which must be signed by at least two persons, seven for a public company, who each agree to take up at least one share in the company.

The Memorandum is the document which defines the nature and objects of the company. It must state the name of the company ; the situation of the registered office ; the objects of the company ; that the liability of the members is limited ; and the amount of the share capital.

A second document, called the Articles of Association, is usually prepared. The Articles contain the regulations for internal management of the company, and deal with such matters as the classes of shares into which the capital is to be divided, procedure at meetings, voting rights, powers and duties of directors, transfer of shares, dividends, and audit

A company limited by shares is not bound to have Articles ; if no Articles are prepared, a set of model Articles (called Table "A") contained in a schedule to the Companies Act of 1948 automatically applies.

Broadly speaking, the Memorandum defines the relationship of the company with the outside world, while the Articles define the rights and duties of members and officers of the company *inter se*, and govern internal procedure.

The Memorandum and Articles, together with a list of the names and addresses of the directors, and a statutory declaration that the requirements of the Companies Act have been complied with, must be filed with the registrar of companies at Bush House, Strand, London. Certain fees, including a stamp duty of 10s. per cent. on the authorised share capital, must be paid, and the registrar may then issue a Certificate of Incorporation. The legal existence of the company commences from the date of the issue of the certificate.

Prospectus

If the directors apply to the public for subscriptions to the share capital, they must publish a document, called a prospectus, which must contain very full information of the company's affairs. In particular, details of the manner in which the shareholders' money is to be used must be given.

If the company is formed to acquire an existing business, a report on the past profits of the undertaking, certified by the auditors, must be included in the prospectus. The nature of any assets to be acquired, and the price to be paid, particulars of all underwriting commission, and any payments to the promoters, must be disclosed.

The share capital authorized by the Memorandum is not usually issued at once in total ; the business of the company may expand in the future, and it is convenient for the directors to have the power to issue further capital without altering the constitution of the company.

Classes of Shares

Shares can be of several classes, of which the following are typical examples : pre-preference, preference, participating preference, preferred ordinary, ordinary, deferred ordinary, and founders' shares.

Fundamentally there are two types of share. The investor may choose security of dividends, combined with a small return on his money, or he may decide to take a greater risk of losing his capital in whole or in part, combined with the *possibility* of receiving a relatively high return on his money. If he makes the first choice, he buys preference shares, which carry the right to receive dividends in priority to the ordinary shareholders.

If the profits are only sufficient to pay the stipulated preference dividend, this dividend will be paid, and the ordinary shareholders will receive nothing. But if the profits are high, the preference shareholder receives nothing beyond his stipulated dividend, and the ordinary shareholder may receive a dividend as high as the profits of the company will allow.

Varieties of Capital

If the company makes no profits at all, the preference shareholder cannot be paid, but if the shares are cumulative preference, the unpaid dividend is carried forward and is a first charge against the profits of subsequent years. The preference shareholder has generally a good chance of a small return on his investment. The ordinary shareholder takes the chance of making handsome profits, but he takes a greater risk of getting nothing at all.

The seven classes of shares previously mentioned are arranged in the order in which they share in profits. The pre-preference shares rank above all others, but the rate of dividend is lowest ; the deferred shares rank after all other classes, and are therefore subject to the highest degree of risk, but they claim the largest dividends if large profits are earned.

Participating preference shares are in the nature of a compromise. A minimum dividend, at a lower rate than the preference share dividend, is promised, and in addition these shares carry the right to a stated share of the surplus profits remaining after all preference dividends have been paid.

The right of participation is usually small ; the ordinary shares carry the right to the greater part of the surplus. When deferred shares are issued, the dividend on ordinary shares is restricted to a certain maximum, and the whole of the balance goes to the deferred shareholders.

The greater security of preference shares is (experience has shown) largely illusory ; the preference shareholder runs all the risks without a prospect of commensurate return. Those who seek security should invest in debentures, which are loans to a company.

The expression "share capital," if used without qualification, may refer to any one of the following varieties of "capital."

1. *Authorised capital* is the maximum capital of the company as defined in the Memorandum.
2. *Issued capital* is equal to the nominal value of all the shares for which the public and other persons have been invited to subscribe.
3. *Subscribed capital* is equal to the nominal value of all the shares subscribed ; all shares issued are not necessarily subscribed.
4. *Paid-up capital* is the total paid up on the shares issued and subscribed, which is frequently less than their nominal value. For instance, the directors may require the shareholders to pay 10s. per share on shares of £1 each, and defer the payment of the balance until it is required. A certificate is issued to each shareholder when he has paid the amount due on his shares, and his name, with particulars of the shares, is entered in the share register of the company.

LESSON 14

Share Issues in Company Accounts

THE published prospectus of a company usually includes a form of application for shares. All applications received are listed on a document called an application and allotment sheet. If the public apply for more shares than the directors propose to issue, the directors will usually make allotments uniformly proportional to applications, though they are entitled to use their discretion.

The amount due on application (frequently 2s. 6d. or 5s. on a £1 share) is entered in detail in the shareholders' cash book, and in total in the general cash book ; the shareholders' cash book is a memorandum record.

An application and allotment account is opened in the ledger, which is debited with the total amount due on application and the total amount becoming due when the directors proceed to allotment. Share capital account is credited with both these amounts.

The cash received on application and allotment is credited in total to the application and allotment account. If the issue is oversubscribed, the credit to the application and allotment account will obviously exceed the debit. The excess application moneys are returned to the subscribers, and application and allotment account is closed by a debit of this amount. The accounts are illustrated by the following example.

On January 1 a company makes an issue of 100,000 ordinary shares of £1 each, 5s. being

payable on application, 5s. on allotment, and 10s. on April 1. Applications are received for 120,000 shares; the directors return the excess application moneys, and proceed to allotment on January 8, all sums due on allotment being received.

On April 1 the directors make the first and the final call of 10s. per share, and all the shareholders pay the amounts due from them, except John Smith, who holds 500 shares.

The entries in the financial books are made in the way shown in pages 1461 and 1462, but it must be remembered that the detailed work is dealt with in subsidiary memorandum records; the shareholders' cash book has been referred to, and a similar book is used for calls.

From these subsidiary books the entries in the share ledger (also memorandum) are made. The share ledger contains a folio for each member, and shows the number and value of shares held by each, the amount paid on the shares, the dates when payments are due, and the dates when the cash is paid.

So long as shares are not fully paid, the liability of the shareholder remains, and in the event of the company's becoming insolvent the liquidator can call upon the shareholders

Application and Allotment Account

Jan. 5		Jan. 5	
To Ord. Share Capital % ..	£50,000	By Cash	£55,000
" Cash (application moneys returned) ..	5,000		
	£55,000		£55,000

Call Account

April 1		April 1	
To Ord. Share Capital % ..	£50,000	By Cash ..	£49,750
		" Balance, c/d ..	250
	£50,000		£50,000
April 1			
To Balance brought down (Calls in arrear) ..	£250		

Cash Book

Jan. 5		Jan. 5	
To Application and Allotment a/c	£55,000	By Application and Allotment a/c	£5,000
April 1			
To Call % ..	£49,750		

Ordinary Share Capital Account

April 1		Jan. 5	
To Balance c/d	£100,000	By Application and Allotment %	£50,000
		April 1	
		" Call % ..	50,000
	£100,000		£100,000
		April 1	
		By Balance, b/d	£100,000

to pay up to the nominal value of their shares. In the example, the debit balance on call account (calls in arrear) is a debt due from the shareholder; any balance of this nature will be shown in the balance sheet as a deduction from share capital account, and not upon the assets side.

Shares at a Premium

Shares may be issued to the public at a price greater or (in certain circumstances) less than their nominal value. If the price is greater than the nominal value, the excess is called a premium. There are no restrictions upon the issue of shares at a premium, but a company may not issue shares at a discount until one year after it opens business, and then only after sanction by the members and the High Court.

For instance, a company issues 50,000 shares of £1 each at a premium of 1s. per share, 11s. being payable on application and allotment, and 10s. one month later. Subscriptions were received for the amount of the issue.

Application and Allotment Account

To Share Capital %	£25,000	By Cash	£275,00
" Premium on Shares % ..	2,500		
	£27,500		£27,500

Call Account

To Share Capital %	£25,000	By Cash	£25,000

Share Capital Account

		By Application and Allotment %	£25,000
		" Call % ..	25,000

Premium on Shares Account

		By Application and Allotment %	£2,500

The share premium may not be credited to the profit and loss account, but is regarded as being part of the capital of the company, and the balance of the premium account appears on the liabilities side of the balance sheet. It may be used to write off the preliminary expenses incurred in order to form the company, and for certain other special purposes.

If it is assumed that the 50,000 shares in the above example were issued at a discount of a shilling, the application and allotment account would appear as follows:

Application and Allotment Account

To Share Capital %	£25,000	By Cash	£22,500
		" Discount on Shares % ..	2,500
	£25,000		£25,000

An account for "discount on shares" would be opened, to which the £2,500 would be debited.

Discount on Shares Account

To Application and Allotment a/c	£2,500		

The discount is usually written off to the profit and loss account over a period of years, by the same method as wasting assets are depreciated.

Forfeiture of Shares

The articles of a company usually give the directors power to forfeit shares for the non-payment of calls. For example, suppose a final call of 5s. per share on 1,000 shares of £1 each remains unpaid. There will be a debit balance of £250 on the final call account, representing the debt due from the shareholder.

If the directors decide to forfeit these shares, the accounts necessary to record this transaction, assuming the full value of the issued share capital to be £50,000, would appear as follows:

Share Capital Account

To Forfeited Shares a/c ..	£1,000	By Balance ..	£50,000
" Balance ..	49,000		
	£50,000		£50,000
		By Balance .	£49,000

Forfeited Shares Account

To Final Call a/c	£250	By Share Capital a/c	£1,000
" Balance ..	750		
	£1,000		£1,000
		By Balance ..	£750

Final Call Account

To Balance ..	£250	By Forfeited Shares a/c ..	£250

The credit balance of £750 remaining on the forfeited shares account is the amount paid up on the shares. The share capital account is reduced by the *full* nominal value of the shares forfeited, because this is the amount that is included in the original credit to share capital. The balance of the forfeited shares account is a capital profit, because the shares have been cancelled and the cash is not returnable. Balances of this nature, however, are more usually shown on the liabilities side of the balance sheet.

Forfeited shares may be re-issued at a price not *less* than the amount unpaid by the defaulting shareholder. In this way the company receives the full nominal value of the shares in cash. If the shares were issued to a new subscriber at a figure below—in this example 5s. per share—this would be equivalent to issuing shares at a discount, which is illegal except in the circumstances just referred to.

But if the shares are re-issued at less than their nominal value, the difference must be made good by transfer from the forfeited shares account, since otherwise the credit balance on this account could be treated as a profit and the full nominal value of the shares would not therefore be capitalised.

It is assumed that the 1,000 shares are re-issued at 5s. per share.

Share Capital Account

		By Balance ..	£49,000
		" Forfeited Shares re-issued a/c ..	1,000
			£50,000

Forfeited Shares Account

To Forfeited Shares Re-issued a/c ..	£750	By Balance ..	£750

Forfeited Shares Re-issued Account

To Share Capital a/c	£1,000	By Forfeited Shares a/c ..	£750
		" Cash (5/- per share) ..	250
	£1,000		£1,000

The share capital account is now brought up to its correct figure of £50,000, all the shares being now fully paid up.

LESSON 15

Profits and Income Tax in Company Accounts

It has been explained that when a sole trader withdraws money from his business for his private purposes, a drawings account is debited, and the balance on this account is transferred direct to the debit of the trader's capital account. The net profit is credited to the capital account, and the final effect of these entries is that the balance of the capital account is increased by the profit but reduced by the withdrawals of the cash representing either the whole or part of the profit. For a number of reasons this procedure cannot be followed by joint-stock companies. The share capital account must always remain in the books at its original figure; it can be increased only by a new issue and can be reduced only in exceptional circumstances. The balance of the profit and loss account cannot be added, and drawings (or dividends) cannot be deducted.

The appropriation account is, in form, a continuation of the profit and loss account. The net profit is brought down to the credit of the appropriation account. The purpose of this account is to show what becomes of the profit—how much is to be used for paying dividends, and how much for other purposes.

The directors may decide that the business will be strengthened by a retention of part of the profits within the business. It must be remembered that a profit is represented by an increase in net assets, and if the directors wish to increase permanently the assets of the business, it follows that the whole of the assets which represent the net profit cannot be used for paying dividends.

In order to make this clear, an entry is made debiting the appropriation account and crediting an account called "general reserve," thus indicating that the assets representing this part of the profits are reserved and are not to be used to pay dividends.

To the appropriation account is also debited the balance of the income tax account. Income tax should not be debited to the profit and loss account proper, because it is not an expense which reduces the amount of the net profit; it is a payment made *out* of profits.

Dividend Account Credited

The purpose of the profit and loss account is to show expenses which reduce the amount of the gross profit, while the appropriation account shows what is done with the net profit which remains after charging all expenses incurred in earning it.

The amount of the dividend which has been agreed upon is debited to the appropriation account, and a dividend account is credited; this credit represents a liability to the shareholders. If any balance remains on the appropriation account, this balance appears as a separate item on the balance sheet as "profit and loss account."

This represents profits which have not been used for dividends, but which may still, unlike the general reserve, be used for dividends in subsequent years. The directors are quite at liberty, if they think fit, to declare dividends out of general reserve, but such a course is unusual.

Three Balances

There will now be three balances in the balance sheet of a limited company, which together correspond to the capital account of the sole trader, viz. share capital account, general reserve account, and profit and loss appropriation account. The general reserve and the final balance of profit and loss are of course, equal to the original net profits, less dividends and income tax (i.e. the sole trader's drawings.)

For example, a limited company has a net profit of £20,000. The directors propose to declare a dividend of 6 per cent on 50,000 preference shares of £1 each, and of 5 per cent. on 100,000 ordinary shares of £1 each, both fully paid up. £10,000 is to be transferred to general reserve, and the balance carried forward. The accounts necessary to record these transactions will appear as follows:

Appropriation Account

To	£	By	£
Preference Dividend ..	3,000	Net Profit *brought down*	20,000
Ordinary Dividend ..	5,000		
General Reserve ..	10,000		
Balance, *c/fd* ..	2,000		
	£20,000		£20,000

General Reserve

		By Appropriation a/c	£10,000

Assessment to Income Tax

Limited companies (and other businesses) are assessed to income tax upon the amount of their profits, calculated according to certain rules. When the tax in respect of these profits is paid, an income tax account is debited with the amount of the payment. Limited companies are empowered by law, however, to deduct income tax from dividends before making payment to the shareholders, who do not, therefore, receive the full amount of their dividend in cash.

The reason for this procedure is that the profits of the company are in final analysis the income of the shareholders, who are liable for tax upon the amount they receive. The tax is collected by the revenue authorities from the company for the sake of convenience, because it is simpler to make one assessment upon the company rather than several thousand assessments upon several thousand shareholders.

6% Preference Dividend Account

To	£	By	£
Cash	1,725	Appropriation Account ..	3,000
Income Tax ..	1,275		
	£3,000		£3,000

Ordinary Dividend Account

To	£	By	£
Cash	2,875	Appropriation Account ..	5,000
Income Tax ..	2,125		
	£5,000		£5,000

Income Tax Account

		By Pref. Div. ..	£1,275
		Ord. Div ..	£2,125

Assuming income tax to be at 8s. 6d. in the £, the individual shareholder receives in cash 11s. 6d. for every £1 of dividend due to him. But the appropriation account is debited with the full amount of the dividend, and the dividend account must be "credited" with the same amount. When the cash is paid to the shareholders, dividend accounts are debited; the balances are closed by a transfer to the credit of income tax account.

When the income tax (which is assessed upon the *whole* of the company's profits) is paid, the total payment will be debited to the income tax account. Because the credit to this account represents tax on only that part of the profits which have been distributed as dividends, the debit will be greater. The final balance of income tax account will be transferred to the debit of the appropriaton account of the following period.

LESSON 16

Preparation of the Balance Sheet

The balance sheet is a list of all the balances remaining in the ledger after the nominal accounts have been closed to the profit and loss account. The debit balances which appear on what is usually called the assets side of the balance sheet consist of assets and also of any expenses which have not been written off to the profit and loss account.

An example of the latter is a discount on the issue of shares, which was mentioned in a previous Lesson. Another example is the preliminary expenses incurred in the formation of a limited company. These expenses are usually so heavy that a false impression would be created if they were charged in total to the profit and loss account of the first year, and the usual procedure is to apportion preliminary expenses over a period of years.

Any part not written off remains in the ledger as a debit balance, and it must therefore appear on the "assets" side of the balance sheet, although not really an asset.

Fixed and Current Assets

Assets are divided into two classes—*fixed* and *current*. Current assets include cash and assets which will be eventually converted into cash. Fixed assets are those which are *not* held with a view to re-sale or conversion into cash but which are necessary in order to enable the business to be carried on, and the typical example of this class is plant and machinery. Fixed assets are in final analysis revenue expenditure paid in advance.

The essential point of difference between fixed assets and current working expenses is that the benefit derived from the expenditure upon fixed assets is of longer duration. By the time a fixed asset has worn out, its cost is just as much an expense as any other profit and loss item such as wages or rent.

The difference between fixed assets (such as plant and machinery) and expenses not written off (such as preliminary expenses of a limited company) is not so great as a first impression might lead one to suppose. Both appear on the assets side of the balance sheet, and both represent an expense incurred in the conduct of the business, the benefit from which is not yet exhausted.

The fact that fixed assets represent something solid and tangible, something which may have a market value, while unexpired expenses have no realizable value at all, is not so important as it might appear. It is true that if the company were wound up, the distinction would be of the utmost importance. The balance sheet, however, is a statement of the position of a going concern, not of a business about to be wound up.

In the latter event the expression "statement of affairs" is used; in the "statement of affairs," but not in the balance sheet, all assets are shown at realizable values.

The liabilities side of the balance sheet includes liabilities in the true sense, that is, liabilities to external creditors, and also the liability of the business to its proprietor or proprietors. The amount due to the proprietors is the original capital plus undistributed profits; in the case of the sole trader these are merged together in one account —the capital account.

In the case of the limited company undistributed profits must be shown separately, apart from the original capital. Since the two sides of the balance sheet agree, it follows that the total of assets minus liabilities to external creditors must be equal to original capital plus undistributed profits.

Dissimilar Items Shown Separately

A balance sheet can be drawn up in such a way as to disclose the true position of the business, or it can be drawn up in such a way as to conceal the true position. The first principle to be observed in the preparation of a balance sheet which is to serve its true purpose is to show dissimilar items separately.

For example, there may appear on a balance sheet "Freehold Land, Plant and Machinery, and Goodwill," only one total figure being

shown. It is impossible to appreciate the true position of the concern if the values of all these assets are not shown separately.

Freehold land is a valuable asset, while the goodwill may be unsaleable. In the absence of any information it is impossible to know what proportion of such a total figure represents a valuable asset, and what proportion represents something of little or no value.

The second principle is that assets should be shown in some logical order, not listed indiscriminately. Current assets and fixed assets should be separately grouped, and all assets should be shown in order of realizability. It is of no importance whether the most liquid or the least liquid assets come first, so long as a logical sequence is preserved. The usual order for a list of assets to appear in the balance sheet is :

A. Fixed Assets.	1. Goodwill.
	2. Patents and Trade Marks.
	3. Land and Buildings.
	4. Plant and Machinery.
	5. Furniture.
B. Current Assets.	6. Investments.
	7. Stock in Trade.
	8. Sundry Debtors.
	9. Bills Receivable.
	10. Cash.
C. Unexpired Expenditure.	11. Preliminary Expenses.
	12. Discount on Debentures.

The third principle to be observed is that the basis upon which assets are valued should be clearly indicated. Plant and machinery at cost is a very different thing from plant and machinery less 20 per cent. depreciation.

The values may be understated or overstated, and for a correct estimate of the position of business to be formed the disclosure of the method of valuation is essential. Furthermore, the valuation of the assets determines the profit, as shown by the profit and loss account.

The gross profit will be higher or lower, according as the value placed upon the closing stock is higher or lower. The greater the amount of depreciation written off fixed assets, the lower is the net profit, and vice versa. The greater the provision for bad and doubtful debts (i.e. the lower the value placed upon sundry debtors), the lower is the net profit.

The balance sheet and the profit and loss account are intimately linked. The balance sheet is, in a sense, the more important document—it is the key to the whole position of the business. The profit and loss account shows how the profit for the period has been earned, while the balance sheet shows what has become of it, i.e. by what assets it is represented. Considerations that should determine the values to be placed upon the assets are as follows.

Because current assets are held with a view to conversion into cash, they should be valued having regard to the amount they may be expected to realize. Thus debtors should be stated not at their nominal value but at a figure which makes appropriate allowances for all losses from bad debts.

Stock in trade should be valued at cost or, if the market price is lower than cost, at the market price ; if the market price is above cost, the higher value should be disregarded, otherwise a profit would be anticipated. It is a principle of accounting that no credit should be taken for a profit that has not been earned (and a profit can be said to be earned only when the goods are sold) ; all provision should be made for expected losses.

The profit of a business should never be overstated ; if the proprietors withdraw too much from the business, its financial position will be weakened. An understatement of profit, though equally incorrect in theory, strengthens the position of the business because it reduces proprietors' withdrawals.

Fixed assets should be written off over a period equal to their working life. In the case, for instance, of leases, the exact life of the asset is known. With furniture and machinery, it must be estimated. It is not usually necessary to depreciate freehold land—it is not a wasting asset ; but if the market price rises it would not normally be written up.

Loose tools are not usually depreciated by a fixed percentage, for it is very difficult to keep an exact record of a large number of small items.

The method is to revalue the stock of loose tools at the end of each accounting period. Thus if new tools are purchased during the year, the cost will be added to the balance brought forward from the previous period. The value placed upon the whole at the end of the year will be credited to loose tools account, and it will be brought down as a balance, and the difference will be written off to the profit and loss account.

Long-term and Short-term Liabilities

Liabilities can be divided into two classes, short- and long-term. Short-term liabilities are obligations which must be met either immediately or within a short space of time. Long-term liabilities are more in the nature of an investment of capital.

Economically, a loan for a period of, say, ten years is an investment of capital just as much as the purchase of shares in a limited company. In both cases funds which can be used for general business purposes are acquired, and in both there is freedom from the anxiety of any sudden demand for repayment.

Legally, a loan is distinct from a subscription to capital and must be separately stated in the balance sheet. In estimating the financial

position of a concern, however, long-term loans and capital may be conveniently classed together. Long-term liabilities may take the form of mortgages and debentures; short-term liabilities consist of credit balances on bought ledger accounts (trade creditors), bills payable, and expenses accrued.

Working Capital

The balance sheet is an interesting and sometimes complicated document. If it is badly drafted, it may be quite impossible to form a correct estimate of the true position of the undertaking. Even if it is properly drawn up, it may require close study before an intelligent opinion can be formed.

It is not sufficient to observe the figure on the profit and loss account. The correctness of that figure depends upon the values that are placed upon the assets. The basis upon which the assets are valued should therefore be closely scrutinised. The existence of a credit balance upon profit and loss account is not proof, in itself, that the business is in a sound position. It is of vital importance to ascertain the amount of the *working capital*.

The word "capital" is used, unfortunately, in a number of different senses; it is used to denote the proprietors' capital account, or, in the case of a company, the share capital account; it is also used to denote the assets which represent the original investment of capital, together with undistributed profits.

Assessment of Working Capital

Working capital denotes that proportion of original capital, plus undistributed profits, which is represented by current as distinct from fixed assets. In precise terms, working capital can be defined as the excess of current assets over current liabilities; that is, the amount by which the assets which will in the near future be converted into cash exceed the claims of creditors which will fall due in the near future.

If the current liabilities exceed the value of the assets which will, in the ordinary course of business, be converted into cash, the undertaking is in a dangerous position; it has no working capital. If current assets exceed current liabilities, the business is in a position to meet its obligations as they fall due, and the amount of the working capital is a measure of the firm's strength.

It is quite possible for a business to be short of working capital even if there is a large credit balance upon the profit and loss account. An excessive volume of funds may have been injudiciously invested in acquiring new fixed assets, and many businesses have literally been ruined by success.

Large profits have sometimes led to an over-rapid expansion and the excessive purchasing of new equipment, with the result that liquid funds are insufficient for immediate liabilities.

Long-term liabilities can be secured or unsecured. A secured creditor is one who has priority as to repayment over unsecured creditors. Certain assets can be legitimately earmarked against a loan, even though considerable amounts are due to trade creditors. In the event of the winding up of the business the proceeds of the earmarked assets would be first applied to paying the secured creditors in full, before the unsecured creditors can claim so much as a penny.

Long-term loans usually take the form of an issue of debentures. A debenture is a loan evidenced by a certain document analogous to a share certificate (which is evidence of the ownership of shares). Like shares, debentures are usually divided into amounts of fixed denominations. Debentures can be issued at their nominal value, at a discount, or at a premium. The book-keeping entries are similar to those which are necessary to record the issue of shares.

Classes of Debentures

Debentures can be of three classes: naked, specific, or floating. A naked debenture is simply an unsecured loan, and this is uncommon. Specific and floating debentures are both secured loans, the security in each case being different in nature.

A specific debenture is one which is secured by the mortgage (or the earmarking) of certain specific and usually fixed assets. The company can continue to use for business purposes the fixed assets which have been mortgaged but, it is not at liberty to dispose of them.

A floating debenture is one which is secured by some or all of the assets of the company, present and future. Under a floating charge, the company can do as it likes with its assets; it can buy and sell, and continually change their nature.

If the floating charge covers (as it invariably does) current assets, this provision is necessary, because a continual change in the constitution of the current assets must take place if the business is to be carried on. Debtors pay their debts, and cash takes the place of book-debts; stock is sold, and book-debts take the place of stock; stock is purchased, and so on.

When a Floating Charge Crystallises

A floating charge is said to crystallise when a specified event happens. The specified event, in practice, is the insolvency of the company, or at least grave danger of insolvency. When a floating charge crystallises, the control of the company over the assets ceases; it can no longer dispose of the assets or change their nature in any way. Control normally

passes to a receiver, who is the agent of the debenture holders, and whose duty it is to realize the assets, so that the debenture holders can be repaid out of the proceeds.

A floating charge is generally wider than a specific or fixed charge, for it usually covers a larger volume of assets, but it carries with it the disadvantage that the company is not restricted as to the use it makes of the assets, and the value of the security therefore depends upon the general position of the company.

Furthermore, specific mortgages of certain assets may be created subsequent to the floating charge, and ranking prior to it (as to the assets specifically mortgaged), even though the floating charge is expressed as covering all assets, present and future. Thus from some points of view a specific debenture is preferable to a floating debenture.

The conditions of an issue of debentures are usually set out in a document called the debenture trust deed. If the proceeds of the secured assets are insufficient to meet the claims of the debenture holder, he can claim for the balance on the same level as the unsecured creditors. All debentures must be registered with the Registrar of Companies at Bush House, Strand, London.

LESSON 17

Balance Sheets and the Companies Act

It is not proposed here to attempt a survey of company law. A very brief outline of the legal procedure in the formation of a limited company is set out in Lesson 14. But as Lesson 16 has been devoted to the examination of the balance sheet, it is appropriate to review the provisions of the 1948 Act which are relevant.

The general policy guiding the drafting of the Act was the insistence upon the need for the disclosure of fuller information in the published accounts of limited companies. Such a reform was by no means premature. The following information must appear on the balance sheet.

(1) A summary of the authorised and of the issued share capital.

(2) A summary of liabilities and assets.

(3) Fixed assets must be distinguished from current assets ; it is illegal to group any fixed and current assets in one composite item.

(4) The basis of valuation of fixed assets must be stated.

(5) The following must be stated under separate headings : (a) preliminary expenses ; (b) expenses in connexion with the issue of shares and debentures and separate sums paid as commissions on shares or debentures, in so far as they are not written off ; (c) goodwill, patents, and trade-marks; (d) the total amounts of (i) capital reserves, (ii) revenue reserves, and (iii) provisions ; and amounts added or deducted during the year.

(6) If any liability of the company is secured by a charge on any of the assets, this fact must be stated, although it is not necessary to disclose the nature of the security.

(7) Any loans to directors must be separately stated in the balance sheet, including any loans repaid during the period of the accounts ; e.g., suppose on Jan. 1 the sum of £1,000 is due from a director to the company. On December 30, £950 is repaid. The balance of £50 would be set out in the balance sheet as follows :

Loans to Directors		
As at January 1	£1,000	
Less Repayments	950	
As at December 31		£50

It would otherwise be possible for the director to borrow the £950 again on January 1 in the next period, to repay it on the following December 30, and thus to remain almost permanently in possession of £1,000 without the disclosure of the fact in the balance sheet.

(8) The total remuneration of the directors must be separately stated in the profit and loss account, with the exception of the managing director's salary.

(9) It is illegal for a company to purchase, either directly or indirectly, its own shares, except by way of loan to employees or to trustees for employees, in order to enable the employees or their trustees to acquire fully-paid shares. Any such loans must be separately stated in the balance sheet.

(10) There are certain provisions governing the balance sheets of holding companies and these are dealt with later in this Course.

Redemption of Preference Capital

The provisions governing the issue of redeemable preference shares will now be described. Before the 1929 Act it was illegal for a company to purchase its own shares or to redeem any of its share capital by repayment to shareholders. Under the 1929 Act it became possible for a company to issue redeemable preference shares. The conditions are :

(1) Such shares can be redeemed only when fully paid. (2) They can be redeemed only (a) out of profits, or (b) out of the proceeds of a new issue of capital.

If the shares are redeemed out of profits, an amount equal to the sum applied in

redemption must be transferred from profit and loss account (or from general reserve) to the credit of an account called "capital redemption reserve fund." The amount of this fund cannot be written back to the profit and loss account, nor can it be used for paying dividends.

In this way a part of the company's profits takes the place of preference share capital which has been redeemed; and the assets representing these profits take the place of the assets representing the capital, in so far as they cannot be used for dividends. Thus the general principle that the capital fund available for creditors may not be deliberately reduced (except in special circumstances) remains unaltered.

Out of General Reserve

For example, a limited company has issued 40,000 redeemable preference shares, redeemable at a premium of one shilling per share. The directors decide to redeem the whole issue out of general reserve, which stands at £70,000. The entries will be:

Redeemable Preference Shares Account

To Sundry Shareholders ..	£40,000	By Balance ..	£40,000

General Reserve

To Capital Redemption Reserve Fund ..	£40,000	By Balance ..	£70,000
To Sundry Shareholders (Premium on redemption) ..	2,000		
To Balance ..	28,000		
	£70,000		£70,000
		By Balance ..	£28,000

Capital Redemption Reserve Fund

		By General Reserve ..	£40,000

Sundry Shareholders

To Cash	£42,000	By Redeemable Preference Shares Account	£40,000
		By General Reserve ..	2,000
	£42,000		£42,000

It will be noticed that the premium on the redemption is provided for out of the undistributed profits, and that an amount equal to the original balance of the redeemable preference shares account is capitalised in the capital redemption reserve fund.

The law has always stressed the importance of preserving intact the capital of limited companies. Such a requirement must logically accompany the privilege of limited liability, for if creditors have no claim upon the private property of members it is only fair that they should be assured of the integrity of the capital that has been subscribed.

Reduction of Capital

In some circumstances, however, reduction of capital is permissible. Reduction can take any one of the following forms: (1) cancellation of unissued capital; (2) forfeiture of shares; (3) cancellation of uncalled liability; (4) return of cash to shareholders; (5) writing off capital on account of losses or permanent decline in asset values.

In the first two instances there are no formalities. A cancellation of unissued capital affects nobody; forfeiture of shares is possible only on the default of the shareholder. In the remaining three instances a general meeting of the company must be called and the reduction sanctioned by a special resolution; furthermore, the consent of the court is required.

In (3) and (4)—the cancellation of uncalled liability, and a return of cash to shareholders—the fund available for creditors is reduced, and before sanctioning the reduction the court will order an inquiry into the liabilities of the company, and *all* the creditors must either consent to the reduction or be paid in full before the reduction can take effect.

Permanently Depreciated Assets

The fifth is the usual form of reduction. In this instance, as the fund available for creditors is not affected, the court makes no inquiry into the liabilities of the company. The transaction is purely a book-keeping one.

If assets have permanently depreciated in value, or if a large debit balance exists on the profit and loss account, the procedure is simply to credit the appropriate asset accounts or the debit balance on the profit and loss account, thus reducing or eliminating them, and debiting share capital account.

The gain from such a procedure may not at first sight be apparent, but the step is a logical one. Part of the original capital has, in fact, been lost, and it is therefore desirable to show the true position by bringing the accounts into line with the actual facts.

If a company has passed through a bad time and entered upon a period of prosperity, the continued existence of a debit balance upon profit and loss, or the slow writing off of assets that have lost a large part of their value, gives a false colour to the accounts.

LESSON 18

Depreciation and Sinking Funds

It is the purpose of this Lesson to examine four of the more important methods by which depreciation is dealt with. The first is called the *Straight Line Method*, by which the cost of the fixed asset (less any residual scrap value) is charged to profit and loss account by equal annual instalments throughout the life of the asset.

This method is certainly the most straightforward, and is in many respects the best, for reasons which will become apparent as the other methods are considered.

The second method is the *Diminishing Balance Method*, by which a fixed percentage, not of the original cost of the asset but of the balance brought forward from the preceding year, is written off annually. The advantage of this method is its simplicity. Theoretically, however, it is indefensible, since its effect is to charge the profit and loss accounts of the earlier years with an unduly heavy proportion of the cost of the asset.

For example, take an asset costing £100, and assume the rate of depreciation to be 10 per cent. In the first year £10 is written off, leaving a balance of £90; in the second year £9 is written off and £81 is carried forward; in the third year £8 2s. 0d. is written off; in the fourth year £7 5s. 9d.; and so on. It is absurd to spread the cost in such an uneven manner. Each year derives a similar benefit from the asset, and each year should bear an equal charge. This method is, however, sometimes used, since the Straight Line Method involves a separate calculation for each machine or individual asset.

The third method is the *Annuity Method*. Under this system the asset account is debited at the end of each financial year with interest on the balance brought forward at the commencement, and this interest is credited to profit and loss account. It must not be thought that this interest is received in cash or that it represents a transaction with any outside individual; it is purely a matter of straightforward book-keeping.

Secondly, the asset account is credited with a *fixed amount* of depreciation each year, and profit and loss account is debited. Because the balance of the asset account diminishes from year to year, the interest which is debited thereto and credited to profit and loss account will diminish also, because this interest is calculated on the value of the balance at the commencement of the year.

And because the depreciation charge which is debited to profit and loss account is fixed and does not diminish, it follows that the *net debit* to profit and loss account (i.e. the fixed debit for depreciation minus the diminishing credit for interest) must increase each year.

The fixed amount of depreciation is so calculated that the asset account will be closed at the end of the life of the asset. Interest on the balance brought forward at the commencement of the last year of the asset's life will bring the asset account up to a figure which exactly equals the fixed amount of depreciation to be written off. The mathematics by which the annual instalment of depreciation is calculated does not concern us here.

This method is made clear by a simple illustration. Take the example of a two years' lease dating from January 1, 1956, costing £205. (This unusually short period is assumed for clarity of exposition). Interest is to be calculated at 5 per cent. per annum. The accounts will be as follows:

Lease Account

1956			1956		
Jan. 1	To Cash ..	£205 0 0	Dec. 31	By Depreciation (to P. and L.) ..	£110 5 0
Dec. 31	To Interest (to P. and L.)	10 5 0		" Balance ..	105 0 0
		£215 5 0			£215 5 0
1957			1957		
Jan. 1	To Balance	£105 0 0	Dec. 31	By Depreciation ..	£110 5 0
Dec. 31	To Interest	5 5 0			
		£110 5 0			£110 5 0

Thus, in 1956, Profit and Loss Account is debited with	£110 5 0
And credited with	10 5 0
NET DEBIT	£100 0 0

In 1957, Profit and Loss Account is debited with	£110 5 0
And credited with	5 5 0
NET DEBIT	£105 0 0

At first sight it seems inequitable that the depreciation charge should increase with each successive year. The theory of the annuity method is this: as depreciation is written off, the capital invested in the asset is gradually recovered.

For instance, if a profit of £1,000, calculated before charging depreciation, is earned, it

follows that the net assets of the business have also been increased by £1,000, If a depreciation charge of £200 is debited to profit and loss account, the net profit is reduced to £800. but the increase in assets (other than the fixed assets which have been depreciated) is still the sum of £1,000.

The fixed asset has been reduced in book value by £200; the net profit is £800; the increase in other assets is £1,000. Of this £1,000, £800 represents the profit and £200 represents part of the capital previously invested in fixed assets.

The fourth method of dealing with the problem of depreciation is the *Sinking Fund Method.* This is used where it is necessary to provide a liquid fund to replace the asset at the end of its working life.

Lease Account

		£			£
1956			1957		
Jan. 1	To Cash ..	205	Dec. 31	By Sinking Fund	205

Sinking Fund Account

		£			£
1957			1956		
Dec. 31	To Lease Account	205	Dec.31	By Profit and Loss Account ..	100
			1957		
			Dec. 31	By Interest (5%)	5
			Dec. 31	By Profit and Loss Account ..	100
		£205			£205

Investment Account

		£			£
1956			1958		
Dec. 31	To Cash ..	100	Jan. 1	By Cash (proceeds of Sale of Investment) ..	205
1957					
Dec. 31	To Interest ..	5			
	" Cash ..	100			
		£205			£205

Profit and loss account is debited each year with a fixed sum for depreciation, which is not credited to the asset account but to a sinking fund account. Each year an amount equal to this fixed sum is invested in some gilt-edged security. The interest on this investment is re-invested in the same security. The investment account is therefore debited with the interest, and the sinking fund account is accordingly credited.

The fixed sum is so calculated that at the end of the working life of the asset the value of the investment and the credit balance on sinking fund account are exactly equal to the cost price of the asset. The asset account is transferred to the debit of the sinking fund account, both accounts (because they are equal) being closed. The investment is realized, and the proceeds are used to purchase a new asset.

Taking the same simplified figures that were used in the example of the annuity method, the accounts would appear as set out in the adjacent column.

The process by which sinking fund is credited, and the asset account transferred thereto at the end of the life of the asset, is not in principle different from the process by which the asset account is directly credited with depreciation. The final result is the same in both instances.

It must be emphasised that there is no *direct* connexion between the credit to sinking fund and the purchase of the investment for cash. The purchase of the investment could be made even if the ordinary direct method of depreciation were used. The purpose of showing the asset at cost price and the sinking fund on the liabilities side is to make it clear that the investment is earmarked and that it cannot be used for general purposes.

A sinking fund may be created for the redemption of a liability (e.g. debentures). The entries are similar: (1) debit profit and loss account and credit sinking fund; (2) credit cash and debit investment account. Interest is treated as before. When the debentures are to be repaid, the investment is realized, and cash is then credited and the debenture account debited.

The sinking fund account in this instance *remains in the books,* and is transferred to general reserve. The reason for this important difference is that the depreciation of a fixed asset is a charge against profits, while the repayment of a liability cannot be a charge against profits. The credit to the sinking fund in the latter instance would amount to an appropriation of profits.

LESSON 19

Company Profits, Provisions, and Reserves

It is a ruling principle of company law that limited companies may not use their capital for the purpose of paying dividends. The capital of the company must, as far as possible, be preserved intact for the protection of the creditors.

It is not always easy to determine whether or not a company may legally declare a dividend, because the law on the subject is very confusing, and some of the decided cases appear to be almost contradictory. The following general principles can be laid down.

(1) A company may distribute current profits without making good past losses. Thus the existence of a debit balance on profit and loss account, representing the losses of past years, does not prevent a company from declaring a dividend, if the profit and loss account of the current year definitely shows a surplus.

It has been held that if a part of the capital has in the past been sunk and lost, then it can be no more used in paying dividends than in paying debts. Therefore a dividend declared on the basis of current profits is not paid out of capital, although it is true that the past losses have not been made good. But the omission to replace capital that has been sunk and lost is a very different thing from using capital to repay dividends.

(2) Before arriving at such current profit it is necessary to provide for depreciation of current assets, held with a view to conversion into cash, and also for depreciation of fixed assets which require replacement. The last clause is very important. Its effect is that it is not legally necessary to provide for depreciation of fixed assets that do not require replacement.

(3) It should be noted that it is possible for profits to be of a capital nature. For example, if a freehold building is sold at a price above its original cost, such an exceptional profit must be distinguished from a normal profit on trading. Capital profits are available for dividend, but only when they have been realized in cash, and provided that any capital losses are fairly offset, and that the results of the year as a whole are taken into account.

(4) Whatever the circumstances, a dividend can never be paid if it leaves a company unable to pay its debts.

Provisions and Reserves Defined

The Companies Act, 1948, draws a clear distinction between provisions and reserves. The expression *provisions* means any amount written off or retained in order to provide for depreciation, renewals, or a diminution in the value of assets, or retained in order to provide for a known liability the exact amount of which is not certain (e.g., assessed income tax, claim for compensation, etc.).

The term *reserve* must not be applied to such a provision but only to any part of it which is thought by the directors to be in excess. Reserves are therefore profits retained or reserved for specific or general purposes other than to make good actual or possible liabilities and losses (e.g., future income tax, or a fall in the value of stocks.)

Capital reserves are those resulting from capital transactions, such as the issue of shares at a premium or the redemption of redeemable preference shares, and must not include any amount that is regarded as free fo distribution through the profit and loss account.

All other reserves are *revenue reserves*. They constitute the fund of retained profit. Capital reserves, revenue reserves, and provisions must be stated separately

Specific Provisions

A provision may be created to provide for expenses accruing, or for the depreciation of an asset. Examples are provisions for outstanding expenses and provision for bad and doubtful debts. In some instances the depreciation of fixed assets is dealt with by crediting a "Provision for Depreciation" account, instead of crediting the asset account directly ; the asset account is closed off to the provision account at the end of the working life of the asset. This procedure is similar to the method by which a sinking fund is created, but in this instance no corresponding investment is made.

Provisions of this type are all for revenue expenditure, and are called specific provisions. They will appear in the balance sheet as liabilities or on the asset side as deductions from assets ; a provision for bad and doubtful debts is a deduction from sundry debtors ; a depreciation provision should be deducted from the appropriate fixed asset

Classes of Reserves

It will be observed that the effect of the creation of specific provisions is to reduce the value of the net assets ; this is quite correct, since revenue expenses must necessarily be accompanied by a decrease of assets or an increase of liabilities.

Reserves are appropriations of profits. General reserves are simply undistributed profits. Reserves for redemption of liabilities, though apparently of a different nature, are nevertheless similar. When a liability is repaid, cash is credited, and the liability account is debited. Such a transaction involves neither profit nor loss.

If a reserve is created (by transfer from the appropriation account) in connexion with the repayment of a liability, the purpose of this transfer is to indicate that part of the assets representing the profits may not be used for dividend purposes, but must be set aside in order to provide a fund for redeeming the liability.

In the same way, a general reserve is created in order to make it clear that it is proposed to increase the working capital of the business by retaining part of the increase in assets produced by the profits.

A secret reserve exists when assets are understated or liabilities are overstated in the balance sheet. If net assets are shown in the

balance sheet at less than their true value, the balance of undistributed profits must also be shown at less than its true figure.

The following are examples of methods by which secret reserves can be created. 1. Excessive depreciation of fixed assets. 2. Undervaluation of stock. 3. Excessive provision for bad debts. 4. Reserves for remote contingencies. 5. Writing down goodwill. 6. Retaining fixed assets such as land and building at the original cost during a period of rising prices.

Purpose of Secret Reserves

The principal object in creating secret reserves is to enable the directors of a company to tone down fluctuations in profit by decreasing the amount of the reserves in bad times, and increasing them in good times. The creation of a reserve has the effect of diminishing the apparent profit ; if asset values are increased, the apparent profit is also increased. Thus the manipulation of asset values makes it possible to show an apparent profit that is greater or less than the profit actually earned.

It is undeniably true that the modification of fluctuations in earnings is an important factor in the maintenance of confidence. Many a business has averted a panic in an exceptionally bad year by drawing on its secret reserves. The most efficiently managed concern can at times suffer reverses, but its reputation might suffer severely if the fact were known.

The utilisation of secret reserves is unquestionably a very useful means of averting a loss of confidence that might have consequences entirely out of proportion to the extent of the losses actually incurred. It should be noted that in no instance does the balance sheet overstate the true position. The real value of the assets is understated to a greater extent at some times than at others.

But there is always the danger that secret reserves may be used for improper purposes. It is the duty of the auditors to see that this is not done.

Whatever may be the justification for the proper use of secret reserves, the fact remains that when they exist, the balance sheet does not disclose fully the actual position of the company. It has been laid down that "the purpose of the balance sheet is primarily to show that the financial position of the company is *at least as good as* there stated, and not to show that it is not, or may not be, better."

The auditor always has the power to report the existence of secret reserves, and he must be satisfied that their existence is for the benefit of the company as a whole.

The power of boards of directors to create and manipulate secret reserves has been somewhat lessened by the detailed requirements relating to accounts, set out in the eighth schedule of the Companies Act, 1948, and those concerning the form of auditor's certificate, in the ninth schedule.

For example, depreciation of assets has now to be shown separately, and the corresponding figures of the previous balance sheet must be stated for comparison. It still remains true that although auditors have to certify that the balance sheet and profit and loss account give "a true and fair view," balance sheets seldom tell the whole truth, and have to be interpreted both wisely and carefully.

LESSON 20

Amalgamations and Holding Companies

One of the most distinctive features in recent years has been the marked growth in the size of the business unit. The process of integration takes many forms, but the most typical are : amalgamations of two or more companies ; absorption of small by large undertakings ; and the creation of "holding companies," which own either the whole or the greater part of the shares of other companies, called subsidiaries.

Accountancy of Absorptions

The accounting aspect of a case of absorption will be first considered. The accountancy of amalgamations is almost identical with that of absorptions, since in the former case a new company is usually formed to take over the assets and liabilities of the amalgamating companies. The entries in the books of the companies that are taken over are analogous to the entries for a dissolution of partnership, because the old companies are wound up.

As an illustration of the accounting necessary in such circumstances, suppose that X Ltd. agrees to absorb the undertaking of Y Ltd. The condensed balance sheets of the two companies are :

X Limited

	£		£
Ordinary Share Capital ..	200,000	Sundry Assets ..	370,000
5% Debentures	50,000	Cash at Bank ..	80,000
Sundry Creditors	160,000		
Profit and Loss Account ..	40,000		
	£450,000		£450,000

Y Limited

	£		£
Ordinary Share Capital	100,000	Sundry Assets	140,000
Preference Share Capital	25,000	Profit and Loss Account	50,000
5% Debentures	40,000	Goodwill	10,000
Overdraft at Bankers	5,000		
Sundry Creditors	30,000		
	£200,000		£200,000

The directors of X Ltd. agree to pay off the debenture holders of Y, and to pay a further £95,000 for the assets, to be discharged as to £30,000 in cash, and as to the balance by the issue of 65,000 fully paid ordinary shares of £1 each in X Ltd. The creditors are to be paid out of the proceeds of the sale of the undertaking, and, with the exception of the bank, agree to accept 16s. 8d. in the £ in settlement.

The preference shareholders agree to accept 3 shares in X for every 5 shares in Y, and the ordinary shareholders agree to accept 1 share in X for every 2 shares in Y.

The market value of the shares of X is assumed to be equal to their nominal value. The entries in the books of Y will be as follows:

Realization Account

To—	£	By—	£
Sundry Assets	140,000	5% Debentures	40,000
Goodwill	10,000	X Ltd.	95,000
Profit and Loss Account	50,000	Discount on Creditors	5,000
		Sundry Shareholders:	
		Preference	10,000
		Ordinary	50,000
	£200,000		£200,000

X Limited

To—	£	By—	£
Realization Account	95,000	Cash	30,000
		Shares in X at par	65,000
	£95,000		£95,000

5 per cent. Debentures Account

To—	£	By—	£
Realization Account (taken over by X Ltd.)	40,000	Balance	40,000

Sundry Creditors (*including Bank Overdraft*)

To—	£	By—	£
Realization Account (discount)	5,000	Balance	35,000
Cash	30,000		
	£35,000		£35,000

Preference Share Capital Account

To—	£	By—	£
Preference Shareholders	25,000	Balance	25,000

Ordinary Share Capital Account

To—	£	By—	£
Ordinary Shareholders	100,000	Balance	100,000

Preference Shareholders

To—	£	By—	£
Realization Account	10,000	Preference Share Capital Account	25,000
Shares in X Ltd.	15,000		
	£25,000		£25 000

Ordinary Shareholders

To—	£	By—	£
Realization Account	50,000	Ordinary Share Capital Account	100,000
Shares in X	50,000		
	£100,000		£100,000

Shares in X

To—	£	By—	£
X Ltd.	65,000	Preference Shareholders	15,000
		Ordinary Shareholders	50,000
	£65,000		£65,000

All assets (including debit balance on profit and loss account) are transferred to the realization account, which is credited with the proceeds (in total, £135,000) since the assumption by X Ltd. of the debentures is equivalent to a cash payment to Y, because Y Ltd. would have been bound to discharge this liability.

The realization account is also credited with the discount on creditors, leaving a balance of £60,000 to be written off. In a partnership, this amount would be debited to partners' capital accounts; in the case of a limited company, the share capital accounts must, first, be transferred in total to the shareholders' accounts, and the loss is then debited to these accounts.

The proceeds (shares in X Ltd.) are distributed to the shareholders, and this is recorded by crediting the shares in X account and debiting the shareholders' accounts.

The entries in the books of X Ltd. are simple. The assets acquired from Y Ltd. are debited to the appropriate accounts at purchase price, i.e. £135,000. Thus goodwill disappears, and the assets which formerly stood in the books of Y at £140,000 will be reduced to £135,000. This double entry will be completed by credits to cash £70,000 (including discharge of debentures) and to share capital £65,000.

The balance sheet of X Ltd., after these transactions, will appear as follows:

X Limited

	£		£
Share Capital	265,000	Sundry Assets	505,000
Debentures	50,000	Cash at Bank ..	10,000
Sundry Creditors ..	160,000		
Profit and Loss Account ..	40,000		
	£515,000		£515,000

The term *holding company* is often used to denote a company which has been formed merely to hold the shares of other companies and to control their activities. But the Companies Act, 1948, is much more specific :

"A company shall be deemed to be another's holding company if, but only if, that other is its 'subsidiary'; and a company is a subsidiary of another if that other is a member of it (i.e., holds its shares) and controls the composition of its board of directors, or holds more than half in nominal value of its equity share capital, or the first company is a subsidiary of any company which is that other's subsidiary."

A company which controls any other company is therefore to that extent a holding company.

The purpose of the complicated provisions in the Companies Act about the accounts of holding companies is to make it impossible for the directors of a holding company to conceal the general nature of the relationship between it and the subsidiaries.

Thus the value of the shares owned in subsidiaries and the amounts owing by the subsidiaries to the holding company or to other subsidiaries, and the indebtedness of the holding company to subsidiaries, all have to be shown on the balance sheet or explained in notes. Aggregate net profits must be stated.

The intention is that the holding company shall submit consolidated accounts, expressing the combined effect of its own balance sheet and profit and loss account and those of all the subsidiaries. If that is not done, the reason has to be explained ; and if the financial year of a subsidiary does not correspond with that of the holding company, the difference has to be stated and justified. Further, significant points in the reports of the auditors of the subsidiary companies have to be repeated as notes in the consolidated accounts.

Accountants had anticipated some of the requirements of the Companies Act, 1948, by evolving consolidated balance sheets. The principle is simply to substitute for the item in the balance sheet of the holding company "Shares in Subsidiaries," the assets and liabilities representing these shares. The following is a simple illustration of a consolidated balance sheet.

Holding Company

	£		£
Share Capital	200,000	Cash	5,000
General Reserve ..	40,000	Stock	110,000
Sundry Creditors ..	250,000	Debtors ..	135,000
Profit and Loss Account ..	10,000	Fixed Assets ..	150,000
		Shares in Subsidiary (at cost)	100,000
	£500,000		£500,000

Subsidiary Company

	£		£
Share Capital	100,000	Cash	2,000
Debentures ..	50,000	Stock	48,000
Sundry Creditors	150,000	Debtors ..	110,000
		Fixed Assets ..	140,000
	£300,000		£300,000

Consolidated Balance Sheet

	£		£
Share Capital	200,000	Cash	7,000
Debentures ..	50,000	Stock	158,000
General Reserve ..	40,000	Debtors ..	245,000
Sundry Creditors	400,000	Fixed Assets ..	290,000
Profit and Loss Account ..	10,000		
	£700,000		£700,000

It will be observed that the share capital of the subsidiary cancels out with shares in subsidiary on the assets side of the holding company's balance sheet.

This is the consolidated balance sheet in its simplest form. If the cost price of the shares of the subsidiary is not equal to their nominal value, or if the holding company does not own all the shares, the construction of a consolidated balance sheet becomes a much more complicated matter.

LESSON 21

Accounting Aspect of Income Tax

INCOME Tax is a difficult and complex subject, and in this Course it will be considered only in relation to business profits. It is not proposed to attempt any survey of the whole field, but an examination of the main principles upon which profits are taxed will be instructive.

For convenience of administration, the income or profit arising in any year is deemed, for income tax purposes, to be the income of the following year. It would be impossible to levy tax during any year upon the profits of that year, because the amount of the profit is not

known until the year is ended. Assessments are therefore based upon the preceding year's profits.

The fiscal year runs from April 6 to April 5. Because it rarely happens that business accounts are made up to the same date, it becomes necessary to base the assessment, in practice, upon the profits arising during the accountancy period ending within the preceding year. Thus, the profits for the accountancy year ending on December 31, 1956, become the basis of assessment for the fiscal year April 1957 to April 1958.

The tax is paid, by individuals and partners, in two instalments, on January 1, in the year of assessment, and on July 1, in the following *fiscal* year (i.e. six months later). Thus the actual profit for the calendar year 1956 becomes the statutory profit for 1957–58, and the tax on this profit is paid on January 1 and July 1, 1958. Limited companies are required to pay the whole of the tax on January 1 of the year of assessment.

Accruing Due by Instalments

For accounting purposes, income tax is regarded as accruing due by equal instalments during the *fiscal* year to which it relates. For example, the profit for the year to December 31, 1956, is the statutory income for the year 1957–58, and income tax on this statutory income is deemed to accrue during the period April 1957 to April 1958.

Thus, when the accounts for the year ending December 31, 1957, are made up, approximately three-quarters of the income tax liability for the year 1957–58 is deemed to have accrued due, although payment does not fall to be made until January and July, 1958. Three-quarters of the tax payable for 1957–58 is reserved on December 31, 1957, and appears in the balance sheet for that date as a liability.

For the purposes of illustration we assume that the profits of a company are as follows:

Year to December 31, 1955	£4,000
„ „ „ „ 1956	£6,400
Assessment for 1956–57 is	£4,000
„ „ 1957–58 is	£6,400
Tax Payable for 1956–57 (at 10s. in £)	£2,000
„ „ „ 1957–58 .. „ „ ..	£3,200

In accounts for the year to December 31, 1956, it is necessary to reserve three-quarters of the tax for 1956–57, that is, £1,500. This sum is debited to income tax account, and brought down as a credit balance, and appears in the balance sheet as a liability. During 1957 the whole of the £2,000 is paid, and debited to income tax account. On December 31, 1957, three-quarters of the liability for 1957–58, £2,400, must be reserved.

If we assume that the general meeting of the company was held on March 31, 1957, and a dividend of £4,000 was declared, out of the profit of £6,400 earned during 1956, income tax account will be credited on that date with £2,000, leaving only £900 to be charged to the appropriation account at December 31, 1957.

It will be seen that the amount written off to appropriation account is equal to one-quarter of the 1956–57 liability (£500) plus three-quarters of the 1957–58 liability (£2,400) minus tax deducted from a dividend declared and paid during 1957.

With regard to the foregoing method of reserving for income tax liability in the accounts, it is becoming more customary to provide for the estimated liability to income tax on the profits of the period as shown by the accounts.

Relief Claimed for Losses

It is only fair that losses should be offset against profits in computing income tax liability. There are three important clauses under which relief can be claimed for losses.

1. Rule 13, Cases 1 and 2, Schedule D (Income Tax Act, 1918) by which a person who carries on, or is a partner in, more than one trade may offset a statutory loss in one trade against a statutory profit for the same year in the other trade.

2. Section 34 (Income Tax Act, 1918), by which a person may offset an *actual* loss in any one year against any statutory income for that year. Thus if a business showed a profit of £500 for the accounting year to December 31, 1955, this would be the statutory profit for 1956–57. If the accounts to December 31, 1956, showed a loss of £400, this loss could be offset against the statutory income of £500, and tax on £100 only would be payable for the period 1956–57.

3. Section 33, Finance Act, 1926, by which business losses, for which relief has not been claimed under either of the preceding clauses, may be offset against the profits of subsequent years. Losses cannot, however, be carried forward for more than six years; thus if a statutory loss for the year 1951–52 were greater than the total of the statutory profits for the years 1952–53 to 1957–58, the excess could not be set off against any profit for 1958–59, and such a profit would be liable to tax in full.

Annual Interest and Other Charges

In Lesson 15 the procedure by which income tax is deducted from dividends was explained. The same method is applied to annual interest and other charges, such as ground rent. Any person making such a payment deducts income tax from the gross amount due.

Thus if the ground rent on a leasehold house is £8, and income tax is 10s. in the £,

the landlord receives only £4 in cash. His position is for tax purposes the same as the shareholder who receives a dividend less tax, in that he is not called upon to pay any further tax on that income. The person making the payment must, of course, account to the revenue authorities for the full amount of the tax he has deducted.

Net Profit and Statutory Profit

The position is complicated by the fact that annual interest and charges are allowed as deductions from gross income in the calculation of taxable income. Thus if the net profit of a company, before charging debenture interest, is £1,000, and the interest is £100 gross, the income of the company is £900.

The company must also hand over to the revenue authorities the £50 tax deducted from the gross amount of the debenture interest, since the debenture holders received only £50 in cash. In order to avoid making two assessments, the £50 is collected not by a separate assessment but by "adding back" the £100 to the £900 net profit of the company, thus making the statutory profit of the company the sum of £1,000.

The letter of the law is to the effect that charges from which tax has been deducted are not allowable as deductions from profits.

Assuming the foregoing figures, the statutory profit of the company is £1,000, upon which £500 income tax is paid. In reality, the profit of the company is £900, tax upon which is £450; in addition, the company hands over the £50 deducted in paying interest.

Income Tax Account

1957		1957	
Jan. 1		Jan. 1	
To Cash ..	£2,000	By Provision .. ($\frac{3}{4}$ of 1942–43 liability)	£1,500
Dec. 31		Mar. 31	
To Provision .. ($\frac{3}{4}$ of 1943–44 liability)	2,400	By Dividend ..	2,000
		Dec. 31	
		By Profit and Loss Account (Appropriation Section) ..	900
	£4,400		£4,400
		1958	
		Jan. 1	
		By Balance ..	£2,400

The procedure works well enough when the profits are sufficient to cover the charges. Supposing, however, the profit and loss account of the company to be as follows:

Profit and Loss Account

To—		By—	
Debenture Interest ..	£100	Balance	£20
		Net Loss	80
	£100		£100

Under the normal method of assessment the statutory profit is:

Net Loss per Accounts	£80
Less Debenture Interest	100
Adjusted Profit	£20

If the company paid tax only on £20, it would not be handing over to the revenue the full amount of the tax deducted from the debenture interest. In order that the tax on the £100 may be recovered, Rule 21 of the General Rules (Income Tax Act, 1918) makes provision for additional assessments in such cases.

In this example there would be a Rule 21 assessment on £80. The company would pay tax on £20 (viz. £10) under Schedule D, and on £80 (viz. £40) under Rule 21. Any assessment made under Rule 21 may be carried forward in the same way as losses under Section 33 (Finance Act, 1926). This is equitable, because the true loss of the company is £80.

The profit shown by the profit and loss account is not necessarily the statutory profit for income tax purposes. Interest and charges from which tax has been deducted are not allowable as deductions from profits, but if debited in the profit and loss account, they must be added back.

Deductions Not Allowed

Profit is ascertained for income tax purposes according to certain well defined rules, by which some specific expenses are not allowed as deductions from profits, while other deductions and allowances may be claimed, whether they are debited on the accounts of the business or not. Some of the more important "deductions not allowed" are as follows:

(1) Provisions for bad and doubtful debts (but specific accounts written off are allowed). (2) Royalties. (3) Premiums paid for leases. (4) Company preliminary expenses. (5) Discounts on issue of shares or debentures. (6) Underwriting commission. (7) Certain type of legal charges. (8) Any losses not connected with or arising out of the trade, such as the loss on a sale of investments. (9) Income tax. (10) Depreciation.

In lieu of depreciation, a wear and tear allowance is granted on plant and machinery according to an official schedule of rates. A wear and tear allowance is also granted on industrial buildings such as factories and warehouses, but not on retail shops. An investment allowance of one-fifth of expenditure on new plant and machinery and of one-tenth on industrial buildings may be granted.

When old plant and machinery is sold, a balancing charge can be levied if the amount realized is greater than the written-down value; if it is less, a balancing allowance can be

claimed. A similar obsolescence relief can be claimed when obsolete machinery is scrapped and replaced by new plant.

Limited companies and other corporate bodies have to pay an additional tax called a profits tax. This is a permanent extension of the National Defence Contribution, first imposed in 1937. Profits tax is charged at a fixed rate on the profits of each accounting year, adjusted as for income tax purposes with certain modifications ; and if some of the profit is not distributed to the shareholders as dividend, a slight non-distribution relief is granted. The ordinary rate of profits tax is 22½ per cent. On non-distributed profits it is 20 per cent.

Since 1944 employers have been responsible for deducting income tax from the payments of wages and salaries. This virtually makes each employer an unpaid agent of the tax collector, and it entails the keeping of additional records, though no new accounting methods are involved. Each employee is given a code number by the tax office ; according to this code number tax has to be deducted by the employer and periodically paid to the local tax office.

Tax deduction cards, showing cumulative remuneration and tax deductions, have to be maintained and returned to the tax office at the end of the year. A certificate showing the total remuneration and the total tax deducted is then issued. The aggregate of the tax deducted week by week should be debited to the wages or salaries account and credited to the P.A.Y.E. (Pay As You Earn) tax account. This in turn should be debited and the cash book credited when the payments are made.

LESSON 22

Punched Card Accounting

Punched card accounting is a method of recording in which holes punched in cards replace figures and sometimes words. Sorting machines can then sort the cards with extreme rapidity (up to 40,000 cards an hour) according to the holes punched in them. Tabulating machines can then print in column form the information punched into the cards, and simultaneously add and subtract the various quantities to give totals and sub-totals required in accounting statements.

Other machines can cross-multiply the numbers represented by holes in two sections of a card to provide a product. The latest machines of the electronic computer type can perform an astonishing variety of additions, subtractions, multiplications, and divisions at extraordinary speed.

Preparation for Statistics

Punched card machines were first used on a large scale in Britain to prepare the various analyses required for the tables of the 1911 population census reports. A card was punched for each entry on each census schedule form. These cards were then successively sorted and automatically totalled to provide the great variety of classifications necessary.

Since then the machines have been very much elaborated, particularly in regard to the printing and the recording of words as well as figures. To-day they are the normal means of dealing with the mass of detail of a census or other large-scale statistical inquiry.

For example, they are used by the statistical department of the Customs and Excise to prepare the monthly statistics of imports and exports, the so-called "Board of Trade Returns," a card being punched for each entry on each customs specification of goods which are imported or exported.

Substitute for Book-keeping

Punched card accounting is now extensively used in commerce and industry as a substitute for ordinary book-keeping. It may be suitable where there is a great volume of the same kind of detail (e.g. items sold to customers, items of stores purchased and issued to a factory, items of stock sent to branches, items of wages or salary to be calculated and paid), especially when each item has to be recorded or included in three or four different types of tabulation.

For example, an item in an invoice may also have to be recorded in a stock account, a geographical classification of sales, and a commodity classification of sales. The great advantage of punched cards lies in the fact that the machines can re-sort the cards so rapidly and then quickly print a line of detail and simultaneously include it in various additions or subtractions.

Division of Cards

The cards used are specially made by the companies manufacturing the machines, and they are of various sizes, ranging from 21-column to 80-column. In each column there are printed the figures 0 to 9. Holes can be punched by a punching machine in any of these positions, so there are ten ordinary punching positions in each column.

In addition, it is possible to "over-punch" in three additional positions for 10, 11, and 12.

From left to right the card is divided into sections called "fields," each field containing an

appropriate number of columns to deal with a type of information ; thus a date field will normally require two columns for the day, because there may be 31 days in the month, and one column for the month, since months can be indicated by the numbers 1 to 12.

Numerical Coding

If a firm has between 1,000 and 10,000 customers, each of whom can be given a distinctive number, four columns would be needed in the customers' number field. A £ s. d. field to accommodate up to £10,000 will require four columns for the £s, two columns for the shillings, and one column for the pence—seven columns in all. The cards have to be designed by the machine companies to suit the special needs of the user.

Before the cards can be designed, numerical codes have to be devised to represent the various kinds of detail ; e.g. customers' accounts, the different kinds of commodities sold, etc. Numerical coding is the basis of punched card accounting. Representatives of the punched card accounting machine companies undertake the preliminary organization of the detail and instal the system. Its subsequent operation requires a full understanding of the ways in which it contributes in the detailed work of double-entry book-keeping.

Basic Operations

The basic operations in punched card accounting, once the system has been installed, are briefly as follows :

1. The numerical coding of all the items that are to be " put on punched cards " if they are not already numbers on the documents.
2. The grouping of the coded documents into batches ready for punching.
3. The rapid totalling of each batch by means of an adding machine to provide a control total.
4. The punching of cards for each batch of documents. This operation is done by a punch operator with a machine that somewhat resembles a typewriter. A skilled operator is able to punch about 10,000 holes an hour.
5. Verifying the punched cards, that is, making certain that the holes have been punched in the right places. This operation is done on another machine, usually by a different operator. The punched card is the basis of all the subsequent work, and it is necessary to ensure as far as possible that it is completely accurate.
6. The punched and verified cards can then be put into a balancing machine, which with extreme rapidity adds the amounts punched into them to ensure that the total is the same as the control total (*see* 3).
7. Cards can then be suitably stacked until they are required for further operations. The first of these is usually sorting.
8. The sorting machine sorts on one column at a time with amazing speed, and can thus quickly (and accurately) arrange piles of cards in any desired order.
9. When they have been appropriately sorted, the cards can be put into a tabulating machine, which will print, one line for each card, as much as may be desired of the information punched into the cards, and provide subtotals and totals and grand totals where these are necessary.

Reclassifications

In this way there can be produced such documents as invoices, statements of accounts, departmental trading accounts, stock lists, pay rolls, expenses accounts, etc. For full advantage, each punched card should be used for at least three different purposes : hence punched card accounting is most suitable for statistical work involving a number of reclassifications of the same detail.

Besides the basic operations of punching, verifying, sorting, and tabulating, there are numerous subsidiary activities, as follows:

Printing and Interpreting

Alpha punching. This method represents the letters of the alphabet by two holes in various positions in one column ; the tabulating machine, "sensing" these two holes, prints the corresponding letters so that tabulations containing words can be produced.

Interpreting. Experienced punched card operators can read with ease the information punched on the cards. But there are machines that print on the card itself the words or figures indicated by the holes punched into it, so that anybody can understand it. Some firms issue interpreted punched cards to employees as wages dockets, and sometimes even as cheques to pay into the bank.

Rapid Development

Cross-adding punch. This is a machine which automatically multiplies the figures punched into two fields of a card and punches the product into another field.

Summary punch. This machine is coupled to a tabulator in order to produce a fresh punched card for each total printed on the tabulator.

Gang-punch or Reproducer. This machine can produce rapidly any required number of cards punched identically with a pattern card placed into it. It may be required to punch 10,000 cards with the same date.

The art of punched card accounting is developing so rapidly that there is hardly any quantitative aspect of business or manufacture in which punched cards are not being used. The development of the electronic computer

has further extended their application, both for accounting and for planning and production control, and many undertakings now rely primarily on punched cards for most of the detailed work incidental to financial and manufacturing control.

But it cannot be too strongly emphasised that punched card accounting is not a substitute for double-entry book-keeping; it is merely a way of carrying it out on a large scale. Only those who can see clearly the double-entry framework behind the punched card operations and the stacks of cards can rely confidently on this mechanical substitute for the ordinary ledgers and books of first entry.

BOOK LIST

The Principles and Interpretation of Accounts, Ellis (Pitman); *Carter's Advanced Accounts* (Pitman); *Management Accounting* (British Productivity Council); *Accounts for Management* (British Institute of Management); *Bookkeeping and Accounts*, Cropper, Morris, and Fison (Macdonald and Evans); *Balance Sheets and Accounts under the Companies Act, 1948*, Kettle (Gee); *Guide to Income Practice*, Carter (Gee); *Machine Accounting*, Sutton (Macdonald and Evans); *Elements of Punched Card Accounting*, Cemach (Pitman).

PSYCHOLOGY

PSYCHOLOGY has been the subject of close study by many scientific minds. The results of their work are presented in a thoroughly comprehensive, though necessarily compressed, study in this Course. The observations and theories of leading psychologists are explained, and those of the most modern schools of analytical psychology are examined and appraised. Acquaintance with the Courses on PHYSIOLOGY, *in Vol. 2;* SOCIAL ANTHROPOLOGY, *in Vol. 2; and* BIOLOGY, *in Vol. 1, will assist the student.*

27 LESSONS

LESSON 1

The Definition and Purpose of Psychology

It is usual to open a course of teaching in a particular science with a brief definition of its subject-matter. With psychology, this task of preliminary definition is unusually difficult. It is easy enough to specify some of the topics with which psychology is concerned, and most text-books of psychology agree in their main chapter headings. There will be chapters on memory, emotion, learning, thinking, habit formation, instinct and intelligence, perceiving, imagining, and so on, and we all understand quite well what is meant by these words. Difficulty arises as soon as one tries to summarise these topics under one comprehensive heading.

If psychology is thought of as the study of consciousness or of mind, can one be as sure of the meaning of these words as of words like star and chemical compound? Could the student say in half a dozen lines just what he understands by mind? But some sort of preliminary definition is essential, so let us examine some of the available alternatives.

In doing so, it will be necessary to touch upon questions which will be more fully dealt with at a later stage. If, as a consequence, the student feels a little confused, he is advised to re-read this preliminary discussion after finishing the Course. The discussion will serve to bring out certain special features of psychology which make it at once more interesting and more difficult than some other sciences.

We all know what it is to be conscious. When in a dreamless sleep or in a faint, we cease, it is said, to be conscious. Our conscious life is the life in which we are aware of objects and persons around us and of our own thoughts and feelings and desires. But what is consciousness? It is not a stuff or thing, but seems to be a special quality which attaches to those "inner," mental activities of ours which are called perceiving, thinking, feeling, and so on.

The New Psychology

When psychologists of an older generation spoke of psychology as the study of consciousness, the expression meant, in practice, merely that psychologists studied these activities of human beings.

Suppose that there is somebody who has done you an injury in the past. Whenever you see this person or think of him you will probably experience a feeling of dislike and animosity; at other times you forget all about him. But even when you are completely forgetful of him, it would still be true to say that you dislike him. Your dislike for him is a characteristic of yours, due to past experience, which determines from time to time the quality of your feelings without being itself something of which you are conscious *all the time*. Yet it is a mental characteristic of yours, and not a bodily characteristic like brown eyes or dark hair.

There are many factors of this kind which must be taken into account before one can fully understand the conscious activities of human beings, and owing to the existence of these factors most psychologists have ceased to define their science as the study of consciousness. An adequate definition of psychology must cover not merely the facts of consciousness but also the conditions which determine these facts.

Another objection to defining psychology as the study of consciousness is the fact that we can study consciousness directly only in ourselves. Our knowledge that other people have conscious experiences like our own is an inference from their behaviour. On reflection, this will be seen to be true even when they describe their emotions or thoughts to us in words; for language, or the making of a special series of noises with the mouth, is simply a form of behaviour peculiar to human beings.

What is Meant by Behaviour

Many modern psychologists define their science as the study of behaviour; and there is much to be said for this definition, if only because it makes the observation of other people and of small children and animals a legitimate part of psychological method. The psychologist who insists that psychology is concerned only with the direct study of consciousness is really confining himself to the study of his own conscious processes, and it is difficult to see how he can hope by this means to reach conclusions which will be true of others as well as of himself.

It is important to understand what is meant by behaviour. We naturally tend to divide the things which constitute our world into two classes, living and non-living, and we confine our use of the word behaviour to the former class. Non-living things change only in so far as other things act upon them, but living things seem usually, in the words of Professor McDougall (1871–1938), "to pursue actively or with effort their own welfare or their own ends or purposes."

Examples of this are the nest-building of birds, the salmon swimming up-stream and leaping over rocks to deposit her spawn at a particular place, and the migration of swallows at the approach of autumn. It is this apparent purposiveness which distinguishes a living creature

from an inert piece of matter, and it is precisely this kind of activity which is described as behaviour, in contrast to the merely mechanical movement of inanimate things.

Deliberation and Foresight

Here there are two points which call for special mention. In the first place when we say that a human being acts purposively, we usually mean that he consciously sets out to do something or to achieve some end which he has decided upon beforehand. If it is said that all living creatures, as opposed to sticks and stones, appear to act purposively, does this mean that all living creatures think out their course of action and foresee the result they desire to achieve? Should one attribute deliberation and foresight in this way to snails and worms?

The only possible way to answer this question is to study carefully the behaviour of all kinds of living creatures and to compare it with the behaviour of human beings, who do actually deliberate and foresee. If the behaviour of all living creatures in certain circumstances is exactly like the behaviour of human beings acting under the conscious guidance of a purpose, then it will be reasonable to conclude that all living creatures do sometimes have conscious purposes.

The behaviour of various kinds of animals resembles the behaviour of human beings in some respects but differs from it in others. Moreover, not all human behaviour is guided by conscious purpose. You may walk along the road deep in thought and pay no attention to your direction, and yet reach your destination. Many forms of human behaviour have this habitual, mechanical character and seem to involve little, if any, conscious activity. In addition, as will be shown later, a great deal of human behaviour is motivated by unconscious (i.e. not known to the person himself) motives and thoughts.

The fact that a great deal of human behaviour seems to involve little, if any, conscious activity suggests that if one talks of all living creatures as acting purposively, the word "purposively" must not be used in such a way as to imply that all living creatures are capable of conscious deliberation and foresight. Purposive activity or behaviour is simply bodily activity which is adapted to achieve some end, as opposed to mere random movements. Furthermore, it does not involve any assumption about "soul" or "spirit."

Behaviour and Conscious Purpose

This leads to the second point which arises out of the consideration of behaviour. If behaviour is defined as "purposive" activity, and if, while all living creatures "behave" in this sense, not all behaviour necessarily involves *conscious* purpose, what is the relationship between behaviour and conscious purpose? This question is only part of the larger problem of the relationship between bodily and mental happenings in general.

Consider some examples. If someone has a bad stomach-ache and feels depressed, something which is happening in his body seems to determine his conscious mental activity of feeling. If one decides to go for a walk, conscious decision seems to determine the movement of arms and legs. If one walks along deep in thought, the bodily activity of walking and the conscious mental activity of thinking seem to go on independently. As a further complexity, those mental factors previously referred to which condition conscious activity without themselves being conscious, must also be remembered.

The "Mind-Body" Problem

The problem of the relationship between mental activities and processes and events in the body is known as the "mind-body" problem, and it carries us beyond the realm of psychology proper. This problem will be discussed again, but here it is sufficient to point out that it is precisely this intimate association of the mental and bodily sides of life which makes the definition of psychology so difficult. Some of the objections to defining it as the study of consciousness have been given. If, now, we consider an activity such as remembering, an activity with which psychology is bound to deal, we shall see some of the objections to defining it as the study of behaviour.

If I remember something which happened to me yesterday, my remembering seems to be a purely mental activity; but if I play a tune on the piano from memory, my remembering takes the form of making a series of movements with my fingers.

If psychology is defined as the study of behaviour or bodily activity, this limits the psychologist to the study of the second kind of remembering only. Most contemporary psychologists therefore endeavour to define the subject matter of the science in such a manner as to include within it both bodily happenings and the mental activities and processes associated with them.

Behaviourism

For centuries psychology was intimately associated with philosophy and religion, and it was as a revolt against this that the movement called "Behaviourism" started in the 1920s. The Behaviourists, led by John B. Watson (b. 1878), argued that if psychology was to be a science in its own right it should not deal with philosophical problems which could not be resolved by scientific methods, nor even with

such things as "consciousness" and "feeling" which were personal experiences which could not be observed by others. All that psychology should study is that which can be seen by all.

But Watson and his followers were extremists; they even went so far as to deny the existence of conscious experiences because these were not observable by others. Thus the early behaviourists would never accept such a statement as "He is feeling angry." They would insist on saying, "He is behaving in a way which can be called 'angry' behaviour."

Need for Careful Scientific Observation

Behaviourism has exerted a profound effect on the development of modern psychology, for although the views of Watson and his followers were extreme, they have drawn attention to the need for careful scientific observation in psychological work, and have given a great impetus to the development of modern experimental psychology. In particular, they have drawn attention to the profound influence of learning (e.g. by trial and error or by reward and punishment) on later behaviour. The modern Behaviourists, such as E. C. Tolman, and Clark Hull (1884-1952), have made outstanding contributions to the theory of learning.

Most psychologists are not Behaviourists. They are inclined to agree that one can study mental activities directly only in oneself and that it is not known for certain how far animals can be credited with similar activities; but they contend, the psychologist is not primarily interested in behaviour but in the mental life of individuals, and he studies behaviour because of the light which it throws on mental life.

Mental Processes

In these Lessons psychology is regarded as the *study of behaviour and of mental life.* The study of behaviour, or the bodily activity of human beings, will be able to throw light on the mental processes associated with such behaviour, though the student must be careful not to allow himself to be drawn into philosophical and moral judgments which cannot be investigated scientifically. These judgments are important in themselves, but are not part of the science of psychology. We shall therefore ignore the question of the ultimate relationship between mental life and the body, although by so doing we shall no doubt frequently use language which seems to prejudge the answer to this question.

If this course seems to require justification, it can only be said that the psychologist is likely to contribute more to the solution of the mind-body problem if, during his study of mental life and behaviour, he allows his language to be governed by the particular set of facts he is considering than if he begins with some preconceived theory and adapts his analysis of the facts to it.

LESSON 2

Observation and Experimental Methods

How does the psychologist set about the task of studying mental life according to our definition of psychology? How does he investigate such inner mental activities as remembering, thinking, feeling, on the one hand, and the many forms of bodily behaviour on the other?

Like any other scientist, the psychologist has two ends in view. He desires to make generalisations and to explain. In generalising, he aims at stating the conditions which determine the particular activity he is considering. If he is investigating memory, for example, he wishes, if possible, to state precisely what conditions determine the fact that one remembers some things and not others, and forgets some things quickly and others more slowly.

Introspection

In explaining, the psychologist tries to show just how the general conclusions he has reached apply in particular instances. The psychologist's generalisations and explanations are based upon the customary scientific methods of *observation* and *experiment*, and we will glance at some of the forms which observation and experiment take in the psychologist's hands.

When we say that we feel angry, or tired, or depressed, or that we are thinking rapidly or remembering vividly, our own mental processes are being observed. Self-observation of this kind is called introspection, and it is much used by psychologists, who are able, through training and practice, to introspect more accurately than most people. As practised in daily life, introspection is far from accurate. For example, someone may say and even believe that he is not angry, when an impartial observer is able to tell from his gestures and expression that he is very angry indeed.

"Mind's Eye" and "Mind's Ear"

Comparison of the results obtained by different persons from introspection is often of great psychological value. In the 19th century the psychologist Sir Francis Galton (1822–1911) was interested in the fact that one is able to see things "with the mind's eye," and to hear

sounds with "the mind's ear," as when, for example, we picture to ourselves some landscape we have seen, or run over a familiar tune in our mind.

He sent out lists of questions to a number of people asking them to write down how vividly and with what accuracy they could do these things. The results were extremely interesting and revealed astonishing differences in the capacity of different persons to visualise, and so on.

Objective Observation

This procedure is valuable enough in the case of [illegible] But let us suppose that we are dealing with the mental life of very young children. It is not possible to ask them to introspect and to report the results of their introspection. The psychologist is therefore thrown back upon objective observation or observation from the outside. He may note, for example, just how a particular child learns to talk, the circumstances in which it shows signs of anger, the time during which it shows interest in a toy before throwing it aside for something else, and so on. He may study the child at different ages in order to note the development of different interests, of powers of concentration, of memory, and the capacity to recognize and distinguish different objects.

Whenever possible, the psychologist will devise special means of testing the child's abilities. He will place it in one special set of circumstances and then in another, and note just how its activities differ in the two cases. To alter and control in this way the conditions under which the child exercises its activities is what is meant by experimenting.

Accurate Checking

In mere observation one takes such conditions as circumstances offer and simply notes what happens. The psychologist uses both observation and experiment, but like other scientists he prefers to experiment if he can. The great advantage of carefully devised experiment is that it enables results obtained by one investigator to be accurately checked by another.

Suppose the psychologist wishes to find out how quickly a person can memorise. He may pick up a volume of poetry, choose a poem at random, ask the person to learn it by heart, and note the time he takes. It will not be possible to test the person again with that particular poem, for he will know it ; and another poem, even if it is of the same length may present features which make it more easy or more difficult to learn. But if the psychologist tests a learner with a series of meaningless syllables such as Tij, Las, Tob, Jit, Res, another investigator can devise a similar series and the results obtained with the two series will be strictly comparable.

At first sight it may seem strange to talk of measuring mental activities such as remembering ; and one cannot, of course, measure them literally and directly as one measures a garden or a wall. But by using the experimental method the psychologist is able to measure certain aspects of these activities which are of great interest.

Measuring Mental Activities

Consider the following questions, each of which involves a reference to some mental activity or other. How soon does Smith learn to repeat without error a series of nonsense syllables ? How quickly does he forget them ? At what distance can Jones, Smith, and Brown read letters of a certain size ? The answers to the first two questions can be stated in terms of time, i.e. in seconds, minutes, or hours, and the answer to the last in terms of feet or inches.

By asking himself questions of this kind and by devising different ways of testing people in order to provide himself with the answers, the psychologist is able to find out a great number of highly important and useful facts. It turns out, for example, that some methods of learning enable one to remember what one has learnt much longer than other methods do—a fact which is of obvious value to school teachers.

The ability of animals to form new habits and of human beings to acquire forms of skill such as that involved in typewriting can be similarly measured by noting the number of times the task must be performed before it can be executed without mistakes, the number of errors made in each successive attempt, and so on.

It must not be thought that introspection and the kind of observation and experiment referred to exhaust the psychologist's resources. All kinds of apparently unpromising material are grist to his mill. A great deal can be learnt about the mental activities of human beings even when direct observation and experiment are impossible. We can no longer directly observe the prehistoric inhabitants of the British Isles, for instance, yet by examining the tools and other things which they have left behind them we can infer a great deal as to their interests and mental activities generally.

Methods Common to all Sciences

In the same way, we can no longer observe Shakespeare, let us say, or Chaucer, but from their books we can, if we know how to go about it, frame quite a number of conclusions as to their ability, their interests, their emotional peculiarities, and their habits.

By using observation and measurement as well as systematic experimentation the psychologist is using methods which are common to all sciences, although their particular application in psychological work may be quite

different. In recent years the use of statistical methods has been a valuable aid to psychological investigation. A psychologist might, for instance, be able to demonstrate that two mental measurements are associated (e.g. the time taken to solve two different types of problem), and this provides a valuable clue to the extent to which they involve a common mental ability. In the next Lesson a brief description is given of the different fields in which these various methods can be applied by the psychologist.

LESSON 3

Importance of Psychology in Modern Life

It has been seen that the psychologist, in investigating mental life, relies upon the usual scientific methods of observation and experiment. Some psychologists are more interested in one aspect of mental life than in another, and therefore psychology tends to break up into a number of different branches. We will now glance briefly at the more important of these, but it must be borne in mind that they are not separate, independent studies, but merely subdivisions of one science representing the special interests of different investigators.

Branches of Psychology

The most important branch of psychology is that which seeks to frame generalisations concerning ordinary adult human beings. It studies such activities as perceiving, thinking, remembering, and imagining. It also deals with the emotional life of normal human beings, and tries to ascertain the nature and causes of the different emotions and the way in which they are related to one another and to behaviour and conduct. Such psychology is called normal human psychology.

But it is impossible to understand the ordinary human adult without knowing something of the way in which he has grown up. The character and special abilities which he shows as an adult are the outcome of the characteristics and aptitudes which he brought into the world with him as a child, the influences to which he was exposed during his early years, the profession or trade to which he has been trained, and so on.

In endeavouring to understand the adult, therefore, the psychologist finds it essential to study his development from childhood upwards and to regard his activities in adult life as the result of a continuous process of change and growth. Psychologists, in consequence, sometimes speak of genetic psychology, or psychology which studies mental activities with particular reference to their growth and development. Because all living creatures grow and develop, the word genetic refers not so much to a special branch of psychology as to a special method of approach to all psychological problems.

The adult human being does not live in isolation but is constantly influencing and being influenced by other people. We are all members of various social groups, such as the family, the nation, a trade union or profession, and so on. As members of such groups we have habits and beliefs and sentiments in common. The study of the various mental activities and forms of behaviour which find their chief expression in social or group life is called social psychology. The social psychologist is specially interested in mental activities such as emotion and impulse, and in the way in which human beings behave to each other.

Animal Psychology

Another branch of psychology is known as comparative or animal psychology. Psychology, like all the other sciences which deal with living creatures, has been much influenced by the theory of evolution. The psychologist accepts the view that human beings are descended from an animal ancestry and believes that they resemble other animals in important respects, although they have also special characteristics of their own. He therefore studies the psychology of animals partly for its own sake but mostly for the light which it throws on the psychology of human beings. Much interesting work has been done in this field within recent years, and it will be frequently referred to in subsequent Lessons.

Abnormal Psychology

Insane persons are mentally abnormal, and so frequently are men and women of genius and child prodigies. Abnormality, in fact, is a question of degree. We all tend to regard as abnormal persons who differ very much from ourselves. Those persons whom we regard as insane are, roughly speaking, those who differ so much from their neighbours that they are unable to look after themselves, or live in any sort of harmony with their fellow beings. It has been found that some forms of mental disorder which used to be regarded as incurable can be cured if dealt with by very special psychological methods.

A great deal of attention has been given in consequence by some students of psychology to the study of mental disorder in all its forms. Many such disorders turn out to be the exaggerated expression, in a few persons, of mental

characteristics which are shared by us all. In these persons these characteristics have, owing to special circumstances, been warped and distorted or forced into undue prominence.

As a result of the study of mental disorders many things have been found out about the psychology of normal persons, particularly about their emotions, and the part played by emotions in mental development, which might not otherwise have been discovered. This branch of psychology is called abnormal psychology or psychopathology.

Problems of Everyday Life

Psychology is applied in various ways to the solution of problems of everyday, practical life. Suppose you are an employer and wish to choose someone to fill a particular post, or that you are a parent and wish to decide upon a trade or career for your son. The psychologist is ready to help you; what is called *vocational* psychology is an important and growing branch of applied psychology. It is a subdivision of *industrial* psychology.

Industrial psychology is concerned not only with the selection of employees, but also with such problems as the reduction of effort involved in any particular job, the effect of fatigue on the worker, and industrial accidents. It also deals with what might be called the "social psychology of industry," problems of the relationship between foreman and worker, and the whole question of working as a social activity.

In education, too, psychology has found useful applications. Methods of teaching have been reconsidered in the light of the findings of experimental psychology; children have been graded according to their capacity to learn different school subjects; and the findings of abnormal psychology have been applied to problems of the relationship between the child and his teacher.

LESSON 4

Classifying the Facts of Mental Life

THERE are three broad features of conscious experience which are used by psychologists as a basis of classification in dealing with the facts of mental life, and which may be illustrated from any trivial incident of conscious experience we choose to take. In writing these words, for example, I notice suddenly that my fountain pen has run dry. I experience, in consequence, a feeling of vexation and I look round for the ink bottle. If I do not immediately find it, my feeling of vexation grows and I continue my search. As soon as the ink bottle is found, my vexation disappears and I fill my pen and return to my writing.

Here, then, is a definite sequence: first, I am *aware* of something (the empty pen); second, I *feel* something (vexation), and thereby I am prompted to *do* something (find the ink). These three features of consciousness—being aware of a situation, feeling somehow in relation to it, and being prompted to do something about it—are termed by psychologists *cognition*, feeling or *affection*, and *conation* respectively.

Cognition

As regards the first, cognition covers every kind of awareness. As I sit at my table I am aware of the paper in front of me, of the sunlight falling through the window, and so on. This awareness of the objects present to our senses is known as perception. In memory we are aware of something experienced in the past, such as the conversation we had with a friend yesterday, or the scenery we saw during our holiday. In imagination, as when we picture to ourselves the scenery we hope to see when the holidays come round, and in thinking, as when we grapple with a problem requiring solution, we are aware of what we may provisionally term our own ideas.

Within cognition, however, we must distinguish the act of awareness from its object. This distinction is of great importance, and failure to make it has led in the past to an immense amount of bad psychology. Cognition is an activity directed upon objects of various kinds, and is one aspect, as feeling and conation are others, of our responsiveness as conscious beings to the world in which we live.

What "Feeling" Means

The term feeling has a much narrower meaning in psychology than it has in everyday speech; here it refers simply to the experience of being affected pleasantly or unpleasantly by some situation or of experiencing some emotion in relation to it.

In everyday speech we say that we feel hungry or thirsty, or that a piece of ice feels cold. In the experiences to which we thus refer, cognition and feeling are blended. In hunger and thirst we are *aware* of certain bodily sensations, and in touching ice, of characteristics belonging to the ice, not to ourselves. In psychological terminology the feeling involved in these experiences is the pleasantness or unpleasantness of the bodily sensations and of the sensation of coldness. It is important to grasp this limited meaning of the word feeling, as it is so much at variance with ordinary usage

Feeling and conation are not so easily to be distinguished as are feeling and cognition. The example at the beginning of this lesson will make the distinction clear. When I realized that my fountain pen was empty, not only did I feel vexation but my vexation ran over, as it were, into an impulse to do something about it. This impulse persisted throughout the bodily activity involved in searching, until I found the ink. It is this conscious striving in relation to some given situation, this urge from within outwards, that the psychologist describes as conation.

But conation is not the bodily activity involved in, say, searching for the ink bottle ; it is the mental activity which seems to sustain and guide it. There may be, and frequently is, conative activity where bodily activity is at a minimum. The longing to do something which one is powerless to do is an example of this.

Conation as well as cognition is involved in perception, memory, imagination, and thought. The effort to see more clearly some small object I cannot quite identify, the longing to revisit past scenes which arise before my mind, the desire to reach the end of a train of thought and to find the solution of the problem worrying me, are all examples of conation.

Mental Striving

Two features of conation, or mental striving, are highly important. In the first place, conation is always directed to an end, and ceases when the end is attained. Once I identify the perplexing small object, my effort to see it more clearly terminates. Secondly, while conation which runs to its end is pleasant, thwarted conation is highly unpleasant. The student will be able to find examples of this unpleasantness if he thinks of those occasions on which he has desired to undo something he has already done and has realized that for various reasons this is impossible. The desire, the conation, is there, but in the nature of the case it cannot achieve its end. Remorse, grief, and disappointment, which are intensely unpleasant emotions, are all bound up with thwarted conation.

So far we have considered only those features of conscious experience which can be distinguished within any momentary fragment of it. But the conscious experience of any one moment is continuous with the conscious experience of the past and leads on to that of the future. An examination of conscious experience from this point of view brings to light three other features, which, following Dr. James Drever (1873–1950), we will call *retention*, *cohesion*, and *selection*. These three features are all illustrated in memory. The mere fact that we can recall past incidents in our mental life shows that conscious experience is, in some sense, retained. The important ways in which retention operates to control and guide present experience will become clearer later.

Cohesion and Selection

Cohesion is illustrated by the fact that when we try to recall a series of incidents, they tend to recur in their original order. We remember going into the water to bathe last summer and then coming out to dress. We cannot, even if we wished, remember these events as happening in the reverse order. Nevertheless, some incidents in the past stand out in our memory more than others, just as some features in our present surroundings catch our attention while others are ignored

These are illustrations of selection, which, as we shall see, is closely connected with feeling and conation. Metaphorically, and so long as we remember that consciousness is not really a "stuff," we may think of retention, cohesion, and selection as the woof of the changing tissue of conscious experience and of cognition, feeling and conation as its warp.

LESSON 5

The Physical Basis of Behaviour

As the psychologist is concerned not only with consciousness but with behaviour, and as behaviour is bodily activity, it is desirable that the student of psychology should know something of the organization of the body. A very brief account must suffice here.

Everyone knows that the human body is a very complicated structure, containing many specialised organs, such as the heart, the lungs, the digestive organs, the muscles, and so on, each of which discharges a special function. The activities of the various special organs are co-ordinated in such a way as to ensure that each of them subserves the well-being and activity of the body as a whole, and the chief part in the business of co-ordination is played by the nervous system.

A brief sketch of the three main parts of the nervous system will be given in this Lesson, but it should be stressed that this division is somewhat arbitrary, though it is a very convenient one. There are other ways of classifying nerves and their activity, but the student will find the classification given here useful.

Somatic nerves comprise, first of all, those nerves which run inwards from the various sense organs to the central nervous system. The sense organs themselves are specialised

nerve endings and include, besides the eyes and the ears, special organs of taste in the tongue, organs of smell in the nose, organs in the skin which, for the time being, may be called organs of touch, and a series of organs in the muscles, tendons, and joints.

These last enable us, among other things, to realize the position of our limbs when, for example, we are in bed and cannot see them. They are called *proprioceptors* (inner receptors), as opposed to the eyes, ears, etc., which are *exteroceptors* (outer receptors). The second set of somatic nerves runs outwards from the central nervous system to the muscles concerned in bodily movements.

Autonomic System

The autonomic system is specially connected with the internal organs of the body, and derives its name from the fact that it is more or less independent of the central nervous system. It is not entirely so, but is linked through the central nervous system with the sense organs. It is important to the psychologist because of the role it plays in causing various internal changes in the body associated with the emotions.

Central Nervous System

The central nervous system is much the most important part of the nervous system. It comprises the spinal cord, running through a central cavity in the spine, and the brain, contained within the skull. The central nervous system provides the link between incoming nervous impulses from the sense organs (sensory impulses) and outgoing impulses to the muscles (motor impulses). Sensory and motor nerves never connect directly, but always through the central nervous system.

The sensory nerves keep us in touch with changes in the world around us; the motor nerves initiate the changes in our muscles involved in bodily movement, and the central nervous system ensures that our bodily movements shall be appropriate to the various situations with which our sense organs bring us into touch.

It is convenient to think of the nervous system as a very complicated kind of telephone system. The sensory and motor nerves are the lines of the various subscribers and the central nervous system is the exchange which connects one subscriber to another. The manner in which incoming sensory impulses and outgoing motor impulses are connected is immensely complicated.

Consider the different kinds of bodily movements a man may make. Suppose a wasp stings his hand and, feeling the pain, he withdraws his hand sharply. This is an involuntary movement. Suppose that he then looks round, sees that the wasp has settled near him, and after moving nearer to it and judging its distance, strikes out with his hand and kills it. This is a voluntary movement, or, rather, a series of voluntary movements. It involves the interplay of many more muscles and much more sensory and muscular co-ordination than an involuntary one, and this sensory and muscular co-ordination involves a correspondingly more complex system of linkages and connexions in the nervous system.

Linkages

The student must realize that incoming sensory impulses will connect with different sets of outgoing motor impulses at different times, and that all kinds of complex "pluggings in" and "pluggings out" are going on in the nervous system at every moment. Separate impulses, sensory or motor, travel by separate nerve fibres, and every nerve is really a bundle of fibres. The number of fibres in a nerve may vary; the optic nerve, which connects the eye to the brain, contains 400,000 separate fibres.

Linkages between incoming and outgoing nervous impulses can be graded according to the complexity and the number of the linkages involved. Different parts of the central nervous system are concerned according to whether the linkages are relatively simple or relatively complex. The simplest linkages are those involved in what are called reflex actions.

The contraction of the pupil of the eye in strong light; the reddening of the skin after scratching; sneezing; and watering of the mouth, known as salivation, at the taste of food, are examples of reflex actions or reflexes. (Reflex action, reflex, and reflex response are synonymous expressions.) They are simple, involuntary movements or bodily changes of which we may not even be conscious. In reflex actions the linkage between sensory and motor impulses is usually made more or less simply and directly in the spinal cord.

Hormones and the Nervous System

Complex voluntary movements, of which we are fully conscious, involve the activity of the brain and particularly of a part of the brain called the cerebral cortex. This is the outside, wrinkled coat of the brain, wrapping part of the brain as the peel wraps an orange. In man it is much more extensive and complicated in structure than in any other animal, and its presence is essential to mental life. Children born without it may live for several years, but never show signs of conscious experience, although various kinds of activity, internal and external, may go on in their bodies under the control of the lower parts of the nervous system.

Hormones are chemical substances released by the so-called endocrine or ductless glands

all over the body. Examples of such glands are the thyroid gland in the neck, the pituitary gland at the base of the brain, and the adrenal glands situated just above the kidneys. The hormones act in a complementary way to the nervous impulses, and they have been called "chemical messengers." An example of a hormone is adrenalin, which when liberated by the adrenal gland causes the blood pressure to rise, the heart to beat faster, and, in short, prepares the body in many different ways [illegible] meet an emergency.

The many hormones and the [illegible]us system work together to achieve the [illegible] balance which physiologists call *homeosta*[illegible]-the maintenance of "a constant internal [illegible]onment." Hormones are also important [illegible]lation to emotions; they can exert an [illegible]nse effect on mood and personality. Some [illegible]he activities of hormones are mentioned [illegible] Lessons.

LESSON 6

The Influence of Heredity and Environment

ARE we the product of heredity or of environment? The answer is, of both. The problem is to disentangle the relative importance of the two factors, and it is important to avoid thinking of either factor in too simple terms. The individual begins life as a jelly-like speck, or egg, and it is tempting to assume that this egg contains somehow within it all the characteristics of the adult in miniature.

Modern researches in biology have shown this to be untrue. The egg itself is simply a bundle of potentialities, and is capable from the beginning of developing very differently according to the influences to which it is exposed.

If, for example, the egg of a frog is placed at a very early stage of development in a solution of certain chemical substances, the embryo will develop a single large eye in the front of its head instead of two in the usual positions; and what is true of bodily is presumably true also of mental characteristics.

We must therefore enlarge our notion of environment to include the earliest influences to which the developing egg is exposed, and this means, in human beings, the pre-natal environment afforded by the mother's body. As a consequence, we must be on our guard against assuming that the appearance in a child of characteristics similar to those of its parents necessarily means that these characteristics are passed directly from the parents to the child through the egg.

Differences Perpetuated by Heredity

The psychologist is interested in the problem of heredity and environment because individuals resemble each other in certain respects in their mental life while differing in others. Different persons have similar interests and habits, while differing in intelligence or in emotional susceptibility, or in capacity for concentration. Much more is known about heredity and environment in relation to bodily than to mental characteristics. Eye colour, for example, is hereditary, and a good deal is known about the way in which differences in eye colour are likely to be transmitted to the descendants of people who themselves differ in this respect.

Differences of sex and the primary bodily differences which this entails are also due to heredity. The sex of an individual is determined in the fertilized egg. But when we come to the question of hereditary mental differences between the sexes, we are on much less certain ground. There is some evidence that girls have better capacity to deal with words than boys have, and it is probable that this is a hereditary difference. But any comparison between the sexes in this and similar respects can be based only on averages; and sex differences are, in any case, small in comparison with the range of individual variation to be observed within either sex.

Differences and Resemblances

Hereditary mental differences between individuals may be divided into two broad categories—differences in intelligence and differences in temperament—and these differences may be very marked indeed. There is a very considerable gap in intelligence, for example, between a scientist and the village idiot.

The mental resemblances between individuals are as striking as the differences, and here the problem of disentangling the hereditary and environmental factors presents immense difficulties. The interrelations between the two sets of factors are endless and subtle, and each set constantly tends to counterbalance the other. A boy at a grammar school, for example, may acquire much more knowledge in certain directions than another boy of equal innate intelligence at a technical school, and yet, owing to an equal innate capacity for forgetting, they may both, at the age of 40, show an equal ignorance of all the subjects concerned. Here the initial

difference between the two is due to environment and the final resemblance to heredity.

Uniformity of mental outlook and habit among individuals is in the main the result of a common social environment. This is obviously true in primitive societies, in which every detail of daily life is apt to be regulated by custom and tradition, but it is equally true in more flexible societies like our own. People read the same advertisements, wear the same kind of clothes, learn much the same things at school, and are squeezed, more or less unresistingly, into the same mental moulds.

Differences within the home circle, or within the social class, are largely compensated for by common national traditions and standards, so that, in contrast to an average Englishman, any two Frenchmen tend to be mentally very much alike, though one may be a shopkeeper from the Pas de Calais and the other a schoolmaster from Burgundy.

Controversial Problems

But granting the importance of the social environment in producing mental resemblances among individuals, are there not hereditary resemblances also? Are not human beings alike mentally as well as physically, primarily because they are human beings? Do we not tend to think and feel and behave alike in virtue of a specific human nature common to us all? This question raises one of the most difficult and controversial problems in modern psychology. While individuals within a given society show a striking uniformity in their mental life, individuals from social groups widely separated in time and space differ enormously in this respect.

We can perhaps say that all human beings, irrespective of time and place, show a common capacity for becoming aware of the world in which they live, for feeling in various ways in relation to this world, and for being prompted to strive in various ways to modify it, or to accommodate themselves to it, but as soon as we go beyond this very general statement, we encounter difficulties.

There are certain innate behaviour patterns which are part of the endowment of all human beings. The simple reflexes are an example of these, and it is through the modification of these initial patterns by environment that the adult personality is formed. Hereditary differences in mental function are differences in *potentiality*, but innate potentiality can be profoundly modified through the environment. And one must also recognize the fact of inborn differences in the strength of the instinctual drives (such as the sexual drive) in different human beings.

LESSON 7

Reflex Actions and Learning

In Lesson 5 reference was made to reflex actions, and it was pointed out that simple bodily movements of this kind involve a relatively simple and direct connexion within the nervous system, usually in the spinal cord, between incoming sensory impulses and outgoing motor impulses. If, for example, my foot is pricked, I shall withdraw it sharply, and a similar action will occur in persons whose spinal cord has been severed and in whom, therefore, the sensory and motor nerves of the foot are completely separated from the brain. They are unconscious both of the prick and of their own movement, but the movement occurs nevertheless.

Another example of a reflex action is the knee-jerk, which may be obtained by laying one leg over the other, while comfortably seated, and then striking the upper leg sharply just below the knee-cap.

Interrelated Reflexes

Reflex actions are part of the inborn equipment both of human beings and of animals, although a dog has reflexes which a human being lacks and vice versa. The essential points to note are (1) that a reflex action is a direct, automatic response to some sensory stimulus which will occur whenever the stimulus is applied, and (2) that although one may be conscious both of the stimulus and of the resultant movement, a reflex action involves nothing in the way of conation.

When the knee jerks, or when the foot is withdrawn from a prick, there is no sense of conscious striving to perform the action; it is purely automatic. It should also be added that one may be completely unconscious of the fact that a stimulus evoking a reflex action has occurred. The prick to the skin may be so slight that one is not aware of it, yet the foot is withdrawn. The whole process has gone on unconsciously. Reflexes which adjust the internal environment (such as the reflex which slows the heart when the blood pressure is momentarily raised) are quite unconscious.

Some reflexes are closely interrelated. When a dog scratches its flank with its hind leg, the movement is purely reflex. If a dog's foot is pricked, it will quite automatically withdraw its leg—another reflex. If, however, its left flank is tickled and its left hind foot pricked simultaneously, the dog withdraws its leg but

makes no effort to scratch its flank. In other words, if two reflex responses are provoked simultaneously, one is inhibited, or prevented from occurring, by the other.

A series of reflex actions may be so connected that the muscular movements of the first reflex, by stimulating the proprioceptors, cause the second to occur, and so on. A dog whose brain has been removed by an operation will, if the soles of its feet are stimulated appropriately, go through the motions of walking, each movement in the series of movements involved being a reflex action which leads on in the way described to the next movement.

It is held by many psychologists that much of the behaviour of insects consists in activities of this kind, which are called chain reflexes. Much the most important aspect of reflex action is the fact that in certain circumstances the sensory stimulus which normally provokes a given reflex response may be replaced by an entirely different stimulus.

Conditioned Stimulus and Reflex

The conditioned reflex was investigated by the Russian physiologist J. P. Pavlov (1849–1936). If food is placed before a hungry dog (or a human being, for that matter), saliva immediately starts to flow from the salivary glands within the mouth. This is a reflex response. Having accustomed a hungry dog to the surroundings in the laboratory, Pavlov found that if a bell were rung for a short time and food given to the dog before the ringing ceased and if this were repeated several times, the dog would begin to salivate as soon as ever the bell began to ring.

It was also found that fewer repetitions of the double stimulus (bell and food) were necessary on the second day in order to provoke salivation in response to the bell alone, and fewer still on the next day, and that after a few days the dog would salivate immediately the bell began to ring. The flow of saliva, which at first was a reflex response to food in the mouth, was now also provoked by a completely different stimulus, i.e. the sound of a bell. The sound of the bell Pavlov calls the conditioned stimulus, and the salivation which it provokes he calls a conditioned reflex.

Elaborate Precautions

Further experiments produced still more interesting results. If, for example, the conditioned stimulus alone is given on several occasions to a dog which has acquired the conditioned response, i.e. if the bell is rung but no food given subsequently, the conditioned response finally disappears, and, provided the conditioned stimulus alone has been given sufficiently often, it is difficult to re-establish it.

Pavlov also experimented with two tuning forks giving different notes, food being given to the dog after one fork had been sounded but none after the other. It was found that by this procedure a conditioned response could be attached to the first fork only, even when the notes of the two forks differed by less than a semitone of the scale.

It was discovered that the establishment of a conditioned reflex was difficult unless the dog were shielded from distracting stimuli, such as other noises, lights, and so on, and that the best results were obtained when elaborate precautions were taken to eliminate distracting stimuli by conducting the experiments in a sound-proof laboratory and under conditions which prevented the dog from seeing the experimenter.

Behaviourist Theory of Reflex Action

Other experimenters have obtained similar results to Pavlov's in other directions. Professor J. B. Watson, the exponent of Behaviourism, experimented with young children. A baby spontaneously shows fear if a loud noise is suddenly made near to it. This seems to be an innate response, and comparable to this extent with a reflex action.

If a white rat is shown to the baby and a loud noise made near it at the same time, the baby will shrink away from the rat in apparent fear, and will finally respond in this way to the rat alone, just as the dog will salivate in response to the bell alone. The baby may also come to show fear of other furry objects, i.e. objects which resemble the rat to some extent.

It is important to note that experiments of this kind are not equally successful with all babies. Some babies, although startled by the noise, return to the rat without showing signs of fear, as if they realized that the noise and the rat were separate. Babies in whom a conditioned fear response is established do not easily lose it. The oftener the rat is shown, even without the noise, the more frightened the child seems. The conditioned fear response thus differs from the conditioned reflexes investigated by Pavlov, which, as already pointed out, disappear if the bell is sounded sufficiently frequently without the giving of food afterwards.

The Behaviourists held that conditioned responses are wholly explicable in terms of the nervous system. It has been established by experiment that an animal whose brain has been removed by operation cannot acquire conditioned responses, although still capable of purely unconditioned reflex actions. Pavlov thought, in consequence, that the special function of the brain is to establish conditioned responses by sorting out the various nervous impulses coming in from the sense organs and connecting them with different outgoing motor impulses.

Given this sorting process in the brain, all behaviour, Pavlov considered, is simply a mechanical response, varying in nature and complexity, to the multitude of stimuli which impinge on the animal's sense organs. The dog salivates in response to a bell because, through the operation of the brain, the sensory impulses coming in from the ear as it is stimulated by the sound of the bell are connected up, in course of time, with the impulses which excite the salivary glands to secrete. The Behaviourists accepted this conclusion and regarded the conditioned reflex, thus interpreted, as the key to all behaviour.

The conditioned reflex has formed the basis for much work by psychologists, and there is a great deal to be said for the view that all human behaviour is built up on a gigantic base of millions of conditioned reflexes. The student may find it a useful exercise to try to trace to what extent any item of behaviour may be accounted for on the basis of chains of conditioned reflex activities. It will soon be evident that not all human behaviour can be brought within this scheme.

Acquired Meaning

On an alternative view, the conditioned reflex is an illustration at a very simple level of what is called acquired meaning. In the case of the dog, the bell has come to stand for, or to mean, the coming food, and causes the dog's mouth to water in consequence. Most people quite fail to realize the extent to which our understanding of, and familiarity with, the world in which we live are due to acquired meanings of this kind. We hear a motor horn in the street, and we say, "There goes a car." Yet the sound of a horn is not itself a motor car, but merely something which enables us to know that a car is passing, although we cannot see it. In other words, that kind of sound means, for us, a motor car. Illustrations might be multiplied indefinitely. How is it that one thing comes to mean another in this way ?

Let us return to the dog salivating in response to the sound of the bell, and note the conditions necessary for this to occur. In the first place, the dog must be awake and hungry ; secondly, the combination " bell and food " must occur several times ; thirdly, the bell must start ringing before the food is given ; and fourthly, distracting stimuli must be eliminated if the process of conditioning is to be successful.

Given these facts, it is possible to give a perfectly reasonable account of a conditioned reflex in psychological terms. A hungry dog is naturally interested in the smell and taste of food. Hunger is a bodily condition which evokes in the dog a natural tendency to attend to stimuli of this kind. Again, a dog certainly does not naturally attend to the ringing of a bell, but if the dog is awake and alert, and other stimuli are eliminated, the sound of a bell will attract its attention simply because there is nothing else to do so. When food is given, this involuntary attention to the bell passes over into, and is absorbed by, eager attention to the food, which, in its turn, passes over into the series of bodily activities, including salivation, involved in eating.

If we call the sound of the bell A and the food B, the final situation which absorbs the dog's attention and leads to salivation and eating is not merely B but A B, because A runs on and coincides with B. The naturally interesting, or meaningful, element in this situation is B ; but, as the dog's attention overlaps and includes A, part of the meaning of the situation to the dog becomes attached to A. As pointed out in Lesson 4, past experience is retained. Hence, after several repetitions, A alone comes to stand for the whole situation A B, and to provoke behaviour appropriate to it. A has acquired the meaning for the dog which A B originally possessed, in virtue of the natural, unacquired meaning of B.

It is not necessary to assume that the dog consciously recalls the experience of eating food when it hears the bell, any more than we ourselves consciously recall the felt sensation of coldness when, for example, we say ice looks cold. The dog merely responds to the bell by salivating, just as we may respond to the visual appearance of the ice with an incipient shiver. The important point to notice is that behaviour which has originally been evoked by one situation only may come to be evoked by another, in virtue of the presence of some element that is common to both.

LESSON 8

Instinct and Behaviour

A REFLEX action is, as already pointed out, a simple, automatic response by means of a muscle or gland to the stimulation of a sense-organ. Contrast this with the behaviour of a hungry cat which, confronted with a small bird, adopts a crouching position, creeps nearer, and then springs upon the bird and devours it. Here is a response on the part of the whole animal as a living, striving creature to a situation of a particular kind. Many animals have the capacity of responding to certain situations with special behaviour, appropriate to the situation

and useful to the animal, although they are without any previous experience of such situations.

Unlearned behaviour of this kind is usually called instinctive, and it is seen in its most striking form in insects, the solitary wasps, for example. One of these insects will dig a hole in the ground, then leave the hole and proceed in search of prey ; when it brings the prey back, the wasp deposits it in the hole and, after laying an egg on top of the prey, closes the hole and departs to go through the same series of activities elsewhere.

Flexibility in Detail

The details of the procedure vary according to the species of wasp. In some instances the chosen prey is a caterpillar, in others a spider, and in others a grasshopper. The prey is invariably stung and reduced to a condition of paralysis, and it serves, in due course, as food for the grub which emerges from the egg laid on it. The wasp does not need to learn to act in this highly complex manner but goes through the whole series of activities apparently spontaneously soon after it emerges from the pupa. Moreover, it never sees the grub in whose interests it is acting, and must therefore be assumed to be quite unconscious of the ultimate effects of its actions.

Instinctive behaviour often exhibits a certain flexibility in detail. A wasp of one species, for example, which preys upon spiders, was observed to grip the paralysed spider by the under surface with its jaws and back with it into the hole it had made. When the spider's legs caught upon the surface of the surrounding ground and prevented further backing, the wasp stopped, re-emerged, turned the spider round, took it by the upper surface, and backed into the hole with the spider in this reversed position. The spider's legs now closed like the spokes of an umbrella and entered the hole easily. It amounts to this : the wasp's behaviour, though instinctive, may be spontaneously modified to meet unusual circumstances.

Unlearned and Learned Behaviour

Advancing up the scale of animal life, the capacity for spontaneous modification of instinctive behaviour in unusual circumstances increases. Birds, in whom nest-building is instinctive, show great flexibility of behaviour in their choice of nesting materials and sites, when circumstances render this necessary. Instinctive behaviour is apt to be overlaid by learned behaviour, so that it becomes increasingly difficult to distinguish between the two. Singing among birds is partly instinctive and partly learned. The tendency to sing and the capacity to sing certain notes are instinctive or innate, but the actual sequence and combination of notes in the song of the adult bird is learned from other birds of the same species

The intermingling of unlearned and learned elements in behaviour is still [illegible] noticeable among mammals. The hunting behaviour of a hungry cat or dog left to its own devices is largely instinctive, as is the special hunting behaviour of a setter or a pointer

The Hungry Cat

Consider the following experiment. A hungry cat is placed in a cage from which it can see food outside, but from which it can escape only by manipulating a simple catch and then pushing a bolt. The cat will at first make a series of random and unsuccessful attempts to escape and reach the food. It will thrust its paws and nose through the wires, bite the wires, claw here and there, and shake everything in turn. Soon or late it will hit upon the actions necessary to lift the catch and push the bolt, and will escape and reach the food. If the experiment is repeated several times, the cat will finally come to perform these actions immediately the cage door is closed.

Innate Endowment

This kind of experiment will be referred to again when we discuss "learning." The important point to notice for the moment is the remarkable extent to which instinctive or unlearned behaviour may be modified by experience in animals fairly high up in the evolutionary scale. It is important to remember that instinctive behaviour is not merely unlearned behaviour but unlearned behaviour evoked by a particular situation. The hungry cat confronted by a small bird or mouse will crouch and creep and spring, but the same cat confronted by a dog will arch its back, extend its claws, and spit.

In order to account for instinctive behaviour, it is necessary to assume the animal exhibiting it to be equipped not only with a repertory of unlearned movements, but with some additional form of innate endowment which enables it to bring different items from this repertory into play in different situations. Most psychologists regard this additional innate endowment as consisting in specific dispositions or tendencies to behaviour, each of which is evoked by a special kind of situation and finds expression in a special kind of behaviour.

Result of Experience

A great deal of controversy has taken place over the question of the most useful designation for these tendencies. At one time they were called "instincts," and many psychologists still use this terminology, speaking, for example, of the food-seeking instinct, the mating instinct, and so on. But this usage is unfortunate, for it

inevitably suggests that the food-seeking and mating, etc., activities of animals are wholly unlearned; whereas unlearned behaviour in the higher animals is rapidly overlaid by learned. This does not necessarily imply that the innate tendencies underlying the behaviour of the animal have themselves changed, but simply that the original repertory of movements which are called into play by these tendencies has been modified and supplemented as the result of experience.

The hungry cat in the cage, for example, which lifts the latch, pushes the bolt, and walks out, has obviously not lost its innate tendency to make for food when it sees it, but has simply learned to modify in the service of this tendency its original repertory of actions. On the whole, therefore, while unlearned behaviour can legitimately be called "instinctive" in contrast to learned behaviour, it seems best to avoid the noun "instinct" altogether and to speak simply of "innate tendencies to behaviour."

The discussion on the influence of heredity and environment (Lesson 6) will have shown that the human personality can be considered to be the outcome of the effects of environment or upbringing on these original innate tendencies.

LESSON 9

Innate Tendencies in Human Beings

Anthropologists and sociologists often stress the astonishing diversity of human activities in different parts of the world and at different periods of history. Beneath all this diversity it is possible to trace certain unchanging motives which have actuated human beings at all times and in all places. A Chinese may eat with chopsticks, a savage with his fingers, a European with a knife and fork. Marriage customs may differ; and the white man may live in centrally-heated buildings, while the Australian Bushman builds a crazy lean-to to protect him from the weather. Yet all men and women, at all times and places, eat and drink, seek shelter from the heat or cold, mate and bring children into the world, seek and enjoy in various ways the society of their fellows, and so on.

Human Motives and Propensities

Some of these underlying motives which determine human behaviour, e.g. the desire for food and drink and shelter, are obviously recurrent organic needs: others seem to be innate ways of responding to some external situation, such as the tendency to flee when confronted by an obviously dangerous situation, or the tendency to resent injury, particularly direct physical injury, from others.

The most famous list of innate tendencies to behaviour is that made by Professor McDougall (1871–1938). Each tendency, he said, is, when aroused, accompanied by a specific emotion. The more important human tendencies and their accompanying emotions, according to McDougall, are as follows:

Flight (fear); food-seeking (appetite); repulsion (disgust); curiosity (wonder); pugnacity (anger); self-assertion (elation or the feeling of superiority); submission (the feeling of inferiority); parental (tender emotion); reproduction (sexual excitement); gregariousness (the feeling of isolation).

In addition to the foregoing, McDougall gives certain other tendencies to which no specific emotion is attached, e.g. the acquisitive and constructive tendencies and the laughter tendency. In his book *Outline of Psychology* these tendencies are referred to as "instincts." In a later book McDougall calls them "propensities."

This list has caused a good deal of discussion. Criticism has been in particular directed against the contention that each of the more important tendencies is associated with a specific emotion, and most students of psychology have now come to regard McDougall's list as being of descriptive value only.

Impulse and Emotion

Each innate tendency to behaviour is experienced as an *impulse* (i.e. as conation or striving), and it may be evoked by a wide range of situations and lead to a great variety of behaviour. Strong emotion seems, as a rule, only to accompany the impulse when the appropriate behaviour is for some reason impossible. If, for example, a man who is a good runner is chased by a bull, he will certainly flee, but he may not feel fear acutely unless he finds the bull overtaking him, or his further flight barred by a ditch or hedge which he cannot cross. This fact is important for the light which it throws on the evolutionary process whereby the plasticity of behaviour characteristic of human beings has probably been achieved. Dr. Drever, who in the main accepted Professor McDougall's position, said:

> The emotional tendencies may be regarded as representing what were originally series of simple responses following one another with definiteness and regularity, quite analogous to series of responses we still find in animal life; for example, in the nest-building instincts of birds. The possibility of the adaptation to a complex and changing environment of such relatively definite series of specific reactions is obviously limited. Hence, in the course of evolutionary history, the specific character of the individual responses, and the unvarying succession of the one

response to the other, have tended to disappear and to be replaced by the emotional disturbance, which secures that those kinds of behaviour only that are effective towards the appropriate end shall appeal to the organism.

Emotion, in other words, prompts to "trial and error" behaviour, like that of the cat in the cage under the impulsion of hunger, and so sustains the process whereby the behaviour "effective towards the appropriate end" is learned. If the student will think of those occasions on which he has learned to perform some task requiring skilled muscular adjustment under the impulsion of the desire to "shine" among his fellows (i.e. under the impulsion of the self-assertive tendency), he will see precisely what is meant.

Imitation and Suggestion

The innate tendencies to behaviour in human beings are initially evoked by certain situations, quite apart from previous experience of such situations. Pugnacity and the emotion of anger, for example, may be evoked in very young children by restraining their limbs, but each tendency rapidly becomes capable of being called forth by a wide range of situations. The process involved is that referred to as "acquired meaning" in Lesson 7. Some element present in the situation or situations which first evoked the tendency recurs in a different setting, and this setting in its turn calls the tendency into activity. In this way a given situation may come to arouse more than one tendency at once. An emotion such as admiration, for example, seems to be a blend of wonder and the feeling of inferiority, due to the simultaneous excitement of the two tendencies named by McDougall as curiosity and submission.

Freud's theories of the instinctual motivation behind behaviour will be dealt with in a later lesson when psycho-analysis is discussed. It is worth stressing, as a general point, that we are very rarely aware of the motives which cause us to behave in one way or another. These motives remain for the most part quite unconscious.

The analysis and explanation of human behaviour in terms of innate tendencies are usually dealt with under the heading of "social psychology," and it is impossible to pursue the subject further here. Something must be said, however, about "imitation" and "suggestion," which play so large a part in determining uniformities of behaviour within a given social group. In gregarious animals—among which must be included human beings—behaviour in one animal expressive of an innate tendency is apt to evoke similar behaviour in others. A panic, in which everyone runs blindly because they see others running, is an example of this.

This process of transference of emotion is designated by McDougall "primitive passive sympathy." But deliberate imitation seems to be limited to human beings, and it is prompted primarily by the tendency to submission. We imitate those whom for various reasons we consider to be our superiors or whom we love. What is known as "suggestion" is closely allied to imitation ; but whereas the latter leads directly to uniformity of behaviour, suggestion leads to it indirectly through uniformity of belief and opinion. Suggestion has been defined as

> that particular method of exerting personal influence by which one individual is brought to accept from another . . . an opinion or belief . . . without having . . . adequate logical grounds for its acceptance.

Here, again, the tendency to submission is an important factor. The child at school accepts unhesitatingly the beliefs and opinions placed before it by its teachers, because it is encouraged to regard them as its superiors in knowledge and wisdom, or because it fears rejection or of withdrawal of love.

LESSON 10

Analysis of the Emotions

In Lesson 4 attention was drawn to the fact that at every moment conscious experience has three distinguishable aspects, viz. cognition, affection or feeling, and conation. An emotion is a state or condition of mind in which the affective or feeling element is especially prominent. Ordinary language, by giving names to the emotions, such as fear, anger and so on, stresses this fact, but tends also to suggest that an emotion is nothing but feeling.

It is important, therefore, to remember that when we are emotionally excited, our total experience includes cognitive and conative elements. We not only feel fear, for example, but do so because we are aware of, say, impending danger. Moreover, our fear prompts us to run or to lie still, according to the nature of the dangerous situation.

Major Emotions

The major emotions are closely bound up, as already pointed out, with those innate tendencies to behaviour which constitute the core of human nature ; and the fact that an emotion such as fear may, in the adult, be roused by a variety of situations is simply one aspect of the more fundamental fact that by means of association, conditioning (*see* Lesson 7), and the grasp of

meaning, the innate tendencies to behaviour may themselves be called into activity by a wide range of situations. Most of our emotions are compound, e.g. admiration (referred to in Lesson 9), which are dependent upon the simultaneous action of more than one innate tendency.

From the point of view of the external observer, emotion is invariably associated with various kinds of bodily activity, which include actual behaviour such as running away in fear or striking in anger, certain facial expressions, and certain complex changes within the body. Facial expression is a less sure guide to the nature of the emotion felt than is usually assumed to be the case. Woodworth, in *Contemporary Schools of Psychology*, writes :

> A favourite experiment on emotional expression is that of presenting photographs of facial poses for various emotions and asking your subjects to judge what emotion is intended by each pose. The experiment brings out the fact very clearly that the emotion cannot be recognized with certainty from a still picture alone. A smiling or laughing face may, indeed, give almost 100 per cent. of judgments of amusement or happiness, and a face intended to register pain may be so judged by as high as 85 per cent. of the judges. Surprise can also be successfully registered and sometimes disgust. But fear is apt to be misinterpreted, unless it takes on the quality of horror ; and anger has given only 30–40 per cent. of right judgments.

Darwin (1809–82) maintained that some facial movements under the stress of emotion are relics of movements that once possessed survival value. Sneering in contempt, for example, is a relic of a primitive movement to bare the teeth preparatory to attack ; and the expressive movement of the nostrils in disgust is a vestigial form of a movement which was originally defensive against bad odours.

Internal Bodily Changes

Experimental work has shown that emotional excitement is accompanied by complex and well-marked changes within the body. If a well-fed cat is placed on a table and examined by X-rays, its stomach is seen to be making rhythmical churning movements. A dog is then brought into the room and encouraged to bark at the cat. The cat displays the usual signs of anger, and at the same time the X-rays show that the movements of its stomach abruptly cease and may not be resumed until a quarter of an hour after the dog has gone. Anger completely inhibits the processes of digestion.

Still more subtle internal bodily changes have been observed. There are within the body endocrine glands (*see* Lesson 5), which secrete into the blood complex chemical substances called hormones, which affect the activity of various other organs. Two of these glands, situated near the kidneys and called the adrenals, secrete a substance called adrenalin, which has a tonic effect upon the heart and muscles and is essential to life. In fear and anger the secretion of adrenalin is increased.

As a consequence, the heart beats more rapidly ; the veins squeeze the blood more quickly into the heart, so that the circulation is accelerated ; and the liver releases quantities of stored sugar into the blood to serve as fuel for the muscles. All these changes (including the inhibition of digestion mentioned above) facilitate the violent bodily exertion involved in the behaviour of running or fighting, in which fear and anger would, apart from the restraining influences of custom and habit, naturally find expression.

Endeavours have been made to find different internal bodily states corresponding to the different emotions, but investigation along these lines has not been very successful. The internal bodily changes which occur in fear and anger have been found to occur also as the result of violent pain, in strenuous muscular activity, in football players before a game, and in students just before an examination ; yet it is clear that the emotion of a student or a football player in such circumstances is not necessarily either fear or anger. Emotional conditions such as amusement and curiosity do not seem to be associated with any very specific internal bodily changes.

Theory of the Nature of Emotion

These facts lead naturally to the consideration of a famous theory of the nature of emotion called the James-Lange theory, put forward simultaneously (about 1887) by the American psychologist William James (1842–1910) and the Danish physiologist C. Lange. The gist of the theory is that an emotion is simply a confused mass of sensory experience arising from the sensory impressions coming in from the various parts of the body, including the internal organs, and that the difference in quality between the various emotions is due to the different combinations of sensory impressions arising from different bodily changes. James expressed this, rather paradoxically, by saying that we do not cry because we are sorry or strike because we are angry, but that we are sorry because we cry and angry because we strike. His most cogent argument in support of the theory was that if we think away all the bodily accompaniments of an emotion, there is no emotion left. James said :

> Emotion dissociated from all bodily feeling is inconceivable. The more closely I scrutinize my states, the more persuaded I become that whatever moods, affections, and passions I have are in very truth constituted by, and made up of, those bodily changes which we ordinarily call their expression or consequence ; and the more it seems to me that if I were to become corporeally anaesthetic I should be excluded from the life of the affections, harsh and tender alike, and drag out an existence of merely cognitive or intellectual form.

James was discussing here, not so much emotional excitement, which includes cognitive

and conative elements, but merely its *affective aspect* ; and later experimental work discredits James's theory as anything approaching a complete explanation of emotion.

Experiments with Adrenalin

Experiments have been performed to test the adequacy of the James-Lange theory of emotion. Professor Sherrington (1861–1952), by cutting certain nerves in a dog, deprived it of almost all sensation from the interior of the body, yet no apparent change occurred in the dog's behaviour. This remained of the kind which in a normal dog would unhesitatingly have been interpreted as expressive of the emotion appropriate to the situations. Cannon (1871–1945) performed an even more drastic experiment by cutting certain other nerves in a cat, and thereby rendering impossible the organic changes which usually occur with emotion. Yet the cat continued to show all the overt signs of emotion, such as hissing, flattening its ears, etc.

These experiments are not conclusive, as there is no means of knowing what the dog or cat *felt*. Experiments have been performed with a view to inducing artificially in human beings—e.g. by giving adrenalin—the internal bodily changes which normally accompany excitement of an emotional nature.

The results of these experiments show fairly conclusively that it is impossible to reproduce all the qualities of a particular emotion simply by giving hormones such as adrenalin. This may be due to the fact that there is not as yet sufficient knowledge about the particular physiological mechanisms brought into play when one experiences emotion, or (and this seems the more likely theory) that there is some nervous activity in the brain which is an integral part of any emotional experience. It is known that the part of the brain called the *hypothalamus* plays a very important role in the experiencing of any emotional excitement.

Pleasure and Pain

Another element in the affective aspect of emotional excitement is " feeling," in the narrow technical sense referred to in Lesson 4. Feeling, according to most psychologists, is of two kinds only : pleasure, and pain or " unpleasure." (The latter word is used to avoid confusion with *sensed* pain due to the stimulation of specific nervous end-organs.) Feeling in this sense is something of a mystery. It may be experienced in connexion with any kind of mental experience. Pleasure may be felt in the mere sensing of certain colours or sounds.

Moreover, while we can " image," or recall imaginatively, both cognitive experiences and those elements in emotion, for example, which are due to bodily sensations, this does not seem to be true of feeling in the strict sense. Whether feeling is itself conditioned or accompanied in any way by internal bodily changes is a disputed point, but emotions may be either pleasant or unpleasant in regard to their " feeling tone," and emotional excitement above a certain degree of intensity is, of course, nearly always unpleasant.

If we regard emotion as bound up with the primary conative tendencies, and accept the view that emotional excitement is specially strong when conation is thwarted or held up in some way, the unpleasantness of very strong emotion is explained by the well-established fact that feeling is conditioned by the success or failure of conation, success being accompanied by pleasure and failure by unpleasure.

Derived Emotions

In human beings the development of intelligence and imaginative foresight permits the fulfilment or otherwise of conation to be imaginatively anticipated. This fact led Professor McDougall to suggest that certain well-known states of mind, such as confidence, hope, anxiety, despondency, and despair on the one hand, and regret, remorse, and sorrow on the other, are differentiations of the fundamental forms of feeling—pleasure and unpleasure. These states of mind he calls "derived emotions," using the word " derived " to

> denote the fact that an emotion of this class is not constantly correlated with any one impulse or tendency, but rather may arise in the course of the operation of any strong impulse or tendency.

Our emotional experiences of fear, anger, wonder, and so on may be tinged not merely with pure pleasure or unpleasure, but with subtle variations of these elements, according to the degree to which we anticipate the probable future course of events ; for example, the " anxious " fear of a mother who, when her child fails to return from school at the usual hour, not only fears a possible accident but half contemplates that her fear may be realized. Her " anxiety " is that particular modification of unpleasure which normally accompanies fearful anticipation.

Disposition and Moods

Following A. F. Shand (1858–1936), McDougall classified confidence, hope, anxiety, despondency, and despair as " prospective emotions of desire," and regret, remorse, and sorrow as " retrospective emotions of desire." The former accompany a conative tendency whose goal is in the future, while we experience the latter when our impulses are directed towards the past, e.g., when we regret what might have been.

Touching upon the subject of " disposition " and " moods," when a person is spoken of as being of a timid, or irascible, or inquisitive

disposition, the expression denotes primarily that the person in question is specially liable to evince fear, anger, or curiosity, as the case may be. A person's disposition, therefore, is dependent upon the fact that the various innate tendencies to behaviour and their accompanying emotions seem to be inherited in different degrees of strength in different persons, and it seems at least possible that racial differences are partly of this kind.

At present this suggestion is little more than speculation, as anthropologists are by no means agreed as to what is really meant by race. Many differences between peoples popularly regarded as racial and therefore as innate are really acquired differences due to different cultural traditions.

"Moods" are related to the emotions ; this is shown by the fact that in everyday speech the moods are described by adjectives derived from the names of the various emotions, e.g., an angry or pugnacious mood, etc. Moods are commonly due to the evocation of an emotion which fails to find complete expression. A man provoked to anger at his work, for example, may find it expedient to conceal his anger for the time being, but will return home, in consequence, in an angry mood.

The persistence of moods seems to be due—at least in part—to the persistence of the bodily conditions which accompany the emotion, and a mood renders the person suffering from it especially susceptible to a recrudescence of the appropriate emotion. Kicking the cat because some time earlier someone to whom one cannot retort has aroused one's anger, is an example of a mood initiated by frustrated emotion passing into open emotion once more.

LESSON 11

Theories of Human Temperament

The classification of men according to temperament dates back to classical antiquity. Hippocrates, the Greek physician (*c.* 460–377 B.C.), taught that the body was composed of four "humours," and that health was due to a harmonious blending of these. The four humours were blood, phlegm, and yellow and black bile.

About five hundred years later another great physician, Galen, distinguished four varieties of temperament, each due to a preponderance of one or other of the four humours, viz., the sanguine, the choleric, the phlegmatic, and the bilious. A man of sanguine temperament was characterised by superficiality of emotion and inconstancy ; a bilious temperament manifested itself in stability of emotion and inflexibility of purpose ; the words choleric and phlegmatic had roughly the same meaning as at the present day.

Endocrine Theory of Temperament

Recent discoveries concerning the endocrine glands have caused some investigators to conclude that temperament is that aspect of personality determined by the influence on the general chemical processes of the body and upon the nervous system in particular of the endocrine glands considered as an interrelated system. It is doubtful, however, whether the ascertained facts really justify some of the theories which have been elaborated.

It is well known that inadequate secretion from the thyroid, a small gland in the throat, leads to general sluggishness, and that, in the very young, thyroid deficiency results in imbecility and dwarfism ; and also that equally drastic consequences ensue from the malfunctioning of the pituitary gland at the base of the brain. Excess of the secretion from this gland in youth leads to giantism, while a deficiency results in a dwarfish development.

Modern advocates of the endocrine theory of temperament see in these and similar facts the final key to all problems of personality. The subject is too technical and controversial to pursue here, but those who are interested are recommended to read Dr. E. Kretschmer's *Physique and Character*. Dr. Kretschmer's view, briefly, is that temperament is the expression in mental life, not merely of the endocrine glands, but of the whole bodily chemistry.

Asthenics and Pyknics

On the basis of measurement Dr. Kretschmer distinguished three physical types, which he called the asthenic, the pyknic, and the athletic. Asthenics are lean, and of average height, but have narrow shoulders, flat chests, and poor muscular development. Pyknics are medium in height or short, with rounded figures, thick limbs and prominent abdomens. Athletics are well developed and above the average in height and strength, but are regarded by Dr. Kretschmer as a sub-variety of the asthenics rather than as a distinct type.

There are two sharply differentiated forms of insanity known respectively as manic-depressive insanity (characterised by alternation between extreme exaltation and extreme depression) and schizophrenia (in which the patient lives entirely in a fantastic dream-world of his own). From a study of the physique of asylum patients Dr. Kretschmer was led to the

conclusion that there is an affinity between manic-depressive insanity and the pyknic type of physique on the one hand, and between schizophrenia and the asthenic and athletic types on physique of the other.

Cyclothymes and Schizothymes

Following up this clue, Dr. Kretschmer distinguished two extreme types of temperament, the cyclothyme and the schizothyme. The individual of cyclothymic temperament tends to be sociable and genial, but is apt to oscillate between moods of exaltation and depression. Physically he is of the pyknic type. The schizothyme is apt to be of asthenic or athletic physique ; in temperament he is unsociable and reserved, capable of deep emotion, but outwardly cold and unresponsive.

In the majority of people temperament is a blend in varying degrees of cyclothymic and echizothymic characteristics, and physique a corresponding blend of pyknic and asthenic factors. Sufferers from manic-depressive insanity and schizophrenia can be regarded as representing the extreme ends of a scale of temperamental difference, the centre of which is occupied by normal, well-balanced individuals, while more or less well-marked cyclothymes and schizothymes fill the gaps at each end.

More recently W. H. Sheldon, in the U.S.A., developed a system of classifying people according to their physique, using a photographic method. He distinguishes three components of physical build, and considers that these physical dimensions are related to definite psychological types. His theories are related to those of Kretschmer, but constitute an advance on earlier work in that he provides ways of measuring and estimating the degree to which each physical component is present in any one person.

Much work has been done both in Britain and in the U.S.A. on the statistical analysis of psychological tests which have been specially designed to measure temperamental differences. By using mathematical techniques, a number of factors (or dimensions) of temperament have been described. Thus Eysenck, in London, considers that there is an important factor of neuroticism, and that people vary in the extent to which they have this factor in their personality. He has also found a factor on introversion-extraversion, similar to that described by Jung (*see* Lesson 24).

While these factors are of great descriptive value, it should be remembered that they are statistical constructions, and may not exist as real entities.

LESSON 12

Role of Sense-Organs in Mental Life

PRECEDING Lessons have dealt primarily with affection and conation, while cognition, the third of the distinguishable features of mental life referred to in Lesson 4, has been more or less taken for granted. It is now time to deal with cognition in more detail.

An important part is played in cognition by the sense-organs. Very simple organisms such as the minute, jelly-like creature, Amoeba, seem to be sensitive all over to environmental stimuli such as light, temperature, etc., but in the higher organisms responsiveness to external stimuli becomes concentrated in the various special sense-organs. In addition to the external sense-organs, moreover, there are internal sense-organs associated with the muscles and joints, the gullet, lungs, stomach, etc. The experiences of sounds, colours, smells, etc., of which we become aware as the result of the stimulation of our sense-organs, are known technically as *sensations*.

Cutaneous Sense Organs

The skin contains four distinct kinds of sense-organ, each of which gives us a different kind of sensation. If the back of the hand is explored carefully with the point of a pencil at the temperature of the surrounding air, the pencil will evoke at most places a simple sensation of contact, but here and there a distinct cold sensation will be felt. This is due to the presence of "cold spots," at each of which the sensory nerve-endings within the skin are specialised to give that particular response only to stimulation. There are several cold spots within a half-inch square on the back of the hand.

The four kinds of sense-organ within the skin give the sensations of warmth, cold, touch, and pain. Most so-called touch sensations are really blends of some or all of these four kinds of sensation. The sensation of burning heat, for example, is a blend of warmth and pain. Each kind of spot responds in its own way irrespective of the stimulus. A cold spot will give a sensation of coldness even if stimulated with a warm object. Most objects which come into contact with our skin stimulate more than one kind of spot. A sensation of moistness is a combination of touch and cold. A sensation of great coldness is a sensation of pain, due to stimulation of pain spots as well as cold spots.

The sense-organs in the muscles, tendons, and joints give information regarding the movement

and position of our limbs, and also make us aware of the resistance of objects to our muscular efforts. The muscle sense-organs and the sense-organs in the skin work together to inform us of the hardness or softness of objects. Hardness is partly touch, but mainly a muscular sensation of resistance.

The sensation of pressure is due to the stimulation of nerve endings in the muscles when the skin is pressed down on them. If pressure increases, the resulting sensation is one of what is called "deep pain," i.e. the peculiar pain sensation felt, for instance, when another person grips one's arm tightly. Sensations of pressure and of deep pain are unobtainable when the skin over the portion of the body concerned has been rendered anaesthetic, so that the sense-organs in the skin itself are no longer responsive to stimulation.

Sense of Smell

The sense-organs of smell are situated well back inside the nose, and it is not possible, in consequence, to apply separate stimuli to the different kinds of nerve cell found there in order to ascertain whether they are separate, simple sensations of odour comparable with the simple sensations of touch, pain, etc., given by the skin. But by assembling collections of objects giving as many different odours as possible, and then endeavouring to classify the various odours on the basis of the resemblances and differences between them, it has been found that odours can be arranged in series grading from one "salient" odour to another. H. Henning, a German investigator, distinguishes six salient odours as follows, the words in brackets indicating the substances which have these odours :

Spicy (pepper, cloves, nutmegs, etc.) ; flowery (heliotrope, etc.) ; fruity (apple, vinegar, orange juice, etc.) ; resinous (turpentine, pine needles, etc.) ; foul (hydrogen sulphide, etc.) ; scorched (tarry substances).

The student will note that the adjectives used to designate the odours are mainly derived from the names of *things*. The fact that while there is an extensive vocabulary to describe colours, sounds, and tastes, we have for smells to make shift with derivative words, brings out clearly the relatively unimportant role played by smell in human mental life. Recent investigation has shown that there is a relation between the chemical structure of the substance and the sensation of smell evoked by it.

The acuteness of the sense of smell differs greatly from one individual to another. Some people are unable to smell certain odours at all ; on the other hand, there is a frequently quoted instance of a woman in charge of a boarding school who sorted the boys' linen, *after the wash*, by the smells alone ! Many so-called smells are really not smells at all. Ammonia, for example, is, strictly speaking, odourless ; the sensations derived from sniffing it are due to its irritant action on the sensitive membranes of the nose and throat, etc.

Sense of Taste

The interior of the mouth yields sensations of touch, temperature, and pain, as well as taste sensations proper, and most "tastes" are really combinations of taste, odour, and other sensations. Investigation has reduced the number of pure taste sensations to four : bitter, sweet, sour, and salty. The organs of taste are situated in protuberances on the tongue and soft palate, called papillae. Some papillae give only one taste, others two or three or all four.

The taste organs for bitter are situated principally at the back of the tongue, those for sweetness at the tip, those for sour at the sides, those for salty at the tip and sides. Substances yield sensations of taste only when dissolved. If the tongue is dried and a pinch of dry salt placed on it, only a sensation of contact is felt until saliva accumulates again on the tongue and dissolves the salt.

Many tastes are largely smells. If the nose is plugged with cotton wool, a piece of apple and a piece of onion will be found to taste alike. As examples of "tastes" which are blends of several kinds of sensation, tea and lemonade may be cited. Analysis shows that tea gives a combination of sweet and sour tastes, a sensation of temperature, a smell, and a weak pain sensation, while the characteristic "flavour" of lemonade is composed of a blend of a bitter taste, a temperature sensation, an astringent sensation, and a smell.

Awareness of Colour

Our awareness of the colours of objects is dependent upon the stimulation of the retina of the eye by light of varying wavelengths. Light of a particular wavelength will stimulate special sense-organs to send a nervous impulse to the brain, and the impulse from any one receptor in the eye will always give rise to the perception of the same colour. Stimulation of the brain in human beings undergoing brain operations under local anaesthesia can give rise to various sensations of "colour." The psychologist is concerned with the colours which we actually experience ; these, like smells and tastes, are sensations. It is not possible here to deal adequately with all the complexities of colour experiences, but two of the most important points must be touched upon, i.e. the so-called complementary colours and "after-sensations."

If a disk divided into differently coloured sectors is rotated rapidly, different colour effects will be experienced according to the colours used. If only red and yellow sectors are used, the rotating card will appear to be orange ; the

two colours blend to produce an intermediate colour. If red and green are used, both colours will disappear and the rotating card will appear to be grey.

Colours which cancel each other in this way instead of blending are called "complementary" colours, and every colour has a complementary colour in this sense. The complementary colour to blue is yellow. (The mixing of pigments does not yield the same results, as the factors involved in this case are much more complex.) Complementary colours are also known as "contrast" colours. For example, red and green are complementary. Suppose that we place a red disk on a white background and cover the whole with tissue paper so as to blur the edge between the two colours. The white background in the neighbourhood of the red patch will assume a greenish hue. Red and green are thus said to be "contrast" colours, because when in the circumstances stated one appears, it suggests or causes the other colour to appear also.

After-sensations

After-sensations can be classified as negative and positive. If the student will look steadily at some brightly coloured object for 30 seconds or so, and will then turn his eyes to a white surface (the ceiling, for instance), he will see a patch similar in shape to the coloured object but complementary to it in colour. This experience is called a negative after-sensation; after-sensations of this kind are constantly complicating our colour experiences. Every woman must have felt how difficult it is to judge the precise colour of a piece of material after having examined a number of differently coloured pieces in succession. Difficulties of this kind are due in part to the persistence of negative after-sensations of the colours previously examined.

A positive after-sensation can be obtained by looking steadily at an electric light for a few seconds and then interposing a book between one's eyes and the light. For a short but appreciable period one will seem to see the light faintly shining through the book.

Vision

Novelists frequently speak of "sweeping glances," but a little attention on the part of the student to the eyes of his friends will show him that this expression has no justification. The eye sees only while it is at rest; thus when we are looking round a room, our eyes move not continuously but in little jerks, jumping from one point to another. The same thing occurs in reading, the eyes jumping from one point to another along the line and then jumping back to the beginning of the next line, the actual seeing taking place between the jumps.

One's usual appreciation of the solidity of objects is due to the co-operation of both eyes. Each eye, taken separately, yields a slightly different impression of the object, but the two impressions together combine to give a "stereoscopic effect," i.e. a three-dimensional effect. But this is not true of objects which are more than about a hundred feet from the eyes.

Hearing

The sounds of which we are aware are due to the stimulation of our ears by vibrations in the air. Sounds may be classified as musical tones and noises. A pure musical tone is due to the stimulation of the ear by a succession of uniform vibrations. An irregular, unsteady medley of vibrations gives a noise. Tones differ in loudness, in pitch, and in timbre. Differences of pitch are correlated with differences of vibration rate; differences of timbre are due to the presence with the fundamental tone of what are called "overtones." When a violin string vibrates, it not only vibrates as a whole, there is also a separate vibration of the half, the third, and so on. It is these subordinate vibrations which give the overtones, and the differences between the overtones given along with the fundamental tone afford one of the chief means of distinguishing between the sounds of different musical instruments.

Complex Speech Sounds

Speech sounds are very complex. There are not only tones and overtones due to the vibration of the vocal cords, but additional high tones due to the mouth and nose. In pronouncing *ee*, for instance, mouth tones are prominent which involve a vibration rate ten times that of middle C of the piano. The pronunciation of *s* involves a mouth tone of which the vibration rate is three times higher. These high mouth tones are audible only to a very sensitive ear. Hence the difficulty even the partially deaf person experiences in understanding speech; he can hear sounds, but he misses the proper values of vowels and consonants.

Other Functions of the Ear

The ear analyses sounds much more delicately than the eye analyses colours. The range of colour variation to which the eye is sensitive is comparatively limited. Audible tones range (in terms of air vibrations) from 20 to 20,000 vibrations a second, and most people can distinguish tones only four vibrations apart. It is interesting to speculate how different human life would be if the reverse were true and we discriminated colours more accurately than we discriminate sounds. Colour art would be as complex as music is now, and music, if it existed at all, would be a very simple and crude affair.

The ear is not merely the organ of hearing.

Certain portions of the inner ear, called the vestibule and the semicircular canals, play an important role in the maintenance of posture and equilibrium. Inclining the head in various directions leads to the stimulation of sensory nerves in the vestibule, as a result of which we are able to judge the position of our head and correct it if necessary.

The semicircular canals lie in different planes, and contain a watery fluid which rotates when the head is rotated. The rotation of this fluid stimulates hair-like nerve cells within the canals, and it is from these cells that we derive those sensations which enable us, even when we are blindfolded, to judge accurately in which direction the head is rotating.

LESSON 13

The Nature of Perception

A LITTLE practice in introspection will enable the student to isolate sensations of the various kinds described in Lesson 12, but we do not live in a world of " pure " sensations. A prick in the arm is experienced not merely as a combination of contact with something and pain, but as the point of a needle or a thorn. A patch of blue colour is seen as the sky, or a flower, or a woman's frock. Our sensations point beyond themselves to a world of objects and events and persons which we *perceive* by means of them.

Perception in adults is complicated by thought, imagination, and the influence of language, but in the higher animals, and probably in very young children, these complicating factors are absent. The nature of perception (i.e. the process of apprehending and responding to colours, sounds, and so on, as having meaning) is only imperfectly understood, and psychological theory on the subject has changed considerably in recent years.

An Active Process

Perception is essentially an active process of interpreting our sensory experiences. Let the student listen for a moment to noises reaching him from the street outside. Considered simply as noises, each of them is a sensation, but this one, he realizes, is the horn of a passing car, that a dog barking, while that distant rhythm of musical notes is a band in a neighbouring road. Underlying these distinct and prominent sounds there is possibly a background of other noises, among which nothing special can be discriminated.

A simple experiment of this kind illustrates several of the factors involved in perception. At any one moment the world around us which impinges upon our sense-organs provides us with a multitude of different sensations. To our eyes the world appears as a variegated manifold of coloured patterns. In the Tate Gallery, London, there is a striking picture by Pissarro of the Boulevard des Italiens at night. At a distance of a few yards one sees the gaily-coloured kiosks, the shop-fronts, the long rows of lights, the passing cars, all the colourful animation of a Parisian thoroughfare. Go nearer and examine the picture closely, and it suddenly becomes an almost meaningless medley of patches and daubs of differently coloured paint.

The Visual Field

Analysed into its component sensations, our visual field at any moment is like the picture seen close at hand. When we *perceive* it, as a world of things, it is like the picture seen from a distance. In looking at the picture from a distance, one sees more than is actually *given* on the canvas. The cunning arrangement of pigments merely serves as a cue ; the perception of the boulevard at night is the response of the percipient to that cue. To a savage contemplating the same picture at the correct distance it would appear entirely different. Never having experienced motor-cars or newspaper kiosks or arc-lights or shop-fronts or even a street, he could not now see them in the picture.

Precisely the same would be true of the sounds heard from the street. To the student they are significant ; they are indicative of the presence of this or that real object in the world, but they can be so only in virtue of his past experience. In the case of a person who had had no previous experience of dogs or motor-cars or brass bands, the various prominent sounds which impinge upon his ears might still catch his attention, but they would not be meaningful ; he would hear them, but he would no longer *perceive* a world of objects by means of them.

Patterns in Perception

How is it that sensations such as colours, sounds, etc., come to serve as cues to things ? Past experience is obviously important, but how does the process of learning which is involved begin ? Into what sort of world is the child, who is without past experience, born ? The first point to notice is that the baby's world is not a mere medley of sensations pouring in upon him in inextricable confusion. Sensations differ in quality, fall into groups, exhibit

orderly arrangement. The blind man, restored suddenly to sight and seeing a landscape for the first time, would be aware not merely of a multitude of coloured points, but of coloured shapes—here an expanse of green, above it an expanse of blue, and so on. Colours are not merely given as so much crude colour, but are given in *patterns*.

The importance of patterns of this kind in perception has been much stressed by a group of German psychologists known as the *Gestalt* school. Gestalt is a German word meaning shape, form, or configuration. A melody is a good example of a Gestalt or pattern. The various notes constituting the melody are arranged in a particular way. In a melody, moreover, the arrangement is more important than the separate notes, as can be realized from the fact that a change of key changes each of the notes without changing the melody itself.

Complex Sensory Pattern

The Gestalt school maintains that patterns, rather than isolated sensations, form the basis from which the perceptual interpretation of sensory experience starts. A now classical experiment performed by Professor Köhler is frequently cited in support of this view. Two boxes A and B, each painted in a different shade of grey, are placed side by side, and an animal is trained to take food from the lighter-coloured box B. The box A is then replaced by another box C of a grey still lighter than B, and it is found that the animal now goes to C for its food instead of to B.

This result is inexplicable if, as would once have been assumed, the animal's food-seeking impulse had merely become attached, by "conditioning" (*see* Lesson 7) to a particular visual sensation, i.e. the grey of the box B. The animal has in fact been responding to the whole complex sensory pattern "two-shades-of-grey-one-lighter-than-the-other," and continues to do so. Similar experiments performed with young children have produced similar results. An extremely important kind of pattern is that afforded, e.g. by a compact, closed figure of definite outline against a relatively vague background. To the student this may sound mere common sense, but insistence upon the fact marks, nevertheless, a complete break from a long-established and respectable tradition in psychology.

The second point to note is that perception is a process which subserves practical ends. The ability to hear a noise just round the corner as proceeding from a rapidly approaching motor-car may be a matter of life or death if one wishes to cross the road.

LESSON 14

The Processes of Perception and Conation

As previously pointed out, perception is a process which subserves practical ends. Perceptual interpretation is carried on under the guidance of conation organized in the innate, conative tendencies already discussed. The hungry animal singles out and attends spontaneously to the smell of food, and the human infant—who, of course, is a gregarious animal—singles out and attends to those visual and auditory patterns which are faces and voices, within 25 days of birth. Thus not all the sensations resulting from the stimulation of the sense-organs receive equal attention. Some are discriminated, others are more or less ignored from the beginning.

Various factors facilitate interest and discrimination. Of these the most important is change. Our sensory experience is always changing, but sometimes it may change abruptly. Rapidly moving objects, sudden noises, and so on are instances of this kind of change, and provoke interest and attention.

Sensory patterns do not occur in isolation. They are discriminated within a wider context, and they recur in different contexts. Consider the visual pattern in the baby's world which is its mother's face. The child is hungry and its mother feeds it ; it is in discomfort and its mother relieves this. If the student bears in mind the account which was given in Lesson 7 of Pavlov's dogs and the process of "conditioning" he will realize that a similar process takes place in the baby.

Meaning and Retention

The whole situation for the child is one which terminates in conative satisfaction, and the mother's face is a recurrent visual sensory pattern in the satisfying situation. Her face thus comes to mean the satisfaction of hunger or the removal of discomfort to the child, just as the note of the bell comes to mean food to the dog. The mother's voice is a similarly recurrent pattern, and acquires meaning for the child in the same way. A part of the total situation (i.e. the face or voice as a single visual or auditory pattern) comes to stand for the whole, and to evoke in the child behaviour appropriate to the whole. Thus when it sees its mother's face, it stretches its arms out and crows, just as the dog salivates.

With the occurrence of different situations—

as, for example, when the child is reprimanded or restrained and its conative impulses are thereby thwarted—new elements are added to the meaning of the visual and auditory patterns. To take an instance of another kind: the child sees a round, compact something, reaches out its hand and grasps a ball.

Tactual sensations of smoothness and so on follow, and the next time the ball is seen this previous experience of its qualities as felt gives a new meaning to the visual pattern. In both examples the principle is the same; past experience persists in virtue of *retention*, a basic fact in mental life (*see* Lesson 4), and it enriches the present. Endless instances might be given.

Progressive Discrimination

The world, however, is a complicated place, and perceptual meanings acquired in this way are at first inadequate to its complexity. The baby to whom the household black cat is a warm, soft thing to be stroked, strokes one day a tabby interloper and is scratched. Discrimination between black and tabby patterns soon follows. Failure to discriminate is shown in the early stages of talking when, in virtue of having learned to call the dog "bow-wow," the child calls all small animals "bow-wow." Children's drawings are interesting in this connexion; it has been found, for example, that very young children shown drawings of human heads without ears or eyes, etc., completely failed to notice the missing parts. Discrimination between closely similar sensory patterns takes place, because failure to make it leads to practical difficulties and therefore to thwarted conation.

Perception is thus a process in which we apprehend the presence and meaning of some total situation through the effect on our senses of some element within that situation which is *recognized* in virtue of past experience of it. Perception so defined is only possible because the world is an orderly place, in which things do not happen without sufficient cause.

The active interpretative nature of perception is best seen in cases in which the interpretation of the given sensory pattern is erroneous. The timid person sees a coat hanging on the wall in a dark room as a person; and to the apprehensive mother a dozen noises seem to be the crying of her baby upstairs. Conation is a powerful determining factor here.

A subtler example is seen in misprints which occur even in the most carefully printed books. The proof-reader jumps to the conclusion that the word is rightly printed because he expects it to be so. The actual visual pattern of the printed letters resembles sufficiently the expected familiar word to be mistaken for it.

The fact that such "perceptual illusions" occur leads to a problem with which this brief account of perception must close. To state the problem as a question: What is the relation between the world we apprehend in perception and the "real" world as it is, apart from our apprehension of it? The role of conation and past experience in perception, and the dependence of our sensory experience upon our possession of highly specialised sense-organs, inevitably suggest that the perceptual world of each individual is a private world, and the conclusions of the physical sciences seem to support this view.

Philosophical Problem

There is no resemblance between the physicist's world of space-time occupied by electrons and protons and the world of coloured and tangible "things," located in space, enduring through time, and shot through and through with emotional and conative values and meanings, in which each of us lives.

The problem is a philosophical one, and philosophers differ considerably in their views on the subject, which cannot, therefore, be pursued here, because a full appreciation of its significance and complexity requires a considerable knowledge of philosophy. The student should bear in mind that the nature of perception is not only a question for the psychologist but one for the philosopher also.

LESSON 15

The Process of Learning

Everyone understands, in a rough and ready way, what learning means; yet the problem of the nature of the learning process, and of the precise factors involved when a man, or a child, or an animal learns to perform some new act, is one of the most difficult and controversial questions in psychology. Perception involves learning. The child who says "Gee-gee" on seeing a horse has learned to interpret accurately a particular sensory pattern. In the first part of this Lesson are given some instances of learning in the sphere of behaviour. These are followed by a brief discussion of the more important contemporary theories on the subject.

The point has already been stressed that all behaviour is adaptive; it is directed to achieving an end which subserves the well-being of the creature concerned. In any example of learning, some new set of

circumstances, at first baffling, is mastered, and the capacity for adaptive behaviour, for achieving ends, is extended and enriched. The problem for the psychologist is to analyse and understand, if he can, the process or processes by means of which this enrichment of adaptive behaviour is attained.

Maze Experiments

An immense amount of experimental work has been done in connexion with the investigation of the learning process, but only a few typical instances can be cited. One experiment is that of placing a rat in a maze, from which it can escape only by following an intricate path. A hungry animal is used, and the reward for escaping is food. At first the rat, after going through every inch of the maze, turning and doubling and exploring and going over some paths several times in succession, hits upon the way out by chance. In successive experiments blind alleys are progressively avoided, and eventually the rat travels unhesitatingly by an invariable and well-chosen path from the inlet to the outlet.

Roughly similar results are obtained with blindfolded children and adult human beings. The children and adults—particularly the latter—learn the maze more rapidly than rats, and make fewer mistakes, i.e. enter blind alleys or turn back on their course less frequently. Improvement with the rats is only gradual ; with human beings there is apt to be a sudden improvement, marked by an abrupt drop in the number of errors, after the first few trials.

Rat's Problem

The puzzle box experiment is comparable with that of the cat in the cage (*see* Lesson 8). Food is placed in a box which the animal—a rat, say—can open only by manipulating a simple latch. After a period of trial and error behaviour like that of the cat, the animal hits upon the right method of doing this, and opens the box and obtains the food. A second latch is then added, which holds the first in position. Professor McDougall has said :

> Again the rat attacks the problem , pushes and pulls at L1 (the first latch) with teeth and paws, but in vain. Can he be taught by showing him how to move L2 ? Let us try. After he has struggled vainly for some time, we push his nose or his paw gently against L2, thus releasing L1, and leave him to his task. He quickly pulls down L1 and obtains his food. After a few repetitions of this teaching process he quickly deals effectively with L1 and L2.

A third latch is then added, and the same procedure followed. Other latches are then added successively up to 14, each one being so arranged as to lock the preceding one. According to McDougall, the rat requires to be assisted up to the seventh latch, learning more rapidly however, with each attempt.

> From the seventh latch onward the rat needs no assistance, no guidance ; he deals effectively with each new latch, mastering it within a very few seconds of the outset of his attack on the box. His facility becomes so great that he opens the whole series of 14 latches in about three seconds, repeating this again and again with sure deft movements and moving so rapidly that the eye can hardly follow. But sometimes he is a little careless, and fails to move a latch far enough ; then he attempts to raise the lid and finds it fast ; at once he runs back to L14, rapidly reviews the latches, finds the one on which he fumbled, opens it and the remaining latches up to L1, and so obtains the food.

Similar experiments were performed by McDougall and his son with baboons, which were found to master all the latches in a series of 24 without assistance

Chimpanzee Experiment

A different kind of experiment has been performed by Professor Köhler with chimpanzees. A chimpanzee which had acquired the habit of using a stick to reach and drag towards him a banana lying on the floor outside his cage, and thus out of his reach without the stick, was given two sticks, one of which could be fitted into the end of the other to make a single long stick, and the banana was placed sufficiently far away for it to be impossible for him to reach it with either stick alone. For some time the chimpanzee attempted in various ways to reach the banana by using the sticks separately, but at last gave the problem up.

A few minutes later, while playing with the two sticks, the chimpanzee happened by chance to fit one into the other, whereupon he immediately jumped up and ran to pull the banana in with the jointed stick. When the sticks fell apart, he put them together again, obtained the banana, and without stopping to eat it raked in everything else he could reach. On the following day, when the experiment was repeated, the chimpanzee began by making merely random movements with the sticks, but in a few seconds fitted them together and repeated his previous performance.

Child and the Toy

The foregoing experiment has been performed with small children with very similar results. An example is quoted by Professor Woodworth. A small girl just over three years of age learned in two tests on successive days to use a single stick to reach a toy placed just out of her reach outside her playing pen. On the first day she made various unsuccessful attempts to reach the toy, but although she handled the stick she did not hit upon the device of using it. When she did so on the second day, she did so quite suddenly, pouncing upon the stick after accidentally treading upon it, and reaching the toy with its aid in a few seconds. Four days were necessary for the child to learn to use two jointed sticks. During the tests on the first three days she attempted to reach the toy in

various days, using each stick separately, but on the fourth day, after two minutes, she said :

> "Let's try big stick on little one," picked up the other stick, examined ends carefully and succeeded in fitting them, with a shout of "Bang !" In a few seconds she angled for the toy, reached it exultantly, and repeated the stunt several times.

Significance of "Learning" Experiments

The experiments show that learning is sometimes a gradual process and at other times occurs relatively suddenly. The rat in the maze masters its intricacies only slowly ; human beings usually show a sudden improvement in this kind of test, the number of errors they make dropping markedly from one attempt to the next.

Sudden learning, in which random fumbling gives place abruptly to mastery of the situation, is most clearly seen in the experiment with the jointed stick. The chimpanzee seems to "tumble" all at once to the essential point of the problem ; and in the similar experiment with the child, the child's words give explicit expression to a sudden appreciation of the key to the problem.

Theories of the nature of the learning process fall into two main groups, one of which stresses the gradual, trial and error character of the process, while the other stresses the element of suddenness and insists that all learning takes place through "insight." The most celebrated advocate of the "trial and error" theory was an American psychologist named Thorndike (1874-1949). Insistence upon "insight" comes from the Gestalt school, referred to in Lesson 13.

Trial and Error

On the "trial and error" view, learning is fundamentally blind. The animal confronted with a novel situation responds to it with random movements, some of which, by chance, are successful. The rat in the maze escapes from it only because it covers every inch of the maze in its random doublings and turnings. Why, then, is it that on subsequent trials the animal traverses the maze more rapidly, until eventually it runs unhesitatingly from inlet to outlet, avoiding blind alleys and ceasing to double on its tracks ?

In answer to this question the school of psychologists under consideration appeals to the "law of effect." Those movements which lead to success are "stamped in" by the conative satisfaction of reaching the food ; unsuccessful movements are eliminated through the "unpleasure" of thwarted conation. (The law is frequently stated in more behaviouristic terms, but the foregoing is the more plausible form of it.) The behaviour of the rat which has learned to run the maze thus consists of a series of actions selected, by the operation of the law of effect, from the wider series of random and purely impulsive actions with which the rat first responds to the situation of finding itself in the maze.

Learning theory has become one of the most important fields of study in contemporary psychology, and it can be applied to many spheres of behaviour which do not at first sight seem to be connected with it. The student is referred to the work of Clark Hull and E. C. Tolman for more detailed study of the process of learning and its possible application. It will suffice to say here that many psychologists believe that even the behaviour of mentally disturbed patients can be accounted for in terms of learning theory.

Insight and Perception

Professor R. S. Woodworth, in *Contemporary Schools of Psychology*, expresses the following view :

> The stress of Gestalt psychology is on the perceptual factor in learning. Learning means doing something new. The newness cannot be understood by examining the motor performance alone, for the newness consists in a reorganization of the situation, so as to bridge the gap between the situation as it is and the goal. The gap is bridged by seeing the situation as a pattern including and leading to the goal.

According to this view, all learning rests upon "insight" into the relevant features of the situation to be coped with. The chimpanzee, and more clearly the child, suddenly *grasps* the fact that the two sticks can be joined to make one. This "insight" need not involve reasoning ; it amounts, essentially, to *perceiving* a combination of elements as offering the key to the problem. The rat in the puzzle box perceives the location of the latch and its relevance to the opening of the lid. Rats are capable of very little perceptual insight and need to be helped several times ; racoons, standing at a higher stage of mental evolution, are able to dispense with help.

It is this element of insight, according to the Gestalt school, which accounts for the fact that an animal confronted with a novel situation learns suddenly to deal with it. Even in maze experiments, where improvement seems most gradual, psychologists of this school contend that careful examination of the process reveals sudden advances, which are inexplicable unless some perceptual grasp of the situation, some recognition of "landmarks" as pointing towards the outlet, is assumed.

Insight is an obvious element in human learning. In solving a puzzle, a child or an adult will try first this method and then that, but soon or late, according to the care with which the puzzle is scrutinised and the degree of attention devoted to it, i.e. the strength of the conative impulse determining interest in it, the solution will be seen, and thereafter the puzzle presents no difficulty and "trial and

error" fumbling ceases. The merit of the Gestalt school has been to draw attention to the presence of insight even in those cases of animal learning which were formerly assumed to be most blind and to be most easily explicable on other lines.

Not all the nervous impulses reaching the central nervous system are perceived as sensations. They may be quite unconscious, and in recent years it has been possible to show that a great deal of learning and conditioning can take place quite unconsciously. In particular, changes in the autonomic nervous system (which may lead, for instance, to a quickening of the heart rate) may be conditioned to stimuli which have never become conscious.

LESSON 16

The Nature and Measure of Intelligence

When we speak of one individual as intelligent and another as stupid, we usually mean that the first shows a greater capacity than the second for dealing effectively with novel situations, for applying the results of past experience to a present problem, and for understanding new material. The psychologist means much the same thing when he speaks, for example, of a rat as behaving more intelligently than a solitary wasp, except that, as he is concerned with facts rather than with passing judgments upon them, he does not call the wasp stupid.

The study of the differences between individual human beings in regard to their capacity for behaving intelligently, in the sense just defined, has led to much controversy as to what intelligence is. Is it a special "faculty" or does intelligent behaviour depend upon a number of factors?

Intelligence Tests

A Frenchman, Alfred Binet, early in the 20th century devised the first set of what are called "intelligence tests." Other investigators have refined and improved upon Binet's tests, and at present there are about 30 different sets in use. An intelligence test consists essentially of a number of questions and tasks which a child, say, is set to answer or perform. The questions and tasks are graded in difficulty according to the age of the child.

If a child of eight is able only to perform tasks or answer questions which experiment has found to be within the capacity of the average child of six, then the child is said to have a *mental age* of six, although its chronological age is eight. The mental age divided by the chronological age gives what is called the child's Intelligence Quotient. For convenience in calculations this fraction (the said quotient) is usually multiplied by 100 and expressed to the nearest figure. The intelligence quotient (or I.Q.) of the child of eight under discussion would thus be 75. The I.Q. of the average child is, of course, 100, as the "average" child is simply the child able to perform those tasks which, so far as investigation shows, most children of his age are able to perform. The intelligence quotient is thus a purely statistical concept. It is important that the student should realise this and not imagine that I.Q. stands for some mythical entity or thing. The intelligence quotients of different children represent the extent of their deviation from or conformity to a calculated norm.

It has been found that the results of intelligence tests agree fairly closely with the estimates made by teachers of the "intelligence" of children, and that a child's I.Q. remains roughly constant when it is tested at different ages. The tests certainly measure something definite in the child's make-up, but it remains an estimate, albeit a good estimate, of the child's intelligence. It does, however, vary quite readily (though not perhaps more than one or two points) from day to day, and from year to year. Children who are "slow starters" may actually have a higher I.Q. in later life. It should also be stressed that different tests may provide different I.Q.s for the same child, because they may measure slightly different things.

Nature of Intelligence

Controversies as to the nature of intelligence turn upon the fact that the different questions and tasks included in an intelligence test call upon a wide range of different abilities. Some of them involve memory, others imagination, others reasoning power, and so on. The "intelligent" person tends, on the whole, to give good results over the entire range of questions and tasks. But does this fact, it is asked, imply that intelligence consists in a collection of separate abilities, e.g. the ability to memorise, the ability to grasp relations—such as the relation of cause and effect—the ability to reason, etc., or must some general factor be assumed, comparable with, say, the general muscular strength involved in weight-lifting and wrestling, etc.?

This problem has been attacked by what is called the "method of correlation." Special and separate tests for memorising, reasoning, etc., are used, and the results obtained are compared by the use of mathematical methods.

It is not possible to discuss the matter in detail here; but, roughly speaking, two different interpretations of the results obtained from this line of study have been put forward.

Two-Factor Theory

One of these two interpretations was the "two-factor" theory of intelligence of Professor Charles Spearman (1863–1945). Prof. Spearman held that the achievement of a given individual in some special test, e.g. a memory test or a reasoning test, depends upon two factors, one specific to that kind of performance and the other common to all performances. The various specific factors involved in different performances are mutually independent, and are called by Spearman "special abilities"; the general or common factor he calls *g*.

In an "intelligence test," which involves many dissimilar mental performances, the influences of the various special abilities (memory, reasoning, mechanical aptitude, etc.) tend to cancel out, and the result of the test is thus to give a measure of *g*. The special abilities, according to Spearman, can be improved by training; *g*, however, cannot.

What is *g*? Like I.Q., it is a purely statistical concept. "But," said Spearman, it is "a measured value which constantly recurs, and therefore must have some constant basis in fact," and he suggested that the special abilities may be regarded as so many mental "engines," while *g* is a supply of mental "energy" which may be directed into this engine or that. He contended also that mathematical reasoning discloses other measured values, that is, *p*, or the "degree of inertia" of *g*, and *o*, or the "degree of oscillation" of the supply of *g*. He concluded that *g*, *p*, and *o* together "furnish the chief features which distinguish the abilities of one individual from those of another."

Professor Spearman's view of the nature of "intelligence" has been hotly attacked, and the alternative hypothesis of "group factors" is now generally accepted by psychologists. According to this school of thought, many different kinds of mental performance have a common factor, but there is no factor common to *all* mental performances. Certain special abilities are closely related to each other and constitute a group, each ability in such a group having a factor in common with the others; other abilities form a second group, and so on.

One of Spearman's critics, Professor T. L. Kelley, finds evidence of the existence of seven separate group factors. Five of these have to do, variously, with verbal material, number or quantitative concepts, memory, spatial concepts, and speed. The other two are "ebullience" or "vivacity," and a spatial factor involved in "the manipulation of spatial relations as distinguished from their apprehension and retention."

Ascending the Evolutionary Scale

More recently the work of Sir Cyril Burt in London and Professor L. L. Thurstone in Chicago has shown that while there is certainly a *g*, there are definitely group factors which affect a person's final I.Q. Thus someone may be intelligent with a verbal bias, or his intelligence may be mainly due to his numerical or spatial abilities.

The views of the nature of intelligence discussed differ, it will be noted, over the question of the existence of a general factor as completely determining intelligent behaviour. The existence of special abilities is now generally accepted. This suggests that the increasing capacity to behave intelligently exhibited by different animals as one ascends the evolutionary scale of animal life is correlated with an increase in the variety and richness of the special abilities possessed by the different species. From this point of view, the immense superiority in "intelligence" displayed by man in comparison with the lower animals consists primarily in the much greater range and number of his special abilities.

LESSON 17

Theories of Memory, and Memory Images

MEMORY, as ordinarily understood, covers a number of mental processes, which are best considered separately. The most fundamental of these processes is *retention*. The conditioned reflex is a simple instance of retention. The dog salivates in response to the bell in virtue of its past experience (*see* Lesson 7). Its experience during the course of the experiment has modified it in some way and, as a result, it behaves in a manner of which it was not previously capable.

The fact that our past experience is retained and modifies our present experience is also exemplified in perception and learning. Precisely how past experience is thus retained is a mystery, although, according to some psychologists, it is understandable to some extent if we assume that every experience we have causes microscopical changes in the structure of the nervous system.

A question frequently discussed by psychologists is whether the capacity for retention

can be improved by practice. Advertisements for systems for improving "the memory" usually imply that it can, but positive evidence on the point is difficult to obtain.

No amount of culture (wrote William James) would seem capable of modifying a man's general retentiveness. This is a physiological quality, given once for all with his organization, and which he can never hope to change.

James's view is still held by many psychologists, but certain experimental evidence seems to point the other way. It is well known that something which has been learned and subsequently forgotten can be re-learned in much less time than was required for the original learning. In the experiments in question it was found that practice over many months in learning and re-learning rows of syllables resulted in an improvement as regards rapidity and accuracy in both learning and re-learning; the improvement in re-learning, however, was much greater in the case of three out of the six persons experimented on than the improvement in learning. This result suggested that the experiment led to an improvement of the capacity for retention in the three persons concerned; but, for reasons which cannot be fully discussed here, the evidence is not conclusive.

Memorising

Most apparent improvements of the memory consist in increased efficiency in memorising and, to some extent, in an increased capacity to recollect material learned. A vast amount of experimental evidence proves beyond a doubt that the power of memorising can be immensely improved by practice. Memorising is an active process, and it has been found that certain methods of memorising yield much better results than others.

If, for example, a student working for an examination desires to commit to memory a long poem, or a list of mathematical formulae, which is the best method of setting to work? Recitation, i.e. repeating the material over to oneself without the book, is important, and investigation shows that the best results are obtained when from three- to four-fifths of the available study time is devoted to recitation, as opposed to mere silent reading and re-reading. The point to notice here is that recitation constitutes actual practice in the task to be learned, i.e. in repeating the chosen material from memory.

Another important result established by experiment is that whole learning is more productive of results than part learning. A poem of 240 lines, for instance, can be learned more easily by reading it through as a whole three times a day for ten days, than by memorising 30 lines a day and then reviewing the whole until it can be recited without error.

The superiority of whole learning to part learning is largely due to the fact that the general meaning of the material to be learned and the relations between its various parts are better grasped when it is learned as a whole. Part learning, moreover, tends to establish irrelevant associations between the beginning and end of each part, whereas the right associations are between the end of one part and the beginning of the next.

Recollections

Recollection may be of two kinds. A child questioned by his teacher recollects that Dublin is in Ireland. In answering the question he may recollect not merely the geographical fact, but the further fact that he first learned it on such and such a day. The latter type of recollection is an instance of "pure" memory, in the somewhat restricted sense in which the word memory is most appropriately used in psychology, i.e. as awareness of some situation or event *as past*.

It is important to notice that one may recollect in the first sense and not in the second, and vice versa. In the performance of any skilled action such as writing, cycling, reading, and so on, we perform a series of actions or apprehend a series of meanings, and our performance in each case is dependent upon the fact that our past experience is brought to bear upon the present. In reading we understand what we read because we recollect the meaning of the various words. But it by no means follows that we recollect the occasions on which we first learned these meanings, although we may do so. The present writer recollects perfectly and in detail the occasion, many years ago, on which he first learned the meaning of the French word "robinet" (water-tap), but he has no similar recollections in relation to other French words which he understands.

Memory and Anticipation

Pure memory is probably, though by no means certainly, a prerogative of human beings. From an evolutionary point of view it seems likely that the capacity to anticipate the future precedes the ability to recall and dwell upon the past. In anticipation, the present suggests what will be, on the basis of what has been. The dog that begins to jump and bark when its master puts his hat on has a "preperception" of the coming outing, i.e. it is aware of the coming situation before that is definitely presented to its senses.

In human beings anticipation is accompanied by imagery and, arguing from analogy, it seems probable that this is also true of preperception in animals. An image is the kind of thing of which we are aware when we see something "with the mind's eye" or hear sounds

"with the mind's ear"; and the difference between the preperceptions of animals and human anticipation, which may relate to matters in a comparatively remote future, is largely due to the greater freedom of imagery in human beings. In the preperceptions of an animal, such imagery as occurs is strictly and narrowly relevant to its present perceptions.

The young child uses imagery extensively, both for practical purposes, e.g. in solving problems, and in fantasy, as an illusory means of satisfying needs and impulses which circumstances debar him from satisfying in reality. Both uses are important, but it is the latter (fantasy) which probably forms the stepping-stone to memory proper, in which past events are recalled by means of imagery and explicitly dwelt on and realised to be past. From anticipating, with the help of imagery, the kind of meal he would like to have if he feels hungry when dinner is still a long way off, the child passes to definite recollection, also aided by imagery, of yesterday's tea-party as something over and done and to be dwelt on with regret.

Dating of Memories

From what has been said the student will have noted that imagery is present in both anticipation and memory. But its function differs in the two cases. In anticipation it sustains and accompanies conative impulses which are directed towards the future; in memory conation is directed towards the past. As is to be expected, memory in children is very indefinite, for a child's whole life is essentially a striving towards the future.

Anna Freud, as a result of her experience with nursery children, has shown that there is a definite change in the child's memory almost exactly at the age of five. After this age nearly all the experiences undergone by the child are forgotten, and few adults have genuine memories antedating their fifth year. She has also been able to show that much of this forgetting is the result of repression, which will be described later in this Lesson.

Even for the mature adult, who stands "midway in the path of life" and looks backwards and forwards with equal frequency, the dating of memories is difficult and inaccurate. We recollect A as following B, but we are doubtful about the time-gap between them unless we can fill in the intervening period with other recollections.

Forgetting is primarily failure to recollect rather than a failure to retain. While it is doubtful whether everything learned or experienced is retained indefinitely, as is sometimes contended, it is certain that we retain much more than we can recollect. Some past experiences seem to be completely obliterated with the lapse of time. Trivial and uninteresting experiences fall into this category, and the process whereby the memory of them gradually fades and disappears is called *oblivescence*.

Association by Contiguity

This is only a partial explanation of forgetting, for we obviously forget many things that we should like to be able to recollect. Recollection is largely a matter of association, and forgetting is usually due to faulty or hampered association. If two experiences have occurred together in the past, the re-occurrence of one of them will tend to lead to the recollection of the other. This is the so-called law of *association by contiguity*.

As was pointed out in relation to the conditioned reflex (Lesson 7), the re-occurrence of part of an experience tends to reinstate the whole experience, and the essential factor in determining what shall form a whole for this purpose is conative interest. C. K. Ogden, in his *A.B.C. of Psychology*, writes:

Contiguity operates only subject to the guidance of interest. If we are thinking about niblicks, the ideas which are associated with the idea of grass will be quite different from those which will gather if we are thinking about Nebuchadnezzar. The governing principle in association is the direction of interest, and contiguity only works inside this principle.

Thus, if we are keenly interested in some branch of knowledge, any fact at all relevant to it will tend to lead us to recollect other similar facts, and the greater and more enduring our interest, the greater our capacity for relevant recollection will be. In so far as other interests are temporarily dominant in our minds, our ability to recollect in relation to this special interest will be weakened.

Forgetting or failure to recollect is primarily conditioned by a conflict of interests, the operation of one interest inhibiting associative processes connected with the other. In extreme instances of conflict between interests or, which is the same thing, between the conative impulses determining interests, the resultant forgetting is known as *repression*. Repression is an active process whereby memories which are distasteful or out of harmony with the dominant interests of the person concerned are thrust out of mind.

Memories which are repressed are still retained although they cannot be recollected, and may continue to influence conduct in important and unexpected ways.

Nature of Images

The images which accompany our memories of past events stand for or mean those events in the same way as that in which sensations stand for or mean the objects which we perceive. Our views as to the precise nature of memory will depend very largely upon our views as to the nature of images. Images are of various kinds, corresponding to the various possible kinds of

sensation. Thus we may not only see with the mind's eye and hear with the mind's ear, but taste with the mind's palate and smell with the mind's nose.

Individuals differ greatly in their capacity for the different kinds of imagery, some people being predominantly visualisers, others experiencing more vividly auditory imagery and even tactile imagery. In adults who read much, verbal imagery often predominates, i.e. images of words as seen on the printed page or as heard.

Images differ from sensations in several ways, which are not easy to describe. The most important difference, perhaps, is that an image does not vary in relation to bodily movement, as a sensation does. A mental picture of a landscape does not change or vanish when one turns one's head ; the sensory patterns perceived as a landscape change constantly with the movements of one's eyes.

Alternative Theories

There are many theories as to the nature of images. According to one, sensations are special events or existents which come into being as the result of the stimulation of our brains through our sense-organs. Images are events or existents of a similar order which occur when those parts of our brains which have been involved in the production of sensations, and which have been microscopically modified in consequence, are re-stimulated into activity "centrally," i.e. by the activity of other parts of the brain, instead of "peripherally," i.e. through the sense-organs. The objection to this theory is that it completely fails to explain how sensations and images which exist only as the result of events *inside* our heads enable us to apprehend a real world *outside* our heads.

An alternative theory runs somewhat as follows. Sensations are not special events or existents resulting from the stimulation of our brains through our sense-organs, but are aspects of the external world of which the stimulation of our sense-organs makes us aware. In anticipation and memory our past acts of awareness are re-lived, and we have, in consequence, the illusory experience of re-apprehending those events in, or characteristics of, the external world upon which our acts of awareness were originally directed.

Illusions

On this view, images are illusions. Ordinarily, we recognize them as such, for in remembering or anticipating we are also aware of our present surroundings, with which our images fail to harmonise. In dreams and in hallucinations this test fails us, and we then believe our images to be genuine sensations. This view is attended by its own peculiar difficulties, but it must suffice here to say that on the whole the objections to it are probably less grave than the objections to the alternative theory previously referred to. Here again, as with the problem of perception, philosophy and psychology meet, and to pursue the subject farther would take us too far afield.

LESSON 18

The Imaginative Process

In actual mental life, memory, imagination, and thought are inextricably interwoven, but for the purposes of psychology it is desirable to consider them separately, even at the cost of some artificiality of treatment. Both memory and imagination involve imagery, but there are two essential differences between them.

The first difference concerns their respective relations to the real world. In remembrance, our imagery stands for some series of past real events, and it is our awareness of this fact which makes us say that we are remembering. Accurate memory conforms to the past series of events, and even when we remember inaccurately we believe that our memories reproduce past events in the manner and order in which they occurred, although in fact this is not so.

Imagination is either directed towards the future or is free, in varying degrees, from any reference to the actual course of events, past or prospective. In the second place, the stuff or material of the imagery used in imaginative processes, although derived in the last resort from past experience, is manipulated, rearranged, and combined in new ways. It is important to note that imagination is not, so far as its material is concerned, independent of past experience, and is creative only in a limited sense.

A congenitally blind man could never, for instance, construct imaginary colour schemes, because he has never experienced colours ; nor, for a similar reason, could a congenitally deaf man construct, in imagination, combinations of sounds. The most extravagant trains of imagination are in this sense tied down to the world of real events.

Kinds of Imagination

Two kinds of imaginative process may be distinguished, viz. "controlled" imagination and "free" imagination. The latter is exemplified in day-dreaming, but the word "free" is to some extent inappropriate. The real difference between the two kinds of imagination

consists in the nature of the control exercised. In what is usually called controlled imagination, the process of rearranging and combining elements derived from past experience is directed towards some specific and consciously realized end. In listening to a description by a friend of some unusual or otherwise interesting series of events in which he has taken part, we reconstruct, in imagination, with the help of the clues supplied by his words, the various incidents as he recounts, and our desire to do this as fully and accurately as possible sustains and guides our imagery. Any effort to realize scenes or incidents which we have not ourselves directly experienced, or to enter sympathetically into the experiences of others, involves this kind of imagination.

Anticipation of Future Events

Controlled imaginative processes occur most frequently in the service of thought. An explorer, turning over in his mind the relative merits of two alternative routes across unknown and dangerous country, may endeavour to picture to himself the various situations he is likely to encounter, and this imaginative anticipation of future events facilitates his final choice of route. In the same way, an inventor will endeavour to picture to himself what the apparatus he is trying to devise will look like. Here imagination is exercised in the service of deliberate planning, which, of course, requires thought as well as imagination.

In thinking, we discriminate and attend only to certain aspects of concrete situations, e.g. those aspects in virtue of which we regard situation A as the cause of situation B, or situation C as resembling or differing from situation D. Thinking is concerned with the *relations* which subsist between situations or the constituent parts of situations, whereas imagination, in the strict sense, is the process of picturing the situations themselves.

Play-World and Real World

Day-dreaming is a form of play. If we contrast the play-world of the child with the world of the adult, the most important distinction between them is that the child is master in his play-world and moulds things to his desires, whereas the adult must conform his desires to the demands of the world. For the very young child the distinction between the play-world and the real world does not yet exist. In his *A B C of Psychology* C. K. Ogden observes :

> The young child perceives things *only* in their relation to his own needs and desires, and his world is a reflection of his own inner activities . . . . Thus a shift in interest has an effect upon his world which to the adult is difficult to comprehend. The block which ten minutes ago was an automobile suddenly becomes something to throw about, and the next moment may turn into a tree . . . . We regard the block consistently as a wooden cube, but to the child it is only something which favours now one and now another of his interests.

Wish Fulfilments

The process by which the child's world gradually breaks into two—a real world and a play-world—and the manner in which the claims of the former gradually oust the latter are too complex to be described in detail here. Day-dreaming is a process in which the adult regresses for the time being from the real world, with its insistent demands, to the freer, more plastic world of childish play. Owing to the fluidity of his perceptions, the child is able to play with real things ; the adult, debarred from this activity partly by his greater knowledge of the objective characteristics of things and events and partly by the greater complexity of his interests, conducts his play in terms of imagery.

Day-dreams are often compensatory. In the language of the psycho-analysts they are "wish fulfilments," and the function of imagery in a day-dream is to provide an illusory gratification for some impulse which it is not possible to gratify in reality. The town-dweller, confined to an office or a factory, day-dreams of pleasant meadows and tranquil streams ; the clerk day-dreams of the life he would lead if he were managing director ; the rejected lover day-dreams of the bliss which might have been his if the lady who was the object of his hopes had been kinder.

Castles in the Air

In day-dreams the conscious, voluntary control operative in the kind of imagination discussed earlier in this Lesson is replaced by the control of the impulse seeking gratification. Of the existence of this impulse the day-dreamer may be largely unconscious, and associative processes which would disturb the momentary domination of this impulse are inhibited. Hence the coherence of day-dreams, which are never merely haphazard trains of imagery, but genuine "castles in the air," differing in structure according to the nature of the impulse at work.

Novel-reading is frequently a form of day-dreaming. Instead of constructing our own cloud castles we allow the novelist to do the job for us, by identifying ourselves with his characters and living imaginatively their more colourful lives. The prevalence of novel-reading is, in fact, a sure indication of the extent to which modern conditions thwart the natural play of human impulses.

LESSON 19

Nature of Thinking and Reasoning

THINKING is the mental manipulation, in the service of our purposes and interests, of the knowledge of events and things and persons which we have derived from past experience. At one time thinking was regarded as a rather mysterious activity, performed by human beings by virtue of their possession of a special "faculty" called reason, but this view has long been abandoned by psychologists.

Thinking does not involve the exercise of any special faculty or *new* form of mental activity, but turns out, on analysis, to consist in a mobile combination of discrimination, recollection, association, trial and error, all of which are known to occur in some form or other at lower levels of mental life.

Trial and Error Thinking

Let us consider the problem of finding in a crossword puzzle a word of eight letters which is the name of a famous character in antiquity, and let us assume that it can be seen that the third letter of the word is *c*. This or that likely word is recalled ; the process of recall involves association under the guidance of conation, represented here by the desire to find the solution. Thucydides ? No, *c* here is the fourth letter and there are ten letters in the name.

The puzzle is considered further. This other word already filled in in the puzzle suggests that the first letter of the missing word may be *s*. Further possible words are recalled. Each attempt at solution is checked against the requirements of the problem, and as the essential features of the latter are more clearly grasped, e.g. that the word must not only contain eight letters, two of which are *s* and *c*, but must fit in, let us say, as regards all its letters, with its context in the puzzle, the line of solution may shift. Can the second letter of the word be obtained by finding this other, neighbouring word ? Yes, the second letter is *o* ; *s-o-c*—Socrates.

In this instance the trial and error nature of thinking and its fundamental resemblance to the trial and error behaviour of animals are clear. The difference between the two lies partly in the degree of insight involved and partly in the greater efficiency with which, in thinking, past experience is mobilised in relation to the problem in hand. In animal trial and error behaviour the solution is hit upon by accident ; in thinking the essential features of the problem are selected by a process of insight. Most important of all, past experience is brought to bear in the form of explicit recall.

Essential elements in all thinking are called *concepts*. The meaning of the word "dog" is a concept. The same meaning can, of course, be conveyed by another word in another language, e.g. by *chien* in French or *Hund* in German. The concept is not the word itself but the *meaning* of the word or that to which the word refers. Any meaning which refers, not to a particular object, but to a class of objects or to some feature common to many objects, is a concept. Sweetness, hardness, and so on are concepts and so are horse, man, etc.

The capacity to form concepts is essentially a capacity to analyse some given concrete situation and to discriminate within it resemblances to and differences from other concrete situations. This collie dog and that terrier are different animals, but they have certain characteristics in common, and the concept "dog" refers to these common characteristics in virtue of which both animals are recognized as members of one class.

The process of concept formation, i.e. the process of discriminating common features and relations in the world of events and things and persons, is not an easy one, and the history of human thought is largely a history of the errors to which this process is subject. The savage discriminates with difficulty, if at all, between the animate and the inanimate ; and the queer, animistic superstitions which make savage thinking so alien to the civilized mind are consequences of this faulty discrimination.

Scientific thinking, broadly speaking, is a trial and error process whereby adequate concepts, which correspond to real articulations in the world of events, are substituted for the crude concepts of primitive man. Here again there is no question of some new, mysterious mental "faculty." Sir Isaac Newton (1642–1727), in the process of formulating the concept of gravitation, was only exercising more efficiently a capacity already present in a lesser degree at very lowly levels of mental life. The chick, rejecting hairy caterpillars as uneatable after once tasting one, already shows a capacity for discrimination and for responding to a series of situations in virtue of some feature common to them all. In Newton this capacity operated to immensely greater effect ; but that is all.

Process of Reasoning

This capacity to discriminate differences and to recognize similarities, which leads to the formation of concepts, underlies also the process of reasoning. Reasoning is of two kinds :

inductive, which proceeds from particular instances to some general principle ; and deductive, which proceeds from a general principle to particular instances.

The germ of inductive reasoning is present whenever some single experience leads to confident anticipation of its recurrence under similar circumstances. The burnt child who avoids further contact with fire is making an implicit inductive generalisation. The generalisation is not, of course, formulated ; the child does not say "This fire burnt me, therefore all fire burns," but the process of verbal formulation when in due course it occurs, adds nothing essentially new. As Professor McDougall said :

> The tendency to inductive generalisation is . . . fundamental and is exhibited at all levels of mental life. At the lower levels, it is merely the tendency to react to . . . things presenting similar sensory cues, as though they were essentially the same thing over again, and because the world is so full of a number of things which do fall into natural classes, the members of each of which present similar sensory cues . . . this tendency in the main serves us well . . . and, in spite of the errors to which it gives rise, it is the source of our highest scientific generalisations or laws.

Deductive reasoning is implicit in recognition. On a country ramble the walker sees a large, brownish bird, with blue bands on its wings. "A jay," he says. At the back of his mind is the general principle that all large, brownish birds with such wing bands seen in British woods are jays, and the principle is applied in this instance. The classic formula for deduction is "all *a* is *b* ; this is *a* ; therefore, this is *b*." This formula merely lays bare the structure of a process found at all levels of mental life, whereby concrete experiences are interrelated in virtue of some feature common to them all. Wherever there are discrimination and recognition of similarity, reasoning is implicit; from a psychological and evolutionary point of view, human thinking only exhibits these factors of mental life at the highest level they have so far attained.

LESSON 20

The Psychology of Language

OTHER animals besides human beings make noises. But while the whining of dogs, the bellowing of cattle, the songs of birds, etc., are merely expressive of emotion, the use of noises for the additional purpose of naming or referring to objects and events in the external world seems to be a purely human achievement. C. K. Ogden, in *A.B.C. of Psychology*, says :

> The difference, though simple, is fundamental. A merely expressive cry arises directly from the animal's need, his want, his desire, his joy or fear, his interest in general, and *it varies with this activity*. But a naming cry arises from the perception of a given state of affairs and *varies with this state of affairs*. Briefly we express ourselves alike because *we* are alike ; we name things alike because *they* are alike.

The simplest use of a sound as a name is to refer to some single, concrete object, but only proper names such as Tom or Dick or words like "this" and "that" can be used in this way. The meaning of most words, e.g. common nouns, adjectives, prepositions, etc., is conceptual. Language is a vehicle of thought. It provides an elaborate system of conventional signs in which the results of the thinking of countless generations is embodied and preserved, and in learning to speak each of us enters into this thought heritage. Words break up for us the complex flow of events, and enable us to grasp easily articulations in nature which it has taken centuries of arduous, and almost wholly anonymous, thinking to discover.

Consider a word like "bird." Confront the unaided mind of the ordinary individual with the wealth of bird life of even a temperate country like Britain, and the discovery that all these apparently diverse creatures, from the wren to the seagull, were fundamentally of the same kind would probably never be made.

Exploration and Discovery

There is another side to the picture. Thinking is essentially a process of exploration and discovery, and although words enable the results of past thinking to be preserved and passed on to later generations, the very efficiency with which they do this tends to hamper the advance of thought into new fields. We fail to distinguish between things for which we use the same word, and overlook the resemblances between things which we designate differently. We group together—under one word, such as "memory"—mental processes which it is essential to distinguish if we are to think clearly on the subject, while we draw distinctions, e.g. that between "instinct" and "intelligence," which have little or no justification in actual fact.

Language perpetuates slipshod and inaccurate, as well as accurate, thinking, and the child learning to speak is probably learning quite as often to think badly as to think well. Half the difficulty which the average man experiences, for instance, in making sense of contemporary physics, which sees the universe as a continuum of events, describable accurately only in mathematical formulae, is due to the habits of mind induced by ordinary language, which with its nouns and adjectives suggests a universe

of static "things" with fixed and enduring "qualities."

Language has other functions, besides serving as a vehicle, good or bad, for thought, and it retains much of the expressive character which it must have had when man was still barely differentiated from his pre-human ancestors. Words like "nasty," "horrible," "thrilling," "disgusting," "delightful," are obviously indicative of the emotional attitude of the speaker rather than of characteristics of the objects or events which we describe in such terms, and language plays an important part in determining that members of the same social group shall react emotionally in the same way to the same things.

Aspects of Language

This effect of language is specially obvious in the case of moral judgments, perhaps three-fourths of which are merely verbal formulations of emotional attitudes of approval or disapproval, acquired almost as mechanically—under the influence of social pressure embodied in language—as have been the "conditioned reflexes" of Pavlov's dogs (*see* Lesson 7).

Few people ever learn wholly to cease to regard as "bad" or "good" actions to which they were taught to apply these words in early childhood ; yet the history of ethics shows that it is almost impossible to attach any consistent and objective meaning to them which will survive logical scrutiny. An interesting point to note is that the meaning of a word, as expressive of an emotional attitude, may vary with intonation. A number of adjectives, e.g. "clever," "nice," can be made to convey a variety of subtly different meanings according to the tone in which they are pronounced. There are many pairs of words, moreover, which refer to the same qualities, but of which one in each pair expresses approval and the other disapproval, e.g. "thrifty" and "mean," "prudent" and "cowardly," "courageous" and "rash."

Other aspects of language, such as the changing significance of words, expressive and referential, according to their context in sentences, and the subtle associative links between words which enrich their meanings, are too complicated to be dealt with here.

Enough has been said to enable the student to realize the important part played by language in the mental development of us all. Language is a highly complex product of a long period of social evolution, and each of us assimilates by its means a sum of knowledge and experience far too vast for any individual to acquire unaided. There is nothing comparable among animals to this accumulation and transmission of traditional knowledge ; hence the vast gap between the most primitive peoples and even such intelligent animals as anthropoid apes.

Unique Instrument

Man's capacity to use and elaborate the unique instrument of language has its disadvantages as well as its advantages. It enables him to perpetuate error and folly as well as truth and wisdom, and each fresh advance in knowledge is, in consequence, achieved almost in spite of language. This is largely because language, which is a vehicle of emotional expression as well as of conceptual meaning, is equally a vehicle of subjective prejudice as well as of objective insight ; and it is a significant fact, on which the psychology of language throws much light, that the greatest historical advances in knowledge have been made in the mathematical sciences, in which thinking is carried on, not in words, but in specialised symbols, which have been stripped of irrelevant emotional connotations.

Language is only one aspect of the whole process of communication between individuals and groups. Other methods of communication include the written word, facial expression, gestures, etc. Modern communication theory embraces all these modes of expression.

LESSON 21

Projection Tests

In previous Lessons various types of psychological test have been mentioned. These have included tests of intelligence, whereby it is possible to give a child (or an adult) a mental age or an I.Q. Tests of intelligence are rather like the end-of-term tests given at schools. The child either passes or fails each item in the test, and his I.Q. is calculated on the basis of how many items he has passed, compared with the average child of his own age.

There are a great many tests being used by psychologists to-day which do not involve passing or failing. These tests are for the most part tests of temperament and personality, and foremost among these are the so-called projection tests. They are being used daily by clinical psychologists (psychologists who work in mental hospitals or psychiatric clinics) as a means of understanding the nature of their patients' problems, especially where the exact diagnosis of the mental condition of a patient is in doubt.

In all these tests the subject is presented with material of some sort (e.g. inkblots, vague

pictures, etc.), and is asked, without necessarily being given specific instructions, to respond to them. Thus he might be asked to tell a story about the picture he is given, and it is this story (or set of stories, if he is given more than one picture) which forms the basis for the psychologist's assessment. The theory is that the subject will put a great deal of himself into his imaginative productions, and that he may even reveal his most secret or unconscious drives and wishes, by attributing them to the characters in his story. In other words, he *projects* into the stimulus-material (the pictures) his own personal problems and conflicts.

Ink-blot Tests

Herman Rorschach (1884–1922), a Swiss psychiatrist, published in 1921 a monograph in which he described a test consisting of ten standard ink-blots printed on cards. Some were in colour, others in black and white only. These cards are given to the subject of the investigation, one at a time, and he is asked to say what he can make out of them—what things he can see in the blots if he allows his imagination some freedom. Rorschach put forward the theory (and there has been a great amount of subsequent research to support his contention) that the way in which people varied in their responses to the blots reflected fundamental differences in their personalities.

He was able to show that extroverted, emotional people, for instance, were freer in their use of colour in their answers than more introverted people, in whom the use of form predominated. Rorschach suggested a way of counting up and scoring of the responses to the ink-blots, and this has remained practically unchanged as the method in use to-day. The Rorschach test is particularly valuable in the diagnosis of the mental state of *schizophrenia*, and is useful also in the diagnosis of cases where the mental condition is the result of organic damages to the brain.

The interpretation of the results of a Rorschach test is a matter of considerable skill, and those psychologists who make use of it in clinical work have usually undergone a specialised course of training lasting some years. So much work has been done with this test that in 1954 more space was taken up in psychological journals by articles on the Rorschach test than on any other single topic.

Thematic Apperception Test

The thematic apperception test, called TAT for short, was devised by Professor Murray in 1935. It consists of 30 photographs of rather ambiguous situations (or rather, situations which are open to a large number of possible interpretations). Twenty of these photographs are used for men, 20 for women, 10 being common to both sexes. The subject is asked to make up a story about each of the pictures in turn, and these stories are then studied by the psychologist, who finally makes his report upon the subject's personality. The stories can be most revealing, and are an extremely ingenious way of getting at hidden or repressed material. Thus a patient may have an unconscious hatred of his father, which he would never admit to himself or to anyone else. Yet in the stories he may constantly be referring to the hero's dislike of his father or of some figure in authority.

It has long been known that an artist may put a lot of himself into his drawings and paintings. Many psychologists, particularly those who work with children, make use of free drawing or of such techniques as finger-painting, to try to find out what is going on inside their patients. These artistic efforts can often be very revealing, though as yet there is no generally accepted way of systematically interpreting artistic productions.

LESSON 22

Introduction to Mental Pathology

The student who has worked steadily through the preceding Lessons in this Course, and who has supplemented his studies from the suggestions for further reading which are given in the book list, should now be in possession of a fairly serviceable working knowledge of some aspects of what may be called "orthodox" contemporary psychology.

In the last 60 or 70 years certain schools of thought have grown up which have tackled the problems of mental life from the side of medicine, and which, as the result of the study of mental disorders, have elaborated and put forward theories as to the fundamental nature of mental processes which differ radically in many important respects from those to which the student has so far been introduced.

The most influential of these schools are those associated with the names of Dr. Sigmund Freud (1856–1939), of Vienna; Dr. C. G. Jung (born 1875), of Zürich; and Dr. Alfred Adler (1870–1937), of Vienna.

Psychoses and Psycho-neuroses

Mental disorders are generally classified in two groups, i.e. the so-called psychoses or insanities, which are very profound disturbances of the normal flow of mental life, and the psycho-

neuroses, in which the disturbance is less profound and much more amenable to treatment. The conclusions of the three schools of thought mentioned are based primarily upon the study of the latter, though more recent developments in psycho-analysis owe a great deal to the study of the psychoses.

Hysterical Symptoms

The first attempt to elaborate a psychological explanation of hysterical symptoms was made by Professor Pierre Janet (1859-1947), of Paris. Hysterical symptoms are those in which a patient undergoes a change in his body or in its functions, and for which there is no organic basis. The commonest hysterical symptoms at the time of Janet were paralyses and anaesthesias (loss of sensation). Janet noted that hysterical anaesthesias, for example, corresponded in distribution, not so much with any possible injury to the nervous system, as with the idea which the patient himself had of the part of his body concerned. He also noticed that patients suffering from hysterical anaesthesias rarely sustained accidental injury to the limb involved, whereas in organic anaesthesia, i.e. anaesthesia due to a lesion of the nervous system, such injuries are frequent. As a result of his study Janet put forward his theory of "dissociation of consciousness."

According to Janet, the stream of conscious events, which in normal people is a unity, is split up in hysterical patients into two or more independent currents. In the case of hysterical anaesthesia of a limb all the sensations arising from the limb are diverted into a separate current of consciousness, so that the patient himself is unaware of them. These sensations can still influence the motor side of the nervous system, so that in a dangerous situation the limb is moved and thus avoids injury.

Multiple Personality

Janet's theory of dissociation of consciousness can be successfully applied to a very wide range of abnormal mental phenomena, of which automatic writing may serve as an example. Dr. Bernard Hart, in his *Psychology of Insanity*, writes :

> This curious condition, although occasionally exhibited by comparatively normal people, attains its most perfect development in . . . hysteria. Suppose that we engage an hysterical patient in conversation and, while his attention is thus diverted, insert a pencil between the fingers of his right hand. If a third person now whispers some questions into the patient's ear, it may be possible to induce him to write answers to these questions, although he continues all the time to discuss with us some totally different subject. Under such circumstances it will be found that the patient is entirely unconscious of what his hand is doing, and is, moreover, often altogether ignorant of the events which the writing describes. These events frequently relate to episodes in the patient's past life which he appears to have completely forgotten . . .

According to Janet, there is here a case of dissociation. For some reason or other the patient's mental processes have split into two separate streams, one of which is involved in the conversation and the other in the automatic writing, each stream being independent and unaware of the other. Dr. Hart, in the book cited, applies this conception of dissociation to a series of apparently dissimilar mental disorders, and shows how well it fits the facts.

The most famous and elaborate case of dissociation is that of Miss Sally Beauchamp, described in minute detail by Dr. Morton Prince (1854–1929) in *The Dissociation of a Personality* (1906). Miss Beauchamp had three distinct personalities, designated by Prince as BI, BIII and BIV. BIII knew the thoughts of BI, but had no knowledge of the thoughts of BIV, while BI knew directly only her own experience. BI was a nervous, thoughtful conscientious person, BIII mischievous and lively, but good-natured, and BIV proud, assertive, and egotistical. BI knew French well, BIII knew nothing of the language whatever, while BIV had only a smattering of it. The alternations of these personalities were at times astoundingly rapid, each of them in turn controlling Miss Beauchamp's body and directing her actions within a period of an hour or two.

Partial Integration

The notion of dissociation, although at first perhaps a little bewildering, is nevertheless quite compatible with the view of mental life which has been suggested in previous Lessons. According to our present view, wishes, thoughts, desires, and so on are the expression in consciousness of an underlying structure or organization which, beginning in the child as a relatively simple system of more or less independent innate tendencies, gradually develops, as sentiments are built up and knowledge is acquired, into the elaborate and highly integrated intellect and character of the adult.

The degree of integration achieved varies widely, and when for some reason or other integration in a particular individual is only partial, there will be, instead of a single, closely-knit mental structure, a series of relatively independent structural systems. These find expression in consciousness in more or less independent series of mental events.

The Notion of Dissociation

We have only to push this line of explanation a little further and assume an actual cleavage or disintegration of mental structure, and we shall have, so far as conscious mental events are concerned, just that "dissociation of consciousness" which we have been considering. Instead of the slightly different business and home and social personalities of the ordinary man, we shall

have sharply disparate and mutually exclusive personalities. If hysterical symptoms and cases of multiple personality are due to dissociation, why does dissociation itself occur? Janet's own answer to this question, which need not be described in detail here, is not very satisfactory; and for the next step in the understanding of pathological mental phenomena we turn to Freud, who substituted for the notion of dissociation his theory of the unconscious.

LESSON 23

Theories and Technique of Freud

THE views of Sigmund Freud as to the nature and genesis of mental disorders arose initially, like those of Professor Janet, from the study of hysteria. Freud began his scientific career as a physiologist, specialising in the study of the nervous system and its diseases, but subsequently abandoned these studies for medical practice owing to the need for earning a livelihood. As a student of neurology he was naturally interested in the psycho-neuroses, but his attitude towards them seems to have been influenced by two facts.

One of these was a remark by Charcot, a French neurologist, of whom Janet was a pupil, to the effect that in all cases of neurosis there was some trouble in the individual's sex life. The other was an observation made by Dr. Josef Breuer (1842–1925), who, like Freud, was a physician in Vienna. Breuer found that when he hypnotised one of his women patients who suffered from hysteria, she was able to recollect certain otherwise forgotten and highly emotional incidents of her past life, and that when by suitable methods he enabled her to recall those incidents during her ordinary waking state, her symptoms disappeared.

Mental Catharsis

Freud and Breuer were old friends, and on the basis of Breuer's discovery they elaborated a special method for the treatment of hysteria which they called " mental catharsis " and used with considerable success. This method consisted essentially in recovering by the use of hypnotism certain memories of which the patient was otherwise unconscious, and then encouraging the patient to talk these over freely and so give them normal emotional expression.

The co-operation of Breuer and Freud soon came to an end and, left to himself, Freud (who found that many patients were not susceptible to hypnotic treatment) dropped the use of hypnotism and continued the talking-over method alone. This method gradually developed into the " free association method," which is the foundation-stone of the present Freudian technique for the treatment of mental disorders. The patient refrains from concentrating on any particular subject and simply allows his thoughts to wander freely, relating to the physician whatever happens to come into his mind.

Freud found that this method was more successful than hypnotism and led to better results. Later he supplemented it with dream analysis. The patient recounts his last night's dream and, prompted by the physician, lets his mind play freely about the various items of it, each of which is considered with a view to disclosing the forgotten memories which underlie the patient's symptoms.

Neuroses and Sex

While using these two methods, i.e. free association and dream analysis, Freud found that his patients required a good deal of urging before they could reproduce the relevant memories. It seemed as if some active force in the patient's mind were opposing their resuscitation. When the memories were disinterred, they were always of a predominantly sexual nature and out of harmony with the dominant moral and other standards of the patient's conscious personality. The patient's mind seemed to be the seat of incompatible and warring processes, some of which found expression in his ordinary, conscious mental life, while the others, driven down and denied conscious expression, found indirect vent through the production of neurotic symptoms.

The next step in the elaboration of Freud's views is of fundamental importance. Patients whom he had treated by the two methods referred to and who had been dismissed as cured, began to come back to him suffering from slightly different forms of neurosis. He therefore assumed that he had not hitherto carried his analysis far enough, and that behind the memories which he had so far succeeded in resuscitating in his patients' minds there must be other, earlier memories which were now producing new symptoms.

Day-dreams from Childhood

In searching for these earlier memories, Freud made a startling discovery. Some of his women patients began apparently to recollect incidents from their childhood which, when corroboration was sought from other sources, turned out to be quite fictitious. These spurious memories were always of sexual attacks which, according to the patient, had been made upon her by her older male relatives.

Freud's interpretation of this curious state of affairs is one of the keystones to his general psychological theory. As the memories did not relate to actual fact, they must, he concluded, relate to some childish day-dream of the patient which was expressive of an unfulfilled wish or desire. *The root cause of the patient's symptoms was not the buried memory of a particular incident, but an unsatisfied childish wish of a sexual nature.*

This line of approach was confirmed by other evidence. Breuer had broken his partnership with Freud owing to the fact that one of his woman patients, towards the end of his treatment of her, had fallen violently in love with him. Freud himself had had similar experiences, and he therefore came to the conclusion that the result of analysis was to revive the buried sexual impulse or desire which underlay the neurotic symptoms. The essence of a successful cure would thus consist in helping the patient to find some outlet for this impulse which would not conflict with the accepted standards of his or her adult life.

The irrational feelings which patients developed towards their psycho-analyst were seen by Freud to be not only obstacles in analysis, but also of vital importance to the progress of the analysis. It soon became evident that the patient tended to *transfer* to the analyst the feelings he had had towards the important figures in his life (e.g. the parents). The analysis of this *transference* itself is now considered to be one of the most important parts of psycho-analytical technique.

Freud's General Theory

From the starting point afforded by his discoveries concerning the origin of neuroses, Freud carried his investigations into other fields. His use of dream analysis in the treatment of his patients, for example, led him to undertake the study of his own dreams, and his book *The Interpretation of Dreams* (first published in 1900) contains some of his most important and brilliant work. This book was followed by two others: *Psychopathology of Everyday Life* and *Wit in Relation to the Unconscious.*

In these he argues, with a most impressive mass of illustration, that a vast range of everyday mental phenomena, such as lapses of memory, slips of the tongue, humour, and so on, are the expression of repressed impulses and wishes of which the individual himself is quite unaware, and are thus analogous to the symptoms of neurotic patients. Opposing mental forces, that is to say, are not only the source of the psycho-neuroses, but are also present in the minds of normal persons. What distinguishes the neurotic from the ordinary individual, according to Freud, is not the existence within him of a mental conflict, but the particular manner in which this conflict is dealt with. In this way Freud gradually built up an elaborate theory of the nature of mental life in general which is in many respects revolutionary and is still in some circles the subject of heated controversy.

Terminology of Psycho-analysis

Freudian psychology—or psycho-analysis, as it is usually called, by derivation from the methods of treating mental disorders referred to earlier—is complex and, in its subtler ramifications, difficult to grasp without special knowledge of facts which are not easily accessible to the ordinary reader. It is only possible here, therefore, to deal with some of its basic principles. Even this is by no means an easy task, for as Freud and his disciples gradually perfected their technique and extended their observations, the distinctions, classifications, and technical expressions adopted in the early days of psycho-analytical experiment have been supplemented and in some cases partly (but rarely wholly) superseded by others.

As a consequence, the terminology of the subject is still somewhat in a state of flux, the meaning of various expressions which are constantly used by psycho-analysts overlapping to some extent.

Much the most important feature of Freudian psychology is the theory of infantile sexuality. Freud's conception of the nature and number of the basic conative drives or tendencies which constitute the springs of human action is very different from that to which the student has been introduced in previous Lessons. These tendencies, drives, or impulses fall, according to Freud, into two groups: the hostile or aggressive impulses (now called the death-instinct); and the sexual or *libidinal* impulses.

Adult sexuality is not the expression of a single conative tendency which springs rather mysteriously into existence at puberty, but is derived from a number of component impulses which are present from birth, and undergo a complicated process of development.

Impulses Classified

The most important of these components of the sexual impulses may be classified as follows: (1) the oral components, which are connected with the mouth and are expressed in the sucking activities of the child; (2) the anal components, connected with the other end of the alimentary canal, which find expression, at a somewhat later stage, in the child's interest in its own processes of excretion; (3) impulses connected with the genital organs. These are all associated with the so-called erotogenic zones of the body. Then there are (4) certain impulses which find expression in looking (scoptophilic) and being looked at (exhibitionistic); and

(5) impulses connected with the infliction (sadistic) or endurance (masochistic) of pain.

The exercise and gratification by the child of all these impulses is accompanied, the Freudians contend, by pleasure which is definitely erotic in quality, and is the forerunner of, though less intense than, the erotic satisfaction of the adult in sexual intercourse.

These infantile sexual impulses are regarded as so many canalisations of a central fount of erotic energy which Freud terms the *libido*. All the instinctual drives together, in their original undeveloped form, constitute the *Id*. "Id" is the customary English rendering of the German *Es* (= it), a term which is used by Freud to emphasise the fact that the libidinal and ego impulses are at first impersonal, and that the conscious "I" or ego develops only later. Freud also described the fusion of aggressive and libidinal impulses which commonly occurs, as in sadism, in which there is both a sexual and an aggressive satisfaction.

In normal development the various libidinal impulses are gradually grouped into a hierarchy under the dominance of the genital components, the process of integration reaching a climax at about the age of five. The hierarchy falls into two more or less independent parts, one of which is predominantly sensual, while the other gives rise to such emotions as tenderness, devotion, etc. After the age of five a "latency period" occurs, during which the sensual components of the hierarchy are inoperative, to become active once more at the onset of sexual maturity.

While this integration of the child's libidinal impulses is taking place, a number of other factors also become active, and the interplay of these factors with the process of integration is decisive, according to the psycho-analysts, for character formation. The most important of these are *conflict*, *repression*, and *displacement*.

Conflict and Repression

Conflict between conative tendencies has already been discussed in relation to memory and it was pointed out that interests deriving from one conative tendency may inhibit associative processes connected with another. When the conflict of interests is sufficiently acute, this inhibition may be carried to a point at which one of the conative tendencies concerned, and all thoughts and impulses connected with it, are permanently prevented from entering consciousness.

It is this damming up of a conative tendency as a result of conflict which is referred to by Freud as repression, and is regarded by him as the cause of the exclusion from consciousness of those infantile memories which psycho-analysis discloses as underlying the neuroses. It is important to realize that repression is not a conscious process; thoughts and impulses which are repressed are not consciously rejected, but are more or less automatically excluded from consciousness by the growth of the inhibiting tendencies. In an adult, for example, the growth of an interest in science may gradually lead to the repression of all those thoughts and impulses that are connected with religious beliefs without any deliberate and conscious rejection of such thoughts and impulses by the individual.

Displacement Process

Displacement is the process referred to in earlier Lessons in this Course as "conditioning"—a process whereby existing tendencies come to be aroused by new situations and objects, and to find expression in new forms of behaviour. Freud and his followers hold that many of the most important and culturally valuable activities of civilized adults derive, by successive displacements, from behaviour which, in the infant, is simply the gratification of its various libidinal impulses. Displacement which thus leads to activities of ultimate social value is called *sublimation*, a term which has been widely adopted by psychologists of other schools. The student should note that in the Freudian sense, sublimation is applicable only to certain displacements of infantile eroticism.

It is impossible to describe here in detail the complicated way in which repression and displacement co-operate with and influence the gradual integration of the child's libidinal impulses, especially as Freudian theory on this subject has changed considerably in recent years and many points are admittedly still obscure. It must suffice to say that towards the age of five much of the libidinal energy of the infantile sexual impulses has been either displaced or repressed owing, in part at least, to family discipline, while the rest is concentrated in the genital impulses. In the meantime the child's attitude to its parents has gradually assumed a special form, which leads to what Freud calls the Oedipus situation.

The Oedipus Situation

During the period of development briefly described earlier, i.e. during the first four or five years of life, the child has gradually changed from an amoral little animal into a more or less orderly and disciplined human being. From the adult point of view, a succession of regrettable habits, such as finger sucking, dirtiness, apparently thoughtless cruelty to defenceless flies and beetles, destructiveness, obstinate resistance to interference, and so on, have given place to relatively "good behaviour," and something like a dawning capacity to appreciate and submit to the point of view of his elders.

This transformation has naturally been accompanied by an increase in the complexity of the

child's attitude to its parents. Its mother has not only protected it and ministered to its needs, but has thwarted it in the course of the process of inculcating "good habits." The child's emotional attitude to the mother is therefore, in Freudian terminology, *ambivalent*. It both loves her and hates her, but love is the predominant feeling.

A similarly ambivalent, but rather more complex, attitude exists in the child towards the father. If the child is a boy, realization of the father's claims upon the time and love of the mother, claims which conflict with those of the child itself, arouse in the latter a feeling of rivalry, and he wishes his father out of the way. According to Freud, this childish jealousy is definitely erotic, and reaches a culminating point when the child's libidinal impulses become integrated under the genital components. The child now desires to oust his father in his mother's affections. This, in Freud's language, is the Oedipus situation, so named from Oedipus in the old Greek legend, who, ignorant of the relationship, kills his father and marries his mother.

The boy's jealousy towards his father has to contend with other and contrary emotions. The father's own love for the boy, shown in endearments, elicits an answering love, and at the same time the father's apparently limitless knowledge and power come to represent to the boy an ideal on which to some extent he tends to model his own conduct and ambitions. Hence an intense conflict arises in the boy's mind between love and admiration for his father and the wish to get rid of him. The small girl is torn by a somewhat similar conflict, but in her case the respective roles of the two parents are roughly reversed.

The Super-Ego

The conflict arising out of the Oedipus situation is the culminating point in the child's first period of development, and Freud holds that the manner of its solution is a decisive factor in the whole of the individual's later life. If we confine ourselves, for the sake of simplicity, to the case of the boy, the normal solution may be said to involve the triumph of his feelings of love and admiration for his father and the repression of the more sensual, libidinal impulses directed upon his mother. This repression is only achieved by the boy's taking over into himself, as it were, and adopting as his own, the prohibitions and restrictions imposed upon his libidinal impulses by his parents, these restrictions now arising to constitute the inner voice of conscience or, in Freudian language, the super-ego.

The result is a veritable psychological metamorphosis. The repression of the boy's more sensual libidinal impulses not only leads to a change in his attitude towards his mother, as a result of which only feelings of tenderness, or the "aim-inhibited" components of the libidinal hierarchy, as they are called, now remain in consciousness, but to further extensive repression, involving part of the super-ego and nearly all the memories of the preceding period of life. Hence the inability of the ordinary individual to recollect more than a few trivial and disconnected incidents from his early childhood.

All this repressed material is still active and, although debarred from direct access to consciousness, continues, as the most important part of the unconscious, to influence conscious mental activity in a variety of ways, finding conscious expression through devious associative channels, in disguised forms. The neuroses of later life always involve a partial breakdown of the culminating repressions of childhood, and are ultimately traceable to hitches in the complex process of infantile development.

Freud's Theory of Dreams

Freud distinguishes between the manifest content of a dream, i.e. the dream as subsequently recorded by the dreamer, and its latent content, i.e. the thoughts and other mental processes which underlie it and are ascertained by the method of free association. Analysis invariably discloses that the core of the latent content is some repressed infantile wish.

Freud holds that the function of dreams is to preserve sleep, and that this function is achieved by the translation into an imagined wish-fulfilment of anything that threatens to interrupt sleep. The disturbing factor—some worrying thought, let us say, which was on the sleeper's mind when he went to bed—becomes linked in sleep with some repressed infantile wish, the linkage occurring partly because the conative factors controlling association in waking life are temporarily in abeyance, and partly because the repressing forces which ordinarily intervene between the unconscious and conscious mental activity are temporarily weakened.

The manifest content of the dream is the disguised expression of the fulfilment of the repressed wish, the extent of the disguise depending upon the degree to which the wish is out of harmony with the sleeper's waking moral standards. If the repressed wish is too intense, it breaks through in a relatively undisguised form and becomes itself a cause of disturbance; the dream changes to a nightmare, and the sleeper awakes in terror.

This wish-fulfilment theory of dreams is based largely upon a further complicated theory of dream symbolism, the various elements in the manifest content of the dream, which consists largely of visual imagery, being interpreted as symbolical and dramatised representations of the latent content.

Freud regards the analysis of dreams as the royal road to the understanding of the nature of the unconscious, and such analysis has constituted an important part of Freudian therapeutic procedure. The present tendency among psycho-analysts is, however, not to place dream-analysis in the forefront of analytic work, but to treat the dream material in exactly the same way as the other associative matter brought by the patient to the analytic couch.

Detailed comment on Freud's theories must be reserved until we have discussed the widely different views of Jung and Adler. But before closing this Lesson it is worth while to point out that the accusation which occurs on the subject, that Freud attributes everything to sex, is rarely made by competent psychologists. The core of Freud's theories is not sex in the usual acceptation of the word, but *infantile sexuality*, and the distinction is a highly important one.

LESSON 24

Jung's Doctrines of Analytical Psychology

Dr. Carl Gustav Jung (born 1875 at Kesswyl, Thurgau) was originally a disciple of Freud and, like the latter, devoted much of his time to the study and treatment of nervous disorders, studying medicine at Basel and psycho-pathology at Paris. His views soon diverged from those of his master, and he is now known as the exponent of "analytical psychology," a theory of mental life which, in spite of superficial resemblances, differs profoundly from psycho-analysis.

The student should note that the term "psycho-analysis" should be used only in connection with the theories and techniques of Freud. The British Medical Association restricts the use of the title "psycho-analyst" to those who have undergone the full training at a Freudian Institute of Psycho-Analysis.

Jung rejects Freud's theory of infantile sexuality largely on the grounds that it involves the "impossible" notion of a "latency period," to which, he contends, there is no parallel in animals closely related to human beings and which, moreover, seems to be incompatible with fact, there being a good deal of evidence in favour of the view that in many individuals, especially males, incipient stirrings of a definitely sexual impulse occur at as early an age as eight years. In *The Theory of Psycho-analysis* he writes:

> A process of development (i.e. a period of infantile sexuality followed by a latency period such as Freud postulates) would be biologically unique . . . This impossible supposition is a consequence of the assertion that the early infantile activities of the presexual stage are sexual phenomena. . . . What (Freud) calls a disappearance of sexuality is nothing but the real beginning of sexuality, everything preceding was but the fore-stage to which no real sexual character can be imputed.

For Jung, *libido* has a much wider meaning than it had for Freud, and corresponds roughly to conation in general. While abandoning Freud's exclusively sexual conception of the *libido*, Jung retains and elaborates the notion of a central fount of psychic energy, regarding the *libido* as capable, like physical energy, of undergoing transformation from one form to another, and hence not only of finding expression in a vast range of activities but of changing qualitatively when it is diverted from one mode of expression to another.

In this way Jung is able to retain a sexual derivation for certain kinds of mental activity, while denying at the same time that these activities are themselves sexual in quality. He makes no attempt to give a list of the instincts or innate differentiations of the *libido* beyond referring to the sexual and nutritive instincts and to the "instinct for power."

Jung's Theory of the Unconscious

Jung also differs from Freud in his conception of the unconscious. According to Freud the unconscious comes into existence as the result of the complicated interplay of conflict, integration, and repression during the early years of childhood, and consists almost exclusively of repressed infantile wishes and their associated memories, i.e., the Freudian unconscious is acquired.

Jung distinguishes between the *personal* unconscious and the *collective* unconscious. The personal unconscious consists partly of elements repressed from consciousness and partly of knowledge derived from "marginal" impressions and perceptions, i.e. impressions and perceptions which were conscious in the first instance, but were outside the central focus of attention; it is thus, like the Freudian unconscious, acquired during the life history of the individual. The collective unconscious is a racial inheritance, according to Jung, in his *Psychology of the Unconscious*—

> that remnant of ancient humanity and the centuries-old past in all people, which is the common property left behind from all development which is given to all men, like the sunshine and the rain.

The collective unconscious is the ultimate source of the *libido* and, to this extent, corresponds to Freud's "id," but it contains also what Jung designates "archetypes." These are innate tendencies to conceive and explain the world along animistic lines, i.e. in terms of gods, demons, witches and so on, which influence our

mental activities in a variety of ways, not only determining the imagery of our dreams, but prescribing to a certain extent the form of our religious and philosophical beliefs.

Among primitive peoples the archetypes find more direct expression in mythology and folklore than with us, and the striking similarities between the myths and legends of primitive peoples and races widely separated in time or place are regarded by Jung as evidence that certain of the archetypes are the common possession of all men. He holds at the same time that the various great races of mankind differ slightly in their endowment of archetypes, these differences accounting for important racial peculiarities of culture.

Persona and Anima

Jung writes in *Contributions to Analytical Psychology*: "The unconscious, so far as we can now see, has a compensatory function in respect to consciousness." Of the many and varied mental potentialities with which each of us is endowed at birth, some only are developed and integrated into self-conscious personality. These constitute the *persona*, or mask, by which we are known to our fellow-men. Our conscious personality, as recognised by others and acknowledged by ourselves, is only a part of our total personality. All our undeveloped potentialities, which, broadly speaking, are opposed in tendency to those of the *persona*, constitute an unconscious, compensating personality, which stands in the same relation to the collective unconscious as the *persona* does to the external world of men and events.

This secondary, unconscious personality is called by Jung the *anima* (in a man) and the *animus* (in a woman). In the normal, virile man the *anima* is predominantly feminine and tends to figure in dreams in the form of a woman. Dr. Joan Corrie, one of Jung's disciples, writes:

> Every English reader is familiar with Rider Haggard's famous novel *She* and its sequels. *She* is a typical *anima* figure partaking of the characteristics of the images of the collective unconscious; half goddess, half daemonic; alluring, repelling, beautiful, loving, submissive woman, and detestable, jealous murderess. *She* is the *anima*, or personification of Holly's unconscious.

According to the same writer, the *animus* of the typical, monogamously-minded woman is masculine and polygamous. Both the *anima* and *animus* may be projected upon some real person.

> The result is to produce dependence on that person and an emotional tie of an overwhelming and compulsory nature. There can be no indifference to the carrier of the projection. Such a person is bound to be either oved or hated

Jung combines his views regarding the nature of the unconscious with an elaborate theory of psychological types. This theory rests in part upon a novel analysis of mental activity, which differs considerably from those which have hitherto appeared in the Lessons of this Course. Jung distinguishes four principal mental functions, viz. sensation, thinking, feeling, and intuition.

His own definition of these four functions is couched in rather antiquated terminology and need not be quoted here, but apparently sensation and feeling have for him much the same meaning as they have for the orthodox psychologist, while by thinking he means controlled, logical thought, and by intuition the spontaneous expression of the impulses. Jung and his followers speak of sensation and intuition as "irrational" functions, and imply that they represent a lower level of mental activity than thinking and feeling.

Introvert and Extrovert

Jung's theory of psychological types has an earlier and a later form. Originally he distinguished two types only, the *introvert* and the *extrovert*, and maintained that all persons occupy some position on a scale between extreme introversion at one end and extreme extroversion at the other. On this earlier view Jung's introverts and extroverts correspond roughly to Kretschmer's schizothymes and cyclothymes respectively, and in *Analytical Psychology* Jung defines the two types as follows:

> The introverted type is characterised by the fact that his *libido* is turned towards his own personality to a certain extent—he finds within himself the unconditioned value. The extroverted type has his *libido* to a certain extent externally; he finds the unconditioned value outside himself. The introvert regards everything from the aspect of his own personality; the extrovert is dependent upon the value of his object.

Translating this extract into ordinary language, one may say that the extrovert is essentially sociable, making friends with facility and finding pleasure in a variety of social activities. He takes life easily, on the whole, tending to live for the moment and giving free expression to his emotions. The introvert joins groups with difficulty, and is outwardly unemotional. A further contrast is evinced in tastes and interests, the introvert tending to think and read, while the extrovert is probably more interested in receiving impressions of sensuous beauty generally.

Classification of Types

A good deal of work has been done by psychologists with the object of placing people on an introversion-extroversion scale, but Professor Woodworth, commenting on Jung's theory, suggests that in the scale at present used at least two variables, probably independent, are lumped together.

> One variable would be the tendency to immediate overt action as opposed to the tendency to deliberate, ruminate, and perhaps day-dream. The other variable would be the interest in other people and social activity

In his later writings Jung considerably modifies the introvert-extrovert classification of types. The basis of his later classification is the fourfold analysis of mental functions described earlier in this Lesson. In any individual one or other of the four functions will preponderate, and the individual may be either introverted or extroverted in respect of this dominant function. We thus have an introvert-thinking type, an extrovert-thinking type, an introvert-feeling type, an extrovert-feeling type, and the same for sensation and intuition, there being thus eight types in all instead of the original two.

Complications

A further complication is introduced by the fact that, according to Jung, the individual who is consciously introverted is unconsciously an extrovert and vice versa. Moreover the dominant mental function in consciousness is counterbalanced in the unconscious by the opposite function, which Jung calls the co-function. Thus a person who is consciously of the extrovert-thinking type will be, so far as his unconscious is concerned, of the introvert-feeling type, and so on, thinking and feeling constituting one pair of opposing functions, and sensation and intuition another.

A brief description can be given of the extrovert- and introvert-thinking types by way of example. Dr. Joan Corrie writes:

> The extrovert-thinker accumulates facts, for which he has the greatest possible respect; his thought is rational and constructive. He readily finds a role to suit him as natural scientist, politician, financier, advocate, ecclesiastic, architect, engineer. Owing to the inflexibility of his intellectual judgments, when the type is extreme he is apt to be intolerant, fanatical, or even tyrannical . . . Feeling is rigidly repressed; hence there is scant sympathy for others unless they conform to his views of life.

The introvert-thinker is eminently introspective and is more interested in the abstract than in the concrete. His thinking tends to be speculative and theoretical; of such stuff metaphysicians are made. The two types are well exemplified in the personalities of Freud and Jung themselves, Freud showing an intense interest in facts and their analysis, Jung being of a much more speculative, philosophical, and ruminative turn of mind.

Causes of Neuroses

As is to be expected, Jung's views regarding the origin of the neuroses differ in many respects from those of Freud. Whereas Freud finds the essential cause of a neurosis in the past and early history of the patient, Jung finds it in the present. "Only in the actual present," he writes, "are the effective causes, and only here are the possibilities of removing them." Mental disorders, in his view, are primarily due to the over-development of one or other of the four mental functions. Difficulty arises when the individual is confronted by some problem with which his dominant function is unable to deal, and to which the other, neglected functions are now inadequate. If the problem proves insoluble, *regression* occurs, i.e. "the *libido* now gives up the present task and returns to a former and more primitive way of adaptation."

Regression, according to Jung and his followers, is a process of turning away from the actual problems of life, back towards the state of security which the child first experienced in its mother's arms, and may go so far that the patient becomes blind, deaf, dumb, and even completely infantile. Regression may also result in the activation of certain fantasies stored up in a latent and potential form in the collective unconscious, a consequence which Jung has endeavoured to apply to the interpretation of some of the symptoms of *dementia praecox*.

Jung's Theory of Dreams

Jung, like Freud, regards dreams as products of the unconscious, but whereas for Freud a dream is the expression of a repressed infantile wish, dreams for Jung are expressive of the compensatory function of the unconscious. He writes:

> In the conscious process of reflection it is indispensable that, so far as possible, we should realize all the aspects and consequences of a problem, in order to find the right solution. This process is continued automatically in the . . . state of sleep . . . all those other points of view occur to the dreamer (at least by way of allusion) that during the day were under-estimated or even totally ignored.

The dream, that is to say, embodies those aspects of the dreamer's total personality which find inadequate expression in his *persona*. Jung distinguishes equally with Freud between the manifest and the latent content of the dream, but he distinguishes also, in his interpretation of dream imagery, between those images which derive from the personal unconscious and those which arise out of the collective unconscious. The former refer to the objective circumstances of the dreamer's life, while the latter are symbolical representations of qualities or tendencies in the dreamer's psychology.

Hence Jung regards dreams as not merely indicative of the past development of the dreamer, but as pointing the way, when properly understood, to a fuller and more harmonious development of the dreamer's nature in the future.

LESSON 25

Adler's "Individual Psychology"

Dr. Alfred Adler of Vienna was, like Dr. C. G. Jung, at one time a disciple of Freud, but his views soon diverged from those of his master, and he became the leader of an independent school of thought known as the school of "individual psychology." One of its exponents, P. Mairet, says :

> Individual psychology is not psycho-analysis. It is a method . . . of gaining knowledge of individuals, including knowledge of their inner life, but it is a method founded upon a view of the individual as whole in himself, an indivisible unit of human society. It relates everything that the individual does in such a way as to obtain a picture of a single, coherent, and intelligible tendency expressed in most various ways, direct and indirect.

The mental life of each individual is for Adler and his followers the co-ordinated expression of some single, dynamic drive, and full understanding of an individual is only possible in so far as the nature and direction of this drive are fully grasped. This drive, or "style of life," differs necessarily from one individual to another ; but the styles of life adopted by different individuals are all ultimately, according to Adler, variations upon one fundamental conative tendency, the will to power, shared by all individuals alike. As Mairet says,

> The psyche is but the instrument of the will to superiority, the will to remedy one's weakness in the face of Nature and Society.

Feeling of Inferiority

Immensely different expressions of this fundamental will to power are evinced in the life-styles of different persons, and the determining factor underlying these individual differences is a feeling of inferiority. This feeling of inferiority may be rooted either in some actual, organic defect or weakness, or in the environmental conditions of the first few years of life ; whatever its source, it conditions the form which the will to power takes, this being always in the nature of an endeavour to compensate for the initial defect or handicap.

In some cases the compensation is strikingly successful. Thus Demosthenes (*c.* 382–322 B.C.), as the result of his endeavour to overcome his stuttering, became the greatest orator of antiquity, and Theodore Roosevelt (1858–1919), president of the U.S.A. 1901–09, conquered his physical weakness by ranch life and became explorer and rough-rider, apostle of the "strenuous life." Sometimes compensation is less direct, though equally successful, as when the weakly boy concentrates on his studies and so attains an intellectual distinction which outweighs his bodily deficiencies. When the attempt at compensation fails and the individual is unable to achieve any practical superiority, he is apt to withdraw into a style of life which enables him to excuse his failure both to himself and others. He may, for example, set himself an impossible and fictitious goal which, by its sheer unattainability, justifies his failure to reach it, or he may develop neurotic symptoms and excuse himself on the grounds that if he were not so ill he could do as well as others.

In this way he attains an *indirect* position of superiority and power. He secures the sympathy of other people, while evading the need for any genuine effort at assertion and he conceals his failures both from himself and from his fellows because he is judged by more lenient standards. "All forms of neurosis and developmental failure," writes Adler, "are expressions of inferiority and disappointment."

Apart from organic deficiencies, the most important factors determining the nature of the feeling of inferiority which by compensation conditions the life-styles of different individuals are sex, the economic environment of the child, and its position in the family. In a variety of subtle ways girls can be made to feel that they are less important members of the community than boys. The father who comes and goes in an atmosphere of mysterious liberty and freedom stands out in the small child's life as the embodiment of superiority.

In every individual, according to Adler, the will to power is, at least in part, a striving towards masculinity. This striving, carried over into later life, is the "masculine protest," and is of great importance in the mature sex life of every individual. In men it is the explanation of Don Juanism. Women, as the inferior sex, are material for conquest ; to practise fidelity to one woman is in the last resort to admit her to a position of partnership and equality and thus to abandon the essential masculine superiority. In women the "masculine protest" is manifested less crudely. In some women it indirectly nourishes their will to power by leading them to accept openly a position of inferiority for their sex. According to P. Mairet,

> Joining in the cry against woman, they can claim a large amount of irresponsibility as the weaker sex, and in the last resort they throw all the real burden of life's responsibility upon their men-folk.

Unfavourable economic and social conditions in early life have a variety of repercussions on the style of life. The inferiority feeling may be immensely strengthened by such conditions, and

the compensatory striving is apt to take the form of a craving for the material pleasures of life. This, according to Adler and his followers, is a powerful factor in the production of prostitution and crime. The life-style of the individual is determined also by his position in the family. Thus the only child is apt to take it for granted that others will always subserve his needs.

The youngest child of a numerous family (according to E. Wexberg)—

> feels itself the smallest and weakest in the family, and just for this reason strives with heightened ambition to overtake its brothers and sisters and even to outdo them. Thus boundless ambition and a combatant, often a revolutionary, attitude become the characteristic of this

Adler abandons to a very large extent the distinction between conscious and unconscious mental processes which is so much emphasised by psycho-analysis and analytical psychology. Conscious mental processes, he holds, are much less conscious, and unconscious ones much less unconscious, than is usually maintained. Nevertheless the feeling of inferiority is to some extent unconscious, and Adler also talks at times of the neurotic individual transferring the goal of his will to power "into the realm of the unconscious." On the whole, he stresses the unity of mental life and deprecates the elaborate divisions and subdivisions of Freud and Jung.

Both free association and dream analysis are made use of by Adler and his followers, but Adler rejects Freud's theory of dreams, treating them from a point of view which resembles that of Jung. Dreams, he holds, refer to some present or pending problem in the dreamer's life, and reveal, when analysed, his style of life. The dreamer casts his waking problems into allegorical picture form and (again quoting E. Wexberg) "tries with its help to find the solution adapted to his goal-setting (style of life)."

LESSON 26

Divergent Views of Freud, Jung, and Adler

This Lesson is devoted to a brief discussion of the theories of Freud, Jung, and Adler. These three thinkers not only appear to differ in many ways in their conclusions from "orthodox" psychologists, but seem also to differ profoundly from each other. What are we to think of such divergent views? Is the divergence more apparent than real? The problem is a difficult one, and we must confine our attention here to two points of especial interest.

In the first place, it should be noted that Freud, Jung, and Adler do not make much use of the usual distinction between the cognitive and conative aspects of mental life. The comparative abandonment of this distinction has important repercussions on their terminology. Orthodox psychologists, to whom the distinction is vital, have devised a special vocabulary for describing mental processes. The followers of the three thinkers considered here draw largely upon the vocabulary of common life. This is often unfortunate.

To take one example, the everyday word "idea," which bulks largely in analytic literature, is apt to suggest that there are persistent entities or things called "ideas" which are "in" the mind like pebbles in a bag. Other nouns, like thought, wish, memory, etc., tend to convey the same impression, whereas there are various kinds of mental *activity*, e.g. thinking, perceiving, etc., directed upon various kinds of objects.

This use of technical terms which have different meanings in everyday usage is confusing, for one tends to equate the technical usage of such terms with the more common one. This can lead to a great deal of misunderstanding, and it is necessary, though often difficult, to go to the original sources for an accurate definition of exactly what is meant by any particular technical term. It is difficult because Freud, for example, varied over the years in his use of the various psycho-analytical concepts.

The term "notion of the unconscious" is a difficult one to understand, especially as psycho-analysts tend to use such terms as "unconscious feeling" and "an unconscious sense of guilt." How, one may ask, is it possible to have a feeling which is not felt? This apparently paradoxical state of affairs has been the basis of much criticism of psycho-analytic work by philosophers and logicians. Freud's followers would answer that an unconscious feeling is one which was at one time conscious but has been repressed, and if the repressing forces are slackened (e.g. by being pointed out to the patient), the same feeling will emerge as a conscious experience.

The work of Freud and Jung is important for its bearing on the questions, first, of the *kind* of mental structure which human beings inherit and, second, of the factors which condition its growth and the degree of integration achieved in adult life. Freud holds that the innate, conative mental structure is extraordinarily primitive, and that certain conative tendencies assumed by other psychologists to be innate, e.g. the sex tendency, are not really so, but are the product of a post-natal process of integration and growth. Jung holds that the innate

cognitive structure is more complex than is usually assumed, and includes tendencies to think in certain ways, i.e. the archetypes.

It appears that Freud's theories also do what Professor Janet's, for example, fail to do. They show that the chief factor determining dissociation is conflict, repression being a form of partial dissociation. They suggest, moreover, that the growth of mental structure falls into two stages, an infantile and a later stage, and that the mental structure elaborated in the infantile stage underlies, in a partially dissociated form, the mental structure developed in later life, influencing it in various important ways.

It may be argued that the notion of mental structure is itself obscure, and that in particular its relation to, say, the structure of the brain is not understood. Unlike the notion of the unconscious, it does enable us to distinguish between what is a mental event and what is not, and if the student will perform the necessary work of translation, he will find the conclusions of the psycho-analysts more comprehensible and less at variance with those of other psychologists than appears at first sight.

Dynamic Interaction

When, for example, the student reads of a repressed memory as being "revived," he need not think that the memory, as an event, has been there all the time, merely asleep, so to speak. What has happened is that a relatively dissociated part of the mental structure has acquired new associative links with the main body and that in consequence a memory of an incident hitherto unrecollected has now occurred. The incident remembered is past and old, but the memory is new and present. If the student remembers his yesterday's dinner on several occasions, it is the dinner which is the same, not the several memories.

The present-day Freudian view of mental structure refers to the "id" (the reservoir of primitive instinctual impulses), the ego (the more organized part of the personality concerned with perception and with controlling the demands of the instincts), and the super-ego (containing the conscience and moral standards of right and wrong). But these parts of the mental organization are not considered to be real things which can be found on microscopic examination of the nervous system. Rather are they concepts which have been found useful in describing and analysing behaviour.

Fundamental to the Freudian view is, however, the idea of a dynamic interaction between these various structures of the mind. The ego, which is, roughly speaking, the *self*, has to effect a compromise between the demands of the instinctual wishes and the moral standards of the super-ego. It also mediates between the id and the outside world. Furthermore, it does so in the most economical way—it follows the principle of least effort in dealing with all the problems presented to it.

In this struggle of the ego to cope with the demands of id, super-ego, and reality, use is made of the concept of *defence-mechanisms of the ego*. Repression, previously described, is one of these defences. So is displacement. Another is projection (the attributing to others of unconscious feelings in oneself). The analysis of these defence-mechanisms, which are fairly characteristic for any one person, is now an integral part of Freudian technique.

When the notion of the unconscious is translated into terms of mental structure, the really vital divergence between the various psycho-analytic schools and between them and other psychologists becomes immediately clear. This divergence, which is the second point for discussion, *is a divergence of view as to the nature of the innate, conative structure of the mind.*

Two important facts are relevant here. In the first place, innate conative tendencies are not observed: they are inferred from the facts of introspection and behaviour. Secondly, the nature of the inference made is dependent upon the facts observed, and this in its turn depends partly upon the direction of investigation and partly upon the capacity of the observer. In these circumstances the existence of differences of opinion as to the nature and number of the innate conative tendencies is not surprising.

None Fits all the Facts

It is puzzling, nevertheless, to find investigators of the calibre of Freud and Adler, who have concentrated on the study of the same material, e.g. the neuroses, differing as widely as they do. (Jung's views on this particular point are obscure, as has already been pointed out, and they may, therefore, be ignored.) Adler stresses self-assertiveness almost to the exclusion of any other tendency, while Freud stresses sex, in his own special sense of the word, i.e. as a bundle of component tendencies and not as a single tendency.

Which of the various theories as to the innate conative structure of the mind is to be preferred? Freud's? McDougall's? The most probable answer is that each of them fits a special range of facts, while none of them fits all the facts. A great deal more investigation is needed to enable anyone to say on precisely what points each theory is deficient. For the purposes of this Course, the question must be left open. The student should realize that the solution of this problem is one of the most important tasks for the psychologists of the future. An excellent discussion of the views of Freud, Jung, and Adler, from a different point of view, is in Woodworth's *Contemporary Schools of Psychology*.

LESSON 27

Occupational Psychology

Of late years there has been a marked development in the application of psychological knowledge to the solving of practical problems. The psychologist has reacted strongly against the intensive study of mental processes as detected in individual minds. He has moved from the study into the laboratory, and from the laboratory into the home, the school, the hospital, the church and the cinema, the factory and the office.

Indeed a whole new department of psychological study has been marked out and explored what some have called industrial psychology but what may be more properly styled occupational psychology.

Briefly defined, the job of the occupational psychologist is the successful fitting of round pegs into round holes. That this is eminently worth while can be demonstrated easily enough on economic grounds. A worker who has the abilities and temperament required for a particular occupation, whether it is a "white-collar job" or one which is performed in dungarees; who spends his working life in well-lit, well-ventilated, and well-equipped premises, under expert and considerate direction, and whose relations with his employers and his fellow employees are of a friendly description—such a man is likely to produce more in a given time than the man who is condemned to toil amid unhygienic surroundings, hating his job and everything and everyone connected with it, and stimulated only by the fear of "the sack."

Occupational psychology makes for happier and more contented workers, for men and women who take a pride and intelligent interest in their work. Work, however necessary, is not the be-all-and-end-all of life: men work in order to live, and there is no reason why the hours spent earning one's living should be so many hours subtracted from those that are really lived. Occupational psychology, by removing causes of friction, by improving the environment, by making unnecessary much of the really hard toil, is making a very distinct contribution to human well-being.

Of the many branches into which occupational psychology may be divided, one of the most important and promising in its results is that known as vocational guidance. This is based on the close and careful study of an individual's innate abilities, temperamental characteristics, and acquired knowledge. From the composite picture that is thus built up it is possible to suggest the occupational frame to which it is most suited. Vocational selection is a further stage, in which the individual is submitted to tests intended to see whether or not he has the qualifications that would fit him for a particular job. In both guidance and selection specially devised tests are used, and a great mass of evidence, accumulated e.g. by the National Institute of Industrial Psychology, goes to show that they are more than justified.

Not the least of the services rendered by the workers in this field of psychological technique is the development of child guidance clinics, where young people about to embark upon their life work are advised and encouraged as to the fields for which they are most suited.

BOOK LIST

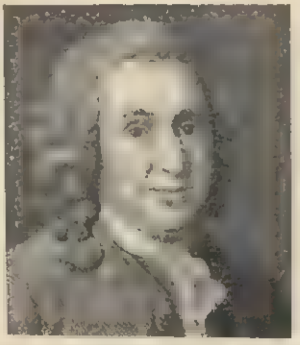
General.—*General and Social Psychology*, R. H. Thouless (University Tutorial Press); *General Psychology*, Gardner Murphy (Harper); *Psychology*, Norman L. Munn (Harrap); *An Introduction to Modern Psychology*, O. L. Zangwill (Methuen); *A.B.C. of Psychology*, C. K. Ogden (Penguin).

Lesson

4. *Outline of Psychology*, W. McDougall (Methuen).
5. *Physiological Psychology*, Morgan and Stellar (McGraw-Hill); *Human Physiology*, Kenneth Walker (Penguin).
6. *Animals and Men*, David Katz (Penguin); *Introduction to the Study of Heredity*, McBride (Home University Library).
7. *Lectures on Conditioned Reflexes*, I. P. Pavlov (Lawrence & Wishart); *Behaviourism*, Watson (Kegan Paul); *The Energies of Men*, W. McDougall (Methuen).
10. *Emotion in Man and Animal*, P. T. Young (Chapman & Hall).
11. *Physique and Character*, E. Kretschmer (Kegan Paul); *Personality*, G. Allport (Henry Holt); *The Varieties of Human Physique* and *The Varieties of Temperament*, W. H. Sheldon (Harper).
13. Any of the general text books.
15. *Principles of Behaviour*, C. L. Hull (Appleton-Century); *Purposive Behaviour in Animals and Men*, E. C. Tolman (Appleton-Century).
16. *Uses and Abuses of Psychology*, H. J. Eysenck (Penguin); *A Guide to Mental Testing*, R. B. Cattell (University of London Press); *The Meaning of Intelligence*, G. D. Stoddard (Macmillan).
20. *Psychology of Language*, J. Eisonon (Harrap); *Manual of Child Psychology*, L. Carmichael (Wiley).
21. *Modern Clinical Psychology*, T. W. Richards (McGraw-Hill); *The Rorschach Technique*, B. Klopfer and D. M. Kelley (World Book Co.).
22. *Psychiatry To-day*, D. Stafford-Clark (Penguin).
23. Any of Freud's works (Hogarth Press), especially *An Outline of Psycho-Analysis*; *The Ego and the Mechanisms of Defence*, Anna Freud (Hogarth).
24. *An Introduction to Jung's Psychology*, Frieda Fordham (Penguin).
25. *Individual Psychology* and *Problems of Neurosis*, Adler (Kegan Paul); *Alfred Adler*, H. Ogler (Daniel).
26. *Contemporary Schools of Psychology*, R. S. Woodworth (Methuen).
27. *Industrial Psychology*, M. S. Viteles (Norton); *Personnel and Industrial Psychology*, E. E. Ghiseli and C. W. Brown (McGraw-Hill).

John Ray (1627–1705), in whose honour the Ray Society was founded in 1844, made extensive collections and studies of plants.

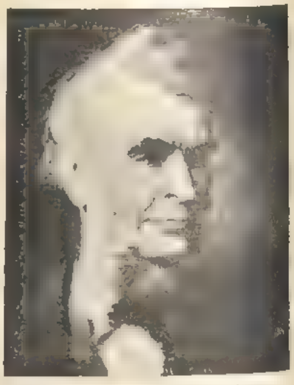
Left, Nehemiah Grew (1641–1712) laid the foundations of the science of plant anatomy. Right, Marcello Malpighi (1628–94) applied his methods of microscopic anatomy to botanical research, and his "Anatomia Plantarum" was published by the Royal Society



Left, Stephen Hales (1677–1761) introduced methods of weighing and measuring into his experiments on living plants. Centre, Carl Linnaeus (1707–78), Swedish botanist, whose real name was Linné. He was the first to define genera and species, orders, and sub-kingdoms. Right, Sir J. E. Smith (1759–1823) who founded the Linnean Society in 1788.

Left, Robert Brown (1773–1858) was the leading systematic botanist of his time in Britain. Right, Sir J. D. Hooker (1817–1911) led a botanical expedition to N. India in 1848, and in 1865 succeeded his father, Sir W. J. Hooker, as director of the Royal Botanic Gardens, Kew.

The portrait of Stephen Hales, by T. Hudson, is in the National Portrait Gallery



George Bentham (1800–84) joint author with Sir J. D. Hooker of "Genera Plantarum" (3 vols., published 1862–83).

BOTANY

ONE of the divisions of Biology, Botany is the science of plant life. In the following Lessons the student will range systematically over the vegetable kingdom, learning much that is fascinating about the activities of the various forms of plants. Simple experiments are suggested. The student is advised to study also the Course on BIOLOGY, *in Vol. 1.*

34 LESSONS

LESSON 1

Introduction to the Plant Kingdom

BOTANY is the science which deals specifically with plants. Its aim is to investigate and demonstrate the general laws which underlie the development, composition, and activities of the plant kingdom. Because of the vastness of its scope it is divided into a number of subordinate sciences, such as *Systematic Botany* (the classification of plants according to their relationships), *Morphology* (the structure of the different parts of plants), *Ecology* (the relationship between plant and habitat), *Plant Geography* (the distribution of plants), and *Genetics* (which deals with inheritance and variation).

All Animals Dependent upon Plants

Aspects of applied botany include agriculture, horticulture, forestry, pharmacology, bacteriology, and plant pathology. Man has, more or less, achieved dominance over the vegetation that surrounds him. Mismanagement can convert grassland or forest into barren desert, and careful reclamation can bring about the reverse.

Green plants alone can manufacture organic food substances from inorganic raw materials ; and all animals are completely dependent, directly or indirectly, upon plants for their food. Green plants manufacture three main classes of food materials : carbohydrates, proteins, and fats. Carbohydrates are composed of carbon, hydrogen, and oxygen, and they include sugars, starch, cellulose, and lignin (which is the basis of wood). Proteins contain nitrogen and very often phosphorus and sulphur, in addition to carbon, hydrogen, and oxygen.

Food Manufacture

The process of food manufacture by the plant is called *photosynthesis*—literally, manufacture by light. The radiant energy of sunlight is absorbed by the green pigment of plants, called *chlorophyll*. This energy is used in the synthesis of simple sugars from water and carbon dioxide which is absorbed by the plant from the atmosphere. The sugars may be built up into starch, fats, or proteins, and stored in this form.

Importance to Mankind

Ultimately the food substance, by combining with oxygen in the process called *respiration*, can be broken down to its raw materials, liberating energy at the same time. Respiration occurs in all living organisms, but photosynthesis occurs only in plants. So the fundamental importance (to mankind) of green plants lies in the fact that they can use the sun's energy to make sugar from carbon dioxide and water. The chemist can imitate such synthesis in a test-tube, but he cannot harness the sun's energy economically. He has even succeeded in synthesising complex proteins, such as those found in the nucleus of the cell, but whether this will prove possible on a commercial scale remains to be seen.

Saprophytes and Parasites

Plant products include drugs for curing ailments, timber, fibres such as cotton and jute, resin, rubber, spices, and dyes. Peat and coal are compacted plant remains from which energy can be released by burning. Seaweeds yield compounds called *alginates* which are used in foodstuffs (ice-cream, synthetic cream) and in other ways.

The foregoing remarks apply mainly to green plants. Fungi and bacteria have no chlorophyll and cannot photosynthesise. They are either saprophytic (living on decaying plant or animal remains), or parasitic (living at the expense of some other living organism). The saprophytes are mainly beneficial to man. Many occur in the soil, where they cause the breakdown of dead leaves, roots, and stems into simple inorganic substances which other plants can then absorb through their own roots. The farmer or gardener who makes a compost heap or uses animal manure to enrich the soil is making use of the saprophytes to convert his compost or manure into a form in which his crops can absorb it.

Many of the parasitic plants cause diseases, both in plants and in animals. Some diseases, such as pneumonia and cholera, are due to bacteria ; some, such as athlete's foot, are fungal in origin. Most plant diseases are fungal. The class of fungi called yeasts is used extensively in brewing, wine-making, and baking. Various medicinal drugs (or antibiotics, as they are called), including penicillin and streptomycin, are extracted from fungi.

The Four Main Groups

It must be emphasised that the pathogenic, or disease-forming, characteristics of some of the lower forms of plant life are of secondary consideration. The main consideration is that without plants, animals could not exist. Plants are the basis of every food chain. Sheep eat grass, and men eat sheep. Men eat herrings, which feed on smaller fish, which feed on minute animal plankton, which feed on even

smaller, unicellular green algae. The four main groups, or phyla, in the plant kingdom are :

(1) Flowering plants ;
(2) Fernlike plants ;
(3) Mosses and liverworts ;
(4) Algae, fungi, bacteria, and lichens.

Phylum 1 comprises all flowering plants, whether trees, shrubs, or herbs. These are the Phanerogamae. All the phanerogams have root, stem, and leaves which are traversed by tough strands or "veins" (hence they are called *vascular* plants), and all produce seed.

There are two subdivisions to this phylum : (a) Angiospermae, in which the seeds are protected by being enclosed in a special seed case, and (b) Gymnospermae, in which the seeds are not specially protected. The large majority of seed plants belong to the former sub-phylum ; conifers, such as fir, pine, and larch are gymnosperms.

The remaining phyla comprise all the flowerless plants, called Cryptogams. Neither flowers nor seeds are produced, and the typical method of propagation is by means of spores.

Phylum 2 comprises the vascular Cryptogams or Pteridophyta, and includes all ferns, the club-mosses, and the horsetails.

Phylum 3 consists of mosses and liverworts, technically called Bryophyta, which have neither roots nor vascular tissue.

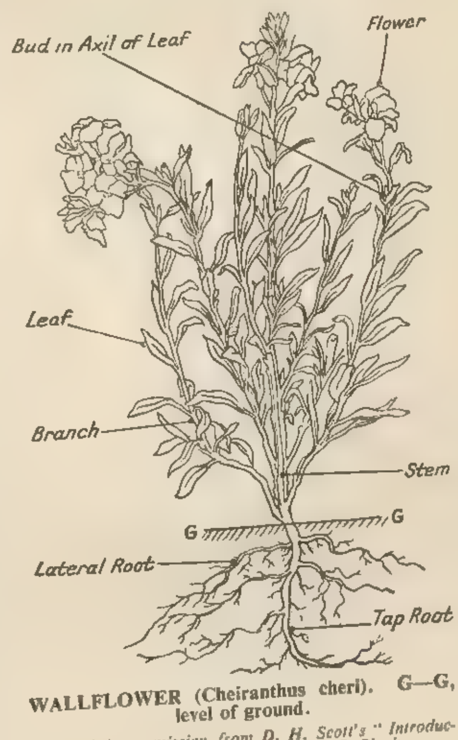
Phylum 4 includes the lowest plant forms, called the Thallophyta. Here the plant body is not differentiated into root, stem, and leaves, but is of a very simple form and is called the *thallus*. Many members of this phylum are of microscopic size. The Thallophyta include the following four groups : (a) algae (e.g. the green slime of ponds), which contain chlorophyll, and all seaweeds ; (b) fungi, which lack chlorophyll (e.g. mushrooms, toadstools, yeasts, and moulds) ; (c) bacteria, which also lack chlorophyll ; and (d) lichens, which are compound plants consisting of an alga living in association with a fungus.

Further classification is given in detail in Lessons 22 and 25.

LESSON 2

External Features of a Plant

Many plant species show remarkable adaptations to a particular environment, or way of life. Competition for rooting space in the soil, for mineral nutrients, for light, or for water is frequently intense, and only those species best able to grow and reproduce in any particular environment will survive. In this way have been evolved all sorts of shapes and forms, differences in size and habit, diversities in colour and odour, and modifications in structural detail, all concerned with maintaining the life of the individual or of the species.



WALLFLOWER (Cheiranthus cheri). G—G, level of ground.

Adapted, by permission, from D. H. Scott's "Introduction to Structural Botany." A. & C. Black

Root and Stem

The wallflower (*Cheiranthus cheri*) is a simple plant to study. Like all flowering plants, it has two distinct parts which live under very different conditions. There is (1) a branched root which is fixed firmly in the ground and lives in darkness ; from this rises (2) an upright shoot, consisting of the stem, bearing leaves and surrounded on all sides by the air and exposed during daytime to the sunlight.

The root has a main portion which grows straight downwards in the same line with the direct axis of the stem. A main root of this kind is called a tap-root. This main axis of the root bears side branches which grow obliquely downwards, forming an acute angle with the tap-root. From these lateral roots

may be borne other branches which penetrate the soil in all directions.

If the plant is dug up for examination, it will be seen that particles of earth adhere firmly to the youngest branches of the root a little behind their extreme tips. This is because these regions of the root are covered with very fine root-hairs (they have withered away from the older parts). Root-hairs are most easily seen on seedlings grown on moist blotting-paper. Examination with a hand-lens will reveal a thimble-shaped cap over the extreme root tip ; here growth takes place, the root cap being a form of protection.

Branch and Leaf Arrangement

The stem is branched, each branch arising from the angle between a leaf and the erect main stem (the *axil* of the leaf). The branches are described as axillary. That part of stem or branch from which a leaf arises is called a *node* ; the portion between two leaves is an *internode*. In the wallflower no two leaves arise at the same level ; every leaf is placed a little above or below its next neighbour, and the arrangement is said to be *alternate*.

Each wallflower leaf is attached to the stem by a narrow base ; it has no distinct leaf-stalk, and therefore is described as *sessile*. The blade, or *lamina*, of the leaf has two surfaces, one directed upwards towards the light, the other facing downwards. The upper surface is the darker green, the under surface is the more hairy. From end to end the leaf is traversed by a stout midrib, or principal vein, which is more prominent on the lower surface, and it gradually tapers towards the tip. From the midrib, branch veins are given off at irregular intervals ; these branch still further to form a network of fine strands, dispersed throughout the leaf.

For some distance above the ground there are no leaves ; they were the oldest and have dropped off, and only scars remain to mark where they were. The vigorous foliage is towards the top. The extreme end of a branch is the growing-point ; it is not protected by a cap as is the root, but by very young leaves which are folded over it.

Production of Flowers

Flowers are borne on the upper part of the stem and its branches, above the leaves. The branches which bear the flowers do not at first differ from the ordinary branches. They arise in the axils of leaves, and for some time bear leaves themselves. Soon the production of leaves ceases, and the branch bears flowers only.

Each flower can be regarded as resembling a modified branch, the leaves of which differ in arrangement, form, structure, and colour from the green foliage leaves. When the flower is fertilized, the petals wither and the fruits develop. Within these the seeds ripen, and when sown produce a new generation of (in this instance) wallflowers.

Organs and Their Functions

To sum up: flowers are the *reproductive* organs ; leaf, stem, and root are the *vegetative* organs. The root of a wallflower, as in the higher plants generally, has two chief functions : it fixes the plant in the ground, and it takes up mineral salts and water from the soil. The main function of the leaves is to absorb carbon dioxide from the air ; carbon dioxide together with the mineral salts and water taken up by the root represents the raw material upon which the plant " feeds." The leaf also provides a large green surface for the absorption of radiant energy from sunlight. The stem conducts food substances from the root and the leaves to other parts of the plant, and it supports the leaves and flowers in a suitable position. The flowers produce seeds ; and thus the species continues to exist.

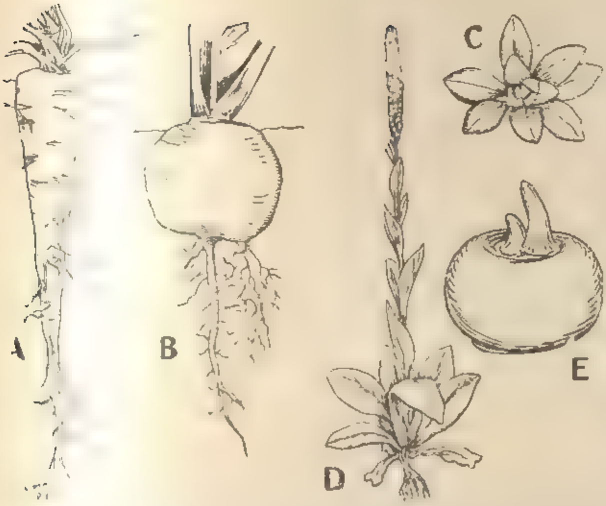
LESSON 3

Duration of Plant Life

SOME common plants appear above ground in spring from seeds shed during the previous summer or autumn, which have remained dormant in the soil throughout the winter ; or the seed has germinated in autumn and the small plants have survived the winter. With the advance of spring the plants grow rapidly, produce flowers and ripen their seeds, and then die. They complete their whole life history within 12 months, and they are called *annuals*. Examples are cornflower (*Centaurea cyanus*) and opium poppy (*Papaver somniferum*).

Other annuals, such as groundsel (*Senecio vulgaris*) and shepherd's purse (*Capsella bursa-pastoris*), complete their life-cycle in much less time, producing several generations within 12 months. These are referred to as *ephemerals*. Because of this rapid reproduction they are well up in the list of " weeds " and they are among the first to take possession of newly dug ground.

A plant which normally completes its life cycle in two years is called a *biennial*. Sown outdoors in spring or summer, it does not flower and produce seed until the following



THE PLANT'S LARDER. A and B, storage roots of carrot and turnip. C, rosette stage (first year) of the mullein, a biennial. D, mullein, second year stage. E, winter condition of underground stem of crocus, showing two buds which will produce flowering shoots in the following season.
From Fritsch and Salisbury, "Introduction to the Study of Plants," by permission of G. Bell & Sons, Ltd.

year, after which it dies. Examples are carrot (*Daucus carota*), turnip (*Brassica campestris*), and mullein (*Verbascum thapsus*). During their first season of growth these concentrate, as it were, on storing up food to provide for the effort of flowering and seeding in the following summer. Winter is passed in a more or less dormant condition, and the leaves are retained. With the arrival of spring, growth again becomes active, and in summer—mainly at the expense of the stored food—flowers are produced. At the ripening of the seed the biennial dies.

Plants that live for years are called *perennials*. During the seasons of active growth food is being accumulated and stored in the underground organs; and after each resting period growth starts again with renewed vigour. The common dandelion (*Taraxacum officinale*) is an example of an herbaceous perennial—it retains its leaves in winter, in contrast with the perennial crocus whose leaves die away after the flowering. The dandelion's food is stored in a stout tap root, that of the crocus in a *corm*.

In contrast, again, are the perennials whose stems are covered with bark—the shrubs and trees. Some of these live for very many years, and some are immensely tall. Some lose all their leaves each year at the approach of winter, e.g., the common beech (*Fagus sylvatica*) and the horse-chestnut (*Aesculus hippocastanum*). Such trees are described as *deciduous*. They start each year anew with an entirely new set of leaves.

Others, such as holly (*Ilex*) and laurel (*Prunus*), are evergreen. They carry leaves at all times of the year, old ones being displaced gradually by new ones throughout spring and summer, so that the change goes on unnoticed (apart from the accumulation of old leaves on the ground round about). All trees and shrubs store their food throughout the woody portions.

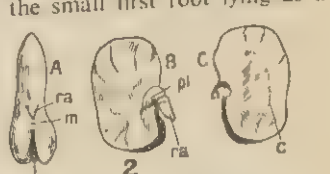
LESSON 4

Structure and Germination of Seeds

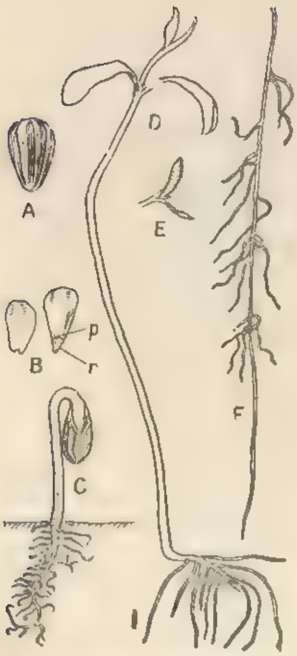
The broad bean seed is easy to study on account of its large size. It is a flattened, somewhat oval body, covered by a tough pale-brown skin, the seed coat or *testa*. Along one edge is an elongated black scar called the *hilum*, which marks where the seed was attached to the inside of the pod.

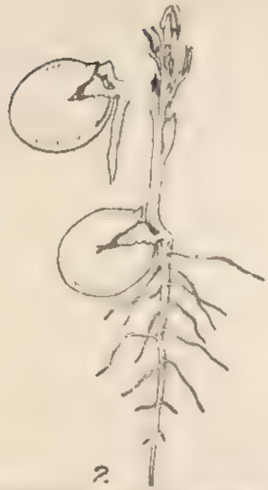
Near to the hilum, on the slightly concave edge of the seed, is a small triangular swelling caused by the tip of the first root or *radicle* which is under the seed coat, and between this and the hilum is a very small hole called the *micropyle*. The micropyle is a channel for the absorption of water by the dry seed, enabling it to germinate.

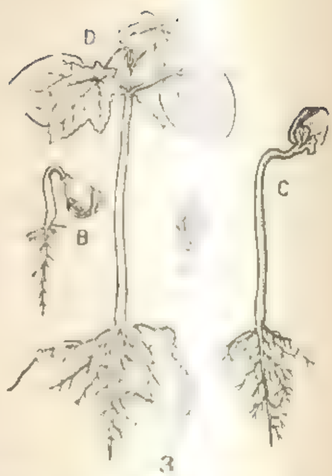
For the examination of the seed contents it is best to use seeds which have been soaked in water, as this process causes considerable swelling of the seed coat and contents. On removal of the tough seed coat the embryo is disclosed, filling the whole space within. Its most conspicuous features are two large fleshy cream-coloured lobes, the seed leaves or *cotyledons*, and the small first root lying as a



STRUCTURE OF SEEDS. 1. Bean pod opened, showing attachment of seeds. 2. A, concave edge of soaked bean seed; B and C, testa removed and cotyledons separated; c, cotyledon; h, hilum; ra, radicle; m, micropyle; pl, plumule.







GERMINATION OF SEEDS. 1, **Sunflower**: A, fruit; B, cotyledons separated; r, radicle; p, plumule; C, elongation of the hypocotyl; the cotyledons are still enclosed in the split fruit coat. D, at this stage the smooth green cotyledons perform the work of foliage leaves; E, hairy plumule; F, longest root. 2, Common bean (Vicia faba): successive stages of germination. 3, Castor oil plant (Ricinus communis): A, seed seen from outside; B, seedling with seed-coat burst but cotyledons still enveloped in endosperm; C, hypocotyl straightening, endosperm still adhering to cotyledons; D, established seedling, with expanded cotyledons and first plumular leaves.

Diagrams in this Lesson reproduced by permission from Bower, "Botany of the Living Plant," Macmillan; Kirkwood, "Plant and Flower Forms," Sidgwick & Jackson; Fritsch and Salisbury, "Introduction to the Study of Plants," Bell; and Woodhead, "Study of Plants," Clarendon Press

projection against their edges. If the two cotyledons are separated, a small bud bearing minute yellowish leaves is seen lying between them near to one edge. This bud is the first shoot, or *plumule*. Each cotyledon is attached to the base of the plumule by a very short stalk, and below this point its axis is continuous with the radicle.

Bean Type of Germination

When conditions are suitable for germination, the radicle bursts through the seed coat and grows rapidly downwards. Very soon branch roots arise from it, so that the young plant becomes firmly anchored in the soil. In the meantime the plumule begins to grow upward. The tip of this young shoot is sharply curved, so that the delicate leaves at the apex are not injured by pushing up through the soil. On emerging at the surface of the soil the plumule straightens out, and the leaves expand and become green.

Food Storage

The food material required for the first growth of shoot and root is supplied from the fleshy cotyledons. If seedlings are examined at successive stages of germination, it will be seen that as growth proceeds these cotyledons gradually shrivel, until finally only a shrunken remnant is left. By the time the food store in the cotyledons is exhausted, the young plant is able to obtain its own food from the soil by means of the roots, and from the air by means of the green leaves. In the broad bean seed the cotyledons do not come above the soil; in some plants, e.g. sunflowers, they do.

Sunflower Type of Germination

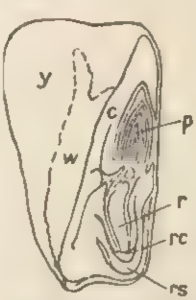
The so-called seed of the sunflower (*Helianthus*) is in reality a fruit—the ovary, in which the seeds are developed, forms the outer protective coat. The ripened sunflower fruit is triangular in shape, and its outer coat is hard, dark, and brittle. When this is removed, a thin white skin, the seed coat, is seen enclosing the embryo, which (as in the bean) consists of two cotyledons, a radicle, and a plumule. When the radicle emerges at germination, the fruit coat splits into two halves along the edge, and the stalk region below the cotyledons, the *hypocotyl*, grows up in the form of a crook and, continuing to elongate, carries the cotyledons and the split fruit coat above the ground. The cotyledons then turn green and act as the first green leaves, obtaining food from the air.

There are thus two types of germination: (1) that in which the cotyledons stay below ground and serve only for food storage, and (2) that in which the hypocotyl carries the cotyledons up into the air, and in which the cotyledons serve first for food storage and later

perform the normal leaf function of assimilation. These two types are known as *hypogeal* and *epigeal* germination respectively. Another common example of hypogeal germination is afforded by the acorn; of epigeal germination, by the winged seeds of sycamore (*Acer pseudoplatanus*).

The Endosperm

In bean and sunflower seeds the food reserve which is drawn upon by the young shoot and root is contained in the cotyledons, that is, in part of the embryo itself. In some seeds the food store is not in the embryo, but in a part called the *endosperm*, which lies with the embryo inside the seed coat; an example is the seed of the castor oil plant (*Ricinus communis*). Its seed coat is hard and brittle, and if this is removed, the white food store, which is oily to the touch, is revealed. On splitting open this endosperm, the embryo can be seen, consisting of plumule, radicle, and two paper-thin cotyledons. On germination, the embryo, lying in close contact with the food store, extracts nutriment from it, and the food store gradually shrivels away. The germination is of the sunflower type, and the cotyledons, endosperm, and seed coat are carried out of the ground by the hypocotyl. As the endosperm shrivels, the seed coat is thrown off and the cotyledons become green and expand. Here the cotyledons first act as suctorial organs and later expand into nourishing leaves.



MAIZE. Longitudinal section of grain. C, cotyledon; p, plumule; r, radicle; rc, root cap; rs, root sheath; w, the starchy endosperm; y, sugary endosperm.

So it is seen that seeds may have the food substances stored up in the embryo itself (as in the bean) or in a food tissue outside the embryo (as in the castor oil plant). The endosperm is sometimes spoken of as *albumen*, and seeds which have this tissue are said to be *albuminous*; those lacking it are described as *exalbuminous*.

The seeds so far considered have two cotyledons. Many plants, e.g. grasses and lilies, have embryos with only one cotyledon. Such plants have certain other characters in common and are grouped together as *monocotyledons*. The majority of monocotyledonous seeds have endosperm.

Maize Type of Germination

The maize grain (or Indian corn) is really a fruit, the fruit coat and seed coat being united. If a soaked maize grain is cut through at right angles to its broader surface, the yellowish endosperm and white embryo, consisting of one cotyledon, plumule, and radicle, can be easily distinguished. The cotyledon lies in close contact with the food store and extracts the necessary nourishment from it for the growth of radicle and plumule. The plumule grows straight up through the soil, the tender growing point being protected by the outer sheath of leaves which is hard and pointed and called the *coleoptile*. The radicle bursts through the root sheath and grows downwards. It does not become a large main root, but is rapidly followed by other roots, which arise from the base of the plumule. Such secondary roots from a stem are called *adventitious* roots.

PRACTICAL WORK

With a drop of iodine, test cut seeds of bean, pea, wheat, and maize for the presence of starch (a common form of food reserve in seeds). A blue-black colour results.

Obtain any of the common seeds mentioned and observe their germination in damp soil or sawdust; large seeds, such as bean and acorn, can be suspended in the neck of a glass bottle filled with water. From peas or beans which have started to germinate, cut off the cotyledons. Note the subsequent starvation of the seedling.

LESSON 5

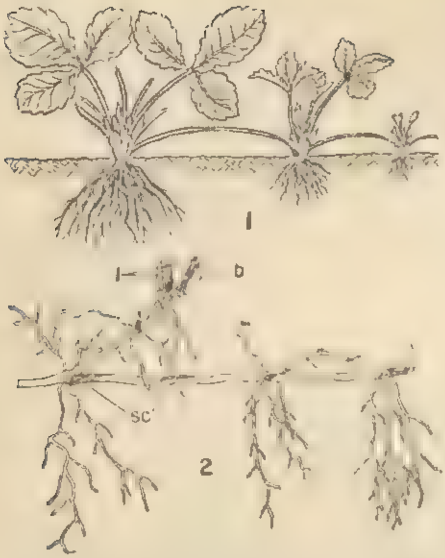
Vegetative Propagation

A BROAD distinction is drawn between sexual reproduction—in which, in the higher plants, fusion of gametes is effected in the flower, resulting in the formation of seed, and vegetative reproduction—in which some part of the more or less mature plant becomes detached as a new young plant.

Runners and Rhizomes

The strawberry plant (*Fragaria*) is an example of vegetative propagation by *runners*. These are long and thin off-shoots that extend along the ground away from (but still attached to) the parent plant. From every node of this runner can be produced roots and a new plant. By eventual decay of the internodes, connexion with the parent is severed and the young plants grow on individually.

Some plants have stems which creep horizontally under, or partly under, the surface of the soil; these stems are called *rhizomes*, and they can be distinguished from roots by the fact that they bear small, scaly leaves, and buds in the axils of these leaves. Adventitious roots are produced from each node (joint), and the end bud of the rhizome grows upward to form a



VEGETATIVE REPRODUCTION. 1. Propagation of the strawberry plant by runners. 2. Rhizome of lily of the valley ; b, end bud emerging from the soil ; l, withered leaf-bases ; s.c. scale leaf.

new aerial shoot, while growth of the underside is continued by an axillary bud. Thus new plants are established some little distance from the parent, as in plants with runners. Some common plants which propagate by means of rhizomes are certain grasses and sedges, lily of the valley (*Convallaria*), and solomon's seal (*Polygonatum multiflorum*).

Underground Stems

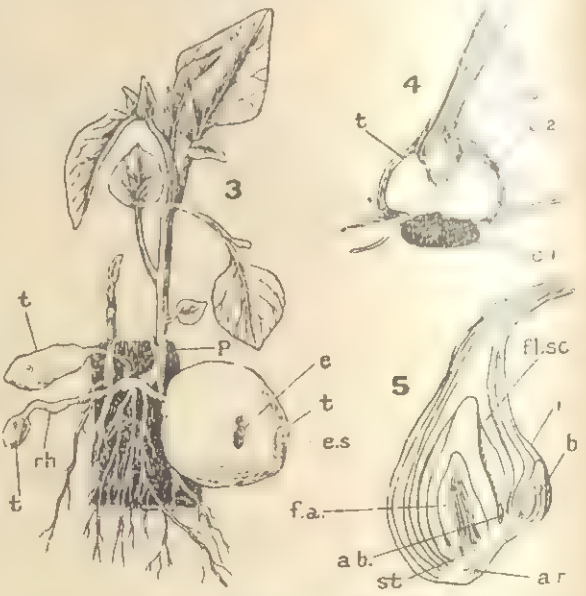
The rhizomes of marram grass (*Ammophila arenaria*) grow to many yards in length, and this makes the plants useful in reclaiming sandy shores and binding together the banks of canals and railways. Long and slender roots are formed which grow deeply in search of water, and these, together with the long rhizomes, form a tangle which binds the sand or loose soil (*see* also Lesson 24).

In solomon's seal the rhizome serves not only for vegetative reproduction but also for food storage. The horizontal stems become greatly thickened with food reserves ; these enable the aerial shoots to grow rapidly in the spring, and detached rhizomes are well equipped for an independent existence. As the plants grow, the rhizomes tend to rise above the surface of the ground. On such rhizomes it is common to find thick unbranched roots which penetrate the soil deeply and then contract, pulling the rhizome downwards.

Other swollen underground stems producing new plants are those of potato (*Solanum tuberosum*) and Jerusalem artichoke (*Helianthus tuberosus*). The large quantity of food materials stored in the tubers makes them valuable as vegetables. The so-called eyes of a potato tuber represent the scale leaves and their axillary buds. If the tubers are left in the ground, the eyes grow out into new shoots and roots. Under cultivation the potato tuber before planting is sometimes cut up into a number of pieces, each including an eye, as the starting point for a new plant.

Corms and Bulbs

The storage organ of the crocus is also an underground stem, called a *corm*. This is formed by the swelling-up of the base of the flowering stem. If the lower part of a flowered crocus corm is cut in halves lengthwise, a new corm is seen as a swollen structure enclosed within the sheathing bases of the lower leaves of the flowered shoot, and below it is the old corm from which the flowers arose. In addition to ordinary adventitious roots, the crocus produces thick contractile roots, which pull the corms of successive seasons deeper into the soil.



TUBER, CORM, AND BULB. 3. Plant grown from a slice of potato : e, eyes ; e.s., shoot growing from an eye of the large tuber ; p, slice of potato ; rh., rhizome ; t, tubers. 4. Longitudinal section of base of crocus corm and flowering shoot : a.s., aerial flowering shoot ; c.1, c.2, c.3, corms of successive years ; t, tunic 5. Longitudinal section showing winter condition of tulip bulb : a.r., adventitious roots ; b, incompletely developed daughter bulb ; a.b., axillary bud ; f.a., flowering axis ; fl.sc., fleshy scales of bulb; i, investment of dry scales; st, flattened stem.

Figs. 2 and 3 reproduced by permission from Woodhead's "Study of Plants," Clarendon Press ; Figs. 4 and 5 from Fritsch and Salisbury's "Introduction to the Study of Plants." Bell & Sons. Ltd.

Another organ of vegetative reproduction is the *bulb.* Bulbs are the basis of numerous flowering plants, such as tulip (*Tulipa*), hyacinth (*Hyacinthus*), snowdrop (*Galenthus nivalis*), and daffodil (*Narcissus*). A tulip bulb cut in halves lengthwise is seen to consist of a short, thickened stem which bears roots on its under surface, and a number of large, fleshy leaves encircling one another. The food reserves are stored in these modified leaves. The bulb is enveloped externally by thin, papery scale leaves, and in the centre of the fleshy leaves is a large bud, in which parts of the future flower and young foliage leaves can be distinguished. Growth of this bud takes place in spring at the expense of the food materials in the fleshy leaves. Food substances manufactured by the green foliage leaves pass back into the small axillary buds of the bulb, which increase greatly in size.

The original tulip bulb is thus replaced by one or more daughter bulbs, which repeat the sequence of events in the following year. In some bulbs, exemplified by the daffodil, the storage organs are not entire modified leaves, but are the swollen bases of the green foliage leaves of the preceding season's growth.

In some plants there is a type of reproduction, called *apomixis*, which has the superficial appearance of sexual reproduction but actually occurs without fertilization and/or meiosis. It includes *parthenogenesis*, in which the ovum develops without being fertilized ; this is common in, for example, the dandelion. Ova which develop in this way are usually diploid (*see* Lessons 33 and 34) in which case all the offspring are genetically identical with the parent. Ordinary sexual reproduction, providing genetic recombination, occurs from time to time. Also included in this group are *apospory*, where an ordinary diploid sporophyte cell substitutes for the spore, meiosis being omitted, so that the gametophyte is diploid ; and *apogamy*, in ferns, in which a gametophyte cell gives rise directly to a sporophyte, the gametophyte being diploid. Where vegetative reproduction has become completely dominant—that is, the plant does not reproduce by means of seed, such plants are, strictly speaking, *apomicts*.

PRACTICAL WORK

Obtain a number of rhizomes, tubers, corms, and bulbs, ascertain that they are really stems (indicated by the presence of small leaves and buds). Cut a corm and a bulb in halves vertically, and compare their structures.

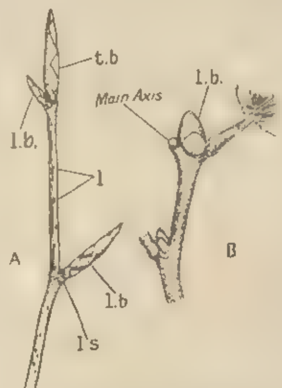
LESSON 6

Branching and Buds

PLANTS higher in the evolutionary scale (trees, shrubs) are usually greatly branched. This increases the surface area over which respiration and photosynthesis can take place. In winter a woody shoot, e.g. of beech, is terminated by a long, slender, and pointed structure, brown in colour. This is the terminal bud and it contains the next continuation of the shoot, both stem and leaves, in miniature. Along the sides of the woody shoots are a number of similar buds, the *lateral* buds, each capable in the next season of growing out into a side branch.

Beneath each lateral bud the bark shows a kidney-shaped smooth scar with a curved band of dots. Each scar marks the former point of attachment of a leaf, and the dots are the broken ends of the veins which ran from the leaf into the stem. The lateral buds are all axillary to leaves of past seasons, the terminal bud (at the tip of the shoot) alone having no leaf scars.

The colour and shape of buds and leaf scars vary so much in different trees and shrubs that it is not difficult to recognize them in winter by such characters. Other interesting external features are the *lenticels*—small, generally lighter-coloured projections of varying shape, which are distributed irregularly on the bark of woody twigs and serve the purpose of gaseous interchange between the interior of the branch and the atmosphere. This process is prevented, except at these points, by the impermeable bark.

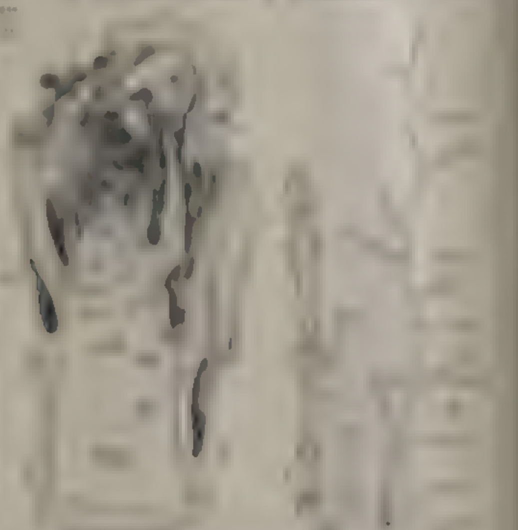


MODE OF BRANCHING. A, portion of a twig of beech in winter time. B, the same of elm. l, lenticels ; l.b., lateral buds ; l.s., leaf-scar ; t.b., terminal bud.

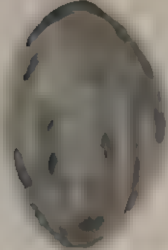
Reproduced by permission from Fritsch and Salisbury, "Introduction to the Study of Plants," George Bell & Sons. Ltd.

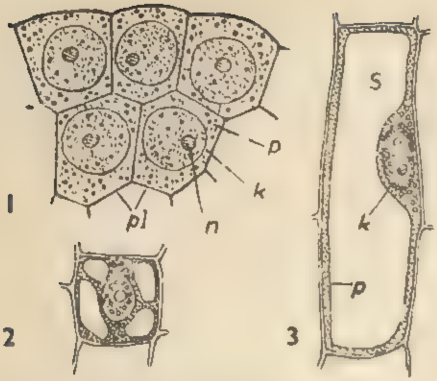
Mode of Branching

The most fundamental point of distinction in the mode of branching lies in the behaviour of the terminal bud. In a considerable number of trees, of which the conifers (spruce, fir, cypress, etc.) are examples, the original plumule of the seedling steadily continues its growth year by year, lengthening in the spring and forming a new terminal bud in the autumn, so that a tall, straight main axis is produced. A number of lateral buds develop into side branches, but these are all subsidiary to the central shaft and again branch in the same manner. This



Plant Cells and Their Structure





PLANT CELLS: STAGES OF DEVELOPMENT. 1. Young thin-walled cells from a growing point; p, protoplasm; k, nucleus; n, nucleolus; pl, plastids. 2. Young cell showing formation of vacuoles. 3. Typical living, fully developed cell; p, protoplasm; s, vacuole filled with sap; k, nucleus.

Reproduced by permission from F. O. Bower, "Botany of the Living Plant." Macmillan

The interior of the young cell is filled with a colourless, finely granular substance, the *protoplasm*, which is the essential living matter of all plant and animal bodies. Protoplasm consists of a mixture of complex proteins, composed of carbon, hydrogen, oxygen, nitrogen, and sulphur, and in the active state it contains a large proportion of water. In consistency it resembles a thin jelly, but it can change to a more liquid form; these are known as the *gel* and *sol* conditions respectively. Protoplasm is present in all cells in which growth is going on, or in which food is assimilated, or any new structure formed. It is, in fact, the seat of all those processes, whether in plants or in animals, which serve to distinguish living organisms from lifeless matter.

The Nucleus

Embedded in the protoplasm of the young cell is a roundish, denser body, which appears more coarsely granular than the protoplasm itself. This body is the *nucleus*. A nucleus is present in all living cells of the higher plants, and probably in all living cells whatsoever, though its presence as a definite body in the bacteria and some low algal forms is not yet proved. The substances of which a nucleus is composed have been detected inside these algae and bacteria.

The nucleus has a complicated structure; under a microscope the granules are seen to be arranged along extremely minute threads which are entangled and give the appearance of a fine network. The nucleus also contains one or more comparatively large round bodies, the *nucleoli*. The nucleus contains the same elements as the protoplasm and some compounds which are called *nucleic acids*. Phosphorus is usually associated with the latter.

The protoplasm, nucleus, and cell wall are the most constant constituents of a living vegetable cell. The cell wall is formed sooner or later in all cells of the higher plants, but it is frequently absent from certain cells of the simpler plants. Most cells contain only one nucleus, but in certain parts of the higher plants, and in a number of the lower forms as well, several nuclei may be found in one cell.

Cell Sap

In the older cells, such as those occurring at a little distance behind the growing-point, the whole cell is larger, and it is no longer filled by the protoplasm. Clear spaces are present in the protoplasm; they are filled with water, which contains various substances, including salts, sugars, and acids, in solution and is called the *cell sap*. The spaces filled with cell sap are called the *vacuoles* (because they appear to be empty). The nucleus has increased very little in size. The cell wall is seen to have become slightly thicker by the addition of cellulose.

Examination of the older parts of a plant reveals the structure of fully grown living cells. The protoplasm is reduced to a thin layer which lines the inside of the wall; the cell is occupied by one large vacuole containing cell sap, the large vacuole having been formed by the running together of the small ones; the nucleus now lies close against the cell wall, embedded in the protoplasmic lining. The cell wall itself has become further thickened by new cellulose added to it from the active protoplasm.

Definite Arrangement of Cells

This is the structure of most of the mature living cells of plants. Some cells, in the older regions of a plant, retain the ability to divide. These are called *meristematic* cells, and they do not have vacuoles in their protoplasm. Some cells lose their protoplasm altogether; they are then incapable of any further growth, though they may still function for the conduction of food substances and for mechanical support. The different kinds of cell are not distributed uniformly throughout the body of the plant but are definitely arranged with similar cells associated together in well-defined regions. Such an association of cells performing the same function is called a *tissue*.

Protoplasmic Granules

Other structures occur in cells besides protoplasm, nucleus, and cell wall. A few of the more important may be mentioned here. Embedded in the ordinary protoplasm there are often round bodies of denser protoplasm. Such protoplasmic granules are called *plastids*, the most important being those which contain the green pigment chlorophyll. Starch grains

frequently occur in cells, and these are easily recognized under the microscope by their characteristic stratified appearance. Oil is often present in the form of small droplets in the protoplasm, and proteins in the form of rather complex granules.

In all vegetable tissues the protoplasm of each cell is in communication with that of its neighbours by means of very fine protoplasmic threads, which pass through the cell wall from one cell to another. The living matter in the plant is thus continuous. The plant, in fact, is a living protoplasmic body supported, but not interrupted, by the skeleton of cell walls.

LESSON 8

Structure and Function of Roots

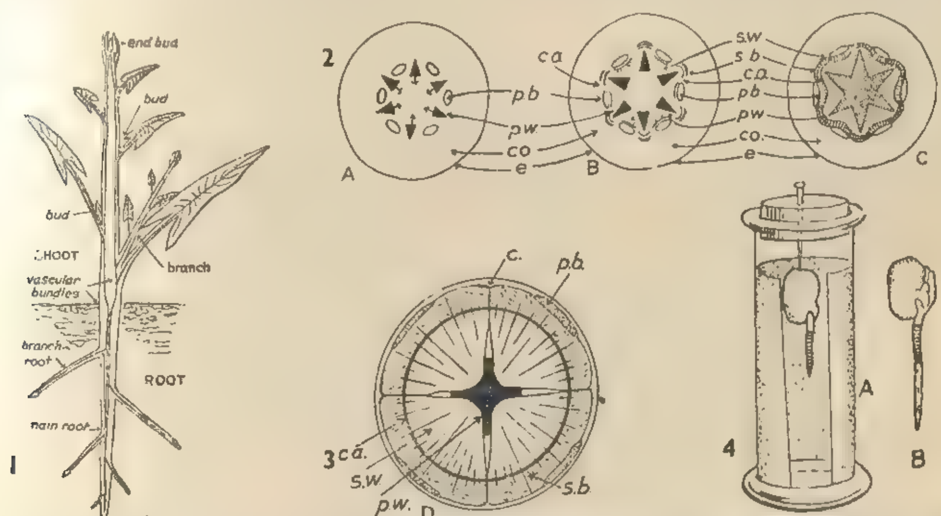
THE root has an apical growing region which is smooth and conical, and its growth activity leads mainly to increase in length. In the majority of cell divisions the new cell wall is laid down in a transverse plane; this results in the formation of longitudinal rows of cells.

On the very outside, at the tip of the root, the cells are less regularly arranged and are vacuolated. This group of cells, called the *root cap*, protects the meristematic region from damage. The root cap is added to from the meristematic region as the root grows and the outer cells wear away. As the cells in the main body of the root are moved farther away from the apex (by the interpolation of new cells), they mature and develop into distinct tissues.

The middle of the root is traversed by a cylindrical strand of vascular tissue which is continuous with the vascular tissue of the stem. The central region of the vascular tissue is occupied by the wood or *xylem*. This consists of two main types of cell: *vessels* and *fibres*. All these cells have very thick, tough walls; they are thickened by deposits of a substance called *lignin*, and are dead—they have lost both protoplasm and nucleus. The vessels are the conducting elements, and have large cavities; the transverse walls between vessel segments have broken down, forming long, continuous tubes in the wood. The fibres are very long and narrow, with extremely thick walls, and their function is mainly mechanical.

Phloem or Bast

Outside the xylem is the *phloem*. This is sometimes, especially in the stem, called *bast*, but this is an ambiguous term since it often includes other tissues as well. In the root there is only a small quantity of phloem, which



ROOT TISSUES. 1. Diagram of plant showing continuous vascular system. 2. A, very young root, showing alternating groups of primary xylem and primary phloem (or bast); B, older root, showing cambium arising on outside of xylem and on inside of phloem; C, old root, showing complete cambium. 3. Old root with fully developed secondary thickening; c., cork; ca., cambium; co., cortex; e., epidermis; p.b. primary phloem; s.p., secondary phloem, formed on outside of cambium; p.w., primary wood; s.w., secondary wood, formed on inside of cambium. 4. Bean seedling marked to determine region of elongation in the root; A, at beginning of experiment; B, at end of 30½ hours.

Reproduced by permission from Bower, "Botany of the Living Plant," Macmillan; and Woodhead, "Study of Plants," Clarendon Press

consists of elongated, thin-walled, living cells. The end walls between longitudinal rows of cells are perforated. The phloem conducts foods such as sugars, which are manufactured in the leaves, through the stem and roots to be used in growth or for storage.

Between the xylem and phloem is a very thin layer, one or two cells thick, of meristematic cells. This is the *cambium*, and it resumes activity in the older root to form *secondary vascular tissues*. The vascular cylinder is surrounded by a layer of thin-walled cells, the *pericycle*, and this is where all the root branches originate. Root branches are therefore said to be *endogenous* in origin (stems are *exigenous*).

Outside the pericycle is a layer of thickened cells, the *endodermis*; this prevents excessive water from getting into or out of the vascular system. Typically these cells have a narrow strip of the radial walls impregnated with both fat and lignin. This is known as the *Casparian strip* (after the discoverer). Sometimes the radial and the inner tangential walls are all completely impregnated with fat and lignin. Certain cells at intervals around the cylinder do not exhibit this structure, and water movement can take place only through or around these cells. They are called *passage cells*.

The endodermis is the innermost layer of the *cortex*, which is composed of many layers of living, thin-walled cells, variable in shape, which usually do not fit closely together but have little spaces between. These *intercellular spaces* contain air and water-vapour, but not water. This allows free gaseous interchange throughout the plant.

The outermost layer of these cells is of great importance, for many of them, within a limited area just behind the apex, grow out to form the long, slender *root hairs* which absorb all the food that the plant obtains from the soil. The layer of cells which give rise to root hairs is called the *piliferous layer*. The root hairs are restricted to a very short region, usually not more than an inch or two in length and starting about half an inch behind the root tip. New root hairs are continually being formed as the root grows. The older ones are sloughed off, and the layer of cells underneath becomes thickened and forms a protective *exodermis*.

As the root ages, a secondary thickening takes place in the vascular tissue; the pericycle begins to divide actively and forms a layer of *suberised* cells called *cork*. The tissues outside this—endodermis, cortex, and exodermis—die and become sloughed off. The cork is commonly called bark.

The great increase in thickness in older roots is due to formation of a large amount of wood. The cambium, which has persisted between the xylem and phloem, resumes its meristematic activity and forms large amounts of secondary xylem to the inside and smaller quantities of secondary phloem to the outside. The cell divisions of the cambium are very uniform, and the secondary wood has cells radiating from the centre in regular rows.

Irregularly distributed in radiating rows in the xylem are bands of thin-walled cells which have their long axis arranged across the plant. These are for lateral transmission of food substances, and they are called *rays*. They may be one or more cells wide and they vary in depth. To these are due the characteristic "grains" of wood.

Chief Functions of a Root

The chief functions of a root are (a) fixation of the plant in the ground, (b) absorption and conduction of food and water. Hence the extensive growth of the root system and increase in the amount of vascular tissue. New branches are continually being formed, and a larger root system is necessary for anchoring the plant securely. Every new branch bears a number of leaves, so the older a plant becomes the greater is its total leaf surface in spite of the loss of old leaves. The plant loses water vapour through its leaves; hence the greater the leaf surface, the more must pass up from the roots; therefore the amount of conducting tissue, the wood, must be increased. Leaves also manufacture foods from the carbon dioxide of the air. The more leaves there are, the greater is the amount of assimilated foods to be conveyed through the plant, hence the increase in phloem to deal with it.

The central position of the tough vascular tissue is most effective in withstanding the stresses and strains imposed, which mainly take the form of an upward pull.

While the activity of the cambium is providing these necessary tissues, the absorptive surface of the root is also increasing with the number of young branch roots being formed. If a young root of a seedling bean or pea plant is examined, it is seen that the root hairs are not borne immediately behind the growing tip, but a little distance from it. This region which is devoid of the hairs is known as the *region of elongation*, for it is here that the cells are becoming greatly lengthened and modified for their future work. If the root hairs were borne on this elongating region, they would be injured as the root worked its way through the soil.

PRACTICAL WORK

Cut across the main root of a bean seedling and examine the cut surface with a hand lens. Note the central vascular core.

The roots of the vegetable marrow plant are excellent material to examine on account of the large size of the wood vessels. On a cut surface (across the root) the cavities of the vessels appear to the naked eye as small holes.

Cut vertically down the root of a bean seedling and note the vascular strand running through the length of the root.

LESSON 9

Plant Stems and Tree Trunks

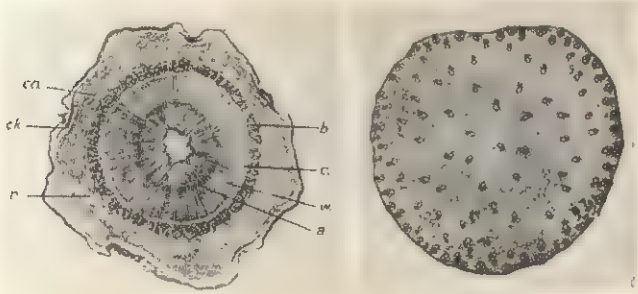
THE main functions of the stem of a plant are (1) to expose the leaves to abundant light and air, and (2) to provide communication between the leaves and the roots. Ordinarily the stem occupies an erect position, while the branches are directed obliquely upwards, exposing the leaves to maximum light. The vascular tissue of the plant is continuous throughout root, stem, and leaves, thus providing communication between these organs.

The growing tip of the stem, like that of the root, is composed of cells filled with protoplasm and containing large nuclei. A longitudinal section through the middle of the growing-point shows a dome-shaped tip with the young leaves arising as projections on the sides of the developing apex. Farther behind the apex the leaves increase in size and assume the typical form, while the tissues in the stem are differentiating.

Epidermis and Cuticle

The distribution of the tissues in a stem differs from that in a root, though the actual types of cell forming the tissues are, on the whole, similar. Examination of a cross-section of a young stem shows that around the outside there is a layer of rather flattened cells, the *epidermis*. These cells are protected by an impervious, transparent layer of non-living material called the *cuticle*. The cuticle and epidermis are perforated by small holes, called *stomata*, through which air can pass to the inner tissues of the stem and waste gases and water vapour can be excreted.

A little distance from the surface is a ring (in young dicotyledons, at least) of small groups of vascular tissue. Each group is called a *vascular bundle*, and it consists of xylem on the inner and phloem on the outer sides.



STEMS OF PINE AND OF MAIZE. Left, transverse section of stem of pine; a, annual ring; b, bast or phloem; co, cortex; c, cambium; ck., cork; r, resin canal; w, wood. Right, transverse section of maize stem, showing numerous scattered vascular bundles.

From Woodhead, "Study of Plants," Clarendon Press

Between the two is a narrow layer of meristematic cells called the *cambium*. The structure and function of these cells are the same as in the root, with which they are continuous, and also with the vascular bundles of veins in the leaves. The rest of the stem is composed of living, thin-walled cells called *parenchyma*. Those in the centre constitute the *pith* or *medulla*, which is relatively large in the stem of an herbaceous plant. Those external to the vascular bundles form the *cortex*, which is comparatively narrow. Similar cells run between the bundles and form the *rays*, which connect the pith and cortex.

Vascular Tissue

The stems of monocotyledons, e.g. the maize plant, differ in several respects. In cross-section the vascular bundles are seen to be scattered in the ground tissue of parenchyma and not arranged in a peripheral ring. The bundles have no cambium between the xylem and phloem, and so no secondary thickening takes place. They are described as closed bundles, to distinguish them from the open bundles of dicotyledons, which have a cambium.

In some dicotyledons the vascular tissue increases greatly in amount, in order to support the weight of new branches and to convey more abundant food supplies through the plant. This gives rise to a "woody" plant, e.g. a tree. In these the cambium in the vascular tissue remains meristematic. Between the vascular bundles, parenchymatous ray cells become secondarily meristematic and link up with the vascular cambium to form a complete ring of actively dividing cells. These form xylem towards the inside and phloem towards the outside. Much more xylem than phloem is developed, giving the stem great strength and hardness. Because most of this tissue is formed from secondary cambium, it is called *secondary thickening*. The xylem is the wood, or timber, of commerce.

Usually only the outer region of the xylem functions as a conducting system, and substances such as tannins and resins are often laid down in the centre of the stem. These give the woods their characteristic colours and smells. The pith, which may be squashed almost out of existence, is always present to some extent in stems, whereas there is usually no pith in roots. Rays run transversely through the secondary tissues but are much narrower than in the young plant.

MONARCHS OF THE WOODS. This photograph of a scene in the redwood forest of Bull Creek Flat, California, shows some of the world's finest trees—the giant sequoias (Sequoia gigantea) and the coast redwoods (Sequoia sempervirens).

Courtesy of Professor Metcalf and the Carnegie Institution of Washington

It is at the ground level that the tissues in the plant become rearranged, thereby giving quite different appearances to cross-sections of root and stem. These changes are complicated, and involve not only a separation of the centrally massed bundles of the root to the peripheral ring in the stem, but also a rearrangement of the groups of xylem and phloem.

Formation of Cork

In stems, branches arise from external tissues and not deep down as in roots. They are said to be exogenous in origin. Old stems are covered externally by layers of cork or bark, which usually arises just under the epidermis of the young stem, that is, the outer part of the cortex. When the cork is formed, the original epidermis shrivels up and dies. Gaseous exchange then takes place through lenticels, which are small areas in the cork where the cells do not fit closely together, thus allowing air to pass between them and thence to the intercellular spaces in the cortex.

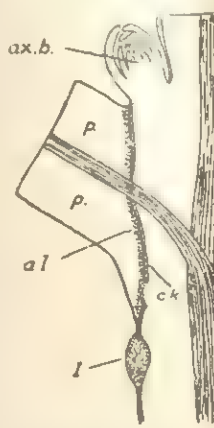
Cork is usually formed from a layer of mature cells which becomes secondarily meristematic. This layer is called a *cork cambium.* It divides to form cells which are regularly rectangular in shape and fit closely together. These cells then become impregnated with a substance called *suberin* (mainly of fatty constitution), which is impervious to water.

Before the fall of leaves in autumn, a layer of cork cells is formed in the stem at the base of each leaf stalk (petiole). When the leaf stalk falls, the surface of the stem is thus provided with a protective covering, not left as a raw surface of thin-walled living cells which might be damaged by insects or infected by spores of bacteria or fungi. Cork is similarly formed very rapidly through the agency of the living cells of the cortex in response to any injury.

Rigidity and Stability

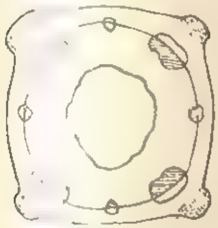
The chief mechanical tissue, i.e. the vascular bundles, is arranged so as to give the best support to the stem, especially in windy weather. Hence the ring-like disposition towards the exterior of the stem, as seen in cross-section. This construction, with isolated bundles separated by soft tissue, gives much greater resisting power than is attained by a massing together at the centre. The stems of the smaller herbaceous plants have to resist stresses different from those imposed on the root, and they are subjected mainly to lateral pressure. They bend very readily to the wind or other influence. In trees, where great size is attained, the enormous development of wood and its central positioning give the rigidity and stability required.

In many young stems of dicotyledons the relation to girder construction is plainly seen, as in the stem of the dead-nettle (*Lamium album*). The arrangement of the four chief vascular bundles at the corners of the square stem is equivalent to two crossed girders. Additional mechanical support is given by the development of a thick-walled tissue, called *collenchyma*, immediately under the epidermis at the four corners of the stem.



PETIOLE. Vertical section through base, showing absciss layer (a.l.) with cork (ck.) beneath it; l, lenticel; ax.b., axillary bud; p, petiole.

From Bower, "Botany of the Living Plant"



DEAD-NETTLE STEM. Left, transverse section showing an arrangement equivalent to two crossed girders. Right, arrangement of veins.

Reproduced from Bower, "Botany of the Living Plant," Macmillan, and Woodland, "Study of Plants," Clarendon Press

Increased firmness is brought about by transverse and oblique bands, which unite the bundles girder-fashion at the nodes. (This method of support by peripheral girders disposed in a ring is seen in the frames supporting large gas-holders.) The girder principle is apt to be overlooked in older stems of dicotyledons, owing to the formation of the secondary tissues and to the fact that the vascular ring is a central column.

Concentric Rings

If the cross-cut surface of a tree trunk is examined, it is seen to be marked with a number of concentric rings. Each distinct ring often consists of two less distinct, differently coloured layers. It shows the amount of secondary wood formed by the cambium in any one growing season, and thus some idea of the age of the tree can be obtained by counting the number of rings, which are called *annual rings.*

In temperate countries such as the British Isles the growing season is an annual event and the rings are therefore truly annular; but sometimes two such rings may be produced in the same year owing to a deterioration in climate half-way through the year. In tropical countries the trees do not show any well-defined rings.

LESSON 10

Structure and Function of Leaves

LEAVES are flattened outgrowths from the nodes of the stem, with which their tissues are completely continuous. A fully developed leaf consists of a flattened blade or lamina, borne upon a stalk or petiole, at the base of which there are sometimes a pair of flattened outgrowths called stipules, e.g. in the rose and the sweet pea. But stipules are often absent, and the leaves of some kinds of plants are stalkless, as in the wallflower. In some plants, e.g. grasses, the attached part of the leaf is in the form of a sheath which wraps around the stem. Leaves present almost endless variations in shape, proportion, and arrangement on the stem.

Arrangement of Leaves

The majority of plants have only one leaf at each node, an arrangement which is described as *alternate*; but there are a large number of plants in which a pair of leaves arise together, when the leaf arrangement is said to be *opposite.* This occurs in, for example, the horse-chestnut, lilac, and privet. Such pairs of leaves nearly always occur at right angles to one another at successive nodes. If more than two leaves occur at a node, the group is spoken of as a *whorl*, an example being common goosegrass or cleavers (*Galium aparine*), in which the leaves of one whorl usually alternate with those above and below. The effect of such alternation between the leaves of successive nodes is to prevent undue overshadowing, and alternate leaves achieve the same end by their spiral arrangement.

In many herbaceous plants, such as dandelion and daisy, a large number of leaves arise, close together, from the base of the stem just above the surface of the soil; such leaves are usually collected in the form of a rosette and are described as *radical* leaves.

On erect shoots and in rosettes, the leaves retain throughout life the position which they occupied at their first development. But on the horizontal branches of trees and shrubs the leaf-

1

2

3

TYPES OF LEAVES. 1. Compound leaf of rose. 2. Simple opposite leaves of privet, alternate pairs being at right angles to each other. 3. Whorled leaves of goosegrass or cleavers.

LEAF MOSAIC. The beech tree presents an example of the disposal of leaves in such a way as to avoid much overshadowing and to make the most of the sunlight.

stalks twist so that the blades are placed more or less at right angles to the light.

There is a great variety of leaf mosaics, in which a number of adjacent leaves, or parts of leaves, fit together to make up an almost continuous sheet of green without much overshadowing (to make the most of the sunlight in a climate where this is not too abundant). The leaf rosettes of daisy, dandelion, etc., and the foliage of many trees furnish examples.

The leaf blade may consist of one continuous undivided surface, and is then described as *simple*, or it may be cut up into a number of lobes connected with one another by an undivided portion, when it is said to be *lobed*. Or it may be completely segmented into a number of separate leaflets, when it is called *compound*.

Arrangement of Veins

The manner in which the veins are arranged in a leaf blade is called the *venation*. In the wallflower each leaf has a prominent main vein or midrib, giving off lateral veins, which by continued branching form a network throughout the lamina. Such *reticulate* venation is characteristic of the leaves of most dicotyledons, though differences occur in the way the prominent veins are distributed.

Most monocotyledons have a different type of venation ; the blade is traversed by several veins of equal size running side by side and connected by small cross-branches. This venation is described as *parallel*. The leaf blade is bounded by a cuticle which is composed of solid fats and prevents evaporation of water. In plants such as holly and laurel there is a very thick cuticle. The cuticle does not cover the *stomata*, through which gaseous exchange takes place. Under the cuticle is the epidermis, which may have thick outer walls for extra protection.

RETICULATE VENATION. In the poplar leaf, typical of most dicotyledons, continuous branching from the midrib forms a network.

Stomata

The leaf stalk closely resembles the stem in structure. As the blade is horizontally expanded, it is possible to distinguish between the upper and the lower epidermis. The latter is usually perforated by a large number of minute holes, by means of which the interior of the leaf is placed in communication with the exterior. The upper epidermis has few or no pores. Each pore is known as a *stoma* (Gk. *stoma*, a mouth), and is bounded by two guard cells, which are capable of altering their shape so as to vary the size of the slit between them. Stomata also occur in the epidermis of young stems, and are not necessarily continued to the undersides of leaves. In the floating leaf of a water-lily (*Nymphaea*), for example, they occur only in the upper epidermis, which is next to the air ; in vertical leaves, such as those of the iris, they are fairly equally distributed on both sides.

Between the two layers of epidermis there is a mass of green cellular tissue, divisible into an upper layer of cells (*palisade* layer) elongated at right angles to the surface, and a lower (*spongy* layer), which shows no such regularity.

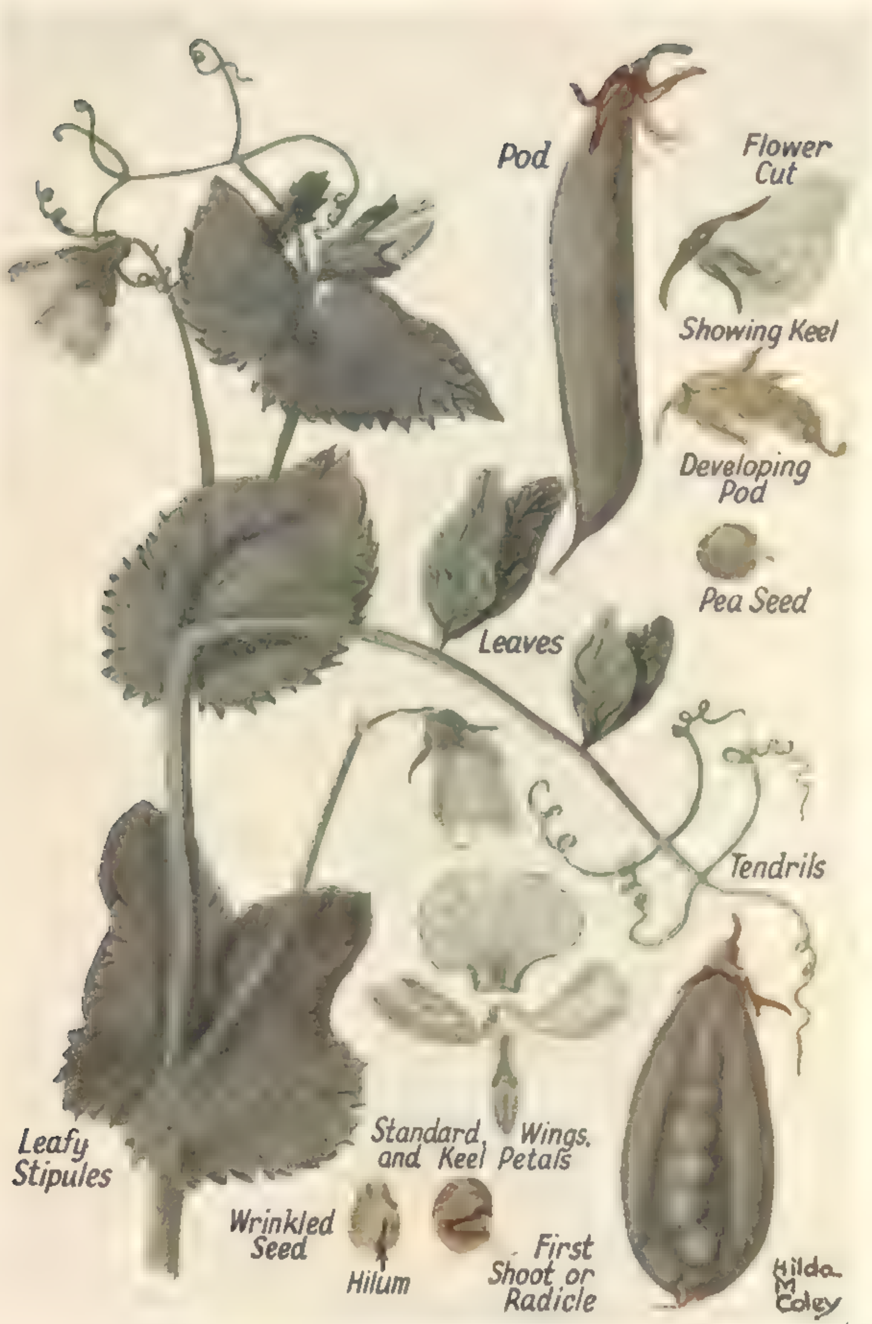
Between the cells of the spongy layer are air spaces, which form part of a continuous system of narrow chinks and passages by which the plant is traversed. The existence of this continuous set of air passages can be demonstrated by immersing the blade of a thick-stalked leaf in water, placing the cut end of the stalk in the mouth, and blowing into it as hard as possible. Minute bubbles of air will be seen to come off from definite points on the leaf blade, having passed through the stomata. The cells of both palisade and spongy layers contain *chloroplasts*—rounded particles of protoplasm, which contain chlorophyll.

Each stoma opens directly into a large air space amongst the spongy tissue of the leaf, and it is through

LIFE HISTORY OF WHEAT. The flowers of wheat are arranged in the ears in groups called spikelets. In a breeze they shake out their pollen which, falling on another flower whose stigma and hairy style are in the right condition, adheres to the minute ovary. The male and female nuclei fuse, and the fertile seeds swell until the whole ear is packed with golden grain.

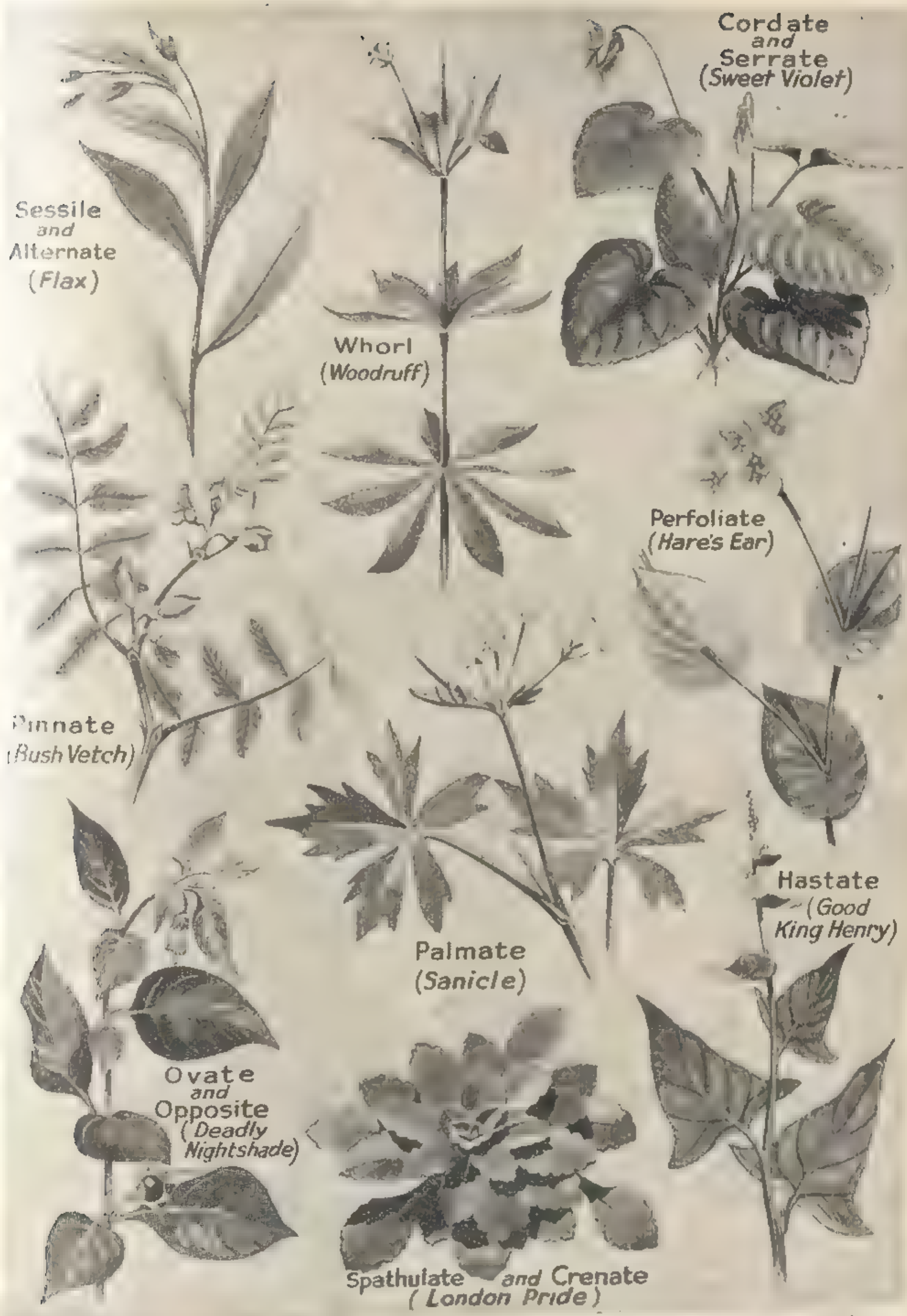
Drawing by Hilda Coley

To face page 1536, Vol. 3



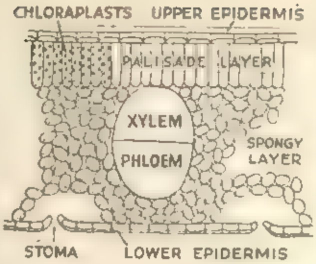
LIFE HISTORY OF THE GARDEN PEA. On one side of the green pea seed there is a line like a scar. This is the "hilum" and at one end of that is the micropyle, or "little gate." In this the fertilizing pollen enters to develop the seed egg in the ovary into a true living seed. This will, in its turn, produce a pea plant, bearing leaves, bracts, flowers, and tendrils, and finally pods. The ovaries are fertilized by insects transferring pollen from the flowers.

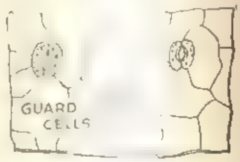
Drawing by Hilda Coley



LEAVES OF VARYING SHAPE AND SIZE. Some of the principal types are represented. Sessile: leaves borne direct on the stem. Perfoliate: leaf surrounds stem. Cordate: heart-shaped. Serrate: with toothed edge. Crenate: with notched edge. Pinnate: the midrib bears leaflets up both sides. Palmate: cut deeply into lobes. Hastate: having two ear-like projections at base. Ovate: pointed, with broad base. Spathulate: spoon-shaped.

these pores that all gaseous exchange between the plant tissues and outer air takes place—that is, the loss of water vapour, the entry of carbon dioxide, and the liberation of oxygen in the process of sugar and starch formation ; and the entry of oxygen and loss of carbon dioxide in the process of breathing. The action of the guard cells regulates the opening and closing of the stomatal pore. The factors which control this action are many and varied, the most important being the constitution of the cell contents.





LEAF STRUCTURE. Left, section showing upper and lower epidermis and cellular tissue enclosed. Below, guard cells.

If insoluble starches collect in the guard cells during photosynthesis, then water is lost to the surrounding cells, the guard cells lose their turgidity, and the pore closes. When the starches are removed and only soluble sugars remain, the guard cells take in water from surrounding cells, become turgid, and the pore opens. If a plant starts to wilt, the stomata close, and by this expedient the plant guards against further loss of water vapour.

Energy Absorbed from the Sun

The veins of the leaf blade are its vascular bundles, consisting of xylem above and phloem below. These bundles are so arranged as to prevent the leaf from tearing easily, and they guard against the collapse of the delicate green cellular tissue by keeping the two layers of epidermis well apart.

The leaves manufacture food from the carbon dioxide of the air, using energy absorbed from the sun by the green chloroplasts which are in many of the cells. The palisade cells, filled with chloroplasts, lie just below the upper epidermis and therefore receive the best illumination, at the same time being adequately protected by the thick-walled epidermis. Carbon dioxide enters through the stomata and circulates freely through the air passages. Water and mineral salts, necessary for food manufacture, are brought up from roots in the wood vessels which continue into the finest veins of the leaf, and the elaborated foods are conveyed to other parts of the plant through the cells of the phloem.

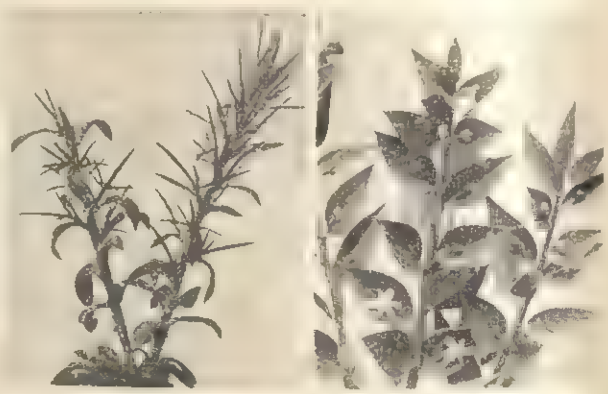
LESSON 11

The Plant and its Environment

The struggle for existence between plant species leads to specialisation, and each species tends to occupy one habitat or *ecological niche* for which it is most fitted by its modifications. These modifications make it possible to group plants in several major classes, according to what is called *life form*. This classification was introduced by a French botanist, Raunkiaer, in 1910.

The classes include Therophytes, annuals which live through the unfavourable season as seeds ; Phanerophytes, trees and shrubs bearing buds on aerial branches ; Chamaephytes, with buds on, or just above, the soil surface, e.g. milkwort (*Polygala vulgaris*) ; Hemicryptophytes, plants with buds at soil level, e.g. dog's mercury (*Mercurialis perennis*) ; Geophytes, with subterranean buds, including most plants with bulbs or tubers ; Helophytes, or marsh plants ; and Hydrophytes, or water plants.

Raunkiaer showed that the life form predominating in any region in the world depended mainly on the climate. Thus, in the arid desert of Nevada many of the species are Therophytes or small Phanerophytes. In a climate with a cold winter such as Labrador, Chamaephytes, Hemicryptophytes, and Geophytes are predominant. In the moist tropics Phanerophytes



MODIFIED STEMS. Left, gorse prickles—branches adapted to form protective structures. Right, butcher's broom, which produces flattened leaf-like branches with greenish flowers that eventually produce berries.

predominate. Life form can therefore be considered as an adaptation to climatic environment.

Functions of a Root

The range of modification of the main plant organs will now be considered. The functions of a typical root are to anchor the plant in the ground and to absorb from the soil water and mineral salts in solution. Some trees, such as the oak, are anchored by a long tap root; others, such as the pine, have a very shallow, spreading root system. Some plants of the Nebraska prairie send their roots 20 feet into the ground.

PLANTS WITH TWINING STEMS. Left, bindweed or convolvulus, which, like most twiners, moves in the anti-clockwise direction. Right, the hop plant, in which the movement is clockwise.

Roots may be modified to perform other functions. The swollen tap root of the carrot stores food. The contractile roots of the crocus pull the corm lower into the ground. Some tropical orchids which grow on the trunks of trees have roots which hang down into the air, and these aerial roots have a sponge-like outer tissue which absorbs moisture from the atmosphere; plants such as these, which grow on other plants but gain therefrom only support and no food substances, are called *epiphytes*. The aerial roots may in some instances contain chlorophyll and be modified to perform photosynthesis, the basic process of food manufacture. Mangrove (*Rhizophora*) trees, which grow in tropical sea-swamps, have buttress roots—knee-shaped roots growing out from the lower stem nodes—which support the stem against wave action and shifting sands and mud.

Functions of a Stem

A typical stem has two main functions. It has the physiological function of allowing the passage of water and dissolved substances between the roots and the leaves and flowers and fruits. It also has the mechanical function of bearing the leaves and flowers and fruits in an erect position where the leaves can get maximum sunlight, where the flowers can be pollinated and the fruit ripened and dispersed. These two functions are normally performed by two different tissues in the same stem. The stem may develop hard woody tissue by a process called *secondary thickening*, which assists its mechanical function. The problem of mechanical support is met in a different way by climbing plants.

Stem and stem branches may be modified to perform other functions. The thin green stem branches of broom (*Cytisus*) are the main assimilating organs, and there photosynthesis takes place. The leaf-like *cladodes* of butcher's broom (*Ruscus aculeatus*) are flattened stem branches, as indicated by the position of the flower in the centre of the cladode. In some species, e.g. furze or gorse (*Ulex*), stems are modified to spines and have a protective function.

In couch grass or twitch (*Agropyrum repens*) the underground rhizomes are a means of vegetative reproduction. A potato is a stem tuber modified for reproduction and food storage and bearing branches on its surface. The bulbs of tulip and daffodil are stems modified for perennation and food storage.

The typical function of a leaf is photosynthesis, and most leaves are thin flat expanses, absorbing the maximum sunlight. Their modifications include spines, as in the holly and in many cacti. Such spines protect the plant against grazing animals, as do the sting glands of the nettle leaf. The leaves of cacti and other succulent plants are modified to hold water and other storage products, and are organs of vegetative reproduction. Bud scales are leaves specially adapted to the protection of the bud.

CLIMBING BY TENDRILS. In some plants, for example, Virginia creeper (left), the tips of the tendrils apply themselves to solid surfaces and then form sucker-like organs which make a close union. A bryony tendril (right) is like an elongated spiral spring, with tremendous holding powers.

An alternative way by which the leaves may be raised up to the light is shown by climbing plants. In a simple example such as ivy (*Hedera*) the main modification is the development of adventitious roots along the stem, these enabling it to cling to a tree or other support.

Another class of climbing plant is the twiners. Here the stem twines around some support. The tip of the stem of most twiners moves in the anti-clockwise direction, e.g. bindweed (*Convolvulus*) and the runner bean (*Phaseolus multiflorus*). In some, e.g. hop (*Humulus lupulus*) and honeysuckle (*Lonicera*), the twining movement is clockwise.

Frequently the climbing is by *tendrils*. In its simplest form the tendril is an organ unaltered apart from its direction of growth. In *Clematis* species the leaflet stalks twist themselves around any firm support. The leaf stalks of the ordinary nasturtium (*Tropaeolum*) behave in the same way. More often the leaves, as in some species of pea (*Pisum*); or some of the leaflets, as in most pea and vetch (*Vicia*) species; or the stipules, as in *Smilax*; or a whole branch, as in the vines, are modified to form narrow spiralling tendrils. In some plants, e.g. the clinging Virginia creeper (*Ampelopsis veitchii*), the tips of the tendrils, instead of twining around the support, apply themselves to solid surfaces and then form sucker-like organs which make an extremely close union.

PRACTICAL WORK

Examine several species of climbing plant: notice the morphological nature of the organs modified. Examine also the cladodes of butcher's broom and determine that the cladodes are stems

LESSON 12

How Plants are Nourished From the Soil

WATER enters a plant's roots by way of root hairs. Examined under a microscope, each root hair is seen to be a slender tubular outgrowth of the wall of an epidermal cell. The cell wall is very delicate and is lined throughout its length with protoplasm: a vacuole extends throughout the cell, and a nucleus is usually present about the middle or end of the hair.

The root-hair region corresponds in position with the region of formation of xylem and phloem in the internal structure of the root, and is constantly being renewed as the lengthening roots penetrate new regions of the soil. The water which enters via the root hairs is then conducted up to the rest of the plant by the newly-formed conducting elements of the xylem.

Absorption and Transpiration

The underground system of a land plant, such as an oak tree (*Quercus*), occupies a large area of soil, with the finest roots disposed around the limits of the area. These fine roots, clothed with root hairs, present a very large surface in close contact with the soil, in the same way as the flat expanded leaves borne by the stem present a large surface in contact with the air. These large surfaces make for great efficiency in absorption of water from the soil and in evaporation of water into the air. This latter process is called *transpiration*, and the stream of water through the plant is called the *transpiration stream*.

Ordinary soil consists of particles of various sizes to which films of water cling, and this water constitutes a large part of the food of the plant. It contains small quantities of various mineral matters in very dilute solution. Water is of the first importance, for not only is it abundantly present as such in the living plant, but it also supplies two chemical elements—hydrogen and oxygen—which enter into the composition of protoplasm and are also essential components of substances which are the products of vital activity, such as wood and starch.

The Root at Work

The plant will not grow and thrive if its roots are supplied with nothing but pure water. There must also be small quantities of simple mineral compounds containing other elements, i.e. nitrogen, sulphur, phosphorus, potassium, calcium, magnesium, and iron. At least the first three of these help to build up protoplasm, and are contained in most proteins. Minute quantities of trace elements are also essential for healthy growth, these including copper, manganese, iodine, and boron.

Species differ considerably as to the exact composition of the food they require, which is one reason why it is desirable for farmers and gardeners to adopt a rotation of crops, instead of growing one kind of plant continuously in the same area. Because a given soil may from the first be deficient in one or other of the essential plant-foods or may have become exhausted of its supplies, the necessity for manuring arises.

A minute quantity of cell sap diffuses out of the root. This is slightly acid, and it helps to dissolve the particles of the soil, i.e. it assists in the preparation of more food. This is facilitated by the fact that the root hairs come into very close contact with the particles in question. By placing a polished slab of limestone in the

bottom of a flowerpot in which a plant is growing, an illustration of this process can be observed; after a time the roots will spread over and corrode the limestone, etching their outline upon it.

Soil Solutions

Oxygen must be available to roots in order that they may respire. Respiration, a characteristic of all living protoplasm, provides the energy used by the roots in taking up mineral salts from the soil solution. A waterlogged soil is injurious to the life of the plant, as no air spaces remain between the soil particles.

The root hairs take in the dilute solution from the surrounding soil by means of the process called *osmosis*. The protoplasmic lining of the root-hair cell constitutes a *semi-permeable membrane*, i.e. a membrane that allows certain substances to pass through it but prevents others from doing so. In root absorption the soil solution is on one side of this membrane, the cell sap on the other. The cell sap contains, in addition to a weak solution of mineral salts, sugar and other dissolved organic substances. These, through osmotic pressure, cause passage of water through the semi-permeable membrane from more dilute soil solution into the more concentrated cell sap. In this way water is taken up until the root hairs become distended (turgid) with water.

Ascending Water

The water passes, by osmosis, from the root hairs through the cells of the cortex of the root, all of which become turgid and in this distended condition exert considerable pressure on their neighbours. This is called *root pressure*, and it is responsible for driving the water into the conducting vessels of the wood, to which it cannot pass by osmosis because the wood cells are dead and empty.

The tallest trees of the world—for example, the eucalyptus trees of Australia—are about 300 feet high. Their roots are very long. Since it is only the young part of roots which absorbs water, the distance through which water is carried in such tall trees can scarcely be less than 200 yards. The question as to how water is pulled up such enormous heights in trees is not yet satisfactorily answered. Osmosis explains its passage through the cortex of the root and through the living green cells of the leaf, but not through the vessels of the wood. Experiments have proved that the water, by some force not fully understood, passes up through these cells. Transpiration from the leaves supplies considerable " pull," and root pressure may assist indirectly in helping to make good the reserve of water in the vessels of the root.

Copious Transpiration

A broad-leaved herbaceous plant, e.g. a sunflower, gives off a pint or more of water on a hot sunny day; a big beech tree transpires very nearly 100 gallons. It has been calculated that a well-grown oak tree gives off during the five months of its active annual life more than 200 tons of water, and that wheat transpires in its season of growth at the rate of 1,000 tons per acre.

PRACTICAL WORK

The course of water through a plant can be followed by placing a white flower (e.g. a lily) in red ink diluted with water: in a few hours the veins of the flower are coloured red. The same experiment can be carried out with any woody shoot; when this is cut transversely, or longitudinally, the red solution is seen to be in the vascular bundles.

An easy experiment to prove that transpiration takes place from the *under* side of a leaf, where the stomata are situated, can be carried out with cobalt paper; blue when dry, the paper turns pink on being moistened. A piece is placed on a glass plate and covered with a dry leaf under side downward. A second piece of cobalt paper is then put on the upper side of the leaf and a second glass plate over all. After exposure for a short time to sunlight it will be found that the paper which has been in contact with the *under* side of the leaf has become pink, while the other piece remains blue.

LESSON 13

The Plant's Food Factory

CHEMICAL analyses of different parts of plants—seeds, stems, tubers, and leaves—show that the same chemical elements are contained in varying proportions in these diverse tissues. They are: carbon, oxygen, hydrogen, nitrogen, and certain mineral elements, of which the chief are sulphur, magnesium, potassium, calcium, phosphorus, and iron. These exist in the plant not in a free state but in a combined form and all, with the exception of carbon, are obtained by the plant from the soil, through the medium of the roots.

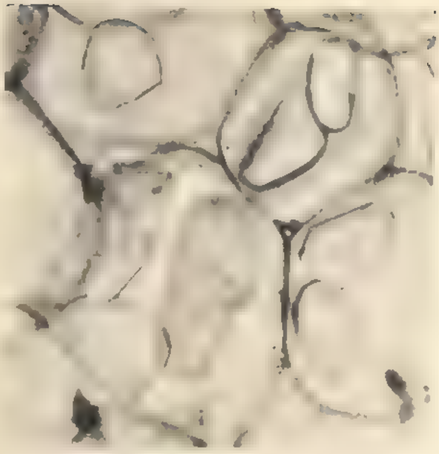
The following solution contains the inorganic elements commonly present in soil:

Distilled water	1,000 cubic centimetres
Potassium nitrate	0·25 grams
Calcium nitrate	1·0 grams
Magnesium sulphate	0·25 grams
Acid potassium phosphate	0·25 grams
Ferric chloride	a trace

This is a normal water-colour solution, and in it plants may be grown up to the flowering and fruiting stages, either in the solution alone or in quartz sand watered with the solution. To show the importance of the various constituents,

plants are grown in incomplete culture solutions from each of which one of the six mineral elements is omitted. The plants grown in the incomplete solutions not only look unhealthy, but die much sooner than plants given complete nutrient solution. Each of the six elements contained in the water-culture is therefore essential; three others commonly present in plants but not essential are sodium, chlorine, and silicon.

Actual amounts necessary for healthy growth differ in the case of different elements. A relatively large quantity of a suitable nitrogen compound must be supplied, but a mere trace of an iron compound is sufficient. Without iron the leaves develop no chlorophyll; they remain a sickly yellow colour and are said to be *chlorotic*. If painted with a weak solution of an iron salt, they turn green and the plant lives. The white patches of variegated leaves do not respond in this way; variegation is due not to lack of iron, but to an internal hereditary cause, which results in undeveloped chloroplasts.



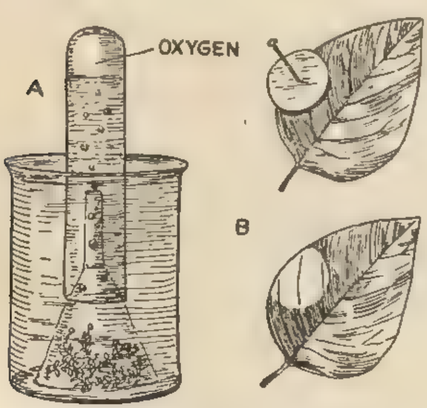
STARCH FORMATION. Photomicrograph (× 400) showing grains of starch contained in the cells of an orchid plant.

Photosynthesis

Nitrogen, phosphorus, sulphur, and potassium all enter into the composition of proteins or their derivatives, which form the chief part of the living matter, protoplasm. Magnesium is present in chlorophyll; iron is necessary for chlorophyll formation; and calcium enters into the composition of cell walls.

The basic process by which plants manufacture proteins and subsequently protoplasm from the elements previously listed is called photosynthesis. During this process, energy in the form of light from the sun is used in the presence of chlorophyll in the plant cell to harness carbon dioxide from the atmosphere to various elements which are in the cell sap, forming chemical compounds. In all there are five requisites: protoplasm, light, chlorophyll, carbon dioxide, and mineral salts (chemical elements).

Carbon forms about one-half of the dry weight of a plant. The only source of carbon available to the plant is from the carbon dioxide of the atmosphere. This passes in through the stomata, dissolves in the small quantity of water in the stomatal cavity, and passes into the cells as a solution of carbonic acid g



PLANT CHEMISTRY. Diagram illustrating (A) demonstration that oxygen is given off during photosynthesis; (B) experiment showing necessity of sunlight for starch formation.

Carbohydrates

The first products are known to be carbohydrates, usually sugars and starches. The process is very complicated and still incompletely understood, so general principles only are given here. The carbon dioxide and water which the leaf receives from the root ultimately form an organic substance composed of carbon, hydrogen, and oxygen, i.e. a carbohydrate. The first visible product is starch, the grains of which can be seen under a microscope

A chemical test will show the presence of starch. A leaf which has been exposed to the sun for some time and has been put into boiling water to kill it, and then soaked in alcohol (methylated spirit), loses its green colour. If the leaf is then put into a solution of iodine in potassium iodide, the leaf will rapidly become dark blue. This is a certain test for starch. The decolorising allows the blue colour to be seen more easily.

Oxygen as a Waste Product

During photosynthesis a certain amount of oxygen is given off as a waste product. Some of it is used in the respiratory process, but much of it passes out through the stomatal pores into the atmosphere. A convenient way of demonstrating this is to put a water-plant, such as the Canadian pond weed (*Elodea canadensis*), into a glass vessel containing tap water. The plant is then covered with a funnel and an inverted test-tube full of water is placed over the

stem of the funnel. If the whole arrangement is exposed to light, small bubbles of gas will be seen to rise from the plant, gradually filling the test-tube. By applying the usual tests it is easy to show that the gas is oxygen. If, for example, a glowing splint of wood is put into the test-tube, it will at once be rekindled.

Variegated leaves are useful for showing that chlorophyll is essential for starch formation. If variegated leaves are exposed to sunlight and then submitted to the iodine test, only the parts which were green will turn blue owing to the presence of starch. No starch is formed in the white part.

Sunlight and Starch

To show that sunlight is necessary for starch formation, it is necessary to use a plant whose leaves are devoid of starch at the beginning of the experiment. If a plant is kept in the dark for a day and its leaves treated for starch, none will be found present. Pieces of cork or opaque paper should then be pinned above and below parts of some of the starch-free leaves and exposed to sunlight. On testing for starch, the parts which were so covered give no reaction, while those exposed to sunlight are deep blue.

The necessity of carbon dioxide can be shown by placing a starch-free potted green plant under a glass bell-jar or similar cover, together with a dish containing caustic potash. This chemical substance absorbs carbon dioxide from the air, leaving none for the green leaves. If the apparatus is exposed to sunlight for some hours and the leaves are then tested for starch, a negative result will be obtained.

In addition to chlorophyll, sunlight, and carbon dioxide, there must be a suitable temperature and adequate water supply for the plant to bring about carbon assimilation. It is the chloroplasts of the palisade cells of the leaf which are chiefly concerned with the photosynthesis of carbohydrates in the higher plants.

It it were possible to destroy all existing chlorophyll and to prevent the formation of more, the organic world would soon be brought to a standstill and life would become extinct. For no more plant-substance could be built up from non-living material, and the vegetable kingdom would therefore soon cease to be ; as animals directly or indirectly depend on plants for food, they too would soon die out.

Apart from this, the breathing of plants and animals and the processes of combustion continually exhaust the oxygen of the air and increase the proportion of carbon dioxide. The composition of the atmosphere is, in fact, kept normal only by the utilisation of carbon dioxide as food by green plants, with concomitant liberation of oxygen. As previously mentioned, this process is absolutely dependent upon the presence of chlorophyll.

Etiolation

The formation of chlorophyll depends on a trace of iron in the nutrient solution and also on light. If seedlings are grown in the dark, the shoots which develop are pale yellow in colour, the stems greatly elongated and weak, and the leaves much smaller than the normal. Plants showing these changes caused by growth in the dark are said to be *etiolated*. If a large stone or plank is placed on a lawn for a few days and then removed, the grass there is seen to be almost white. The etiolated grass recovers its green colour on exposure to light.

PRACTICAL WORK

To determine the conditions necessary for starch formation in green leaves, experiment as described in the text. To notice the characteristics of etiolated plants, grow seedlings, e.g. beans, in the dark.

LESSON 14

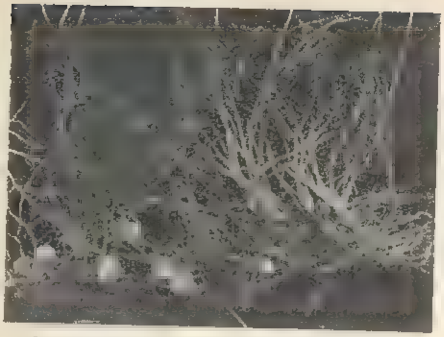
The Nitrogen Cycle

The photosynthesis of carbohydrates is only one of the synthetic processes carried out by the green plant. It must also convert nitrogen, sulphur, phosporus, potash, and other elements from the simple substances in which they occur in the soil, into the complex organic materials which form part of the living substance of the plant. Some of the chemical changes which go on in the plant in these synthetic processes retain their mysteries ; they have still to be explained. Here, the way in which the plant obtains its supply of nitrogen will be considered.

Nitrogen is one of the essential constituents of proteins, which form a great part of protoplasm. The majority of green plants obtain all their supply of nitrogen from the soil. The nitrogen is combined in the soil with other elements in the form of soluble salts called nitrates. Even though nitrogen is one of the gases present in the free state in the atmosphere, the higher plants do not utilise it. The soluble nitrates which are absorbed by the roots are conducted up to the leaves, and they accumulate there until, on exposure of the leaves to light, they disappear and are replaced by other nitrogenous compounds. This change depends in some way on the activity of the green cells ; it does not take place in the colourless cells of variegated leaves. Thus the animal world, which lacks the power of bringing about this change, depends ultimately on the green

plant for its nitrogenous foods as well as for its carbohydrates.

The change which takes place in the green leaves results in combination of the nitrogen with carbon, oxygen, and hydrogen to form substances called amino-acids, and by the chemical union of several of these acids the proteins are formed. Chemically, the proteins are extremely complex substances; some of the simpler forms are used as food material by the growing plant, while the more complex actually enter the composition of protoplasm itself.



STORAGE OF NITROGEN COMPOUNDS. The root nodules are composed of cells which contain nitrogen-fixing bacteria. The compounds thus built up benefit both plant and soil.

Some of the lowliest members of the plant kingdom play an important part in rendering nitrogen available to green plants and hence to the animal kingdom; these inconspicuous plants are certain species of bacteria. When dead animal or plant bodies undergo decay, the complex proteins contained in them are broken down to the simpler amino-acids, from which, by the activity of bacteria, ammonium compounds are set free. These compounds are washed into the soil by rain, and there, by the action of other bacteria, they are chemically changed into a fresh supply of soluble nitrates, which higher plants can absorb through their roots. By means of these important bacteria, therefore, the *nitrogen cycle* of nature is kept going.

Denitrifying Bacteria

Not all soil bacteria are so helpful to the green plant. There are other bacteria species which act on the soluble nitrates and convert them chemically to substances which the green plant cannot utilise, such as ammonia and free nitrogen—that is, they diminish the nitrate content of the soil. These harmful bacteria are active only in a soil which is badly aerated, such as waterlogged ground. They are called the *denitrifying* bacteria, as distinct from the useful ones, which are called *nitrifying* bacteria.

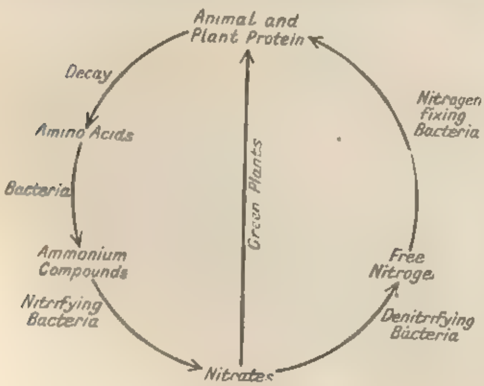


DIAGRAM OF THE NITROGEN CYCLE IN NATURE.

Some kinds of bacteria, also present in the soil, can make direct use of the nitrogen in the air, which green plants are not able to do. These bacteria take in the free gaseous nitrogen and build it up into the protoplasm of their cells. Some of these so-called nitrogen-fixing bacteria are of great importance to soil fertility, because of their association with the roots of all members of the pea and bean family (Leguminosae). A pea plant pulled up and examined will have numerous swellings (nodules) on its roots. Microscopic examination of these nodules shows that they consist of cells similar to those in the cortex of the root, but containing large numbers of living bacteria.

Symbiosis

If a plant of the pea family is grown in water-culture solution, nodules do not develop; the presence of a small quantity of ordinary soil is necessary for infection by the bacteria, which enter the root through the root hairs. Both the pea plant and the bacteria derive benefit from this association. The bacteria obtain their supplies of carbohydrate from the plant in whose cells they are established, and in return pass into the plant the complex nitrogenous compounds which they are able to build up. Such an association for mutual benefit is called *symbiosis*.

Members of the Leguminosae family thus show a peculiar relation to nitrogen. As long as their roots develop nodules, they are able to thrive in a soil which may lack nitrates. Although the discovery of the nodule bacteria and their power of fixing free nitrogen dates from fairly recently, the beneficial results to the soil by the growing of peas, vetches, clovers, etc., has been known for centuries. When land that has carried such crops is ploughed, the nitrogen compounds contained in the roots are distributed throughout the soil and are available for the next crop. Nitrogen fixation occurs also in root nodules of the alder (*Alnus glutinosa*) and the bog myrtle (*Myrica gale*), but here the organism concerned may be a fungus and not a bacterium.

Bacteria also play a part in the circulation of carbon. When dead vegetable matter decays, certain bacteria and fungi are concerned with the breakdown of the carbohydrates, such as the cellulose which composes the cell walls and hence the whole plant framework. Such breakdown produces the *humus* of the soil and improves its water-holding capacity, its aeration, and general fertility. In the chemical processes which take place, carbon dioxide is set free and is thus available to living green plants for further carbohydrate synthesis.

Little is known of the origin of fats and oils in plants. There is no evidence that they are direct products of synthesis as are the carbohydrates and proteins. They are probably produced from previously formed carbohydrates, by chemical alteration.

LESSON 15

How Plants Obtain Energy

The building up of carbohydrates and nitrogenous food materials takes place in the leaves, but this food must be accessible to every living cell of the plant. The materials are often in an insoluble form, but these are made soluble by the activities of enzymes.

Activities of Enzymes

There are numbers of different enzymes present in living cells, and each is responsible for bringing about a particular change. They can effect in living cells at ordinary temperatures chemical changes which the chemist is not able to reproduce in the laboratory or only at a very high temperature. But enzymes can be extracted from plant tissues by various means, and purified, and will then perform in the test-tube the reactions they performed in the living plant cell.

Enzymes not only convert insoluble food material into soluble form for easy diffusion through the cellular tissues of the plant ; they also convert soluble substance back to the insoluble form if the food is being passed into a storage organ.

Constructive Uses of Food Materials

For example, in the potato plant, starch is formed in the green leaves in photosynthesis ; the enzyme called *diastase* converts the insoluble starch into soluble sugar, in which form it is conducted down the phloem of the vascular bundles into the underground tubers, where the diastase reconverts the sugar to insoluble starch, and in this form the food is stored.

By the actions of enzymes all food substances—carbohydrates, proteins, fats—are rendered available to the cells which will utilise them, transported thither by the phloem, and finally by diffusion from one cell to another in those tissues which are not specialised for conduction.

When food materials reach their destination, they are either used at once or stored for future requirements. There are two main uses to which they may be put immediately : construction of plant tissues and liberation of energy for the plant. From a microscopic embryo consisting of one cell there arises in due course a gigantic tree. All the mass of the tree's woody and other tissues is the product of constructive activity of the protoplasm. In order to grow, the plant must carry on chemical operations by means of which proteins and other food materials are incorporated with the protoplasm, thereby augmenting it.

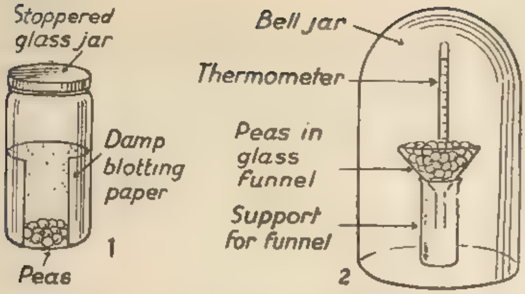
Accumulated Waste Products

Evidence of the varied constructional work of the protoplasm is furnished by the amount and diversity of the waste products which are found deposited in a plant as the result of its chemical activities. These accumulate because plants have no definite excretory system for getting rid of waste products. Some of the latter are of value to mankind as stimulants, drugs, and poisons (nicotine, caffeine, quinine, opium). Certain poisonous waste products are probably useful to the plant in protecting it from attacks by insects, or they may act as natural antiseptics and preserve the wood from decay.

The energy necessary for carrying on the plant's vital activities is derived from the breaking down of the food materials formed in photosynthesis. The process, which is called respiration, occurs in all living bodies, plant or animal. The main supply of the green plant's energy comes from the sugars it contains. The process involves intake of oxygen through the stomata, oxidation of the sugar molecules to water and carbon dioxide, release of the latent energy of the sugar molecules, and liberation (through the stomata) of the carbon dioxide.

Carbon Dioxide Given Off

In daylight it is not possible to demonstrate that green leaves give off carbon dioxide in breathing, because during the period of light this gas is utilised at once for photosynthesis ; but it can be done in darkness in the following manner. The experiment consists in standing a growing pot-plant in a jar or other glass vessel just large enough to contain it, and closing the mouth of the vessel with a greased glass plate. The apparatus is then placed in a dark



EXPERIMENTS WITH GERMINATING SEEDS. 1. Apparatus to show intake of oxygen and evolution of carbon dioxide in germinating peas. 2. Showing evolution of heat in respiration.

cupboard. The following morning, the air in the vessel is tested with a lighted taper, and some lime water is introduced. The taper is extinguished: the air does not support combustion, because the oxygen has been used up. The lime water turns milky, showing that carbon dioxide is present.

Germinating seeds respire very rapidly, and their intake of oxygen and evolution of carbon dioxide can be demonstrated in the light, because no photosynthesis is taking place. For this demonstration, some soaked peas are placed in a jar lined with damp blotting paper, and the jar is closed tightly with a stopper. After a few days the lighted taper and lime-water tests are applied; the results are similar to those in the preceding experiment.

Some of the energy which is released in respiration takes the form of heat. This can be shown by filling a funnel with germinating peas packed around the bulb of a thermometer; this will register a higher temperature (after a few hours) than a thermometer packed around with, say, cotton wool.

Anaerobic Respiration

Respiration entails a breaking down of carbohydrates and loss of substance from the plant; but in periods of light, assimilation is in excess of respiration. This can be shown by soaking two equal quantities (by weight) of peas and placing them aside to germinate, one lot in the dark, the other in the light. After about a fortnight the seedlings grown in the light will have gained weight (because of the assimilation of foods in photosynthesis), whereas those grown in the dark will have lost weight.

In the absence of oxygen a plant does not die immediately, but its normal activities are interrupted. In such circumstances plants carry on for a time a modified breathing process called *anaerobic respiration*, during which the food substances are not broken down as completely as in ordinary respiration. Carbon dioxide is evolved, as under ordinary conditions, and in addition alcohols are produced. These accumulate in the living cells and poison the plant, and it dies.

Among the bacteria which are responsible for some virulent diseases (e.g. anthrax, cholera, typhus) there are organisms which not only ordinarily exist without oxygen but to which the gas, except in minute quantities, is actually poisonous.

LESSON 16

Factors Governing Plant Growth

There are three distinct phases of growth: first, the formation of new cells by the division of pre-existing cells in the growing-points; second, the enlargement of the new cells; third, their modification to perform the different functions which they fulfil in the life of the plant. The second phase is the only one visible externally and it constitutes "growth" in the popular sense.

The growing-points, which are at the extreme tips of the shoots and immediately behind the caps of the roots, consist of numerous small cells which undergo repeated divisions. Some of the cells thus formed become gradually changed into mature cells; others retain their power of division and remain as the growing-point. Apart from growing-points at the tips of roots and stems, some plants—for example, grasses and pinks—have similar growing-point tissue situated along their mature stems near the nodes. These are *intercalary* growing-points. as distinct from the *apical* growing-points. The intercalary points help the plants to raise their shoots when they have been flattened down by rain or wind.

Rate of Growth

This restriction of growth to definite regions constitutes a marked point of difference between the growth of plants and that of higher animals. Another distinction is the continual formation of new organs from the growing-points of plants, whereas in animals the number of organs remains constant after the embryo state is reached.

The region of elongation in roots is restricted to a very short portion just behind the tip; in stems it is spread over a much greater length. In both, the rate of growth of different parts of the elongating zone is not uniform. The intervals near the tip of the root elongate very slightly. Passing farther from the apex, the

intervals become wider and wider, until they reach a maximum ; after this they show gradual decrease up to the mature zone, where no elongation has occurred. This gradual rise and fall in the rate of elongation is called the *grand period of growth* ; it is plainly seen in the growing plumules of seedlings.

Increase in length is greater in the dark than in the light, provided the temperature remains approximately uniform. This is illustrated by seedlings, or by potato tubers, which are allowed to grow in the dark. The plants become etiolated ; that is, they have no chlorophyll, and very small leaves but exceptionally long internodes. Some herbaceous plants have longer internodes when grown in shady places than when grown in the open.

Darkness limits photosynthesis and therefore the supply of organic building material. Thus growth is really increased by light, but elongation of plant cells is increased by darkness. The acceleration of elongation in darkness enables plants rapidly to reach the light. In plants growing from a rhizome or rootstock the absence of light causes great elongation of the petioles.

Phototropism

As well as influencing rate of growth, the intensity of light may also affect the extent of development of plant organs. The leaf buds which are entirely shaded in a large tree usually fail to develop, whereas the majority of those at the margin give rise to branches. Flowers are rarely produced by the yellow archangel or dead-nettle (*Lamium galeobdolon*) if the plant is growing in the deep shade of a wood, but flowering is profuse where there is greater light. In many plants leaf development is very noticeably affected by light intensity, *sun leaves* being smaller and thicker than *shade leaves*.

Light also has an effect on the form or shape of a plant. If an erect plant is placed near a window so that it receives one-sided illumination, the plant will bend towards the light. If the plant is then turned through a right angle, it will endeavour to bend back to the light. Thus the erect growth of a stem is dependent on its being exposed to equal illumination on all sides. The influence of light on the direction of growth of a plant is spoken of as *phototropism*, and stems which curve towards the light are *positively phototropic*. The aerial roots of the ivy (*Hedera*) are common examples of *negative* phototropism, always growing away from the light.

Influences other than light play a part in determining the erect position of a shoot. If a pot-plant is placed horizontally in the dark for some hours, the tip of the shoot gradually curves upwards till it again assumes a vertical direction. Evidently some agency other than light is responsible, and this it is which in stems causes growth in a direction opposite to that in which gravity acts.

Influence of Gravity and Moisture

The influence of the force of gravity on the direction of the growth of the plant is called *geotropism*, and stems are *negatively geotropic*. Main roots, which grow downwards, are *positively geotropic*. Lateral roots, which grow obliquely from the main axis, are not so strongly influenced by gravity, and this force has no influence on the finer ramifications of the root system. These latter spread through the soil in search of water. The influence of one-sided moisture on the growth of roots is called *hydrotropism*, and roots which grow towards the moisture are *positively hydrotropic*. *Negative* hydrotropism is a rare phenomenon, and has not been observed in the higher plants.

Effect of Temperature

Temperature has a marked effect on growth, as on all other vital processes. Plants grow slowly at a low temperature, the rate increasing as the temperature rises, until a degree of heat is attained which is unfavourable to further existence. Apart from light and temperature,

EXAMPLE OF GEOTROPISM. A feature of plant growth is the fact that whatever the position of a seed the roots grow downwards, while the stems grow upwards. This geotropic tendency of the roots is illustrated in these photographs of the radicles of beans that have been turned about when germinating, curving over towards the ground.

the most important factor in growth is the supply of oxygen for respiration and water containing mineral salts.

Injury to any living tissue of a plant causes the surrounding cells to undergo rapid division, and thus the wounded surface becomes covered with a new growth. Injury due to insects or fungus pests is a similar stimulation to growth, and leads to the formation of galls. "Witches' brooms," sometimes seen on birch, cherry, and elm, are the result of excessive branching caused by the presence of minute fungi.

Growth Hormones or Auxins

If a seedling with plumule and radicle marked with ink is placed horizontally in a dark box and examined about 24 hours later, it will be seen that geotropic curvature has occurred in the region of maximum elongation. But the region of curvature is not the region in which the influence of gravity is first felt. The *perceptive* region is at the extreme tip, and in experiments with roots whose tips are cut off no curvature results.

The stimulus which is received at the tip of a root or shoot must therefore be transferred by some means to the zone of elongation. In phototropic curvature the bending also takes place in the region of elongation, though the perceptive power is localised elsewhere. There must be conduction of the stimulus to the region where bending occurs, and recent investigation has shown the existence in the plant of minute traces of growth hormones or *auxins*, controlling growth.

These are formed mainly in the perceptive region and translocated to the responding region. When the stem bends towards the light, it does so because there is a greater concentration of auxin on the side of the stem away from the light. This side elongates faster than the side near to the light, causing the curvature of the stem. It will be remembered that elongation

Fig. 1

Fig. 2

ROOT GROWTH. Fig. 1. Experiment to show the region of geotropic curvature in the radicle and plumule of a pea seedling placed horizontally. Fig. 2. Experiment showing hydrotropism in roots.

takes place faster in the dark. One single auxin, such as indoleacetic acid, may have entirely different effects at differing concentrations and on different plant organs. Almost all the tropisms mentioned can be explained in terms of varying concentrations of auxins. The use of auxins and other plant hormones is now common horticultural practice—for example, to ensure the rooting of cuttings.

PRACTICAL WORK

Place a pot-plant in a window and notice curvature of stem, or stems, towards the light. Soak some pea or bean seeds, allow them to germinate until the radicles have emerged, then place them in damp sawdust with the radicles pointing in different directions. After a few days, notice the downward curvature of all the radicles.

LESSON 17

Parasitic and Insectivorous Plants

Unlike the ordinary green plant which builds up its body from simple chemical compounds obtained directly from the soil, there are some species which (like members of the animal kingdom) are dependent upon elaborated food substances and can thrive only when these are available in the form of either living or dead organic material.

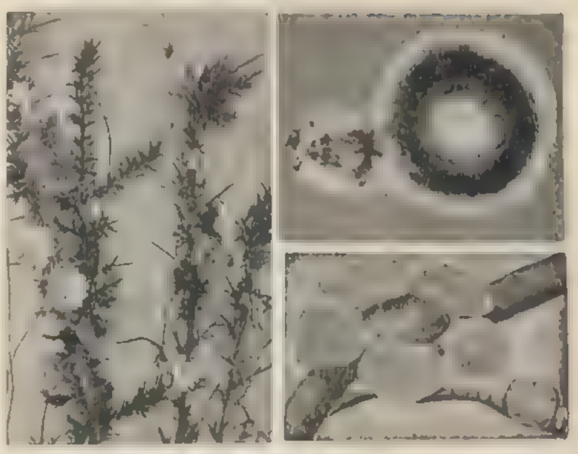
In such plants the leaves are always small and generally without chlorophyll. When some or all of its food is obtained from another living organism, the plant is called a *parasite*, and the organism upon which it feeds is the *host*. When the food is dead organic matter, the plant living on it is called a *saprophyte*.

Some parasites are only partly dependent upon their hosts, and these are described as *semi-parasites*. Their leaves are often green, and consequently the plants can grow independently, though in these conditions growth is not vigorous. Common examples are yellow rattle (*Rhinanthus crista-galli*) and eyebright (*Euphrasia officinalis*), which are parasitic on the roots of other plants, particularly grasses. The roots of the parasite are attached at certain points to those of the host by minute suckers, which appear as slight swellings. These penetrate into the roots of the host as far as the central conducting tissues, and absorb water and mineral salts. Mistletoe is a parasite which supports itself to some extent (*see* next page).

Complete parasites are rare among British flowering plants; the best examples are dodder

(*Cuscuta*), toothwort (*Lathraea squamaria*), and broom-rape (*Orobanche*). The dodder is parasitic chiefly on gorse and heather. The fine reddish-coloured stems of dodder entwine the stems of the host, penetrating it with small suckers through which nourishment is absorbed. The stems bear minute scale-like leaves containing very little, if any, chlorophyll, and pink flowers which produce an enormous amount of seed. The seedling's stem withers below its point of attachment to the host and it no longer has connection with the ground.

Toothwort and broom-rape are root parasites—that is, they attach themselves by suckers to the roots of a suitable host. The toothwort, an early spring plant with pinkish flowers on a pale brown shoot, usually grows at the foot of hazels or elms. The species of broom-rape, which are somewhat similar to toothwort in their overground organs, are usually parasitic on clover. In common with all parasites these plants show a much reduced leaf surface and extensive seed production. Because of reduced transpiring surface, parasites may receive too much water from their hosts, hence some species are provided with special water-exuding glands.

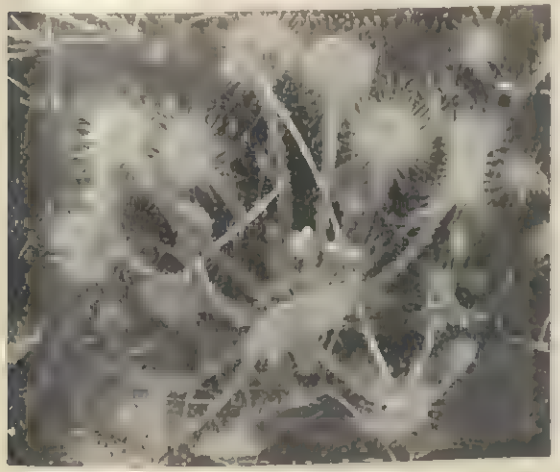


PARASITIC PLANTS. Left, dodder, parasitic on gorse. Top right, microphotograph of dodder on clover. Bottom right, gigantic flower of Rafflesia arnoldi, a tropical parasite which grows on vines.

Rafflesia and Mistletoe

The reduction of the vegetative parts is well seen in a small family of parasites occurring in tropical forests, some of them remarkable for their enormous flowers, which are the only conspicuous part of the plant. An example is *Rafflesia arnoldi*, which is native to Malaya, where it grows upon the roots of vines, and bears flowers over a yard in diameter.

Mistletoe (*Viscum album*) is a true parasite, which grows on several kinds of tree, but it resembles the semi-parasites in having abundant chlorophyll. It absorbs from its host only water and mineral salts. There is no movement of food substances from the mistletoe back to the host—apple, hawthorn, poplar, oak, etc.



SUNDEW SNARE. Here is seen the round-leaved sundew, with its long, sticky-tipped glandular hairs. When an insect happens to touch one of the hairs, it is held by the sticky fluid exuded. The leaf edges then curl inward and the insect is slowly digested by the plant.

Saprophytes

Saprophytes are rare among British flowering plants, the commonest being the bird's-nest orchis (*Neottia nidus-avis*), which grows on the humus in beech woods.

It is a familiar fact that organic substances such as bread, jam, and bruised fruit sometimes become covered with mould. These moulds are saprophytic fungi with thread-like bodies through which they obtain nourishment from decaying organic matter. Some fungi live as saprophytes on the humus of woodlands, where they abound in the autumn. Other fungi are parasites and cause serious diseases in plants of eonomic importance, and occasionally in animals. Bacteria are parasitic or saprophytic plants of simple structure.

All those higher plants which are saprophytes are unable to utilise the humus directly; they require the help of fungi which live intimately associated with the saprophyte. The surrounding or penetrating fungus is referred to as a *mycorrhizal* fungus, and it possibly helps in water absorption by the roots as well as in the taking up of organic matter from the humus. This association may often be of mutual benefit

INSECTIVOROUS PLANTS. Lower left, sundew. The leaves are covered with sticky-tipped hairs which hold alighting insects (see also illus. at foot of the previous page). Lower right, pitcher plant. The mid-rib of the leaf is prolonged and its end forms a pitcher containing a liquid which prepares entrapped insects for digestion by the plant. Top left, bladderwort, a submerged water-plant. Small insects entrapped in the bladders cannot escape. They die, and the plant absorbs the products of their decay. Top right, Venus's fly-trap.

to both organisms (symbiosis), such as the association of nitrogen-fixing bacteria with roots of members of the Leguminosae family.

Small Insects as Food

One other special method of nutrition is that of the insectivorous plants, which feed upon small insects. An example is the sundew (*Drosera rotundifolia*), which grows in peaty bogs. The plant consists of a small rosette of reddish leaves, which have relatively long petioles expanding into a round or oval blade. The upper surface and edges of the leaf blade are studded with reddish tentacles, whose swollen ends glisten with a sticky fluid. This attracts insects, which alight on the leaf and are held by the sticky substance. The more distant tentacles then curve in until they make contact with the insect's body; and digestive juices are exuded which convert the insect's softer parts into a soluble form for absorption by the plant. Later, only the undigested hard portions of the insect remain, and the sundew's tentacles are ready for another [illegible]

The butterwort (*Pinguicula vulgaris*), which grows in peaty bogs captures and digests insects in a similar manner—by means of numerous sticky glands on the upper surfaces of the flattened radical leaves. Another example is the bladderwort (*Utricularia*), a submerged water-plant which bears numerous small bladders on its deeply divided leaves. Each bladder has an aperture, closed by a trap-door controlled by a trigger mechanism. The contact of minute water-creatures sets the mechanism operating and causes an inrush of water which sweeps the creatures within. There they remain imprisoned, and die, and the plant slowly absorbs the products of their decay.

The Venus's fly-trap (*D[illegible]ea muscipula*) is a North American relative of the sundew. Each leaf [illegible] in two lobes, hinged at the mid-rib and fringed with spines. On the s[illegible] of each lobe are sensitive hairs [illegible] a small insect touches one of these, the lobes close over it and the marginal spines interlock, preventing escape. Fluid from the leaf glands dissolves the victim's tissues, from which the plant absorbs food.

In the pitcher plants (family Nepenthaceae), of which several species occur in warmer parts of the globe, the mid-rib of the leaf is prolonged and its end is modified to form a pitcher. The inner walls of the pitcher are very slippery and covered with scales which slant downwards; insects which venture within are unable to retain a foothold and fall into a liquid (excreted by glands on the inner walls) which prepares them for the plant to digest.

LESSON 18

Analysis of Flower and Fruit

The function of a flower is to provide for the formation of seeds which will perpetuate the individual plant. An idea of the nature and function of the different parts of a flower can be gained by considering a "flower pattern," which answers to the general plan of which all flowers are variations.

A flower is a greatly specialised shoot, in which the central stem bears structures crowded together and differing more or less from

ordinary foliage leaves in accordance with their different use. It is borne on a stalk which may also bear simplified and often scale-like leaves, or *bracts*. The central stem part, the *receptacle*, is very short, so that the different flower leaves which it bears are crowded together. These are of four kinds, arranged in two sets, (a) the *perianth* or *covering leaves* externally, and (b) the *reproductive leaves* internally. The two sets of leaves of which the perianth is composed are an external circlet or whorl of five *sepals*—collectively, the *calyx*—and an internal whorl of five *petals*—collectively called the *corolla*—alternating with the sepals.

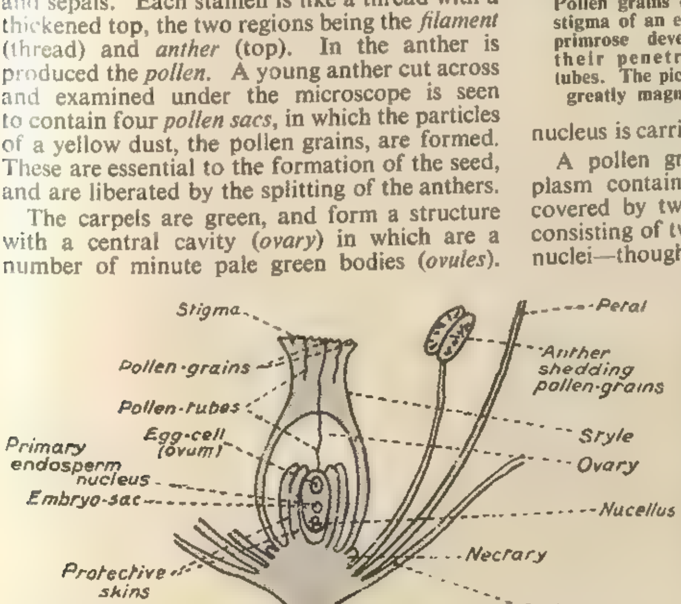
Sepals and Petals

The sepals are firm and green and more like foliage leaves than their associates. They protect the internal parts of the flower, and especially they afford protection in the bud. The petals are larger and more delicate in texture than the sepals, and they are brightly coloured. They help to protect the reproductive leaves, in which there are two sets of structures—two whorls of *stamens*, five in each whorl, and a whorl consisting of five *carpels*. The members of each whorl alternate with those of the preceding one.

Stamens and Carpels

The stamens, the male part of the flower, differ greatly in appearance from the petals and sepals. Each stamen is like a thread with a thickened top, the two regions being the *filament* (thread) and *anther* (top). In the anther is produced the *pollen*. A young anther cut across and examined under the microscope is seen to contain four *pollen sacs*, in which the particles of a yellow dust, the pollen grains, are formed. These are essential to the formation of the seed, and are liberated by the splitting of the anthers.

The carpels are green, and form a structure with a central cavity (*ovary*) in which are a number of minute pale green bodies (*ovules*). The ovules are potential seeds, providing they are fertilized by the pollen grains. The lower part of a carpel, which contains the ovules, is the ovary, and its narrow upper end is the *style*, on the top of which is a sticky patch, the *stigma*. The carpels are the female part of the flower, and collectively are called the *pistil*.



SECTION OF A SIMPLE FLOWER, in which only one carpel is present, containing a single ovule.

How Ovules Become Seeds

In order clearly to understand how ovules become seeds it will be best to leave the pattern flower, and consider a still simpler example where only one carpel is present, in which one ovule is contained. This ovule is covered by two protective skins which are imperfect at one place—the *micropyle* (Greek for " little gateway "). Within these skins is a cellular mass (*nucellus*), of which one cell near the micropyle has developed into a relatively conspicuous structure and is called the *embryo sac*. The young plant develops inside this and is nourished by the nucellus.

FERTILIZATION. Pollen grains on the stigma of an evening primrose developing their penetrating tubes. The picture is greatly magnified.

Part of the contents of this sac is the *egg apparatus*, a group of three small cells next to the micropyle. One of them, much larger than the others, is the *egg cell*, or *ovum* (the female *gamete*), from which the young plant originates when it is fertilized—i.e. when a male nucleus fuses with it. The nucleus is carried in the pollen grain.

A pollen grain is a small mass of protoplasm containing two *nuclei* when ripe, and covered by two skins. It can be regarded as consisting of two cells—as indicated by the two nuclei—though in the higher seed plants these are not separated from each other by a parting wall. The first step towards the attainment of fertilization is the transfer of pollen grains to the stigma. This is called *pollination*.

Fertilization Effected

When pollination takes place, the pollen grain germinates in the sticky fluid of the stigma and sends out a pollen tube, which grows down through the style and into the ovary, where its tip passes through the micropyle. Meanwhile both of the nuclei of the pollen

grain have passed into the pollen tube ; one nucleus here divides into two, which are the male nuclei or male gametes. The tip of the pollen tube is ruptured, and one male nucleus enters the ovum, fuses with the egg nucleus, and thus effects fertilization. The other male nucleus fuses with a nucleus (*primary endosperm* nucleus) which lies centrally in the embryo sac, and causes the development of *endosperm*, a nutritive tissue which nourishes the developing embryo. In some species this nutritive tissue is present in the ripe seeds, which are then described as albuminous.

Self-fertilization

If pollen grains land on the stigma of the same flower, it is said to be *self-pollinated*, and *self-fertilization* may ensue. If the pollen grains land on the stigma of the flower of another plant (of the same kind), it is said to be *cross-pollinated*, and *cross-fertilization* will probably follow (*see* Lesson 19). The fertilized ovum divides repeatedly to produce a small mass of cells, which enlarges and increases to form a minute plantlet. Food reserves in the form of starch, protein, or fats and oils, are deposited in close association with the embryo. These nourish the young plant until it can manufacture food for itself. The delicate investments of the ovule become tough seed-coats.

Formation of Fruit

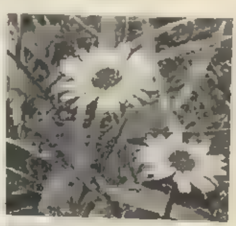
While the seeds are ripening, the ovary enlarges and becomes the fruit. This may be hard and dry, as in buttercup, poppy, and sunflower, or fleshy and succulent, as in plum, orange, and grape. Other parts besides the ovary may undergo changes and contribute to the formation of what is then called a *false fruit*. The red pulp of a strawberry, for instance, is formed by the enlargement of the stem part of the flower—i.e. the receptacle ; the brown pips are the real fruits—in this instance dry, and each formed from an ovary.

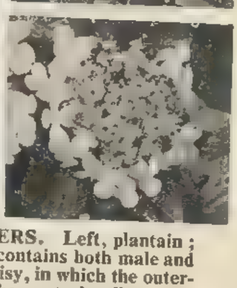
LESSON 19

Methods of Pollination

The transference of pollen from the stamens to the stigma is essential for the production of seeds. If the egg-cells of a flower are fertilized by pollen from the same flower, or from another flower of the same plant, they are said to be self-fertilized ; if by pollen from a flower of a different individual, they are cross-fertilized. In many species a greater number of seeds and more healthy offspring are produced from cross-fertilization ; there are devices by which self-fertilization is hindered and cross-fertilization promoted.

Most flowers contain both stamens and ovaries, and these are described as *hermaphrodite*. In these the male and female organs rarely ripen at the same time. In the Canterbury bell (*Campanula medium*), for example, the stamens ripen first and discharge their pollen before the stigmas are capable of receiving it. In the plantain (*Plantago*) the stigmas in each flower wither before the stamens appear. The small flowers of this common weed grow closely together in a dense spike, sometimes several inches long, and ripening of ovaries and stamens occurs in the basal flowers first. The top of the plantain's flower spike may bear ripe ovaries, while at the bottom the stamens have begun to ripen ; but because the flowers in the female stage are always above those in the male, the falling pollen cannot effect pollination.

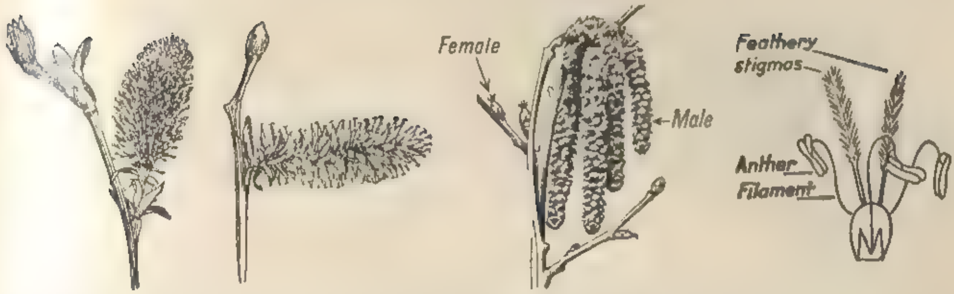




HERMAPHRODITE FLOWERS. Left, plantain ; each flower of the dense spike contains both male and female elements. Top right, daisy, in which the outermost flowers are female and the central yellow ones hermaphrodite. Bottom right, guelder rose, showing an outer ring of sexless flowers surrounding the complete ones.

Unisexual Flowers

Some flowers are *self-sterile*. In these the pollen has no effect at all on the female organs of flowers of the same individual plant. In other instances the stamens and ovaries occur in distinct flowers, which are then said to be *unisexual*. The flowers of different sexes may be found on the same individual, as in hazel, or on different individuals, as in dog's mercury and willow. Unisexual flowers probably arise from the hermaphrodite condition by suppression



WILLOW, HAZEL, AND GRASS FLOWERS. Left, male and female catkins of willow, Salix caprea; they are borne on different trees. Centre, catkins of the hazel; the golden-yellow pendants are the male flowers, the female ones being bud-like with a tiny crimson tuft at the apex. Right, single grass flower, showing the large anthers and feathery stigmas (× 2).

of the organs of one sex. Quite commonly the remains of the other essential organs can be recognized.

The modes of distribution of male, female, and hermaphrodite flowers are very varied. Quite a number of species have unisexual and hermaphrodite flowers side by side. In the daisy the outermost flowers (commonly, but erroneously, called the "petals") are female, while the central yellow flowers are hermaphrodite. In the campion, male, female, and hermaphrodite flowers occur on distinct plants. The ash tree bears all three types on the same individual.

Cleistogamous Flowers

While cross-pollination seems advantageous in many instances, some species are frequently or always self-pollinated. The British species of eyebright (*Euphrasia*) are normally self-pollinated, though cross-pollination may occur. Other plants, e.g. some species of violet, are called *cleistogamous*, because some of the flowers never open, so that self-pollination and fertilization is habitual. Though this reduces variation and vigour in successive generations of the species, it has the advantage that the occurrence of pollination becomes more certain. In apomictic species (*see* Lesson 5) pollination often does not occur.

Because pollen grains are not themselves capable of movement, they must be transferred by some outside agency from the anthers of one flower to the stigma of another. The commonest agencies for conveying the pollen are wind, insects, and (rarely) water.

Wind Pollination

Wind pollination, which occurs among plants with inconspicuous flowers, and among most of the ordinary trees of temperate regions, depends largely on chance, and an enormous amount of pollen has to be produced. Either the flowers contain many stamens (e.g. poplar and elm), or the anthers are very large (e.g. sedges and grasses), or the number of male flowers greatly exceeds that of the female (e.g. Scots pine and hazel). The grasses have a much-branched and feathery stigma, by which the pollen is caught.

Wind-pollinated trees open their flowers in early spring before the new leaves have unfolded, and pollen grains are then most likely to reach the stigmas. Pollen is easily shaken out by wind, especially from long, swaying catkins—for example, hazel (*Corylus avellana*)—and from flowers whose anthers swing freely on long filaments, as in grasses and plantain.

Insect Agency

The vast majority of conspicuous flowers are cross-pollinated by the agency of insects, which are induced in various ways to visit them. These inducements include nectaries or glands which produce nectar. Nectaries vary greatly in size and shape in different flowers, and they may be situated on almost any part of the flower, generally near its base. Quite commonly dark streaks or spots, called *honey guides*, occur on the petals, converging to the point where the nectaries are situated.

But some insect-pollinated flowers produce no nectar and offer only pollen to their visitors. Poppy and broom are examples. They have a large number of stamens, so that pollen is produced in sufficient quantity both for pollination and as food for the insect.

Other Attractions

Colours and odours are (possibly) other attractions. As a general rule it is the perianth which is the showy part of an insect-pollinated flower, but occasionally other parts, such as stamens or bracts, are expanded and brightly coloured. Pale tints or white are characteristic of flowers which open in the evening and are pollinated by moths. Flowers of small size are conspicuous in that they are massed together in large numbers to form a flower-head, as in the daisy. Pollen may occasionally be transferred by mammals, birds, snails, and slugs, but most conspicuous flowers are pollinated by the agency of nectar-seeking insects.

In a large number of small regular flowers, mostly white or yellow in colour, the nectar is fully exposed to view, and these are visited by short-tongued insects. This is so in most members of the parsnip and carrot family (Umbelliferae) and in some saxifrages. Plants whose flowers partly conceal their nectar include most members of the wallflower family (Cruciferae) and the buttercup family (Ranunculaceae). These are visited by insects with slightly longer mouth-parts.

Flowers which completely conceal their nectar in deep recesses or long spurs are visited by bees and butterflies. The mouth-parts of these insects form a long proboscis, which is a most efficient sucking organ. Examples of these flowers include foxglove (*Digitalis*), larkspur (*Delphinium*), monkshood (*Aconitum*), and the orchids.

Water-borne Pollen

In some aquatic plants movements of the surrounding water bear the pollen to its destination. The best example is perhaps that of eel grass (*Vallisneria spiralis*), commonly grown in indoor aquaria. Here self-pollination is prevented by the production of distinct male and female flowers on different plants. The female flower is placed at the end of a spiral stalk, which uncoils and carries it to the surface. Meanwhile the ripe male flowers have become separated from their stalks, and they float about, drifting here and there. Their perianths expand and the stamens project, lifting up their anthers so that the pollen is kept dry. If one of them collides with a female flower, some of the pollen is likely to adhere to the sticky surface of one or other of the three projecting stigmas, and thus cross-pollination is brought about. When the egg-cells are fertilized, the stalk of the female flower coils up again, and the seeds mature in a sheltered situation.

Cross-pollination Mechanism

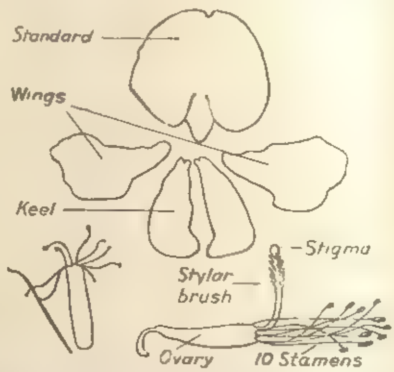
There are four chief kinds of mechanism by which cross-pollination is effected. (1) In clover (*Trifolium*) there is a simple valvular arrangement. When a bee alights on the flower and probes for nectar, its weight causes the stamens and stigma to project and so touch the insect's under-surface. The stamens dust the bee with pollen, while the stigma receives any pollen grains brought from another clover flower. (2) A piston device is present in bird's-foot trefoil (*Lotus corniculatus*) and lupin (*Lupinus*) Here the pollen accumulates in the tip of the keel, and is then pushed forward by the thickened ends of the filaments as the result of pressure. (3) In vetches (*Vicia*), pea (*Pisum*), broad bean (*Vicia*), and runner bean (*Phaseolus*) the style bears a brush of hairs which, on the application of pressure, sweeps out pollen against the body of the insect visitor. (4) The flowers of gorse and broom are in a state of tension, and explode from the pressure brought to bear by the insect visitor. In gorse the underside, in broom the upper side, of the visiting insect is dusted with pollen and touches the stigma.

In the majority of instances the insect alights on the perianth of the flower it is visiting, with the result that some part of its body becomes dusted with pollen. Soon or late the stamens and stigma in an insect-pollinated flower occupy the same positions within the perianth, and therefore the pollen taken from one flower will probably get rubbed on to the stigma of another.

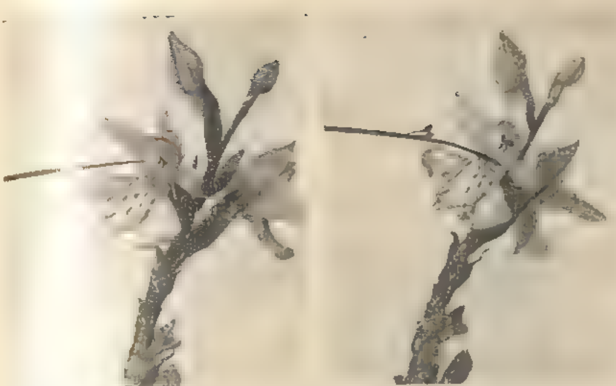
Bees at Work

The dead-nettle family (Labiatae) contains species which are cross-pollinated by bees. In white dead-nettle, for instance, the irregular corolla has a platform on which a bee can alight, and while it is probing for the nectar secreted near the base of the ovary, the four anthers and the forked stigma—which are sheltered under a hood—come into contact with the upper side of its body. Meadow sage and scarlet sage (*Salvia*) have only two stamens, of which the anthers are modified into a sort of curved swinging apparatus. A bee settling upon one of the younger flowers presses against the lower ends of these anthers, and the upper parts of them, containing the pollen, swing down and dust its back. In older flowers the forked stigma bends down so as to touch the back of an insect visitor and take up any pollen sticking to it.

The foxglove (*Digitalis*) can be cross-pollinated only by humble-bees. The four stamens and the stigma lie along the upper side of the bell-shaped corolla, the former maturing first. When a humble-bee creeps into a young foxglove flower, its back is dusted with pollen, and some of this is likely to be received by the stigma



POLLINATION MECHANISM OF PEA. Dissected flower of sweet pea, showing the five petals, and left, ten stamens (one free and nine united). Right, position of essential organs when these are released from the keel.



INSECT-POLLINATION OF AN ORCHIS. Left, spotted orchis (O. maculata) with bristle, representing the tongue of an insect, being inserted into the tube of the nectary. Right, bristle withdrawn bearing two tiny club-shaped masses of pollen; these will adhere to the stigma of the next spotted orchis flower visited.

of the next (older) flower it may chance to visit. A somewhat similar arrangement occurs in the snapdragon (*Antirrhinum*), but here the humble-bee has to force open the curiously shaped corolla in order to secure the nectar which lies within the flower.

Some members of the buttercup family (Ranunculaceae) are bee-flowers. One of the most interesting examples is afforded by monkshood (*Aconitum*), in which the two upper petals are modified into tubular nectaries, which can be rifled only by a humble-bee. The stamens mature first, and after them the stigmas, these successively occupying a position in front of the flower, which has the effect of bringing them into contact with the under side of a visitor alighting there.

Another example of a bee-flower is the spotted orchis (*O. maculata*), whose purple flower spikes are often seen in meadows from May to July. The three sepals and three petals are all brightly coloured, and one of the petals expands into a large lip, which serves as an alighting platform for insects. It is drawn out into a long spur, the inner wall of which is perforated by bees to get at the nectar. Facing the entrance to the spur is a broad stigma, above which is a sticky knob, the *rostellum*. Only one stamen is present, united to the top of the pistil and devoid of filament. Its anther contains two cavities, in each of which the pollen is bound together into a club-shaped mass (*pollinium*) with a slender stalk, which runs down to the rostellum. If a humble-bee alights upon the orchis flower and probes for sap, it will strike against the rostellum, and on withdrawal it will carry off the two pollinia on the top of its head. These bend forward and adhere, in whole or part, to the stigma of the next orchis flower visited. As an extreme example, there are some orchid flowers which resemble the females of definite species of bees; the flowers are pollinated when male bees alight and perform copulation movements.

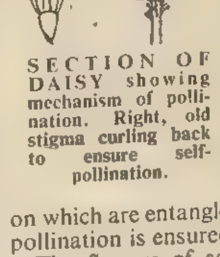
Honeysuckle and Primrose

Honeysuckle (*Lonicera*) is pollinated by hawk-moths, which have a very long proboscis, and they suck nectar while on the wing. This is possible because the corolla of the flower is drawn out into a nectar-containing tube, and there is no alighting platform. The stamens mature first, and project from the blossom in such a way that some of their pollen is shaken on to the visitor, which will be likely to transfer some of it to the stigma of an older honeysuckle flower that occupies the position taken up by the stamens in an earlier stage.

The primrose (*Primula vulgaris*) is visited chiefly by bees and butterflies. The flower is of great interest, because it produces two kinds of blossom. In one ("thrum-eyed") the five anthers occupy the mouth of the corolla tube, and the rounded stigma is borne upon a short style. In the other kind ("pin-eyed") the anthers are deep down in the tube, while the stigma is on a long style and occupies the entrance of the corolla tube. The result is that the pollen from one kind of primrose flower is likely to be transferred to the stigma of the other kind of flower.

Varied Devices

The pollination mechanism in the daisy family (Compositae) is quite different from those already considered. Usually only the hermaphrodite flowers produce seed. The five stamens of these flowers are united to form a tube, inside which the pollen is shed. The stigma grows up as a straight rod through this pollen, which is caught up on the hairy style below the stigma. When mature, the stigma opens out into a Y shape, and the inner receptive surfaces await cross-pollination. If this fails to occur, the stigma lobes, when old, curl right back until they come in contact with the style on which are entangled pollen grains. Thus self-pollination is ensured if cross-pollination fails.



SECTION OF DAISY showing mechanism of pollination. Right, old stigma curling back to ensure self-pollination.

The flowers of some species have traps by which small flies are caught and held until

In a large number of small regular flowers, mostly white or yellow in colour, the nectar is fully exposed to view, and these are visited by short-tongued insects. This is so in most members of the parsnip and carrot family (Umbelliferae) and in some saxifrages. Plants whose flowers partly conceal their nectar include most members of the wallflower family (Cruciferae) and the buttercup family (Ranunculaceae). These are visited by insects with slightly longer mouth-parts.

Flowers which completely conceal their nectar in deep recesses or long spurs are visited by bees and butterflies. The mouth-parts of these insects form a long proboscis, which is a most efficient sucking organ. Examples of these flowers include foxglove (*Digitalis*), larkspur (*Delphinium*), monkshood (*Aconitum*), and the orchids.

Water-borne Pollen

In some aquatic plants movements of the surrounding water bear the pollen to its destination. The best example is perhaps that of eel grass (*Vallisneria spiralis*), commonly grown in indoor aquaria. Here self-pollination is prevented by the production of distinct male and female flowers on different plants. The female flower is placed at the end of a spiral stalk, which uncoils and carries it to the surface. Meanwhile the ripe male flowers have become separated from their stalks, and they float about, drifting here and there. Their perianths expand and the stamens project, lifting up their anthers so that the pollen is kept dry. If one of them collides with a female flower, some of the pollen is likely to adhere to the sticky surface of one or other of the three projecting stigmas, and thus cross-pollination is brought about. When the egg-cells are fertilized, the stalk of the female flower coils up again, and the seeds mature in a sheltered situation.

Cross-pollination Mechanism

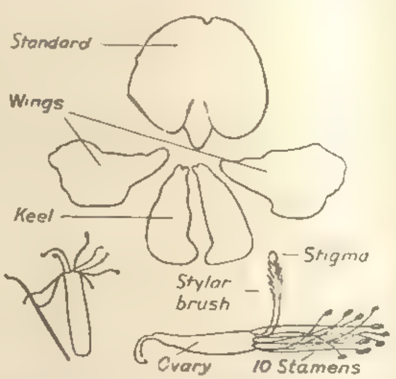
There are four chief kinds of mechanism by which cross-pollination is effected. (1) In clover (*Trifolium*) there is a simple valvular arrangement. When a bee alights on the flower and probes for nectar, its weight causes the stamens and stigma to project and so touch the insect's under-surface. The stamens dust the bee with pollen, while the stigma receives any pollen grains brought from another clover flower. (2) A piston device is present in bird's-foot trefoil (*Lotus corniculatus*) and lupin (*Lupinus*). Here the pollen accumulates in the tip of the keel, and is then pushed forward by the thickened ends of the filaments as the result of pressure. (3) In vetches (*Vicia*), pea (*Pisum*), broad bean (*Vicia*), and runner bean (*Phaseolus*) the style bears a brush of hairs which, on the application of pressure, sweeps out pollen against the body of the insect visitor. (4) The flowers of gorse and broom are in a state of tension, and explode from the pressure brought to bear by the insect visitor. In gorse the underside, in broom the upper side, of the visiting insect is dusted with pollen and touches the stigma.

In the majority of instances the insect alights on the perianth of the flower it is visiting, with the result that some part of its body becomes dusted with pollen. Soon or late the stamens and stigma in an insect-pollinated flower occupy the same positions within the perianth, and therefore the pollen taken from one flower will probably get rubbed on to the stigma of another.

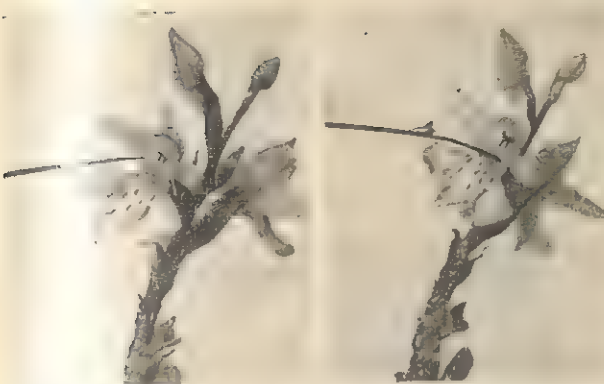
Bees at Work

The dead-nettle family (Labiatae) contains species which are cross-pollinated by bees. In white dead-nettle, for instance, the irregular corolla has a platform on which a bee can alight, and while it is probing for the nectar secreted near the base of the ovary, the four anthers and the forked stigma—which are sheltered under a hood—come into contact with the upper side of its body. Meadow sage and scarlet sage (*Salvia*) have only two stamens, of which the anthers are modified into a sort of curved swinging apparatus. A bee settling upon one of the younger flowers presses against the lower ends of these anthers, and the upper parts of them, containing the pollen, swing down and dust its back. In older flowers the forked stigma bends down so as to touch the back of an insect visitor and take up any pollen sticking to it.

The foxglove (*Digitalis*) can be cross-pollinated only by humble-bees. The four stamens and the stigma lie along the upper side of the bell-shaped corolla, the former maturing first. When a humble-bee creeps into a young foxglove flower, its back is dusted with pollen, and some of this is likely to be received by the stigma



POLLINATION MECHANISM OF PEA. Dissected flower of sweet pea, showing the five petals, and left, ten stamens (one free and nine united). Right, position of essential organs when these are released from the keel.



INSECT-POLLINATION OF AN ORCHIS. Left, spotted orchis (O. maculata) with bristle, representing the tongue of an insect, being inserted into the tube of the nectary. Right, bristle withdrawn bearing two tiny club-shaped masses of pollen; these will adhere to the stigma of the next spotted orchis flower visited.

of the next (older) flower it may chance to visit. A somewhat similar arrangement occurs in the snapdragon (*Antirrhinum*), but here the humble-bee has to force open the curiously shaped corolla in order to secure the nectar which lies within the flower.

Some members of the buttercup family (Ranunculaceae) are bee-flowers. One of the most interesting examples is afforded by monkshood (*Aconitum*), in which the two upper petals are modified into tubular nectaries, which can be rifled only by a humble-bee. The stamens mature first, and after them the stigmas, these successively occupying a position in front of the flower, which has the effect of bringing them into contact with the under side of a visitor alighting there.

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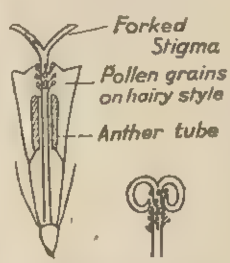
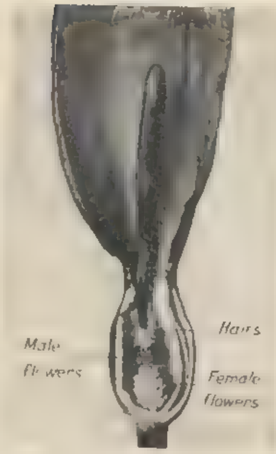


SECTION OF DAISY showing mechanism of pollination. Right, old stigma curling back to ensure self-pollination.

The flowers of some species have traps by which small flies are caught and held until

pollination has occurred. The arum of the British countryside, *A. maculatum* (known variously as wake robin, lords-and-ladies, and cuckoo-pint) shows a modification of this kind. The flowers are enclosed in a chamber formed by the lower part of a large green bract (*spathe*), which opens to display a thickened purple stem (*spadix*). The flowers are arranged in two crowded sets on the base of the spadix; the lower ones are female, the upper ones male. Above the male flowers are a number of downwardly directed trap-hairs; these are modified male flowers which have lost their original function. Small flies are attracted by the dull purple colour of the spadix and by an ammoniacal odour which is exhaled. Creeping down the spadix or inner side of the spathe, they easily make their way past the trap-hairs, into the chamber containing the flowers. At this time the stigmas are mature, and should any arum pollen be sticking to the bodies of insect visitors the stigmas are likely to receive some of it.



WILD ARUM. Section of interior of spathe showing the male and female flowers, and the hairs which serve to imprison insects until pollination has taken place.

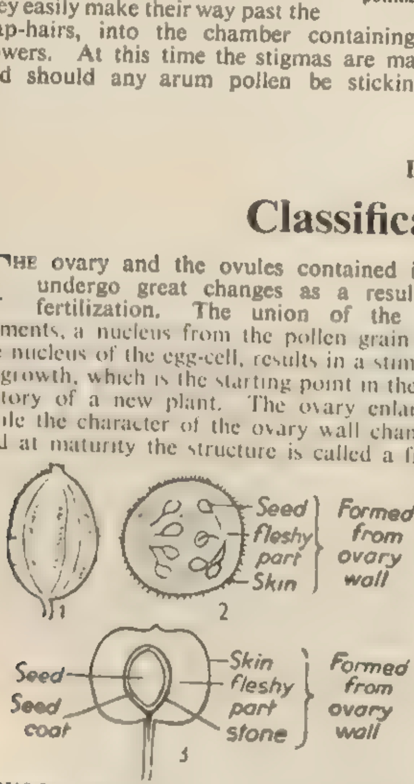
When this takes place, the stigmas of the arum wither, and each of them exudes a drop of nectar. The stamens next shed their pollen, and some of this is likely to stick to the insects, which are at last allowed to escape by the withering of the trap-hairs. The pollen is carried by them to another arum flower.

Some species protect their pollen and nectar from rain and dew, the split anther closing more or less completely when the air is damp. The same purpose is served by the so-called "sleep movements" of flowers, the petals closing at night, as in tulip, crocus, and wood anemone (*A. nemorosa*); by the closing of the compound flower-heads of members of the daisy family (Compositae); and by the hanging position of flowers, e.g. harebell (*Campanula rotundifolia*), heather (*Calluna vulgaris*), and lily of the valley (*Convallaria*).

LESSON 20

Classification of Fruits

The ovary and the ovules contained in it undergo great changes as a result of fertilization. The union of the two elements, a nucleus from the pollen grain and the nucleus of the egg-cell, results in a stimulus to growth, which is the starting point in the life history of a new plant. The ovary enlarges, while the character of the ovary wall changes, and at maturity the structure is called a fruit.



SUCCULENT FRUITS. 1. Fruit of the gooseberry. 2. Transverse section of a berry showing the seeds. 3. Vertical section of a stone fruit or drupe; the seed is enclosed in a thin brown coat within the "stone."

The fertilized ovules within the changing ovary become the mature seeds. A seed can always be distinguished from a fruit by the fact that it exhibits only one scar, the *hilum*, whereas the fruit shows two, one marking the former attachment to the flower stalk, and the other the remains of the style.

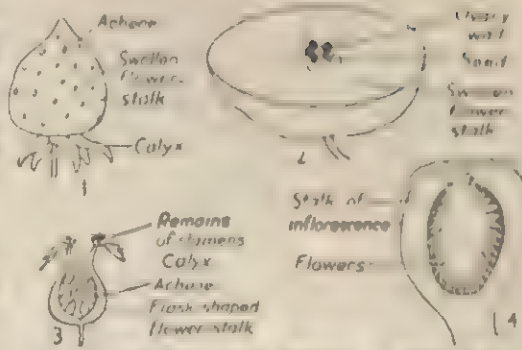
According to the changes undergone by the ovary wall after fertilization, fruits can be classified as *dry* or *succulent*. Some species produce *false fruits*: some part of the flower other than the ovary has contributed to their formation. If the ovary consists of only one carpel (e.g. pea), or of more than one carpel united (e.g. lily), the fruit is said to be simple. If the ovary consists of several separate carpels, the resulting fruit is compound (e.g. buttercup).

Dehiscent and Indehiscent Fruits

Two types of dry fruit can be distinguished: one usually remains attached to the parent plant and splits open to liberate the contained seeds; the other type does not split open, but the fruit wall decays after the fruits reach the ground. These types are described as *dehiscent* and *indehiscent* respectively. Examples of dry dehiscent fruits are the pods of pea, gorse,

and broom, the pod-like follicles of columbine and monkshood, the flattened fruits (*siliquae*) of honesty and wallflower, and the similar fruits (*capsules*) of lily, bluebell, iris, campion, violet, poppy, and pimpernel. Different modes of dehiscence occur among the capsule fruits, some opening by simple splits along the edge, some by teeth, others by valves, pores, or a complete lid.

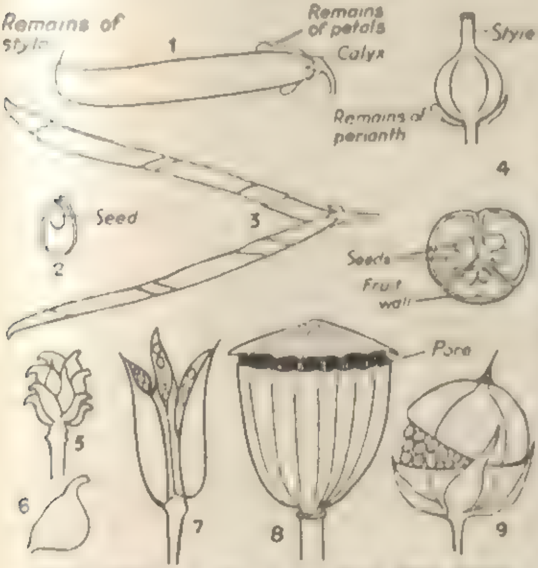
Examples of dry indehiscent fruits are nuts, such as hazel, sweet chestnut, beech, and acorn. Of the same type are the fruits of members of the daisy family (e.g. sunflower and dandelion), the grasses, and the buttercups. Fruits of the indehiscent type often contain only one seed, and thus the plants avoid overcrowding of seedlings at any one spot.



FALSE AND COMPLEX FRUITS. 1 Fruit of the strawberry; it consists of a number of tiny dry fruits (achenes). 2 Apple in transverse section. 3 Vertical section of a rose hip; the true fruit is contained within. The three foregoing are called false fruits. 4 Vertical section of a fig, a complex fruit

Berries

The two chief kinds of fleshy fruit are called *berries* and *drupes*, both types being indehiscent. Examples of berries are currant, gooseberry, tomato, and cucumber—the characteristic feature of this type of fruit being that the wall is fleshy throughout and forms all the succulent portion. The seeds are held within the fleshy middle region. Other examples of berries are orange, lemon, grape, banana, vegetable marrow, melon, and date. In orange and lemon the ovary wall forms the coloured outer skin, the underlying white skin, and membranous skin around the fleshy sections. The fleshy part consists of a great development of juice-containing hairs, which grow from the skin-like lining. The structure commonly called a date "stone" is the true seed, which is made very hard by the store of cellulose in the cell walls of the endosperm



DRY FRUITS. 1 Pod of pea (simple fruit, formed from one carpel). 2. Transverse section of pea. 3 Dehisced pod of peas; the ripe fruits are in a state of tension and the sudden dehiscence of the pod along both edges scatters the seeds. 4 Capsule of lily formed from three carpels united; below, transverse section. 5 Compound fruit of buttercup; it is formed from many separate carpels. 6 One carpel, an achene. 7 Compound fruit of columbine, composed of several follicles which split along their inner edges. 8 Capsule of poppy opening by pores. 9 Capsule of pimpernel opening by lid

Drupes

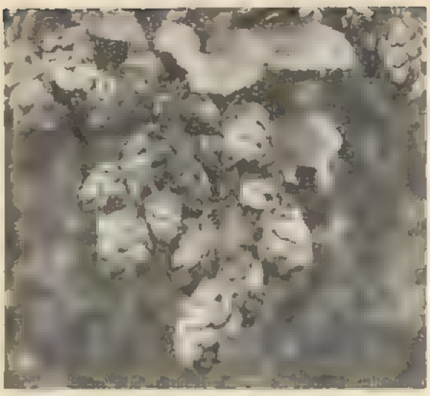
Common examples of drupes are cherry and plum (*Prunus*). The drupe differs from the berry in that the ovary wall of a drupe does not become entirely fleshy, but the innermost region of it forms a hard stony layer in which the seed is enclosed. On cracking open the stone the white seed is revealed, contained in a thin brown seed-coat. Other drupes are damson and peach (*Prunus*), almond ([illegible]), walnut (*Juglans*), and coconut (*Cocos*). Raspberry and blackberry (*Rubus*) are compound fruits composed of several small drupes.

False Fruits

The fruit of some species is not composed of ovary alone, but other parts of the flower (especially the flower-stalk) take part in its formation. This is called a false fruit. In the strawberry (*Fragaria*) the true fruit is compound, consisting of a number of minute dry fruits (called achenes), borne on the much-enlarged and fleshy flower-stalk, which is the actual edible part. In apple and pear (fruits called *pomes*) the flesh is formed by the flower-

stalk, in which the ovary is embedded, the ovary wall forming the core. The rose hip is another false fruit, in which the coloured flask-shaped portion is formed from the flower-stalk and the true fruit is constituted by the dry achenes within.

The fruits of fig (*Ficus*), pineapple (*Ananas*), mulberry (*Morus*), and hop (*Humulus*) differ from those already considered because a complex fruit is formed from a whole inflorescence, i.e. from a group of flowers. The fig is really a swollen stalk bearing numerous small male and female flowers on its inner surface. The female flowers produce minute fruits, commonly regarded as seeds. The composite fruit of the pineapple is formed from a spike of flowers, the fleshy flower-stalk and flowers all fusing together; the areas on the surface of the fruit represent the flowers. Above the flowers the stalk produces a number of leaves forming the *crown*. The mulberry is also formed from a spike of flowers whose perianth members become fleshy and enclose the true fruits, which are achenes. The fruit (the cone) of the hop is formed from a cluster of flowers which are hidden by overlapping bracts. After pollination of the flowers (by the wind) the bracts develop into cone-like scaly heads, and these are the hops of commerce.



HOPS. The complex fruit is formed from the ovaries at the base of the female flowers, which are hidden by overlapping bracts. The latter, after pollination of the flowers by the wind, develop into cone-like scaly heads.

LESSON 21

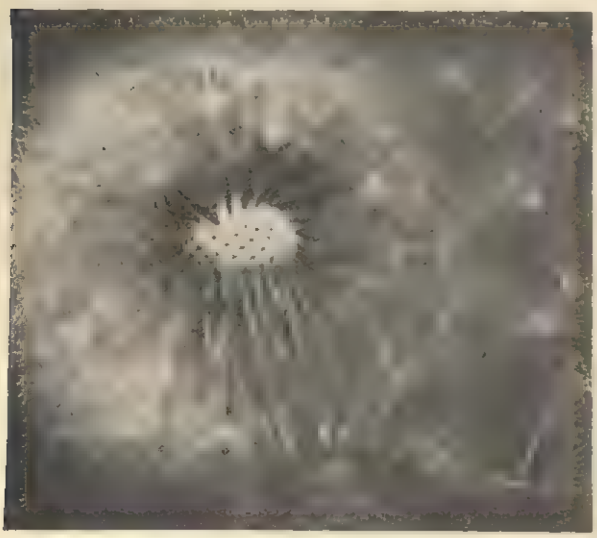
Methods of Seed Dispersal

It is necessary that seeds should be widely dispersed from the parent plant, so that some at least may germinate without overcrowding. Some species are able, by the structure of their fruits, to disperse their own seeds. Others depend more or less entirely on the help of wind, or water, or animals.

The fruits of some plants are in a state of tension when ripe, and they open or split suddenly in such a way as to eject the seeds to a considerable distance. An example is the shrub broom (*Cytisus*), the flat, black pods of which may often be heard exploding in late summer; the two halves curl up suddenly, so that the seeds are scattered in all directions. The ripe capsules of the balsam (*Impatiens balsamina*) split into four twisting strips at the least touch, with the same result: the seeds are distributed.

Violets and pansies throw their seeds to a distance of three or four feet. The ripe fruit breaks open into three boat-shaped portions, in each of which are two rows of seeds, and these are forced out of the "boats" by the sides pressing in to meet each other above the keels.

Some species secure the scattering of the seeds by the elasticity of the dry fruit-stalks when the seeds are ripe. The ripe fruit opens by large splits or small holes, so that the seeds are exposed, but some disturbance is necessary to liberate them. This is effected by gusts of wind, or by the contact of animals or birds which bend the dry stems; directly the bending strain is removed, the stem swings back to the upright position and the seeds are jerked out in various directions well away from the parent plant.



DANDELION "CLOCK." The seeds, each borne by a parachute of down, are shown being driven by the wind from the receptacle.

Examples of this method are the campions, foxglove and bluebell, whose fruits open by wide splits at the upper end ; poppy and snapdragon, which open by pores ; and most members of the dead-nettle family, whose fruits are held in a cup formed by the calyx.

Seeds that are small and light sometimes remain suspended like dust in the air, and may be carried great distances by the wind ; this applies to many orchids, and heaths. Buoyancy is aided in a number of ways. Pines, firs, and some others of the family Coniferae have winged seeds. the wing being a flattened expansion of the seed-coat. The *keys* of maple, sycamore, ash, and elm are winged fruits, the wing being an outgrowth of the ovary wall. Hornbeam and lime have a modified leaf (bract) attached to the fruits. *Parachute* contrivances are on the seeds of willow herb, willow, and poplar, and on the fruits of clematis, wood-anemone, and some members of the daisy family, e.g. dandelion, thistle, and goat's-beard. The fruit of the willow herb is an elongated capsule, which dehisces when ripe, exposing the small seeds, whose seed-coats form a parachute of hairs at one end. Cotton is a growth of this kind. the fibres being long, fluffy parachutes attached to the seeds. Clematis develops a feathery style as the fruit opens, and in wood-anemone the ovary wall becomes woolly.

The " clock " of the dandelion is a group of fruits, each formed from one of the many minute flowers of which the flower-head is composed. The parachute (*pappus*) of each fruit is the modified calyx.

Dispersal by Water

The dispersal of fruits and seeds by water is the method of plants which live and ripen their fruits in the water, and also of some land plants which grow near running water, the fruits and seeds being furnished with contrivances which prevent them from sinking. For example, the fruits of sedges (family Cyperaceae) are surrounded by an inflated sac ; those of some water-plants contain a number of thick-walled air-cells in their outer layers. The dispersal of the coconut is effected by water ; the fruit has a large air-cavity, containing tangled fibrous hair, between its outer tough wall and " stone." Coconuts are sometimes water-borne for thousands of miles without becoming waterlogged.

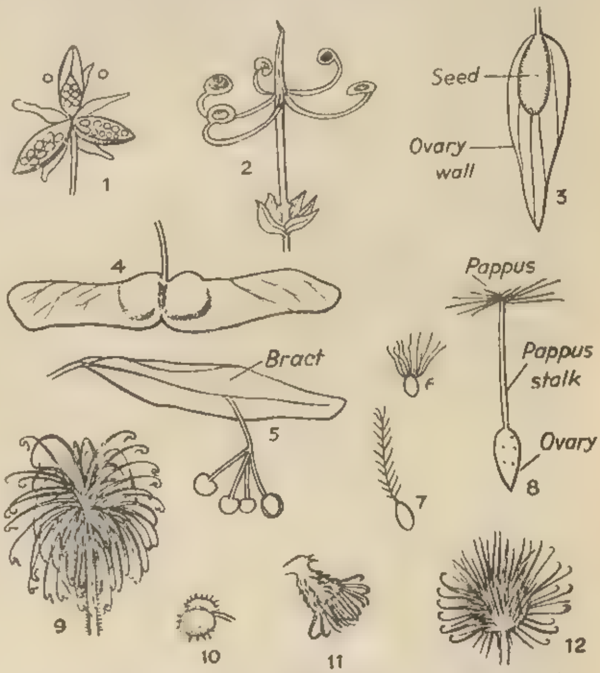
Dispersal by Animals

Seeds of the greater celandine, violet, and gorse have small swellings (*caruncles*) on them near the micropyle, and are often carried by ants to their nests—the caruncles to be used for food, the seed itself remaining uninjured and capable of germinating.

The seeds of succulent fruits are dispersed mainly by birds. Attracted by the bright colours, birds eat the fleshy portions and either scatter the seeds in the process of pecking at the fruit (as in the rose hip, where the achenes are hairy and unpalatable), or swallow the seeds and disperse them more widely in the faeces. Other seeds owe their dispersal to the fact that they adhere to the feet and lower feathers of birds which seek their food in damp places. Some fruits and seeds have hooks or spines, which catch in the hair or fur of passing animals, or in the clothes of human beings ; these are commonly called *burrs*. The hooks are on the ovary in goosegrass, on the tip of the style in avens, on the calyx in agrimony, on the bracts of the flower-head in burdock. A mere touch by a passing animal will carry the ripe fruit away.

PRACTICAL WORK

The student should examine as many as possible of the fruits and seeds mentioned in the text, noting any special adaptations for dispersal. A hand lens should be used for studying the smaller structures.



METHODS OF SEED DISPERSAL. 1. Fruit of violet ejecting its seeds. 2. Dehisced fruit of wild geranium. 3. Ash. 4. Sycamore. 5. Lime. 6. Plumed seed of willow-herb. 7. Feathery style of clematis (old man's beard). 8. One fruit of dandelion " clock." 9. Common avens. 10. Goosegrass or cleavers. 11. Agrimony. 12. Burdock.

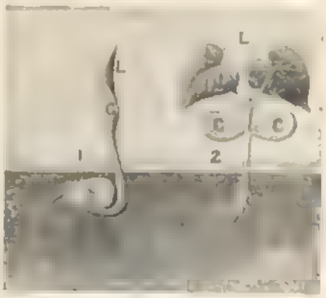
LESSON 22

Classification of Angiosperms

It was explained in Lesson 1 that the seed-plant phylum is divided into Angiospermae (angiosperms), whose seeds are protected in a seed-case, and Gymnosperma (gymnosperms), which are naked-seeded plants. A few details regarding the subdivision of the angiosperms are given in this Lesson, sufficient to serve as an introduction to the subject of classification, the detailed system of which is by no means agreed upon by all botanists.

Angiosperms are divisible into two classes : Dicotyledons and Monocotyledons. The seedlings in the first class have two cotyledons or seed-leaves, and the vascular bundles of the young stem are arranged in a ring, each of them containing *cambium*, which brings about increase in thickness. Perennation is often carried out by large aerial stems, e.g. trees. The veins of leaves are arranged in net-like fashion. Whorls of the perianth are in fours or fives.

The seedlings of the Monocotyledons are each provided with only one cotyledon, and the vascular bundles of the stem are scattered and devoid of cambium. The chief veins of the leaves are generally more or less parallel. Whorls of the perianth are in threes. The Monocotyledons are in the main a herbaceous group with subterranean means for perennation and hibernation (e.g. bulbs, corms, tubers).



Diagrammatic representation of the difference between seedlings of (1) monocotyledon, and (2) of dicotyledon. 1. Germinated seed of maize, S, with single cotyledon, C. 2. Seedling of kidney bean, with two cotyledons (CC). The first true leaves, L, are seen above the cotyledons.

As over 250 plant families are recognized (of which about 90 are British), embracing something like 100,000 species, it is obvious that only the main points can be dealt with here. The characters used in universal scientific classification are mainly in the structure of the flower concerned.

Flower Structure

In the "pattern flower" considered in Lesson 18, the parts of each whorl of the perianth were all alike, and disposed in a star-like manner. Such a flower is *regular* ; if it is cut in two through any diameter, the two halves are exactly similar. Some flowers can be cut only through one plane to give two exactly similar halves, and these are said to be *zygomorphic*, e.g. pea. There are a few flowers which cannot be cut into two exactly similar halves, and these are said to be *irregular*.

In some instances the receptacle (that part of the flower stalk which bears the floral organs) is more or less conical, so that the sepals, petals, and stamens spring from beneath the pistil. The flower is then *hypogynous*. The receptacle may be modified into a sort of cup, from the edge of which sepals, petals, and stamens arise, while the pistil occupies the interior of the cup, as in the rose ; such flowers are *perigynous*. If this cup fuses with the pistil, so that the sepals, etc., appear to grow from the top of the ovary, as in the snowdrop, the flower is *epigynous*. In the first two instances the ovary is attached to the receptacle by its base only, and is said to be *superior* ; in the last instance (epigynous) the attachment is more extensive, and it is termed *inferior*.

Parts of a Flower

There are variations in the number of flower-leaves. They may be relatively numerous, and arranged in a spiral. Such a condition is generally regarded as primitive—that is, early in the evolutionary scale. This is seen in the sepals, petals, and stamens of the common white water lily (*Nymphaea alba*), which pass gradually into one another.

In most flowers the parts are in successive whorls, as in the pattern example, but there are many variations in number. Plants with numerous floral parts are usually regarded as more primitive than those with few. The inner whorl of stamens is often suppressed altogether, and in extreme cases (e.g. most orchids) only one stamen may be present. Such reductions seem to be correlated with increasing certainty of pollination, so that there is no need for a large quantity of pollen to be produced. The carpels, for similar reasons, are often reduced in number, and some flowers, e.g. pea and gorse, have only one. Highly modified methods of pollination are regarded as recent adaptations.

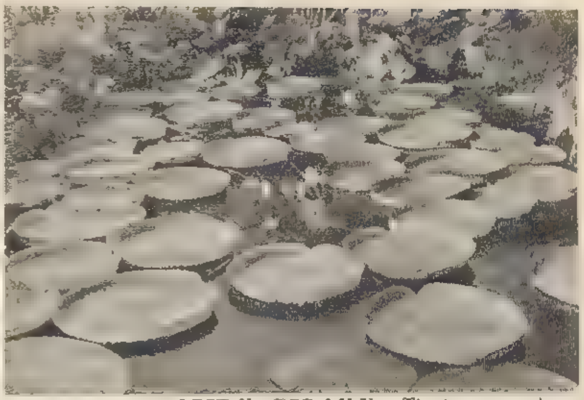
No Colour, Odour, Nectar

The perianth is poorly developed, reduced, and inconspicuous or even absent, where pollination is not effected by insects (or birds), there being no need (in so far as the future of this particular species is concerned) for conspicuousness. In such instances there is usually an absence of colour, odour, and nectar.

In simpler examples the flower-leaves of a particular kind are not united together, i.e. are free. With increasing specialisation, sepals,

petals, stamens and more particularly carpels are respectively united. Flower-leaves of different kinds may also adhere together, e.g. stamens with petals, as in the primrose. There is, further, a great deal of variation in the number of ovules and the way in which these are arranged in the ovary.

Natural classification is an attempt to express the true, natural affinities between plants. In all systems of classification the plants are divided initially into a small number of groups, the components of which all have a greater or lesser degree of similarity in a large number of characters. These groups are gradually subdivided into smaller groups, in which the individuals resemble one another more closely at each successive division. There are several systems, these varying with the opinions of the originators. An outline of the system most widely recognised at the present day is given here.

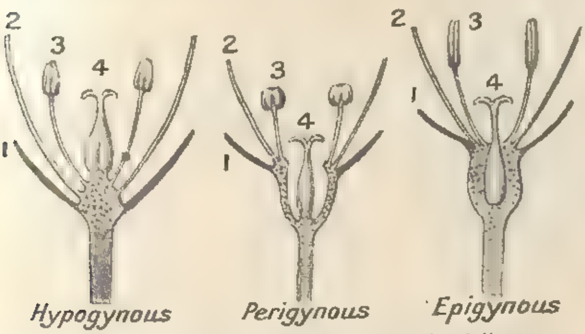


WORLD'S LARGEST WATER LILY. The huge, circular leaves are often over six feet in diameter. They have upturned margins, and on the ribs of the underside are large and strong spines. This majestic plant, Victoria Regia, is native to rivers of the north-eastern part of South America.

Subdivisions

The flowering plants are divided (as previously stated) into Dicotyledons and Monocotyledons. The initial division in the Dicotyledons is based on whether or not the petals are joined. The groups are called divisions. Division 1 has free petals, Division 2 has joined petals. The divisions are subdivided into orders, which differ in the construction and position of the gynaecium. Orders are composed of families, the individuals of which usually show remarkable similarity, e.g. buttercup and marsh marigold in the family Ranunculaceae, and apple and pear in the very extensive family Rosaceae.

The families are divisible into genera, in which the plants differ to a very slight degree. Differences within genera are called *specific* and they differentiate the species within the genus, e.g. *Ranunculus acris* and *R. bulbosus*, which are almost indistinguishable one from the other and would be called buttercup by the layman. Taxonomists and systematists are interested in even smaller differences, called *sub-specific*, which indicate possible lines of adaptation and evolution and which may eventually result in the formation of new species and even genera.



RECEPTACLE OF THE FLOWER. Sectional diagrams showing the three different types of receptacle: 1. sepal; 2, petal; 3, stamen; 4, pistil. The receptacle itself is shown dotted.

Two-part Latin Names

For simplicity, each plant is given a Latin name which is composed of two parts. The first part is the *generic*, and it is written with a capital letter; the second is the *specific*, and it is written with a small letter; and the complete name is generally printed in italics. Latin is used so that the name can be understood the whole world over. For example, *Ranunculus ficaria* is called the lesser celandine in Great Britain, but it has a different vernacular name in other countries.

Some flowers have different names in different parts of a country, and the same name often applies to different plants, so that the necessity for a uniform system is obvious. Classificatory systems undergo changes according to opinions current at the time. It is usual to try to put the (supposedly) more primitive families first, and as the understanding of evolution and its methods becomes greater, previous systems have to be revised. At one time Monocotyledons were thought to be more primitive, owing to the smaller number of floral parts and the form of the leaves. These characters are now thought to be secondary modifications, and therefore Monocotyledons are considered to-day to be more specialised and advanced than the Dicotyledons.

It should always be remembered that

evolution is not a simple process starting at one end of the plant kingdom and working its way through to the other; it is going on in all regions at the same time. Various lines can be traced backwards, and they seem to converge on a common ancestor; other "evolutionary lines" show parallel development; others show divergence. The whole system should therefore be regarded rather as the branches of a tree. Such a system can never be given in true form on the two dimensions of a written page, and the student should try to imagine the many dimensional aspects, e.g. in the Ranunculaceae; although most members of the family are generally regarded as being fairly primitive in flower structure, the larkspur and monkshood have an advanced floral structure which is highly specialised for insect pollination.

Classification of Angiosperms

CLASS I. DICOTYLEDONS

Division I. *Archichlamydeae*

All plants in this division have the petals of the flower free from one another or absent, i.e. the petals are not united in a tube.

Order 1. Ranales. Flowers hypogynous. Carpels free.

(*a*) The buttercup family (Ranunculaceae): leaves generally alternate and exstipulate. The buttercups and others have regular flowers with numerous free stamens and carpels, while some forms, e.g. larkspur and monkshood, have irregular flowers specialised in relation to pollination by the higher insects.

(*b*) The water lily family (Nymphaeaceae) has conspicuous flowers with spiral arrangement. A notable species is *Victoria regia*, a South American form with leaves 6 feet across and flowers 16 inches in diameter.

(*c*) The magnolia family (Magnoliaceae) includes woody plants considered to be the most ancient type of existing dicotyledons.

FLOWERS OF DICOTYLEDONOUS PLANTS. Top left, perpetual flowering carnation, a type grown under glass in large quantities for the florist's trade. Top right, double scarlet geranium, a popular garden plant of the pelargonium family. Bottom left, flowering spikes of different coloured single varieties of hollyhock. Bottom right, Puck, a beautiful variety of rose with cerise petals.

Order 2. Rhoeadales. Flowers hypogynous. Carpels joined.

(*a*) The poppy family (Papaveraceae): herbs which yield a sticky juice; leaves alternate and exstipulate. Sepals, two; petals, four; stamens, numerous. The carpels are united by their edges and the ovules attached to the outer wall of the ovary. This arrangement is also found in the wallflower and violet families.

(*b*) The wallflower family (Cruciferae): leaves alternate, exstipulate. Sepals, four in two whorls; petals, four in the form of a cross; stamens, six—four long and two short; carpels, two. The ovary is divided into two cavities by a partition growing out from the edges of the joined carpels. It is this partition which forms the decorative silver "leaf" of dried honesty (*Lunaria annua*). In this family (Cruciferae) are wallflowers, stocks, shepherd's purse, water- and garden-cress, mustard, radish, turnip, cabbage, etc.

(*c*) The violet family (Violaceae): leaves alternate with large stipules. Flowers, irregular; carpels, three.

Order 3. Centrospermales. Flowers hypogynous. Carpels joined. The pink family (Caryophyllaceae): the ovary contains only one compartment, in the centre of which is a vertical projection to which the ovules are attached. Examples of this family are pinks, carnations, campions, and chickweed. The docks and beet, which have no petals, are in other families of this order.

Order 4. Geraniales. Flowers hypogynous, carpels joined. The geranium family (Geraniaceae): the parts of the flower are mostly in fives; the carpels united at the centre and the ovules borne at the central junction. The cultivated pelargoniums belong to this

family, and the wild crane's-bill and stork's-bill. In a nearly related family are the garden nasturtium (*Tropaeolum*), balsam, wood sorrel, and flax, one species of which (*Linum usitatissimum*) is of economic value in providing fibres for linen, linseed oil, and cattle-cake.

Order 5. Malvales. Flowers hypogynous, carpels joined. The mallow family (Malvaceae): flowers regular, with parts in fives. Stamens variable in number, often united by their filaments. Carpels, three or more, with the ovary in compartments. Well-known examples are the mallows and tree-mallows, hollyhocks, and the economically important cotton plant (*Gossypium*) of which there are several species.

To the same order belongs the lime tree, as do also the plants from which come tea and cocoa.

Order 6. Rosales. Flowers mainly epigynous or perigynous.

(*a*) The rose family (Rosaceae): leaves usually alternate with stipules. Flowers regular with five sepals, five petals, numerous stamens, and a variable number of carpels, which may be free or united. Examples are the roses, hawthorn, plum, cherry, peach, pear, apple, strawberry, and spiraea. Gooseberries and currants are in another family of this order, and nearly allied are the hydrangeas and the fragrant, white flowered mock orange (*Philadelphus*).

(*b*) The pea family (Leguminosae): leaves usually compound and with stipules; flowers irregular with five joined sepals and five free petals. Stamens, ten, either all united by their filaments or nine united and one free. One carpel which ripens into a pod (*legume*): seeds exalbuminous with large embryos. Some of the commonest members are gorse, broom, and clovers. Some are valuable for fodder and human food, e.g. vetches, beans, peas, and lentils.

Many exotic species yield useful timbers and other products, e.g. senna, liquorice, pea-nuts (ground-nuts, monkey-nuts), and indigo dyes.

Order 7. Amentiferae. This order comprises the catkin-bearing families. The plants are mostly shrubs or trees with minute flowers in pendulous catkins or heads. The flowers are usually unisexual, the male and female being on the same or different plants. The perianth is usually wanting, and pollination is effected by wind. Examples are plane, poplar, willow, birch, alder, hazel, oak, elm, beech, sweet chestnut, stinging nettle, and hop.

Order 8. Umbellales. Flowers perigynous or epigynous. The carrot family (Umbelliferae): the small flowers are arranged in umbels, i.e. a number of flower stalks radiate from the same point, and in this way the flowers are rendered conspicuous to nectar-seeking insects for pollination.

Examples are carrot, parsnip, parsley, celery, hemlock, fennel, goutweed, caraway (whose fruits are commonly called caraway seeds), and angelica (long cultivated for use in confectionery). The ivy belongs to another family (Araliaceae) of this order.

Division II. *Sympetalae*

This division contains plants whose petals are united in a tube to form a corolla.

Order 1. Ericales. Flowers hypogynous. Carpels three to ten, united. The heath family (Ericaceae): evergreens with exstipulate leaves. Flowers nearly or quite regular. This family includes all kinds of heather, rhododendrons, and azaleas. Near allies are cranberry and bilberry.

Order 2. Gentianales. Often trees or woody shrubs. Flowers hypogynous.

(*a*) The olive family (Oleaceae): to this family belong the olive tree (of value in yielding olive oil), the ash, and several ornamental shrubs, e.g. jasmine, forsythia, privet, and lilac.

(*b*) The periwinkle family (Apocynaceae): with the exception of the periwinkle, the members of this family are mainly tropical plants.

Order 3. Rubiales. Leaves opposite, usually with stipules. Stamens growing on the corolla tube. Flowers epigynous.

(*a*) The goosegrass family (Rubiaceae): here belong the goosegrass (*Galium aparine*), madder (once important as the source of a red dye), and plants which yield quinine, coffee, and ipecacuanha.

(*b*) The honeysuckle family (Caprifoliaceae). Many ornamental shrubs belong to this family in addition to the sweet-smelling honeysuckle (*Lonicera*), e.g. elder, snowberry, weigelia, and guelder rose.

Order 4. Campanulales. Leaves exstipulate; the flowers are small but, being aggregated together in large numbers, conspicuous. They are epigynous, and the inferior ovary contains only one ovule. The flower heads are usually surrounded by a sheath (*involucre*) of bracts. The daisy family (Compositae): what looks at first sight like a single flower in a member of this family—and entire order—really consists of a number of very small flowers (*florets*) crowded together into a head, round which is a protective covering of bracts likely to be mistaken for the calyx.

The parts of the flower are in fives, except the carpels, of which there are two, united. The stamens are attached to the corolla, and their anthers united into a tube, which surrounds the style. The calyx, sometimes absent, is often represented in the form of a ring of hairs.

Numerous common plants belong to this large family, e.g. sunflower, aster, dahlia, cineraria, chrysanthemum, dandelion, all kinds of daisy, groundsel, thistles, coltsfoot, and hawkweeds. Useful products are obtained from such members of the family as the globe and Jerusalem artichokes, chicory, and lettuce.

Order 5. Primulales. Flowers hypogynous, regular, with five stamens opposite the petals to which they are attached. Carpels, five, joined. The superior ovary has only one cavity, and the ovules are attached to a vertical projection within this. The primrose family (Primulaceae): here belong the primrose and its many varieties in horticulture, cowslip, and cyclamen. Sea-thrift and sea-lavender belong to a nearly related family (Plumbaginaceae).

Order 6. Tubiflorales. Flowers hypogynous. Sepals and petals four to five, stamens two to five, pistil of two to five united carpels. The superior ovary contains two or more compartments.

(*a*) The borage family (Boraginaceae): to this family belong borage, anchusa, comfrey, and forget-me-not.

(*b*) The potato family (Solanaceae): flowers regular. The potato plant (*Solanum tuberosum*) and the tobacco plant (*Nicotiana*) are of importance, also tomato and capsicum. Drugs are obtained from the poisonous henbane (*Hyoscyamus niger*) and deadly nightshade (*Atropa belladonna*). Petunias and salpiglossis also belong here.

(*c*) The foxglove family (Scrophulariaceae): flowers generally irregular and stamens four, sometimes two or five. There are two compartments in the ovary, and many small albuminous seeds. The mulleins (*Verbascum*) possess nearly regular flowers with five stamens, generally of yellow colour arranged in tall handsome

spikes. The conspicuous flowers of the foxglove (*Digitalis purpurea*) are arranged along one side of the main axis. The blue speedwells (*Veronica*) have only four petals and two stamens. Toadflax (*Linaria*) and snapdragon (*Antirrhinum*) have irregular flowers; those of snapdragon have to be forced open by bees in search of the nectar within.

Order 7. Labiatales. One axis of symmetry: leaves exstipulate and usually opposite. Stems four-sided. Flowers markedly irregular, with five sepals, five petals, four stamens (usually), and two united carpels. The ovary is deeply divided into four. Seeds exalbuminous. Includes dead-nettle family (Labiatae). The white and purple dead-nettles (*Lamium album* and *purpureum*) and ground-ivy (*Glechoma hederacea*) are common types. Many members of the order are aromatic, and yield essential oils. Among these are mint (*Mentha*), sage (*Salvia*), thyme (*Thymus*), marjoram (*Origanum*), and basil (*Ocymum*).

CLASS II. MONOCOTYLEDONS

Order 1. Helobieae. Aquatic forms, in which the stamens and carpels are commonly more numerous than in other monocotyledons. The ovary is usually superior. Examples from the families in this order are water-thyme (*Elodea*), water-soldier (*Stratiotes*), frogbit (*Hydrocharis*), flowering rush (*Butomus*), arrowhead (*Sagittaria*), and pondweed (*Potamogeton*).

ARROWHEAD. Leaves and flowers of Sagittaria sagittifolia, a monocotyledonous pond plant. It is native to England.

Order 2. Glumales. Inconspicuous flowers in which the perianth is absent or reduced. Groups of them are enclosed in and protected by scaly bracts.

(*a*) The grass family (Gramineae): herbs with hollow-jointed stems and alternate narrow leaves, the bases of which are in the form of sheaths grasping the stem. There is a little membranous outgrowth (*ligule*) at the junction of blade and sheath. Stamens usually three, and one carpel with two feathery stigmas. The superior ovary contains a single ovule. The seeds are albuminous. All grasses are included here, as well as the cereal crops, wheat, barley, and oats, together with the bamboo and sugar cane.

(*b*) The sedge family (Cyperaceae): these differ from grasses in the possession of solid stems, while the leaf has no ligule. The members of the family generally abound in marshy places, and are common on the margins of streams, ponds, and lakes.

Order 3. Palmales. The stamens and carpels are generally in separate flowers, which are small and crowded together on fleshy axes and protected by a covering sheath (spathe). Ovary superior.

The palm family (Palmaceae) are large woody plants, nearly all of which are tropical. The large leaves commonly form a tuft on the top of a long cylindrical stem. The flowers, in large clusters, are individually small, usually white, greenish, or yellow; stamens six; carpels three, joined. There are about 1,500 species, many of economic importance. Products include edible fruit from the date palm; oil from fruit of the oil palm; sago and starch from the sago palm; vegetable ivory from nuts of the ivory palm; copra (dried kernel of the nut) from the coconut palm.

Order 4. Arales. Arum family (Araceae): mostly tropical plants, with minute flowers borne on a fleshy, brightly coloured axis (spadix) and protected by a spathe, which is often showy. The so-called arum "lily" is an example with white spathe and deep yellow spadix. The native British species, commonly called wake-robin, lords-and-ladies, and cuckoo-pint (*Arum maculatum*), has a green spathe and

MONOCOTYLEDONOUS FRUIT TREES. Left, date palm, Phoenix dactylifera, native to North Africa. The tree attains a height of over 100 feet, and the evergreen leaves are up to 18 feet long. The edible dates hang in immense clusters each weighing from 10 to 40 lb. Right, leaves and fruit of the banana, Musa sapientum. It attains a height of over 12 feet, and the stems are herbaceous.


TWO MONOCOTYLEDONOUS PLANTS. Left, reed mace, commonly but erroneously called bulrush, an aquatic perennial, bearing brown inflorescences. Right, bee orchis, a British orchid.

a dark red or a cream spadix and in autumn a spike of brilliant red berries. The reed mace family can also be placed in this order.

Order 5. Lileales. Here the flowers are regular with parts in three, and the carpels are united. In most instances both sepals and petals are either white or brightly coloured. The seeds are albuminous. There is generally a thickened underground stem.

(*a*) The lily family (Liliaceae): stamens six in number and ovary superior. The order includes many (*Tulipa*), squill (*Scilla*), hyacinth (*Hyacinthus*), lily ornamental plants, such as lily (*Lilium*), tulip of the valley (*Convallaria*), and red-hot poker (*Kniphofia*); also asparagus, onion garlic, leek, and shallot (species of *Allium*). New Zealand flax (*Phormium tenax*) is of considerable economic importance. Nearly all members of the order are herbs, but there are exceptions, such as yucca, and the dragon-tree (*Dracaena*) of the Canary Islands, which grows to an enormous size and is very long-lived.

(*b*) The rush family (Juncaceae): plants with narrow leaves and small brown inconspicuous flowers resembling in structure those of the preceding order. Rushes (*Juncus*) and wood-rushes (*Luzula*) are included in this family.

(*c*) The snowdrop family (Amaryllidaceae): as Liliaceae, but with an inferior ovary. Snowdrop (*Galanthus nivalis*) and daffodil (*Narcissus*) are examples.

(*d*) The iris family (Iridaceae): as last family, but only three stamens. Iris, crocus, gladiolus, and saffron (*Colchicum*) are examples.

Order 6. Scitaminales. Tropical plants with large leaves and irregular flowers, of which the inferior ovary is generally divided into three compartments. Belonging to this group are the following: banana (*Musa sapientum*), plantain (*M. paradisiaca*), ginger (*Zingiber*), Indian shot (*Canna*), and arrowroot (*Maranta*).

Order 7. Orchidales. Flowers zygomorphic and of remarkable form. The stamens are reduced in number and united with the pistil. The orchid family (Orchidaceae): this is the second largest family of seed-plants, excelled in size only by the Compositae, and including over 7,500 known species. There is usually only one stamen present.

PRACTICAL WORK

In relation to this and the preceding Lesson the student would do well to obtain a simple flora with a key to the identification of wild flowers, and then collect as many specimens as possible; with the aid of the key and a hand lens he can track down the specific name of the flower.

LESSON 23

Water Plants

PLANTS which grow submerged or partially submerged in water are referred to as *hydrophytes*. Examples are water lilies, water buttercup, arrowhead, duckweed, water soldier, frogbit, and bladderwort. This Lesson is concerned with the aquatic flowering plants. The Algae, which are simpler, and often unicellular, water plants, will be considered separately.

Fixed and Free-floating Aquatics

In streams the rate of flow markedly affects the character of the flora. Where the current is rapid, all the forms are fixed by their roots in the bed of the stream. In quiet water free-floating (i.e. not fixed) aquatics occur. Some species that grow above the surface in quiet water are completely submerged in a rapid current.

Because water gives support, aquatic plants seldom have an extensively developed vascular system. The chief strain to which their stems and leaf stalks are subjected is a longitudinal pull due to currents, and the small amount of mechanical tissue which the aquatic possesses is usually arranged as a central core (as in the root of a land plant).

The plants do not depend on their roots for the absorption of water and mineral salts. The roots serve chiefly for attachment, and absorption is carried on over the whole of the submerged surface. This is a second reason for the lack of woody tissue.

Large Air Chambers

The oxygen essential for respiration is present in very small amounts in solution in water, but carbon dioxide, which dissolves readily in water, is abundant. There are no pores on the submerged parts of the plants, so that all gaseous

exchange must take place by diffusion through the epidermal cells. The cuticle in aquatics is therefore very thin, or absent. Very thin foliage leaves, or extreme division of their blades, creates a large absorptive surface.

Because of the relatively large amount of carbon dioxide, photosynthesis takes place readily, and the oxygen which is set free in this process is stored up within the plant for use in respiration. This reserve of oxygen is held in the numerous large air spaces which traverse all parts of the aquatic plant ; the air chambers also render the plant buoyant. A similar system of air passages is present in the underground organs of marsh and swamp plants.

Submerged aquatics are exposed to very reduced illumination, and therefore their leaves (the assimilatory organs) need to be as near to the surface as possible. It is the light which determines the depth at which submerged plants can grow.

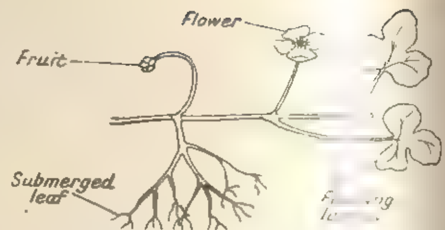
Floating and Submerged Leaves

It has already been mentioned that the submerged foliage often consists of finely divided leaves, as in the water buttercup (*Ranunculus aquatilis*). Such deep division has also a mechanical advantage, in that water flows easily between the segments. The pondweed (*Potamogeton crispus*) has long, undivided leaves which trail out with the current.

The water buttercup illustrates the general rule that when floating leaves are produced they are quite different in form from the submerged leaves. The yellow water lily (*Nuphar lutea*) is an exception to this ; its floating leaves and submerged leaves are similar in form, but the submerged ones are very much thinner. Floating leaves often have an entire edge, with the petiole attached more or less centrally, so that the pull of the leaf stalk keeps the blade flat on the water ; and, unlike submerged leaves, they bear stomata. The latter are confined to the upper surface, which usually has a covering of wax (leaving the pores free).

Peculiarities

Some species have no roots at all, e.g. bladderwort (*Utricularia*). In free-floating forms, such as duckweed (*Lemna*), the roots dangle in the water and not only absorb but also



WATER BUTTERCUP. Diagram showing the contrasting forms of floating and submerged leaves of Ranunculus aquatilis.

act like a weighted keel to keep the minute, flat surface of the plant balanced.

The abundant supply of water and often of mineral salts as well enables aquatic plants to grow rapidly, and vegetative reproduction is generally a marked feature. A simple method is that of the Canadian pondweed (*Elodea canadensis*) ; pieces detached by accident grow on as individual plants. The long slender, and brittle stems grow entirely submerged and send out roots at almost every joint. The plant was brought from Canada to Britain in about 1840 and it increased so rapidly that its spread was phenomenal. It soon became common in ponds and streams all over the country, and it was particularly troublesome in canals. The minute greenish-purple flowers of *E. canadensis* float on the surface and are both male and female. Some species, such as frogbit (*Hydrocharis*), produce winter buds which, when the parent plant dies, sink to the bottom of the water and give rise to new plants in the following spring.

HIBERNATING BUDS. The frogbit, a floating plant, has long stalks which develop winter buds. These break off and hibernate at the bottom of the pool, while the surface plant perishes. Two buds are shown detached.

Submerged Flowers

Flowers are, in the majority of instances, borne above the water level. They may be pollinated by insects, e.g. water buttercup, or by wind, e.g. water milfoil (*Myriophyllum*). Usually the flower stalks bend after flowering is over and carry the ripening fruits under water. A few aquatics such as eel-grass (*Zostera*) produce submerged flowers, and pollen is conveyed to their stigmas by currents of water. To meet the uncertainty of this method of pollination, a large amount of pollen is produced, as with those land plants which are pollinated by wind.

LESSON 24

Plants of Dry Places

PLANTS which inhabit dry situations—heaths, shingle banks, sand dunes, and deserts—are referred to as *xerophytes*. The medium in which they grow retains very little water; it is often so loose that its surface is frequently shifted by the wind, and organic matter is scarce.

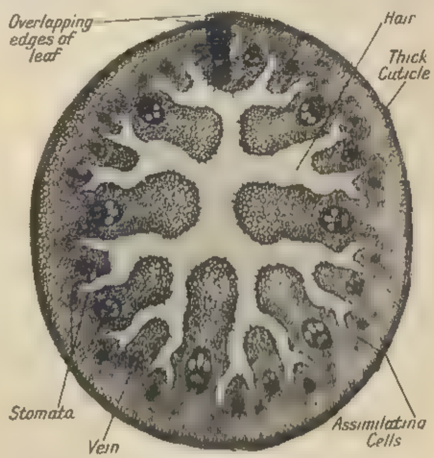
The plants must obtain water from below and retain as much of it as possible. Deep roots are an obvious advantage. With sufficient depth of root a plant can remain fixed even if surface layers of sand are blown away; and at a depth of a few feet the roots will probably come to soil in which there is moisture. Some transpiration must take place, so that use can be made of the water which is absorbed; but transpiration must be reduced to a minimum. The development of special water-storage tissue is another advantage; the plant with reserve supplies is better able to withstand long periods of drought.

Marram Grass

All plants that thrive in dry habitats do not exhibit the same adaptations for water conservation. Marram grass (*Ammophila arenaria*) is an example of a xerophyte with deep underground stems. The plant abounds on sand dunes, and it is often planted to bind together loose shifting sand by means of its long rhizomes and roots. The leaves are about three feet long, with very sharp and hard tips, which can force their way up, without injury, through sand if they become buried. The aerial stems also force their way up, and they frequently send out new rhizomes a few inches below the new surface. This extensive underground system of rhizomes bearing roots is sometimes revealed when a strong wind blows away a lot of the sand. Marram grass rhizomes are sometimes over 200 feet in length.

The leaves of marram grass reduce transpiration in the following manner. In damp weather the blades are flat, like those of any ordinary grass, but in dry weather the edges roll together, so that the leaf appears as a long narrow cylinder. The leaf bears its stomata (through which transpiration takes place) on one surface only, and this surface is on the inside of the "cylinder"; it is deeply grooved, and the stomata are further protected from currents of air by being near the bottom of the grooves and by hairs growing from the intervening ridges. The curling up of the leaf in dry weather and its flattening in moist conditions are caused by changes in turgidity of the thin-walled *hinge cells* situated at the bottom of the grooves. The surface which is exposed to the atmosphere when the leaf is rolled has a very thick cuticle.

Many xerophytes have more or less cylinder-shaped leaves (either permanently or temporarily), e.g. some cacti and other succulents, and plants growing on exposed heaths, e.g. species of *Erica*. Common heather or ling



MARRAM GRASS. Photomicrograph of a rolled leaf of this xerophyte. In dry weather the edges roll together, so that the leaf appears as a long narrow cylinder. Transpiration is reduced in this manner. Details are given in the text.

(*Calluna vulgaris*), has very small leaves which are V-shaped in cross-section and are arranged in four vertical rows on the stem. In dry weather they fold in tightly one upon the other, and thus each is protected by its neighbour immediately below.

Gorse and Broom

Furze or gorse (*Ulex*) is abundant on heaths and commons. It is enabled to withstand dry conditions by having very small leaves, many of which are modified into hard, dry spines. When leaf area is reduced, it follows that the assimilating area is also diminished. To compensate for this, the younger parts of the gorse stem are green and thus help with assimilation. The first foliage leaves produced by the young gorse plant resemble clover leaves. This suggests that gorse once had compound leaves, and its success in dry habitats is due to the suppression of its transpiring organs. In broom (*Cytisus scoparius*) the leaves are very small. The stems are the main assimilating organs and are deeply grooved. The stomata lie in the grooves. Plants

SUCCULENT PLANTS. Water storage tissue is highly developed in desert plants such as cacti. Two examples are shown. In many species the plants bear sharp spines, as a deterrent to browsing animals.

with this structure are called *switch-plants*.

Some species, e.g. butcher's broom (*Ruscus aculeatus*), do not possess leaves but some of the stems are flattened and leaf-like. The true nature of these flattened stems, or *cladodes*, is revealed by the fact that they occur in the axils of scale-leaves and bear flowers in the middle of their upper surfaces.

Phyllodes

A widespread modification among the acacias of Africa and Australia is the formation of a leaf-like petiole. This is called a *phyllode*, and often the first leaves of the plant have the normal form and a transition series exists to the mature phyllodes. The asparagus grown in greenhouses as a decorative "fern" has, apparently, needle-like leaves, but on close inspection these are seen to arise in the axils of scale leaves.

Bulbous plants which are characteristic of semi-arid regions, and from which the general run of garden bulbs are derived, are adapted to survive a long dry summer by completing the major part of their life-cycle in the spring or early summer and then shedding their leaves and remaining dormant until the next rainy season or early spring.

Broad-leaved plants which occur in dry situations have various devices for checking transpiration; the leaves of the horned poppy (*Glaucium luteum*) are covered with a felt of hairs; those of sea holly (*Eryngium maritimum*) have a coating of wax; others have very thick cuticles. These plants grow on shingly beaches and sand dunes.

A CACTUS FLOWER contrasts strikingly with the thick, succulent stems of this xerophyte.

Various parts of plants may be adapted for water-storage. In some, e.g. species of *Tradescantia*, the epidermal cells are large; in some species of *Ficus* the epidermis is several layers thick. All the cells contain abundant water. The development of special water-storage tissue, resulting in succulent fleshy leaves or stems, is common among xerophytes. Succulence is most highly developed in the cactuses of America and the giant *Euphorbias* and *Stapelias* of Africa. These have succulent stems. In species of *Opuntia* the leaves form small blunt projections, which fall off, and then spines appear, which may be modified leaves of side shoots. Frequently a waxy coating is combined with the succulent habit. The most common British xerophytes showing succulence are the stonecrops (*Sedum*) and the house-leeks (*Sempervivum*), often seen growing on rockeries, walls, and roofs.

STONECROP. A common xerophyte, the leaves of which are succulent.

Some plants which do not grow in physically dry habitats have xerophytic characters. These are called *xeromorphs*. Conifers are an example. Plants of salt marshes and bogs are often xeromorphic; the water is not available to them because of the dissolved salts and humic acids. They are said to be in a state of physiological drought as opposed to true xerophytes which are in a state of physical drought.

Examples of salt-marsh plants are glasswort (*Salicornia*), cord grass (*Spartina*), sea-lavender (*Statice*), and sea-plantain (*Plantago maritima*). Salt marshes are areas which are periodically inundated by the tides and are left as large tracts of mud intersected by water channels and pools. There is no shortage of water, but the content of sodium chloride is very high. If the plants were to absorb large quantities of sea water and transpire rapidly, great accumulation of salt would occur in the cells and be injurious to the plant tissues.

PRACTICAL WORK

The student should note the different types of plant growing in different habitats. Uprooting one or two examples, he should note whether they are shallow- or deep-rooted. Observing the general mode of growth, and particularly the leaf characters, he should try to see how the plants are adapted to their environment. He should study the soil character, noting whether it is chiefly clay, sand, gravel, or chalk, etc. A list of the plants found should be made and compared with the flora of other types of soil.

BERRIES OF THE COUNTRYSIDE

MORE BERRIES OF HEDGE AND GARDEN

To face page 1569, Vol. 3

LESSON 25

Classification of Gymnosperms

ALL living seed plants are grouped in the Phanerogamae, which is subdivided into Angiospermae and Gymnospermae, according to the way in which the ovules are born. As previously mentioned, gymnosperms are naked-seeded plants, that is, the ovules are not borne within an ovary but are naked on a scale. Gymnosperms include the following four groups.

Group 1. Ginkgoales, represented by only one living species, the maiden-hair tree (*Ginkgo biloba*), native to eastern Asia.

Group 2. Cycadales, the cycads, a small group of comparatively lowly forms, which in past geological times were very numerous and widely distributed. They are limited to the hotter parts of the globe, especially Central America and Australia. A cycad somewhat resembles a palm in appearance, but has usually a much shorter trunk.

Group 3. Gnetales, a small group of only three genera, which differ greatly from one another in general appearance and are limited to the hotter parts of the globe. *Welwitschia*, which occurs in the deserts of south-western Africa, somewhat resembles a giant radish in having a swollen underground hypocotyl, which may exceed four yards in circumference. The plant bears only two leaves, which are over two yards long and lie flat along the surface of the soil. *Ephedra* may be described as a switch plant, having thin, long-jointed green stems and minute scaly leaves. It occurs in the deserts of western Asia. Members of the third genus, *Gnetum*, are mostly climbers (lianas) of tropical Asia and America; their leaves resemble those of angiosperms in being broad with a network arrangement of the veins. This entire group shows an approach to the angiosperms in the fact that their small flowers have a perianth, which is absent from other gymnosperms.

Group 4. Coniferales, the cone-bearing trees. The great majority of gymnosperms belong here, examples including the monkey puzzle (*Araucaria*), pines (*Pinus*), firs (*Abies*), spruce (*Picea*), cedar (*Cedrus*), larch (*Larix*), cypress (*Cupressus*), juniper (*Juniperus*), and yew (*Taxus*). Some of them are forest trees of considerable size, and in some parts of the world they cover large areas, as in the cooler parts of the northern hemisphere. Among them are the gigantic Wellingtonia (*Sequoia gigantea*) and redwood (*S. sempervirens*) of North America. The former may reach a height of over 350 ft. with a trunk 112 ft. in circumference. Many conifers are of great economic value, furnishing "soft wood" timbers, turpentine, resin, etc. There are about 350 known species, of which more than a fifth belong to the genus *Pinus*.

All the members of Group 4 are woody, and the majority are evergreen trees, with leaves which persist for three or four years. The larch is deciduous, i.e. it is bare of leaves in winter; so also is *Metasequoia glyptostroboides*. The latter was known only as a fossil until 1947, when a few living specimens were discovered in China; seeds were brought to Britain, and numerous young specimens are now growing in gardens. The most distinctive external feature of the group is the foliage, consisting generally of small "needles." The simple flowers are of two kinds, male and female, which in nearly all instances are in the form of cones and have no perianth.

Resin Canals in Conifer Stems

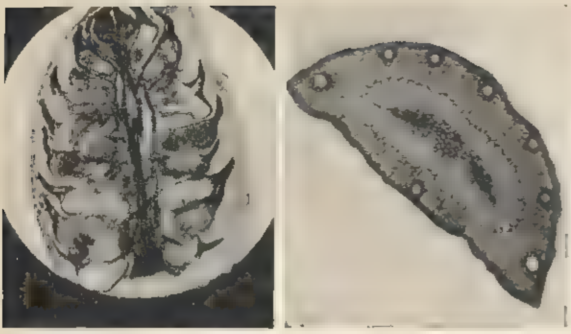
The general anatomy of stem and root of the conifers shows close agreement with that of the dicotyledons. A striking feature in the cortex of the stem of a conifer is a ring of large *resin canals*. Each canal is an intercellular space surrounded by small cells, rich in protoplasm, which secrete the resin into the canal. These canals are in all parts of the plant (except in yew, from which they are absent). The resin is a protection to the plant against animals; it renders the young twigs distasteful.

A detailed examination of the wood and bark shows some marked differences in structure between the conifers and angiosperms. For example, in the wood of the conifers there are no conducting vessels, but conduction of water, etc., is performed by single elongated cells called *tracheids*. Tracheids have holes or pits in their walls through which water passes from one to another. This and other structural differences show that Group 4 is fundamentally distinct from dicotyledons.

Transfusion Tissue

The leaves differ greatly from typical leaves of dicotyledons, being usually of simple structure and often traversed by only one small vein. The needles show characters which are associated with reducing transpiration, e.g. a thick cuticle and protected stomata. The lack of side veins is compensated for by a transfusion tissue which surrounds the vein and is a characteristic feature of the conifers. In several

SCOTS PINE (PINUS SYLVESTRIS). Left, cluster of young female cones, and young dwarf shoots (in centre). Right, young dwarf shoots and male cone, and needle leaves of previous year's growth. See also diagram in next page.



CONE AND LEAF. Left, longitudinal section of male fir-cone. The pollen sacs, which lie beneath the scales, are shown, some full of pollen, some empty. Right, transverse section of leaf; it contains a ring of resin-ducts in the exterior rim.

Photos, H. S. Cheavin

common genera (e.g. *Pinus*, *Cedrus*, *Larix*), the needles are borne on special *dwarf shoots*, which arise in the axils of scale leaves on the ordinary long shoots, and bear a few brown scales below and a variable number of foliage leaves above. In the pines the entire dwarf shoot falls from the tree when the leaves are shed.

Scots Pine Cones

It is in the flowers and the processes of pollination and fertilization that the gymnosperms show the most marked differences from the dicotyledons. For example, in the Scots pine (*Pinus sylvestris*) the male and female flowers are borne in separate cones, but both sexes occur on the same plant. The male cones are small yellow oval structures, and the central axis of each has numerous scales. On the under side of each scale are two large pollen sacs which, when mature, split widely open to liberate the pollen, which is distributed by wind. Each pollen grain is provided with two bladder-like expansions of the cell wall, which act as wings and so enable the pollen to be carried for long distances.

Ovuliferous Scales

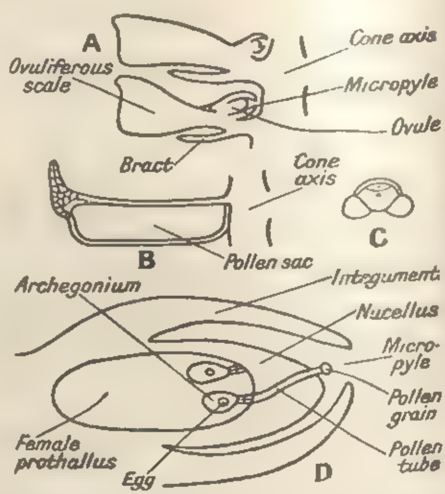
The female cones of the Scots pine are developed laterally in the axils of scale leaves, each on a short stalk. The central axis of the cone bears the *ovuliferous scales*, each of which has two ovules on its upper surface near to the cone axis. Beneath each ovule-bearing scale is a small sterile bract.

Each ovule lies with its micropyle towards the cone axis, is surrounded by a single protective coat or integument, and contains a large embryo sac. At the time of pollination, the scales of the female cones are slightly separated from one another, so that the pollen grains can be blown between the scales and reach the micropyle. A small quantity of liquid is secreted just within the micropyle in which the pollen grains are entangled. A long interval occurs between pollination and fertilization; in the pines pollination takes place in May of one year, but fertilization is not effected until June of the following year. Pronounced growth of the female cones occurs in this interval, with complete development of the ovules.

Cones and Seed

When ready for fertilization, the ovule presents a very different structure from that seen in angiosperms. The large embryo sac is filled with tissue rich in food material. This feature is characteristic of all gymnosperms, and is one of the most important distinctions between this class and all other flowering plants. Embedded in this tissue (the female *prothallus*) at the micropylar end are a few flask-shaped organs called *archegonia*, in which the egg nuclei are held.

A short time before fertilization the pollen grains germinate and put out long tubes which grow down to the archegonia. The nucleus of the pollen grain has undergone several divisions, its most important products being the two male nuclei, one of which fertilizes the egg nucleus in the archegonium, while the other degenerates. The outcome of fertilization is the development of a young embryo within the ripening seeds; the entire cones undergo enlargement, becoming very woody; when ripe, the cone-scales gape apart to set free the winged seeds, which are dispersed by wind.



PINUS SYLVESTRIS: REPRODUCTIVE SYSTEM. A, part of longitudinal section of female cone. B, single scale of male cone. C, winged pollen grain. D, ovule contained in female cone.

The gymnosperms are interesting because they show a transition between the methods of fertilization of other phanerogams and those in the lower types of plant. In the flowering plants the male nuclei are conveyed right to the egg nuclei by the pollination tubes, whereas in the "lower" groups the male nuclei have to reach the female nuclei by their own mobility. Pollen tubes are produced in gymnosperms from the pollen grains, but the male nuclei have to complete their journey to the egg nucleus by their own movements. They are provided with thread-like appendages called *cilia*, and by moving these threads they swim to the archegonia in a slimy fluid which is secreted by the female organ. Such motile gametes are called *spermatozoids*, and fertilization is dependent on the presence of a certain amount of liquid. In the conifers, which have no necessity for the presence of liquid at fertilization, can be seen the loss of the last traces of the probable aquatic ancestry of the vegetable kingdom.

LESSON 26

Short Study of the Algae

THE group of "lowest" plants, Thallophyta, comprises an enormous number of species. The plant body is called a *thallus*: it is an expansion which is not divided into root, stem, and leaf. Thallophytes are conveniently divided into three great groups—algae, or green thallophytes; fungi, or non-green thallophytes; and lichens, plants of a mixed nature consisting of an alga and a fungus living together.

Though all green thallophytes, or algae, have the green colouring matter of chlorophyll, this is in many instances obscured by the presence of brown or red pigment. The group therefore may be conveniently divided into the green, brown, and red algae, according to which pigment is dominant. The seaweeds which cover the rocks between tidemarks are the brown algae. The red algae are the red seaweeds which frequent deeper waters. Green algae are abundant in fresh water, and many of them can thrive on any damp surface, such as soil, rocks, tree-trunks, and wooden fences. Generally speaking, the larger and more complex forms are marine; the fresh-water and terrestrial representatives are smaller and simpler. Among the brown seaweeds there are forms which rank with the most gigantic members of the vegetable kingdom, one (*Macrocystis pyrifera*), native to non-tropical southern seas, attaining the length of several hundred feet. In contrast, other algae are invisible as individuals to the naked eye. The whole plant body of these minute forms consists of a single isolated cell. Two types will be considered as examples of these very simple unicellular plants, both belonging to the green algae.

Green Substances on Tree Trunks

Frequently the trunks of trees, palings, etc., are covered by a bright green, powdery layer, which is particularly conspicuous in damp winter weather. This green substance is chiefly made up of vast numbers of the unicellular alga *Pleurococcus viridis*. Each plant is a single rounded cell, bounded by a thin wall and containing dense protoplasm, a nucleus, and a large chloroplast. The plant reproduces itself by simple division of the original cell, and the resulting cells often remain densely grouped together in clusters for some time. On account of its chlorophyll it is able to form starch in the presence of sunlight; and therefore, given adequate water supply, this single-celled organism is quite capable of leading an independent life.

Bright Green Water

The second example is a plant which passes its ordinary vegetative life in a state of active movement—the green alga *Chlamydomonas*, which is found in ponds and puddles, often in such quantity as to give a bright green colour to the water, though the single cells are invisible to the naked eye, being only about $\frac{1}{50}$ millimetre long. The cells are usually oval and rather pointed at one end; to this end are attached two *flagella*, or fine threads of protoplasm, by means of which the plant swims about.

Within the cell wall at the pointed end there is a red spot called the *eye spot*, which is considered to be sensitive to light; possibly it influences the plant to swim to conditions of greatest illumination. Most of the cell is occupied by a cup-shaped chloroplast; in this is embedded a protein mass called a *pyrenoid* whose function is not definitely known, but around which starch is deposited as it is formed by the chloroplast. The cell contains a nucleus and two small vacuoles near the pointed end. The plant multiplies by the division of its contents into two, four, or eight exactly similar "daughter cells," which are set free by breakage of the cell wall.

In addition to this method, this very simple plant has a definite sexual method of reproduction, important differences being shown by different species. As before, the cell contents divide up, but this time to as many as 64 individual units which, except for their smaller

size, are very similar to the vegetative cell. They swim about by means of their flagella, and when two meet they become entangled and ultimately fuse together. These cells which fuse in pairs are called the *gametes*; and the cell which results from their fusion is the *zygote*. The latter forms a thick wall around itself and can resist cold or drought. When conditions are suitable, its contents divide to form two, four, or more, new individual *Chlamydomonas* plants.

In some species the gametes which fuse are all of similar size; in others, e.g. *C. Braunii*, there is a definite size difference; a large and a small cell always fuse together, the contents of the smaller one passing into the larger. The smaller, more active, is the male element, and the larger receptive is the female.

Mixture of Plant and Animal Characters

It is not easy to draw a hard-and-fast line between the motile unicellular algae such as *Chlamydomonas* and a somewhat similar group of flagellate Protozoa. Some Protozoa show wholly animal characteristics, others are a mixture of plant and animal characters. Such forms probably indicate that plants and animals had a common origin in the very remote past, and their common ancestors were possibly very similar to the flagellate forms which abound in water to-day.

According to this suggestion, plant life began in the water, and there passed through the earliest stages in its evolution, acquiring the essential green pigment and developing forms of many different kinds, all minute, free-swimming, and relatively simple in structure. *Chlamydomonas* may be a present-day survivor of this stage. Consideration of plants which come higher in the evolutionary scale shows that many of them reflect the habits of their aquatic ancestors in that the presence of water is essential to fertilization.

Spirogyra

Some threadlike green algae live in ditches and ponds, some attached by one end, others floating freely. One of the commonest of the floating algae is *Spirogyra*, which consists of a row of cylindrical cells each bounded by a cell wall. The whole filament is enclosed in a sheath of mucilage, which makes the alga feel slimy to the touch. Each cell is lined with protoplasm, and the nucleus is suspended by delicate strands of protoplasm in the centre. The chloroplast is in the shape of a spiral band, and has embedded in it many pyrenoids.

During spring and summer *Spirogyra* is constantly increasing in length by division of the constituent cells, and pieces are frequently separated to form new and distinct plants. On approach of autumn a sexual reproduction takes place, though with no structural difference between male and female cells. The process is called *conjugation*. Two filaments lie together, and their adjacent cells send out processes which fuse together at the tips and form a conjugating tube or canal between the two cells. Meanwhile the cell contents have rounded up to form the gametes. One of these passes through the connecting tube and fuses with the other to form a *resting spore*, or *zygospore*. This remains dormant through the winter and germinates in the following spring.

Desmids

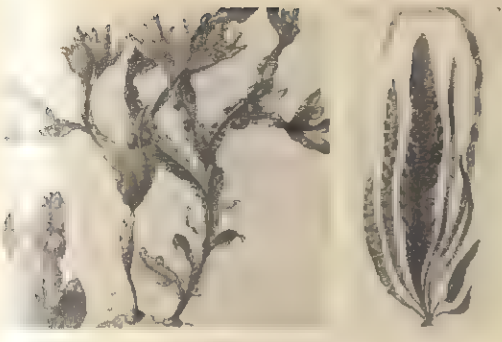
Mud from the bottom of a pond examined under a microscope will almost certainly be found to contain some of the remarkable little plants called *desmids*, each of which consists of a single cell. The desmids show great diversity of form; under the microscope some are very beautiful and some have a formidable exterior of spines and processes which serve as a means of defence against small aquatic animals. Some desmids have the power of movement. The usual method of multiplication is by cell-division, but occasionally the cells conjugate in pairs to form resting spores, which tide over the winter season like those of *Spirogyra*.

Bladder-wrack

The best-known brown algae are those which cover rocks between tidemarks and attract the attention of visitors to the seaside. The common name for them is "wrack," and there are numerous species. One of the most abundant is bladder-wrack (*Fucus vesiculosus*), the forking thallus of which is attached at one end and, when covered by the tide, is buoyed up by numerous air-containing swellings. If a small

TYPES OF ALGAE. Left, Spirogyra nitida (× 45); the zygospores, rugger ball in shape, have formed after conjugation between filaments lying parallel. These are later released and lie dormant through the winter, but in the spring they burst the cell wall and grow to form new plants. Right, Closterium (× 80), a desmid with smooth outline which lives in water at high levels.

Photomicrographs. H. S. Cheavin



TYPES OF SEAWEED. Left, bladder-wrack, Fucus vesiculosus, common on rocks between tidemarks. It is buoyed up by air-containing swellings. Right, Laminaria, larger and broader than bladder-wrack. Both much reduced.

piece of this or any other brown seaweed is placed in alcohol, the brown pigment will rapidly dissolve out, and the seaweed will appear greenish, i.e. the chlorophyll will become visible.

In winter and early spring, the tips of wrack swell up and assume a yellow or orange tint. If such a swelling is held up to the light, a number of little round dots of darker tint will be seen. Each of these is a pit, or *conceptacle*, lined with hairs, some of which are modified into egg-organs, sperm-organs, or both (according to the species). Each egg-organ is an ovoid body on a very short stalk, containing eight egg-cells. The sperm-organs are minute bladder-like structures borne on branched hairs, and giving rise to large numbers of extremely minute sperms. When a ripe egg-cell is liberated, numerous sperms are attracted to it, and one fuses with it, bringing about fertilization.

Seaside "Weather-Glasses"

Beginning near low-water mark and extending some distance into shallow water is the *laminaria zone*, so called after brown seaweeds of that name. They are larger and broader than the wracks, and the thallus is smooth or corrugated, according to the species. These plants are often taken home by seaside visitors to serve as "weather-glasses," because the salt which clings to them causes them to become damp on the approach of rain.

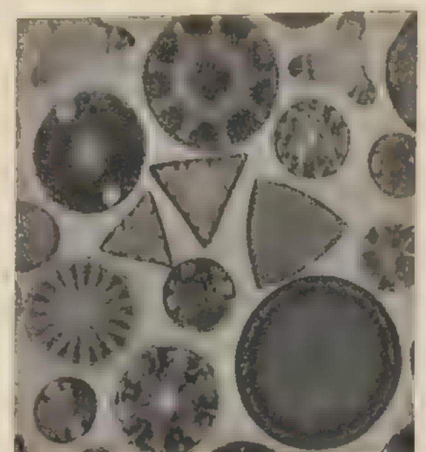
Large masses of seaweed drift about in the ocean, especially in the Sargasso Sea, a huge eddy occupying several thousand square miles of the North Atlantic. The most notable species there is the gulfweed (*Sargassum bacciferum*), which is buoyed up by stalked floats resembling berries in appearance. A huge brown seaweed (*Macrocystis pyrifera*), with pear-shaped floats, native to the non-tropical parts of southern seas, attains the length of several hundred feet. *Fucus* and *Laminaria* are used as manure on the land, and under the name of "kelp" were once used in the manufacture of potash; as a source of iodine they (especially *Laminaria*) are valuable.

Economic Applications

Techniques have been evolved for extracting from seaweed a group of chemicals called alginates; these are used in surgery for swabs and stitches, and also in food preparations, particularly in ice-cream and synthetic cream. Another substance obtained from seaweeds is rather like gelatine in consistency, and is called agar-agar; this is often used as a substitute for gelatine, and in the laboratory as a medium for cultivating bacteria and fungi.

Diatoms

The almost infinitely varied microscopic forms called *diatoms*, which have flinty coverings of great beauty and exquisite geometrical symmetry of pattern, are closely related to the brown seaweeds. They occur in both salt water and fresh water, and even on the surface of damp earth. Large tracts of the ocean floor, especially in the Antarctic regions, are covered with fine ooze principally composed of their remains. Some diatoms are stalked and immobile, but most are free and possess the power of movement. When examined under a microscope, they are seen gliding in a curious fashion. But there is no obvious mechanism causing movement.



DIATOMS. Microscopic forms, closely related to brown seaweed. The coverings show varied and intricate geometrical patterns; highly magnified.

Photomicrograph, John. J. Ward

The surface layers of the sea and of lakes are inhabited by myriads of diatoms, which constitute the chief food of many minute animals, especially the lowly relatives of shrimps and prawns. These creatures in their turn are devoured by herrings and other fish, so that man himself is indirectly indebted to diatoms for an important part of his diet. It has recently been shown that it is possible for a man to survive many weeks on a diet of nothing but *plankton*,

as these minute floating animals and plants are collectively called.

Large numbers of fossil diatoms are known. Not only are these minute plants actively engaged at the present time in forming deposits on the ocean beds, but similar *diatomaceous earths*, found associated with rocks of Tertiary age, are proof of their activities in former eras. Such earths are white or grey in colour, often so soft as to crumble readily in the fingers, and they are composed almost entirely of the flinty remains of diatoms. Some of the deposits are of economic importance, being used as polishing powders and in the manufacture of some dentifrices.

Red Seaweeds

Most red seaweeds grow at moderate depths in the sea, and are notable for beauty of form and colour. Many of them are torn from their moorings and cast up on the shore by storms. The reproductive processes are complex. Some of the red seaweeds are strengthened by calcareous matter, and these are represented on the British coasts by branching forms and pinkish crusts seen on rocks between the tide-marks. Carrageen "moss" (*Chondrus crispus*), a stoutly built, forking red seaweed, is used in much the same way as isinglass; carrageen broth and carrageen mould are delicacies. Laver (*Porphyra laciniata*) is another edible species.

Though the majority of the red algae are seaweeds, a few genera occur only in freshwater streams. These do not usually have the striking red colour of the marine forms, but are nearly black or even green.

Principal Vegetation of Hot Springs

Blue-green algae are of very simple structure and resemble the others in their habit and mode of life, but their real relationships with the algae group are still open to doubt. As their name indicates, they have characteristic colouring. Some are terrestrial, others occur in the sea and in fresh water. These plants must rank amongst the simplest members of the vegetable world, the only others which are equally simple being the *bacteria*. The unicellular forms usually occur as colonies united by a mass of mucilage; a large number of forms have the habit of simple or branched filaments also surrounded by mucilage. The cells have no well-defined nucleus, and sexual reproductive organs are absent. The blue-green algae have a very marked power of existence under adverse conditions. They form the principal vegetation of hot springs, and they flourish in Antarctic lakes. Certain algae are active in rock formation, precipitating carbonate of lime from the water in which they live.

Plant Plankton

It is now commonly supposed that plant life began in the sea, and there went through the earliest stages of evolution. Many different minute and simple forms, all containing chlorophyll, were developed, and all were free to swim or float. Such free microscopic plants are known collectively as plant plankton. *Chlamydomonas* (*see* earlier in this Lesson) serves to illustrate the kind of plants they were, not far removed from organisms with animal characteristics, and yet with green pigment and cell walls.

In the earliest phases of the plankton period there was probably no distinction into plant and animal kingdoms, but in the later plankton phase organisms with plant or animal characteristics had been evolved and were present in the seas together. How long the primitive world of life consisted only of plankton cannot possibly be known, but it must have been for a very long period. In time, the plant world developed a new kind of organism. Some of the primitive plankton plants gave up their free-swimming mode of life and became fixed organisms, or *benthos*, with a specially developed hold-fast at their lower ends which anchored them firmly to the rocks or ooze.

The free end of such a plant grew out into a thallus of threads, or a mass of tissue composed of many cells; in some instances it became differentiated into an axis bearing lateral expansions, perhaps the first foreshadowings of stem and leaves, which later became fixed in the higher plants. This phase in which plankton and benthos shared the marine world must also have lasted for a long period, until land surfaces appeared for plants to inhabit. As portions of the earth's crust arose from the water, plants which could adapt themselves to the drier conditions flourished and became the forerunners of present-day vegetation on the land.

CARRAGEEN "MOSS," Chondrus crispus (left.) This is a stoutly built red seaweed and is shown about actual size. Right, a filamentous blue-green algae (magnified approx. 90 times).

LESSON 27

Fungi

THE fungi group is a subdivision of the Thallophyta. It contains an enormous number of forms, and is by far the largest group of the non-flowering plants. About 50,000 species have been described. All are distinguished by the absence of chlorophyll ; hence all fungi are incapable of assimilating the carbon dioxide of the air and must obtain their carbonaceous food ready-made from other sources : the parasitic fungi from living organisms, the saprophytic forms from dead organic matter.

In addition to the fungi of essentially parasitic and saprophytic habit (so-called *obligate* parasites or saprophytes), there are intermediate forms which are capable of changing their method of nutrition according to circumstances. A form which is usually saprophytic but capable of a parasitic existence is called a *facultative parasite* ; one in which these habits are reversed is a *facultative saprophyte*.

Mutual Benefit

Fungi may also establish a symbiotic relationship with another organism, i.e. a relationship in which the two associated organisms derive mutual benefit from each other. The lichens are an example of such a partnership, the entire lichen plant being composed of the cells of a green or blue-green alga and those of a fungus. The name *mycorrhiza* is given to the structures formed by the association between a fungus and some organ (usually the roots) of the higher plants. When associated with the roots it is probable that the fungus in some way assists the plant to obtain nourishment from the soil ; for example, the mycorrhizal roots of the beech tree increase the tree's uptake of phosphate from the soil.

Dependence on Fungal Infection

Dependence of the higher plants on fungal infection is shown by some orchids. Under natural conditions the germination of the seeds of these orchids depends on the presence of the appropriate fungus. Sterilised seeds can germinate if supplied with concentrated solutions of sugars, which indicates that, in nature, the fungus probably influences the uptake of nourishment by the seeds. Members of the heath family (Ericaceae) show fungal infection in all parts of the plants, and here again the germination of the seeds to healthy and vigorous seedlings is checked if the seeds are deprived of the fungus by sterilisation. The orchid *Gastrodia elata* contains no chlorophyll and obtains all its nourishment from a fungus ; without association with the fungus the orchid produces no flowers and weak tubers, and if isolation is maintained for several seasons the orchid dies.

Destructive and Beneficial Parasites

Some fungi are destructive parasites, causing diseases of field and garden crops and forest trees. The rust, smut, and bunt of wheat, apple scab, and potato blight are examples of fungal diseases of considerable economic significance. Some, such as the dry-rot fungus, injure the timber in buildings. Others, in the form of " mould," destroy articles of food.

But fungi are not entirely injurious to higher organisms. Those which live in symbiotic union with other plants are beneficial to their associates. Several of the larger kinds are good to eat. And the saprophytes are extremely useful in bringing about decay and thus rendering the remains of dead animals and plants available as food for other living plants. The microscopic yeasts, because they form carbon dioxide and alcohol from sugar, are important in brewing and breadmaking.

Spore Production

The vegetative plant body of a fungus is called the *mycelium*, and it is made up of a tangled weft of colourless filaments or *hyphae*. In the protoplasm of these hyphae are contained numerous small nuclei, of the same structure as the nuclei of other plants and animals. Reproduction is brought about by *spores*, of which there are many kinds, some being formed without any sexual process, others as the result of fusion of male and female cells. In " moulds " the actual mycelium is visible as a white or coloured downy mass. In the larger fungi (e.g. mushrooms, puff-balls, bracket fungi), the part commonly called the fungus is really the fruit body (concerned with spore production), the mycelium being beneath the ground or other substratum and therefore not visible.

Zoospores

The fungus *Pythium de Baryanum* causes " damping off " of seedlings, attacking them at ground-level and softening the tissues so that the seedlings fall over. The disease is favoured by excessive moisture and overcrowding. The branched mycelium of this fungus produces spherical swellings (*sporangia*), in which spores are formed. These spores are called *zoospores* ; they have the power of movement, being provided with two minute cilia by means of which they can swim in a drop of moisture. In dry conditions the sporangium can germinate

WHITE MOULD. Growth of Mucor, a mould that often appears on bread in a few hours when conditions are favourable.

Photomicrograph, John J. Ward

directly to a new mycelium without the formation of zoospores. *Pythium* also has well-differentiated sexual organs, the male and female being known as the *antheridia* and *oogonia*. The *oospore* which results from fertilization is a thick-walled resting-spore, and it can tide over unfavourable periods till it germinates, either with or without the formation of zoospores, according to the conditions. *Pythium* is a fungus which starts life as a parasite and continues as a saprophyte after the death of the infected seedlings.

The disease called potato blight is caused by *Phytophthora infestans*. It first became notorious in 1845–46, when it caused a potato famine in Ireland. The mycelium is widely distributed within the host plant, its presence being indicated by black blotches on the stems and leaves, and by rotting of the tubers. Branched hyphae protrude from the host tissues into the air, and bear egg-shaped sporangia, which are detached and carried away by the wind to spread infection. These sporangia may produce numerous motile zoospores, or germinate directly to a new mycelium, when the term *conidium* (instead of sporangium) is usually applied.

Penicillium Mould

Jam, cheese, bread, fruit, as well as leather and some other materials, are liable to become mouldy if kept in a damp place. This is due to infection by the spores of lower fungi. The common form of green mould (*Penicillium glaucum*) is often seen on bruised oranges and on bread. A product of *Penicillium* mould (penicillin) can inhibit bacterial growth, thus assisting in the control and cure of certain diseases. Other fungal antibiotics, such as streptomycin and aureomycin, have been discovered more recently. The mycelium of *Penicillium* is made up of branching threads, some of which grow into the air and give rise to spores that are disseminated by air currents. The blue mould (*Aspergillus*) of cheese is somewhat similar, but the spore-bearing branches end in swellings, from which long chains of spores radiate.

Zygospores

White mould (*Mucor*) is often seen on bread. The mycelium is made up of whitish, cobwebby hyphae, from which long spore-bearing branches rise into the air. Each of these ends in a rounded sporangium, within which numerous spores are produced. There is also a process of conjugation between specialised mycelial branches, in which resting-spores (*zygospores*) with firm investments are produced. These can remain dormant for some time, and thus enable the fungus to combat unfavourable surroundings.

Some species of *Mucor* and its near relatives have two kinds or strains of mycelium. These, when grown apart, produce only sporangia, but when the mycelia of two strains are brought into contact, zygospores are produced. Some species have more than two *compatible* strains. Species in which conjugation depends on the interaction of more than one strain of mycelia are said to be *heterothallic*; those which can form zygospores in a single mycelium are *homothallic*. Strains which when brought together produce zygospores are said to be *compatible*; those which do not are *incompatible*.

The mould fungi are definitely higher in the evolutionary scale than such types as *Pythium* and *Phytophthora*. They are adapted to a terrestrial mode of life, for at no stage in their life history are motile zoospores produced.

Mildew or Blight

The leaves of some common plants, e.g. strawberry, gooseberry, and hop, are subject to attacks by species of fungi which cause the condition called white or powdery mildew, or blight. The fungus *Sphaerotheca morsuvae* causes the destructive disease known as American gooseberry mildew. A spore, on germination, forms a white web-like coating over the leaf and sends hold-fasts into the epidermal cells. During summer, infection is spread by the liberation of oval spores (*conidia*) which are produced in rows on the external hyphae. In late summer, sex organs are formed on the mycelium and, as a result of fertilization, thick-walled spherical fruit bodies arise. When mature, these are dark brown or black, and they can be seen on infected leaves. Within each fruit body are spores which spread the infection in spring.

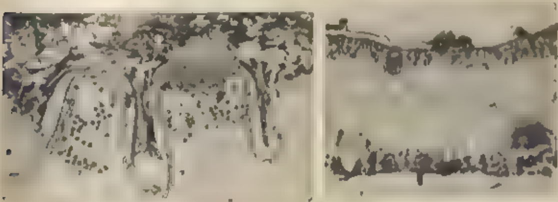
The immediate outcome of fertilization is the formation of a large sac-like cell which is called the *ascus*; in this the *ascospores* (usually eight in number) are produced. The formation of an

ascus is one of the important features in the classification of the fungi, and a large number (over 15,000 species) are grouped together as the Ascomycetes. These vary greatly in habit, but all develop the characteristic mother cell, or ascus, in which the spores are formed. Some of the more familiar Ascomycetes are the edible morell (*Morchella esculenta*) and truffles (*Tuber* species), and the common coral-spot (*Nectria cinnabarina*) so often seen on dead sticks.

The Fungus Called Yeast

Yeast (*Saccharomyces*) is a very simple type of fungus which belongs to the Ascomycetes, though its relationship to other members of the group is somewhat obscure. It has no well-developed mycelium, but occurs as separate cells. They are seen on the surface of fruit, e.g. grapes and apples, and on other plant tissues. Some forms of yeast have been "domesticated," notably those used in the making of beer and bread. A small portion of yeast examined under a microscope is seen to consist of many ovoid yeast plants, each of which is a single cell. These reproduce by the process of "budding," in which a cell gives rise to a small protuberance or bud which gradually grows until it is as large as the mother cell. Occasionally separation is delayed and, by repeated budding, colonies of loosely joined cells are temporarily built up. Under certain conditions some yeasts are able to form spores. The mother cell functions as an ascus, and the protoplasm divides to form four thick-walled ascospores. The cells and spores are so small and light that they are carried long distances by the wind.

When yeast is placed in a solution containing glucose sugar, it breaks up the sugar molecules, and forms carbon dioxide and alcohol. This process of decomposition of sugar results in the liberation of energy which is sufficient for the growth of the yeast plant. Decomposition of sugar by yeast is due to several enzymes (collectively, *zymase*), which are present in the yeast cells. These can be extracted from the cells, and the extract brings about fermentation exactly as do the living yeast plants. The enzyme *invertase* is also present in yeast cells and if the latter are placed in a solution of cane sugar this enzyme must transform the cane sugar into fruit sugar and grape sugar (glucose) before fermentation can start. The process is called *inversion*. The chemical reactions during fermentation are extremely complex, and other substances in addition to carbon dioxide and alcohol are formed as by-products.



FUNGAL STRUCTURE. Left, black rust, Puccinia graminis, showing aecidiospores in cluster cups. Right, section of barberry leaf with pycnospore and aecidiospore clusters caused by rust. Magnification, 46 and 69.

Photomicrographs H. S. Cheavin

Basidiospores

The rust fungi, together with the larger familiar forms (mushrooms, puff balls, etc.), are classified as the Basidiomycetes, and over 13,000 species are included. Their characteristic spores are produced very differently from those of the Ascomycetes. The spores are called *basidiospores* and are produced externally on stalks from the mother cell or *basidium*. There is no member of the Basidiomycetes in which normal fusion of male and female elements occurs. The rust fungi are without exception obligate parasites on the leaves and stems of higher plants. They cause destructive diseases of cereals and grasses, and their name is derived from the fact that at a certain stage they are revealed by the appearance of rust-red streaks on the stems and leaves of the host plants.

Top, basidium from a mushroom gill, bearing stalked basidiospores. Lower, black rust, Puccinia graminis, showing teleutospores. Magnification 435 and 103 respectively.

Puccinia graminis is the black rust of wheat; the mycelium grows within the host tissues, and in summer produces pustules of rust-red spores which break through the epidermis and are exposed to the air. These spores (*uredospores*) are easily detached from their stalks by the wind, and they rapidly infect other plants. Later on in the season the diseased patches turn black, owing to the production of dark-walled spores (*teleutospores*) in the place of the uredospores. These represent the resting stage of the fungus, in which it passes through the winter; and they germinate in spring while still attached to the wheat straw. Each sends out a small hypha (the basidium) which produces four minute basidiospores. These are incapable of infecting any cereal or grass plant, but depend on a totally different host, the shrub barberry (*Berberis vulgaris*), for further development. When the mycelium is

established on a barberry leaf, orange-coloured "cluster cups" appear as swellings on the lower surface. In these still another type of spore (*aecidiospore*) is produced in large numbers. These will develop no further on the barberry, but when transferred by wind or rain to wheat or grasses they produce a mycelium from which the uredospores are formed, the life cycle being thus completed.

In addition to the cluster cups on the lower surface of the barberry leaf, minute yellow specks are formed on the upper surface. These produce spores (*pycnospores*) which are very much smaller than any other kind formed by the rust, and they play an important part in the life history of this *Puccinia*. Like certain of the *Mucors*, the black rust fungus is heterothallic, and unless the two physiologically different strains are united, aecidiospores are not produced. The pycnospores are carried about by flies and other insects, and when deposited by a mycelium of compatible strain they infect it, and so establish the growth of a fertile mycelium.

The fact that barberry was connected with the appearance of black rust on wheat was well known to farmers long before this matter had been explained by botanists. In 1760 a law was passed in Massachusetts ordering the destruction of all barberry bushes, but it was not until 1865 that all stages in the life history of the parasite were demonstrated by the German botanist De Bary. Thus for the full normal life cycle of *P. graminis* two entirely different hosts are necessary. This phenomenon is called *heteroecism*, and it is displayed by several other rusts. But some pass all their life cycle on one host (*autoecism*).

BRACKET FUNGUS on bole of a tree, showing gills on undersurface of the fruit body.
H. S. Chaum

The mushroo[illegible] is a saprophyte growing in pas[illegible]s and in richly manured soil [illegible] The mycelium ("spawn") is [illegible] inconspicuous and remains hi[illegible] It gives rise to the stalked [illegible]ore-producing body, which is se[illegible] above ground and which is m[illegible] up of closely interwoven and com[illegible]cted hyphae. When fully dev[illegible]d, the under side of the ex[illegible]ed top of a mushroom has [illegible] number of radiating plates [illegible]ills," on the surface of which [illegible] numbers of minute basid[illegible] are produced. It has be[illegible]lculated that a fruit body of [illegible]lible mushroom (*Psallio[illegible]stris*) produces about 1,8[illegible]000 spores. The gills vary [illegible] with the fungus species; in the edible [illegible]room they are pink at first, later turnin[illegible]n and then black. The surface layer of [illegible] consists of closely packed club shaped ce[illegible] dia), each with four stalks at its tip, and [illegible]e on each stalk. The spores are disperse[illegible] wind, and they germinate into new myceli[illegible]

Bracket Fungi

Some of the bracket fungi (*Polyporus* and *Polystictus*) do considerable harm to trees. The mycelium ramifies in the wood, and the fruit bodies project as semicircular plates from the trunk. In some of these fungi (and also in some "toadstools") the under surface, instead of being composed of gills, consists of tissue studded with minute holes: these are the ends of tubes which are lined with spore-bearing basidia. The spore-producing body of a puff-ball (*Lycoperdon*) is a structure in which spores develop, to be liberated later by the bursting of the mature wall.

LESSON 28

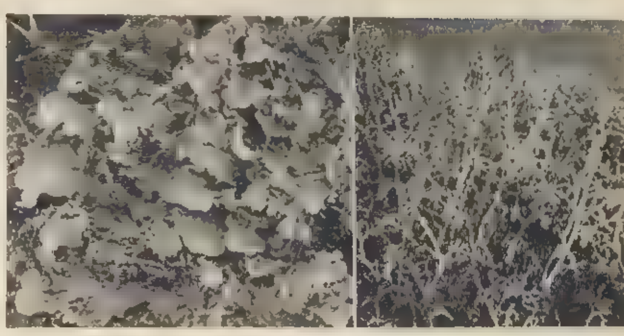
Lichens, Bacteria, and Slime Fungi

The plants called lichens occur in the form of coloured crusts on rocks, roofs, and walls, and as tufted growths on the trunks of trees. A lichen is a compound plant consisting of an alga associated with a fungus. The algal cells, containing chlorophyll, are capable of vegetative multiplication; but the fungus alone is concerned in the development of a definite fruit body. It is the fungus which forms the framework of the plant and, together with the green algal cells, grows into a definite and often conspicuous thallus, very different from the body of an ordinary fungus or alga.

A lichen, as a whole, is neither a parasite nor a saprophyte. The green cells of the alga are capable of carbon assimilation in the light and, being surrounded by the fungal hyphae, are protected from desiccation. The fungus obtains its carbon foods from the alga, and it passes to the algal cells water and dissolved mineral salts, which are absorbed all over the surface of the fungal threads. Many species can withstand extreme conditions of drought, and can exist on barren rocks.

Nearly all of them belong to the Ascomycetes, and produce a fruit body, often cup-shaped and brightly coloured, in which many asci and ascospores are embedded. The ascospores on

germination form a new fungal filament, which must come in contact with the appropriate algal cells to build up a new lichen thallus. In some members of the group, masses of both fungal and algal cells become isolated for reproduction, and when detached from the parent thallus serve at once to reproduce the necessary constituents of alga and fungus.



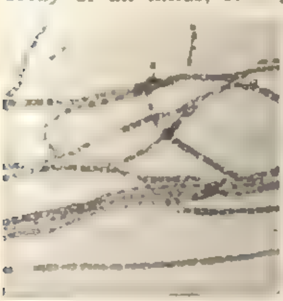
LICHENS. A lichen is a compound plant consisting of an alga associated with a fungus. Left, dog lichen, Peltigera canina. Right, reindeer " moss," Cladonia rangiferina.

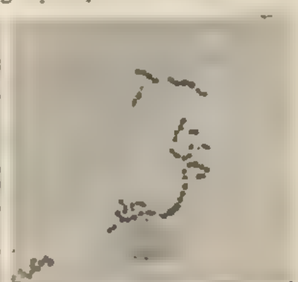
The characteristic colours of many lichen—yellow, orange, or red—are due to acids formed in the thallus as a result of activities of the constituent partners. Over 140 different acids are known to occur in these plants, and from them are obtained litmus and some other pigments used in dyeing. Two lichens especially are of value as food substances : the so-called Iceland "moss" (*Cetraria islandica*), which is eaten in that country ; and the reindeer "moss" (*Cladonia rangiferina*) of high latitudes, which during the winter, when other vegetation is scarce, forms a substantial part of the food of reindeer.

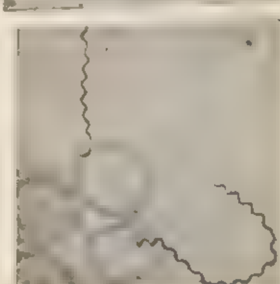
Bacteria

Bacteria form an extensive group of minute and structurally simple organisms, most of which are unicellular and have no well-defined nucleus. They are destitute of chlorophyll, as are the fungi, and are, as a rule, adapted to a parasitic or saprophytic life. Certain parasitic bacteria are the cause of some of the infectious diseases of men, animals, and plants. As saprophytes, they are the great agents of decay of all kinds, setting up rapid chemical changes in the organic substances on which they live. For example, when meat becomes putrefied or milk turns sour or wine is converted into vinegar, the change in each case is due to the action of a definite species of bacterium bringing about chemical alterations. The fermentations set up by these organisms are of great practical importance both to medicine (regarding the parasitic forms) and to those branches of industry in which the saprophytic forms are involved.

The bacterial cell is only about one-thousandth mm. in diameter, and a convenient classification is based on the shape of the individuals. Some are rod-shaped, and are called *baclili* ; others spherical, the *cocci* ; others comma-shaped, the *vibrios* ; and others appear as a spiral, the *spirilli*. By means of prefixes the arrangement of the cells can be indicated ; thus a coccus type in which the cells are attached together in a chain is a *streptococcus* ; if the coccus cells are grouped together in a bunch, they are referred to as *staphylococcus*.







BACTERIA. Above, bacillus of anthrax, a rod-shaped organism. The transparent points are spores. Magnification 1,000. Top right, streptococci, rounded bacteria occurring in " chains." Magnification 1.500. Lower right, spirochaete, a spiral shaped organism. The round objects are blood cells. Magnification approximately 1,500.

How Bacterial Cells Multiply

Each bacterial cell is surrounded by a cell wall of a substance resembling protein. There is no definite nucleus, but the application of suitable stains reveals the presence of granules of nuclear material scattered throughout the protoplasm. Some species of bacteria have cilia (protoplasmic threads protruding through the cell wall) which enable them to swim about in a liquid medium. Others (without cilia) are non-motile.

In suitable conditions bacterial cells multiply by dividing into two parts, i.e. by *simple fission*. In some species the daughter cells do not readily separate, so that dense masses of dividing cells become aggregated together in colonies called *zoogloea*, or masses. Fission can go on with great rapidity, and many kinds of bacteria are known to divide every

20 minutes or half-hour. It is possible for some 17,000,000 individuals to be produced from a single organism in 24 hours by fission occurring once an hour. In other conditions bacteria produce spores. A single thick-walled spore is formed from the protoplasm within the cell, and it has extraordinary powers of resistance. Some will resist a dry heat of 100° C. for a long time, and may even withstand boiling.

Food of Bacteria

The principal food of bacteria is in the substances excreted by living animals, or in the complex organic matter resulting from the decomposition of the dead bodies of animals and plants. Nutrition of plants and animals has a twofold aspect: the material taken in must supply (1) substances for growth of the organism, and (2) a source of energy for vital processes. In animals organic food supplies both these needs, and in green plants the process of photosynthesis yields carbohydrates which are used for both purposes. In some classes of bacteria the case is quite different, and two entirely distinct classes of material are necessary, one type for body-building, the other for respiration and release of energy.

Synthetic Processes

Examples of these classes are the *nitrifying* bacteria, one of which, *Nitrosomonas*, oxidises ammonia to nitrites, while another, *Nitrobacter*, oxidises the nitrites to nitrates. The *sulphur* bacteria use sulphuretted hydrogen for the purpose of obtaining energy, and oxidise the substance to water and sulphur; this reaction takes place within the cells wherein granules of sulphur are deposited. The *iron* bacteria oxidise ferrous salts to ferric salts, and *hydrogen* bacteria oxidise marsh gas or even free hydrogen for the same purpose. In all these reactions sufficient energy is set free for synthetic processes to take place without dependence of the organism on the radiant energy of sunlight. Such synthesis is called *chemosynthesis*, as opposed to the process called photosynthesis.

Some bacteria will thrive only in the presence of oxygen; these are called *obligatory aerobes*. Others grow only in the absence of free oxygen (*obligatory anaerobes*), such as the *denitrifying* bacteria (*see* Lesson 14). Direct sunlight is very harmful to the growth of bacteria, and if sufficiently intense it will kill the cells outright. The rays in the violet end of the spectrum and the ultra-violet rays are most effective in checking the growth of these organisms.

Viruses

Some diseases, e.g. influenza, yellow fever, and foot-and-mouth disease, are caused by "agents" called *viruses*. These are so small that they can pass through porcelain filters (the normal way of "catching" bacteria). Little is known about them. They look like crystals of a chemical substance, and they are capable of reproducing themselves very rapidly once they get into living tissues.

Slime Fungi

The group of organisms called slime fungi lies on the borderland of the animal and vegetable kingdoms, and their place in botany may be questioned. They are certainly of great scientific interest, for in them the behaviour of living protoplasm can be studied more conveniently than in any other creatures. Unlike the fungi and bacteria, the majority are of no practical importance. A few are parasitic on plants. *Plasmodiophora brassicae* causes the club-root (finger-and-toe) disease which affects the roots of cabbages, turnips, etc.

In the vegetative state a typical slime fungus (*Myxomycete*) is a naked mass of protoplasm, sometimes several inches in extent, which creeps slowly about on moist dead leaves, bark, or wood. In dry conditions the protoplasm passes into a resting stage by being partitioned up into numerous hard-walled *cysts*. When moistened, the walls of the cysts are absorbed and active movement recommences. When reproduction is about to take place, the character of the organism is completely changed. Activity ceases, and the protoplasm is converted into a mass of rather complex sporangia, in which the spores are produced. From these spores new active protoplasmic masses originate.

LESSON 29

Liverworts and Mosses

The next subdivision of the vegetable kingdom to be considered is that of the Bryophyta, the moss-like plants. It includes two classes, the true mosses and the liverworts. The mosses have a vegetative growth much like that of small plants, higher in the evolutionary scale, with well-formed stems and leaves but not directly comparable with those of flowering plants. The liverworts sometimes have a habit not unlike that of the true mosses; but many have a much simpler organization, consisting merely of an undifferentiated green thallus. They commonly occur on wet banks and walls, sometimes under water. One of the commonest is *Pellia epiphylla*. The plant of *Pellia epiphylla* in its vegetative

condition is a green, flat, lobed thallus, repeatedly branched, the lobes often overlapping one another. The plants grow together in masses and may cover a considerable patch of ground. The thallus has an upper and an under surface, the former being dark green; from the under surface emerge numerous brown hairlike *rhizoids*, which fix the plant to the ground. The cells of the thallus show scarcely any differentiation; all are thin-walled, except possibly a few central ones, and there are chloroplasts in the superficial cells.

Fertilization of Liverworts

The thallus produces both male and female sex organs on its upper surface. The male organs are called *antheridia*; when mature, they are globular bodies attached to the thallus by a very short stalk. They can be seen with the naked eye as little dots on the thallus near its centre. Each antheridium is enclosed in a sheath of thallus tissue, which leaves only a small opening at the top.

The female organs, or *archegonia*, are in a group just behind the tip of the thallus, and are almost covered by an overlapping sheath of thallus tissue. The archegonia are flask-shaped organs. When mature, there is a long neck consisting of many cells, down the centre of which a canal passes to the swollen basal portion. In this basal part is the egg cell awaiting fertilization.

As in the lower plants generally, water is necessary for fertilization to take place. After rain or dew the surface of the thallus is wet enough for this to be accomplished. When moist, the antheridium bursts, and its contents, a mass of male cells, are set free. Each male cell (*spermatozoid*) is provided with two cilia and can swim through the liquid towards the archegonia. The neck cells of the archegonium secrete substances which attract the spermatozoids. These swim down the canal between the neck cells, and one of them effects fertilization by uniting with the egg cell. The result of this union is the development of a spore-producing fruit called a *sporogonium*.

Spores of Liverworts

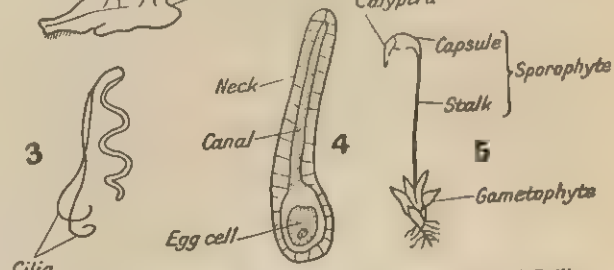
This structure is a dark green ball, about one-sixteenth of an inch in diameter, attached to the thallus by a light green stalk which is fixed tightly in the thallus tissue. The length of the stalk varies greatly according to the habitat of the plant. It may be several inches long. The mature fruit body resembles a long, thin pin with a round, almost black, head. In this head, or capsule, numerous spores are developed, and with them *elaters*, which are long narrow cells with spiral thickening. The wall of the capsule splits open to release the spores, and the elaters are of use in loosening the mass of spores so that they are more easily dispersed by the wind. On germination, a spore develops into a new thallus.

In some liverworts there are special organs for vegetative reproduction. Small green cups appear on the thallus, and in these are born minute *gemmae*, or buds, which grow into distinct plants.

Structure of Mosses

The true mosses are more highly organized than the liverworts, possessing a distinct stem, bearing spirally arranged leaves. *Funaria* is a common moss which usually grows on the ground and is especially abundant in places where there has been a fire. It grows in close, bright green tufts, each single plant consisting of a slender, erect stem densely clothed with small, simple leaves. At the base of the plant are a number of brown rhizoids, which fix the plant in the soil and absorb water. True Bryophyta have no roots.

The tissues of the stem show a simple but well-marked differentiation into three regions: the epidermis, the cortex, and a central cylinder of long, thin-walled cells for water conduction. In some of the larger mosses the tissues are more complex and are possibly analogous to the xylem and phloem of the higher plants. The leaves of *Funaria* are very thin, being only one cell thick, except in the position of the con-



DETAILS OF MOSS-LIKE PLANTS. 1. Plant of Pellia (liverwort) bearing sporogonia. 2. Cross-section of Pellia thallus showing male organs. 3. Spiral-shaped spermatozoid with two cilia. 4. Archegonium (female organ). 5. Fruiting plant of Funaria.

spicuous mid-rib. The cells are densely packed with chloroplasts. The conducting tissue in the stem is not continuous into the leaves, and this is compensated for by the fact that the leaves themselves can absorb water. On this account, mosses which may be completely dried up in hot weather revive rapidly when rain falls.

Male and female organs are borne on the same plant in *Funaria*, but this is not so with all mosses. The antheridia are borne terminally on a shoot in a dense cluster surrounded by a tight rosette of reddish or orange leaves. The rosette strikingly resembles a flower, especially in the larger mosses, such as *Polytrichum* common on heaths. The antheridia are club-shaped, and produce a large number of motile spermatozoids.

Fertilization of Mosses

The archegonia are formed terminally on branches below the male rosette. When mature they have the same form as the archegonia of the liverworts. Here again, fertilization depends on the presence of water, and it is effected by the swimming of the mobile spermatozoids down the neck canal, one of them uniting with the egg cell. In the mosses (as in liverworts) a substance is extruded from the neck which attracts the spermatozoids. The immediate result of fertilization is the development of the sporogonium.

This consists of a long, thin, red-brown stalk attached to the plant and bearing at its top a nodding pear-shaped capsule, green at first, later becoming brown. On the top of the capsule is a conical membraneous hood, the *calyptra*. This hood is really the upper part of the archegonium which has ruptured and had its tip carried up by the developing sporogonium. When the calyptra is removed, the top of the capsule can be seen as a neat conical lid. The internal structure of the capsule is complicated. The upper part is fertile and produces the spores, while the more solid basal portion performs the nutritive function of carbon assimilation. This is rendered possible by the presence of stomata in the epidermis and the layers of cells containing chloroplasts which are just within the epidermis.

How the Spores Escape

As the spores ripen, the capsule begins to dry up, and the lid at its apex becomes detached. But the spores are not all set free at once. Beneath the lid is a double row of "teeth" (formed from thickened pieces of cell wall), which partly close the mouth of the capsule. These "teeth" are sensitive to moisture, and they open only to allow escape of the spores in dry weather.

A spore does not germinate at once into a new moss plant, but forms a very simple branched filamentous growth rather like a green alga ; this is called *protonema*. It is attached to the ground by colourless rhizoids, and the young moss plants arise from it as lateral buds. In this way several new moss plants are eventually produced from the germination of a single spore.

Sphagnum

The leaves of the bog moss or peat moss, *Sphagnum*, contain large, empty cells as well as the green assimilating ones. The empty cells have small holes communicating with the exterior, and can readily fill with water. Dry *Sphagnum* can suck up a great deal of moisture, and because of this and of its acid nature, which makes it almost completely aseptic, it is used as an absorbent in surgery. It is also used in horticulture.

Sphagnum is upright in growth, ranging in length from a few inches to several feet. Where conditions are permanently moist it forms dense, wide-spreading masses. It is common throughout the temperate zones (including the British Isles) and in wet regions of the tropics. It abounds in swampy woodlands, and sometimes forms the chief feature of moors and bogs. Accumulation of the dead growths over the centuries forms peat, and very extensive peat-beds consisting solely of *Sphagnum* are often several feet thick.

Alternating Phases

From the foregoing description of the life cycles of *Pellia* and *Funaria* it will be seen that the life story of the *Bryophyta* includes two distinct phases : the one concerned with the production of the sexual organs (the moss or liverwort plant), the other concerned with the production of asexual spores (the sporogonium). The plant which produces the sexual organs is relatively highly specialised compared with the Thallophyta. It is also self-supporting, absorbing its own water and mineral salts and performing carbon assimilation.

The sporogonium is always in some way dependent for nourishment on the plant to which it remains attached. In most liverworts it is entirely parasitic, and in mosses partly parasitic obtaining water and dissolved salts from the sexual plant while the capsules form their own carbon foods. The sexual phase is called the *gametophyte* the spore-producing phase the *sporophyte*. These two phases normally alternate with one another and arise from one another, and a similar alternation is encountered in all the higher groups of plants, with the sporophyte ultimately becoming the dominant phase.

LESSON 30

Ferns

A GROUP of plants far more highly organized than any of the Bryophyta as regards their vegetative structure is called the Pteridophyta. It includes the ferns, the horsetails, and the club mosses. The ferns are a vast group, comprising about 150 genera and over 6,000 species. In the British flora 17 genera and about 40 species are represented. Ferns abound in hot, damp forests of tropical and sub-tropical regions. In parts of the southern hemisphere, notably Australia, New Zealand, and Ceylon, some species grow to the size of trees, and may even make up forests ; they resemble palms in appearance, with long, bare trunks, and crowns of feathery fronds.

Rhizome and Frond

A fern plant of the kind familiar in Britain generally consists of an underground stem (rhizome), which may creep horizontally at some distance below the surface, as in bracken (*Pteridium aquilinum*), or may be obliquely embedded, as in the male fern (*Dryopteris filix-mas*). Other species are attached to the bark of trees, or occur in crevices of walls or rocks. Brown, branching roots grow out from the stem and serve the double purpose of fixation and absorption of water and mineral salts. The fronds do the same work as the leaves in seed plants. They are sometimes broad and unbranched, as in the hart's-tongue (*Phyllitis scolopendrium*), but generally they are more or less feather-like. Young fern fronds are coiled in such a manner as to be able to force their way up through the soil (in whatever condition it may be) without damage to themselves.

FERN STRUCTURE. Above, transverse section of a rhizome of bracken, Pteridium aquilinum, a polystelic stem (× about 3). Lower right, transverse section of petiole of bracken, showing steles in ground tissue (× 5). Top right, transverse section of a petiole of a royal fern, Osmunda regalis, showing large stele (× 4).

H. S. Chearin

Internal Structure

A transverse section of a fern rhizome examined under a microscope is seen to consist of a large number of vascular strands or *steles* embedded in thin-walled tissue whose cells are filled with starch grains. The steles consist chiefly of the xylem cells almost surrounded by a ring of phloem. Among the steles are seen bands or groups of thick-walled fibres, which are additional strengthening tissue. Other strengthening cells appear near the epidermis, which is thick-walled and bears characteristic brown membranous scales. Such a stem is described as *polystelic*, and it is common to most (not all) ferns. Some species, regarded as the most primitive members, have a single stele in the mature stem, and this ancestral character is usually found in the very young stems of the polystelic forms.

The frond stalks show very much the same structure as do the underground stems, though usually there is less development of extra strengthening fibres. Here again there is variation in the internal structure and some members of the group have only a single (often horseshoe-shaped) stele in the petiole.

The roots and fronds show the same general structure as the analogous organs of the flowering plants. The roots of most ferns have only two xylem groups, and they owe their wiry character to an extremely thick-walled fibrous cortex. The "crowns" of some species are very thick and hard, as in bracken. The xylem itself is not so highly differentiated as in flowering plants.

Life Cycle of a Fern

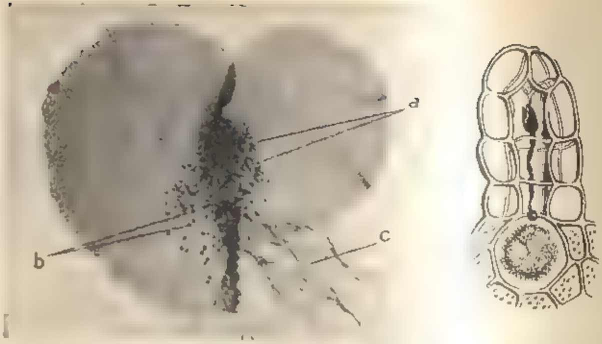
The life cycle of a fern, like that of the Bryophyta, consists of two distinct alternating phases, sporophyte and gametophyte. But in the fern each phase is a separate self-supporting plant. The sporophyte is the familiar fern plant, often of large size, with highly organized internal structure ; this produces the asexual spores which on germination give rise to the very small inconspicuous gametophyte phase.

The regularly arranged brown patches on the backs of fern fronds are called *sori*, and their shape and distribution vary considerably in different species. In bracken they are close to the edges of the frond, and follow its outline ; in the

hart's-tongue (*Phyllitis scolopendrium*) they are long streaks diverging from the middle of the leaf; in the polypody (*Polypodium*), common on tree-trunks, and the male fern (*Dryopteris filix-mas*), they are round patches. A sorus may have no special investment, as in the polypody, or it may be covered by a membrane (*indusium*), as in the male fern. In some ferns there are special fertile fronds of different shape from the others, upon which the sori are borne, e.g. the hard fern (*Blechnum spicant*) and the royal fern (*Osmunda regalis*), the latter being the largest British species.

Examination of a sorus under the microscope shows that it is made up of a number of stalked sporangia, in which are contained a quantity of brown spores. A sporangium is of biconvex shape, with a thickened ring of cells (*annulus*) around the greater part of its margin. Decrease in the water content of these cells causes them to contract, and a split occurs amid the unthickened cells. At a certain tension they suddenly revert back to their normal size, ejecting the majority of the spores from the sporangium in the process.

The spores are cells of simple structure and are not (like a seed) the result of a process of fertilization, but are asexually produced. When one reaches a damp spot, it at once germinates. Its firm coat splits, and a green filament is at first formed. The terminal cell then divides, forming a plate of tissue; colourless rhizoids grow out, down into the soil; and eventually a heart-shaped *prothallus* is formed in which the meristematic region, or area of actively dividing cells, is situated in the notch. Comparatively few prothalli survive to maturity, and underground perennating organs (rhizomes) and methods of vegetative reproduction are prevalent.



PROTHALLUS (left) produced from a fern spore: a, archegonia; b, antheridia; c, rhizoids. Right: an archegonium. It consists of a basal part embedded in the under side of a prothallus and containing an egg cell, and a projecting neck. Greatly magnified.

Birth of a Fern

The prothallus is the gametophyte plant, undergoing a sexual phase. Upon the under side of the prothallus, in its central region (the cushion), which is thicker than its edges, are a group of archegonia, each of which consists of a basal part embedded in the prothallus and containing an egg cell, and a projecting neck.

Scattered about on the same side of the prothallus, but restricted to the thinner part and posterior to the archegonia, are a number of very minute hemispherical projections, the antheridia, in each of which are produced many very small spermatozoids.

Within the mature egg organ certain cells degenerate and produce a slimy mucous substance

FERNS OF THE BRITISH COUNTRYSIDE. 1. Royal fern, Osmunda regalis, the largest British species. 2. Hart's tongue, Phyllitis scolopendrium. 3. Male fern, Dryopteris felix-mas. 4. Hard fern, Blechnum spicant. 5. Spleenwort, Asplenium trichomanes.

which is extended through the cap of the neck, leaving a clear passage to the egg cell. Meanwhile the ripe antheridia have burst open and the liberated spermatozoids swim actively about by means of their numerous cilia. The slime which oozes from the archegonia has a chemical attraction, and usually several sperms reach the egg cell but only one fuses with it. As soon as this happens, the fertilized egg cell begins to divide, and then gives rise to a young fern plant (sporophyte), which for a time remains attached to the prothallus, but ultimately takes root in the ground. The prothallus then dies away.

WATER FERNS. Salvinia is entirely devoid of roots. Finely divided fronds are submerged and these play the part of roots. Oval fronds float on the surface.

Two Alternating Stages

To sum up, the life history of the fern includes two alternating stages: (1) the ordinary fern plant, which produces spores asexually, and (2) the prothallus, with archegonia and antheridia. These two stages are the *sporophyte generation* and the *gametophyte generation*. This is called "alternation of generations," and it occurs to some extent in all plants. It is most obvious in ferns, and certain algae, mosses, and liverworts, and it is modified in flowering plants (*see* Lesson 32).

Unfernlike Ferns

Two small British ferns, the adder's-tongue and the moonwort, have each only a single frond, which divides into a sterile and a fertile part, while each sporangium develops from a group of cells, not from a single one as in an ordinary fern. In the adder's-tongue (*Ophioglossum*) the sterile part of the frond has a simple outline, while the elongated fertile portion is virtually a mass of crowded sporangia. In the moonwort (*Botrychium*) both parts of the frond are branched in a feather-like manner. These are generally considered to be more primitive ferns.

The water ferns comprise a small group which are either purely aquatic or grow, with some exceptions, in ground of a swampy nature. All have sporangia of two kinds, one containing small spores and the other large spores; considering the functions of these, the fronds which produce them may be called male-spore and female-spore fronds. A small spore germinates to produce a minute male prothallus with a single antheridium, while a female spore gives rise to a rather larger female prothallus, which bears a few archegonia. The prothalli are reduced to a few cells, very much smaller than those of land ferns.

One member of this group is *Salvinia*, a small aquatic plant, native to south Europe. It is entirely devoid of roots, and consists of a stem bearing two kinds of frond, some oval and floating on the surface, others finely divided and submerged. The latter play the part of roots, and they are also fertile, the rounded sori at their bases containing large and small sporangia. But *Salvinia* spreads also, and to a great extent, by pieces which become detached from the parent plant and float away and forthwith embark on an independent existence.

Growing in marshy ground, *Marsilea* is represented by European and Australian species. There is a creeping stem, from the under side of which roots are given off. The long-stalked fronds fork into a sterile and a fertile portion, the former terminating in a blade which is divided into four parts. The fertile section ends in a hard bean-shaped structure, called the *sporocarp*, and this contains a number of sporangia, of which some enclose large and others small (female and male) spores. When these are ripe, part of the internal tissue of the sporocarp is converted into mucilage, which swells up and splits open the firm investments along one side. The spores then germinate, giving rise to the two kinds of prothallus, and the fertilized egg cells grow into new plants.

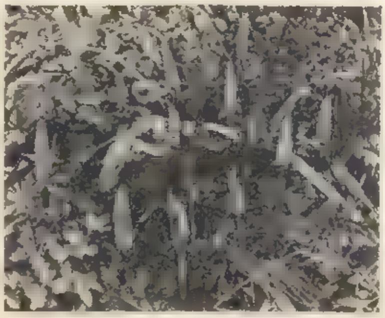
The pillwort, *Pilularia globulifera*, a European species occurring in Britain, grows in the same kind of places as *Marsilea*. Superficially it resembles a delicate grass. It has a creeping stem with roots, and narrow fronds. Parts of their bases are modified into rounded, brown sporocarps, the shape of which has suggested the popular and scientific names. These sporocarps contain a number of sporangia of both kinds; and these sporangia are liberated by the swelling up of mucilage, which bursts open the fruit in a valvular fashion. The rest of the life history of *Pilularia globulifera* is similar to that of *Marsilea*.

ADDER'S-TONGUE fern (Ophioglossum), left. The sterile part of the frond has a simple outline. The elongated fertile portion is a mass of sporangia. Moonwort (Botrychium), right.

LESSON 31

Horsetails and Club Mosses

THE horsetails are not now a very important group, for there is only one living genus, *Equisetum*, comprising about 25 species. But the family is a very ancient one, and it played an important part in the formation of coal. In those early geological times when the coal-beds were being formed, the horsetail family was represented by a number of very diverse forms, some of which were as tall as present-day trees. Most existing species are small, though *E. giganteum*, native to America, reaches a height of some 30 feet. One of the British horsetails (*E. maximum*) is sometimes six feet high.



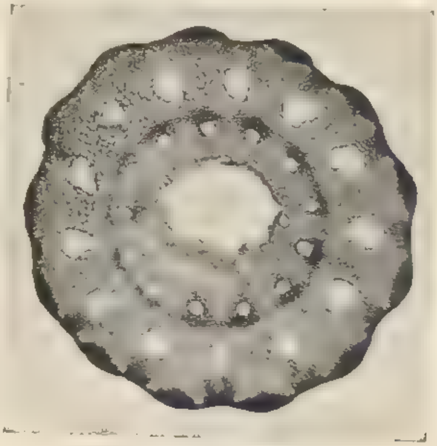
HORSETAILS. Equisetum arvense. The chief features of these odd-looking plants are the stiff, upright stems, these being surmounted by cone-shaped fructifications containing spores.

Structure of Horsetails

Horsetails in general have stiff, upright, jointed green stems, with whorls of small tooth-like leaves, those of each whorl being united to form a sheath round the stem. If the stem is branched, the branches are also in whorls, and the whole plant has a very formal and regular appearance. The spores are borne in cone-shaped fructifications, each cone being at the end of an upright stem or of a branch. In some species, e.g. *E. arvense*, there are stems which bear only cones; these fertile stems do not branch, and they are not green. In others the cones are borne on the ordinary green vegetative stems.

Underground the plants have much-branched rhizomes which penetrate to a great depth, constantly giving rise to new aerial shoots and having numerous slender adventitious roots.

A cross section of an aerial stem of a horsetail examined under the microscope is seen to contain a number of vascular bundles arranged in a ring—an arrangement typical of the bundle system in the stem of a dicotyledonous flowering plant. Larger intercellular spaces are present both in the cortex and on the inner side of the vascular bundles. The stems are ridged, and each ridge is occupied by a strand of fibres giving mechanical support to the aerial shoot, on the principle of girder construction. Additional bands of fibres also occupy the outer part of the cortex between the projecting ridges. In plants whose leaf system is little developed the function of carbon assimilation must be undertaken by the stems. This is rendered possible by the presence of stomata in the epidermis, which connect through to bands of chlorophyll-containing cells in the cortex. Both the underground rhizomes and the fertile shoots are somewhat modified in structure, though the arrangement of the vascular bundles remains constant and approaches nearer to the simple gymnosperms and dicotyledons than does the anatomy of any other living cryptogam.

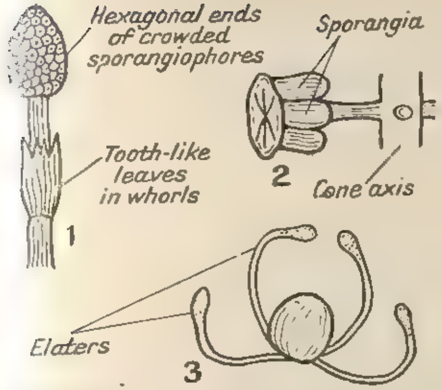


HORSETAIL STEM. Photomicrograph of a transverse section showing the vascular bundles arranged in a circle (× 16 approximately).
H. S. Cheavin

Fertilization and Germination

The horsetails resemble the ferns in having a sharply marked alternation of generations in their life cycle. The normal visible plant is the asexual sporophyte, producing asexual spores in the terminal cones. Each cone consists of a central axis round which are borne densely packed scales, each of which bears from five to ten sporangia. Each scale (*sporangiophore*) has its end expanded into a flat disc, and these are in such close contact that they appear as hexagonal areas on the surface of the cone. The sporangia are borne on these expanded discs.

The spores, all alike, are of complex structure. When ripe, the outer spore coat splits into four spirally coiled strips, which are extremely sensitive to moisture, coiling up when damp and straightening out when dry. There is doubt as to the precise function of these strips (elaters); by their movements they may cause the rupture of the sporangium wall, or



HORSETAIL (Equisetum). Organs of reproduction. 1. Part of fertile branch bearing unripe cone; slightly reduced. 2. Single sporangiophore (× 5). 3. Mature spore in dry condition, highly magnified.

that may keep the liberated spores entangled and thus ensure that they germinate in groups. This is important in the horsetails, as the male and female organs may be produced on distinct gametophyte plants.

The product of germination of the asexual spore is, as in the ferns, a minute green prothallus attached to the ground by rhizoids. The prothalli are the gametophyte generation and, in several species, they produce antheridia (male organs) and archegonia (female organs). In some other species the prothallus produces either antheridia or archegonia but not both. It is believed that environment sometimes determines which shall produce antheridia and which archegonia; the poorly nourished prothalli become males, i.e. produce antheridia and remain much smaller than the well-nourished ones which produce archegonia (females). A similar state of affairs occurs in ferns; when the prothalli grow densely crowded they often bear only antheridia, although unisexuality with the ferns is the exception rather than the rule. It is also thought that unisexuality is more often an inherited character and less often caused by environment.

It is a remarkable fact that while a horsetail plant bears no resemblance whatever to a fern plant, in both the prothallus and sexual organs are very much alike. Fertilization occurs by the union of a motile spermatozoid with the egg cell in the archegonium, and the development of the sporophyte is again initiated.

Club mosses are to-day represented mainly by two genera, *Lycopodium* and *Selaginella*, most species of which inhabit the damp forests of tropical countries. A few may be found growing in Britain on boggy moors or in mountainous districts. They are all that is left of a group which was dominant in the days when the coal measures were being formed, and when club mosses were large forest trees. The quillwort (*Isoetes lacustris*), which grows submerged in mountain tarns, is a peculiar member of this group, differing markedly from the two other genera in habit, yet revealing features in common with the fossil club mosses of the coal measures.

Reproductive Processes in Club Mosses

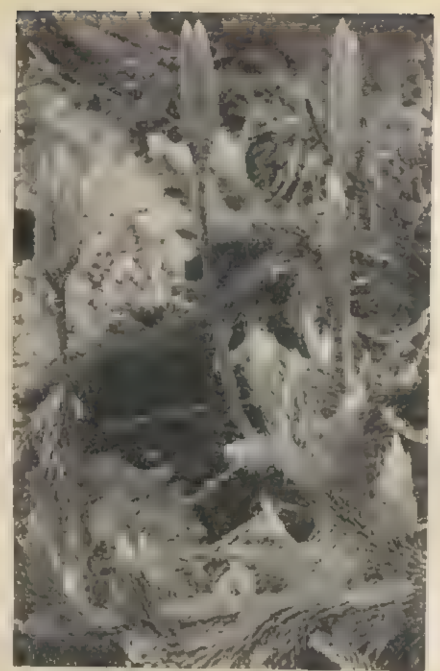
The small, spirally arranged leaves of *Lycopodium* densely clothe the stems, which may be erect, as in the fir club moss (*L. selago*), or creeping, except for the fertile shoots, as in the staghorn club moss (*L. clavatum*). Adventitious roots are produced from the stems, both stems and roots displaying forked branching. Some of the stem branches end in elongated cones comparable to those of the horsetails. The cone axis bears closely-packed leaves similar to the ordinary foliage leaves, except that each has a single large sporangium on its upper surface, near the base.

Each sporangium contains numerous yellow dust-like spores. These spores constitute the inflammable yellow lycopodium powder. On being shed, the spores germinate to give rise to peculiar fleshy prothalli, which in most species grow underground as saprophytes, obtaining their food with the help of a symbiotic fungus (mycorrhiza) from the humus. Both male and female sexual organs are borne on the same prothallus, and the life cycle resembles that of an ordinary fern or horsetail, showing a well-marked alternation of generations.

Selaginella and *Isoetes* produce two different kinds of sporangia, large and small, and two different kinds of spore. Plants which produce spores of two different kinds are said to be *heterosporous*; those whose spores are all alike are *homosporous*. The large spores, called *megaspores*, produce the female prothalli; the small spores, called *microspores*, produce the male gametophyte. The horsetails and most ordinary ferns are homosporous, though conditions of nutrition may on occasion affect the occurrence of sexual organs on the resulting prothalli.

HORSETAIL AND CLUB MOSS. Left, fertile and barren stems of horsetail. Right, quillwort, Isoetes lacustris, which grows submerged in mountain tarns.

Quillwort (previously referred to) has the appearance at first sight of a stoutly built grass, but in summer it has large and small sporangia on the inner sides of the bases of its leaves. The spores produce minute female and male prothalli, and the usual type of life cycle is shown.



STAGHORN CLUB MOSS. The small spirally arranged leaves of Lycopodium clavatum (shown here enlarged) densely clothe the stems, which are creeping, except for the fertile shoots, which stand erect.

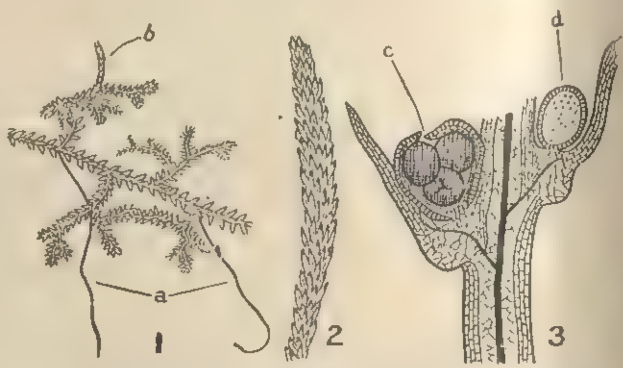
Selaginella Species

Selaginella, in its general course of development and reproduction, comes as near to the flowering plants as any other cryptogam now living. This makes it of special interest, as its reproductive processes illustrate the relation between these two main subdivisions of the vegetable kingdom. The genus is a large one, containing about 500 species, most of which occur in damp forests of tropical countries. Only one species (*S. selaginoides*) occurs in Britain, but several members of the genus are cultivated in greenhouses. The general habit resembles that of *Lycopodium*, the long, usually creeping stems being thickly clothed with small leaves. These are arranged in four rows: two rows of small leaves on the upper side of the stem, two rows of larger ones on the lower side. The British *S. selaginoides* has erect stems and, like other species with the erect habit, has leaves that are all alike. Each leaf bears on its upper surface, close to the base, a small membranous outgrowth, the *ligule*, which is characteristic of the whole genus *Selaginella*, and also of the related *Isoetes*. The function of the ligule has not yet been determined, but it is a very ancient character and is found in a large family of fossil plants of the coal period. From the stems slender leafless branches are given off, from whose tips true roots arise and penetrate the soil.

Sporangia of Two Kinds

The reproductive organs are borne in upright cones as in *Lycopodium*. Each sporangium is borne on the upper side at the base of a cone leaf. *Selaginella* is a heterosporous plant bearing sporangia of two kinds in its cones. The microsporangia usually occupy the upper part of the cone, the megasporangia the lower part. For a time the development of the two kinds of sporangia is identical, but at maturity the microsporangia contain numerous small spores, while the megasporangia have only four large spores each. The microspores are liberated from the ripe sporangium by its dehiscence, and if they fall on damp ground they germinate and give rise to minute colourless male prothalli. These consist almost entirely of a small group of antheridia from which the motile spermatozoids are liberated in wet weather.

The megaspores of *Selaginella* begin to germinate before they are shed from the sporangium, but do not usually mature till they are liberated and lying on the ground. When ready for fertilization, the female prothallus consists of cellular tissue within the spore wall, which is ruptured at one end where the prothallus tissue is consequently exposed. On this exposed part the archegonia are situated; the rest of the prothallus is stored with food material for the nourishment of the young embryo when fertilization is accomplished. The exposed part of the prothallus turns green and may produce a



SELAGINELLA. 1. General view (reduced) of S. kraussiana: a, rhizophore; b, spike or cone. 2. Fertile cone of S. selaginoides, natural size approx. 3. S. helvetica: part of longitudinal section through cone showing two sporangia; c, megasporangium dehiscing; d, microsporangium with numerous microspores (× 11 approx.).

few rhizoids ; but in *Selaginella* both male and female prothalli are much reduced structures compared with the gametophyte generations of the horsetails and ferns.

Following successful fertilization of an egg cell by a swimming spermatozoid, the embryo soon becomes an independent sporophyte plant, ready to repeat the cycle. In a few species of *Selaginella* the megaspores are not liberated from the parent plant until after fertilization, in this respect closely resembling the flowering plants and the gymnosperms.

LESSON 32

Alternation of Generations in Flowering Plants

In order to appreciate the modified status of alternation of generations in the flowering plants it will be necessary to consider the evolutionary changes which have brought this about.

It is generally considered that water is the original habitat of life. Over millions of years plants have migrated, by way of marshes and swamps, to dry land, and at the same time have had to adjust their form and life history to these changing conditions. To a certain extent the life history of an organism recapitulates the evolutionary history of its group.

Gametophyte and Sporophyte

The majority of the Algae show a marked alternation of generations, with the gametophyte usually the more important and obvious. In the true mosses and liverworts the " plant " is the gametophytic generation, while the sporophyte is a wholly or partially parasitic outgrowth upon it, following fertilization.

The Pteridophyta show a great increase in importance of the sporophyte and reduction of the gametophyte. In the life history of a fern, for example, there is a small, relatively insignificant gametophyte (the prothallus), living in very moist surroundings, of which the fertilized egg cell develops into the comparatively large and complex plant that constitutes the sporophyte, and produces spores which germinate into prothalli. The sporophyte usually flourishes best in damp, shady places, but it is far less dependent on moisture than the prothallus ; it is adapted in many ways to comparatively dry conditions.

Remote Aquatic Ancestor

The insignificant gametophyte may be regarded as the much-diminished representative of the remote aquatic ancestor from which the fern has descended, while the fern plant, the sporophyte, is a special development that has arisen as an adaptation to the conquest of the land. Passing up the scale, in seed plants the gametophyte becomes progressively reduced and the sporophyte increasingly important.

An ordinary plant is the sporophyte generation, and its flowers are arrangements for producing spores—in this instance of two kinds, large and small. The carpels are spore leaves, giving rise to megaspores (embryo sacs) contained in sporangia (ovules). The stamens are also spore leaves, which produce microspores (pollen grains), developed in sporangia (pollen sacs), of which four are embedded in each anther—at least, in the angiosperms.

The Flowering Plant

The gametophyte generation in the flowering plant consists of male and female prothalli. The male prothallus is very much reduced, and represented merely by the contents of the germinating pollen grain. Except in the lowest gymnosperms, the male gametes are not motile, but are conveyed to the egg cell by the growth of the pollen tube, and thus they have dispensed with the necessity of water for fertilization.

The female prothallus of the flowering plants is represented by the contents of the embryo sac, and remains safely sheltered within the ovule on the sporophyte plant. In gymnosperms such as the Scots pine, the female prothallus is a tissue of small cells filled with food material and bearing archegonia towards the micropylar end of the ovule. The prothallus of the gymnosperms is often called " endosperm " but it must not be confused with the endosperm of angiosperms, which is formed as a result of, not before, fertilization.

Homology of Organs

The growth and structure of the female prothallus of *Selaginella* exactly corresponds to that in the Scots pine, the differences are that in *Selaginella* the megaspore is usually shed from the sporophyte, while the mature prothallus may protrude slightly from the spore wall, become partly green, and produce a few rhizoids. It is in fact a more independent structure, developing freely on the ground, instead of within the closed tissues of the ovule, as in the gymnosperms. A few species of *Selaginella* show a closer similarity to the gymnosperms by retaining the megaspore on the parent plant until after fertilization.

Organs which resemble each other in their development and place in the life history are said to be homologous with one another, and by tracing the homology of organs it is possible to determine the probable relationships of different groups of plants. The homologies between *Selaginella* and a gymnospermous flowering plant are quite clear, and have been indicated. There is every reason to believe that the flowering plants are descended from heterosporous cryptogams, but it is certain that there is no near affinity between *Selaginella* and the gymnosperms, for fossil evidence suggests that, apart from having spores of two kinds, the actual ancestors of the gymnosperms were quite unlike any existing cryptogam. The lowest gymnosperms (cycads and ginkgo) present a transition between cryptogamic and phanerogamic methods of fertilization. There is no definite evidence for determining whether the relation of angiosperms to gymnosperms, or that of monocotyledons to dicotyledons.

LESSON 33

Nuclear Division of Plant Cells

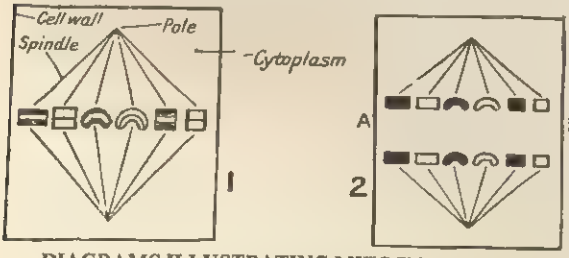
Two kinds of nuclear division take place within all living organisms. First, there is an equational division which is called *mitosis* and is the type of division undergone by the cells in the actively growing, or *meristematic*, regions of the plant, e.g. stem and root tips. Second, there is a reduction division called *meiosis* ; this compensates for the doubling of nuclear material which takes place on gametic fusion (*see also* BIOLOGY).

Chromosomes

The nucleus of a cell which is not actively dividing appears, under the high power of the microscope, to be composed of a fine network of threads on which are scattered many small granules. When an ordinary vegetative cell, e.g. one from the growing point of a root or stem, is about to divide (mitosis), marked changes occur in the nucleus. The network resolves itself into a number of long, thin threads with bead-like projections along their length. These structures have a greater staining capacity than the nucleus in the non-dividing cell, and have been given the name *chromosomes* (Greek, *chroma*, colour, *soma*, body).

The number of chromosomes which appears during nuclear division of any vegetative cell of a given species of plant (or animal) is constant ; for example, in the cabbage there are 18, in the tomato 24. The numbers vary enormously in different plants, some being much lower than those cited, others as high as a hundred or more, but the same number will occur in any vegetative cell of a given plant. Quite often differences in shape and size are shown by the chromosomes of the dividing nucleus, and when they occur the differences are likewise constant, reappearing unfailingly in each successive division of the species in question.

The chromosomes at first lie scattered irregularly in the nucleus and are then said to be at the *prophase* of mitosis. They gradually arrange themselves around the periphery of the nucleus while the membrane which formerly separated the nucleus from the surrounding cytoplasm of the cell disappears. At the same time the cytoplasm outside the nucleus may form fine fibrils converging to two points at opposite ends of the parent cell. This mass of fibrils, which is generally regarded as cytoplasmic strands in the sol form, in contrast to the rest of the cell, which is in the gel, is called the spindle, and its end-points the poles. The spindle is not always present. The chromosomes range themselves around the equatorial plane of this spindle, and each chromosome can be seen to be a double structure, being longitudinally split down its centre. In all probability this split is present in the earlier thread stages, but is less easily observed. This stage is called the *metaphase*.



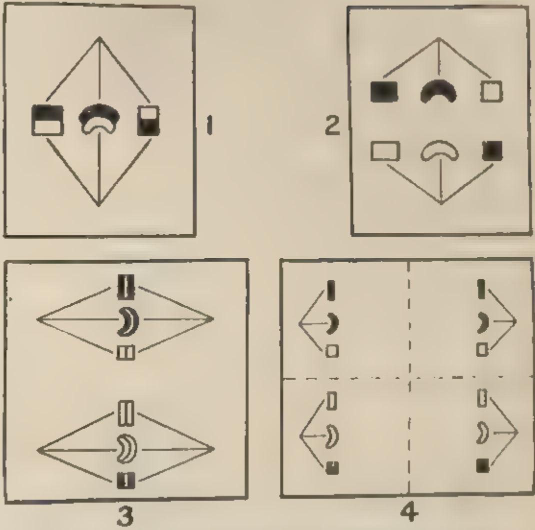
DIAGRAMS ILLUSTRATING MITOSIS. 1. Each chromosome is split longitudinally. 2. Half chromosomes travelling to opposite poles ; AB marks position where new cell wall is later formed.

Anaphase and Telophase

This longitudinal split divides each chromosome into two exactly equal "daughter" halves. The halves of each chromosome separate from one another on the spindle. The chromosomes become attached to the spindle fibres and, by as yet unknown forces, one set of half-chromosomes travels to one pole, the other set to the opposite pole. This is called the *anaphase*. Having reached the poles, the chromosomes enter upon the last phase (called the *telophase*), where they lose their

distinct outline and great capacity for staining, and with the formation of a nuclear membrane each "daughter" nucleus passes into a "resting" stage exactly similar to that of the mother nucleus before it began division.

A new cell wall is formed by deposition of substance across the equator of the vanishing spindle; the wall substance ultimately reaches the sides of the mother cell, and thus division into two similar daughter cells is finally accomplished and the traces of spindle fibres are completely absorbed into the cytoplasm. This method of nuclear division ensures the equal allocation of the chromosome material (*chromatin*) at each vegetative division. It also ensures that the cell of the adult body has the same complement of chromosomal material. Although the term "resting nucleus" has come into use for a nucleus which is not actively dividing, it is misleading because it is during this phase that the chromosomes actually divide longitudinally so that at the beginning of mitosis the chromosomes are already double (i.e. two chromatids).



DIAGRAMS ILLUSTRATING MEIOSIS. 1. Chromosomes in pairs. 2. Whole chromosomes separating. 3. Chromosomes longitudinally split for the second (mitotic) division. 4. Chromosomes about to form four reduced nuclei; dotted lines indicate later division of the cytoplasm.

Process of Meiosis

The behaviour of the chromosomes in mitosis is almost the same in plants as in animals. The differences are: (1) the absence in the higher plants of centrosomes, though these are found in certain lower groups, e.g. fungi; (2) the method of cell division, by constriction of the cytoplasm in animal cells, and by deposition of a new cell wall in plants.

The fact that there is a constant number of chromosomes for any given species is important, and is of special interest in connexion with the formation of the sex cells or gametes, ova, and sperms. In sexual reproduction two cells, the male and female gametes, fuse to form the new individual. If the gametes each possessed the number of chromosomes characteristic of the nucleus of a vegetative cell, the zygote would obviously have twice that number, and so on for each generation. This is prevented by a special kind of nuclear division, meiosis, or the reduction division, occurring in higher plants at the formation of the sex cells and preceding the formation of the gametes, the number of chromosomes present in the resultant daughter nuclei being reduced to half the number of the ordinary vegetative cells. This reduction usually takes place in the spore "mother cells," so that gametophytes have half the number of chromosomes of the sporophyte.

In flowering plants, since gametophytes are so reduced, meiosis takes place in the anther and ovary (pollen mother cells in the former and embryo-sac mother-sac cells in the latter). All nuclei of the mature pollen grains and all nuclei of the ripe sac thus contain half the specific number of chromosomes, and by fusion of male and female gametes the whole number is restored in the zygote.

As a meiotic division is about to take place, the chromosomes differentiate out from the nucleus in the same way as at mitosis. This is again called the prophase. Of the chromosomes in any one nucleus half have been derived from the male parent, the other half from the female parent. Each chromosome from one set has a homologue in the other set which controls the same set of characters. When the chromosomes arrange themselves across the equator of the spindle they do so in homologous pairs, one from each parent, not singly as in mitosis. Each pair then becomes very closely associated and coil around each other, and material is usually exchanged between each pair of chromosomes. This is the metaphase.

Haploid and Diploid

The pairs then separate, and the individuals travel to opposite poles (the anaphase). At the poles the chromosomes lose a little of their individuality (the telophase), but before a nuclear membrane or any cell wall is formed the chromosomes split longitudinally and the two halves, or chromatids, separate in a plane at right angles to the original separation of paired chromosomes. Thus there are four sets of chromosomes, each with *half* the number of

the original cell. This single complement of chromosomes is called *haploid*, as distinct from the double complement or *diploid* number present in all the vegetative cells of the plant. The second nuclear division is followed by division of the mother-cell cytoplasm and the formation of new cell walls.

New Combinations of Genes

This method of nuclear division ensures that the resulting cells will have homologous sets of haploid number of chromosomes and also that these sets need not be identical with the original ones from the parents. Any combination of homologues may occur, and also exchange of material between homologues usually takes place. Thus the resulting nuclei have new combinations of genes, the structures which govern hereditary characters, from those present in the parents. This device promotes variation, and thus increases the adaptability of the offspring produced.

In the anthers four pollen grains are formed from each mother cell, but in the ovaries three of the four reduced cells perish and only one survives to develop into the mature embryo sac. All further nuclear divisions, in both the pollen grain and the embryo sac, are of the mitotic type, and so the male and female gametes each have the haploid number of chromosomes. When the egg is fertilized by the sperm, the zygote will have the full specific diploid number of chromosomes, half of paternal and half of maternal origin.

LESSON 34

Inheritance in the Plant Kingdom

THE idea of evolution was entertained by men of science long before the time of Charles Darwin (1809–82), and attempts had been made to explain how structural differences, by which species are distinguished one from another, might have been established. None of the theories advanced was supported by sufficient evidence to gain general acceptance for the theory of evolution.

Conviction of its truth dates from July 1, 1858, when a paper entitled *On the Tendency of Species to Form Varieties ; and on the Perpetuation of Varieties and Species by Natural Means of Selection*, by Charles Darwin and Alfred Russel Wallace, was read before the Linnean Society. The following year Darwin's *Origin of Species* was published, and this mass of evidence, as to the processes by which evolution could be brought about, was so overwhelming that they won for the evolution theory the wide acceptance that it now has.

Darwin and Wallace conceived that new species arose by the gradual accumulation of favourable variations, inherited and intensified by a weeding-out process, or natural selection, in successive generations. Wide departures from the parental type, sports or mutations as they are called, were considered by them of little importance ; it was thought that these were submerged and suppressed in a few generations. But it has been shown that mutants are capable of transmitting their peculiarities to their offspring, and they are regarded now as probably playing an important part in the origin of new forms, as well as in the more gradual steps of natural selection.

In all the important structural features, and frequently in even most trivial details, offspring resemble their parents, though they always vary from them to a certain extent. During the present century much research has been done by biologists of all nations with a view to ascertaining how variations are brought about and the means by which parental characters are transmitted to offspring by those two single cells, the sperm of the male and the egg of the female parent.

The science of heredity (how heredity works) is called Genetics. The foundation upon which the whole science of genetics rests is the work of Gregor Mendel (1822–84). The distinctive and essential feature in Mendel's methods of attacking the problems is that he studied the inheritance of obvious striking characteristics (" unit characters ") as separate units, whereas other investigators had been confused by the number of differing characters.

Mendel's researches were conducted on the edible pea, a plant very suitable for the purpose ; first, because its flowers are normally self-pollinated, but cross-pollination can easily be effected artificially ; secondly, because it exhibits several well-marked varieties readily distinguishable from one another, e.g. some are tall, some dwarf ; some have seeds with green testas, other with yellow ; some have wrinkled testas, others smooth. Records of the inheritance of these unit characters resulted in the establishment of what are known as Mendel's Laws (*see also* BIOLOGY).

Following Mendel's initiative, others have made similar breeding experiments, and have confirmed and extended his results. A great variety of plant- and animal-characters has been subjected to breeding tests : leaf-shape, flower-colour, resistance and susceptibility to disease, doubleness and singleness of flowers, variegation, stature, fertility sterility, and others. An

example, not taken from Mendel's work but from that of some of his English followers (Bateson and his colleagues), will illustrate Mendel's general conclusions.

The Chinese primula (*Primula sinensis*) is a very variable plant. Some varieties have what are called " palm " leaves, others " fern " leaves, so named on account of their respective shapes. Varieties exist which breed true to the palm-leaf character; self-fertilized or crossed with one another, all the offspring produced have palm-leaves. Other varieties breed true to the fern-leaf character. If one of the true-breeding palm-leaf plants is crossed with one of the fern-leaf plants, all the offspring, called the first filial generation (F_1), are palm-leaf. The fern-leaf character has disappeared, and for this reason it is called a *recessive* character. The palm-leaf character shown is called a *dominant* character.

If plants of the F_1 generation are self-fertilized or crossed with one another, the recessive character (fern-leaf) reappears in some of the second, or F_2, generation, and the number of fern-leaf plants to the number of palm-leaf is in the ratio 1 : 3. The results can be represented thus:

Parents: Palm × Fern

F_1 Palm

F_2 Palm 3 Fern 1

If the F_2 fern-leaf plants are self-pollinated, they breed true generation after generation, giving nothing but fern-leaf plants. The F_2 palm-leaf plants show different behaviour. One third of them produce palm-leaf plants generation after generation; like the F_2 fern-leaf, they are *pure* for the character concerned.

The remaining two-thirds of the palm-leaf F_2 plants prove to be *impure* for the leaf character, for when self-pollinated they yield both palm and fern plants in F_3 in the proportion of three palm to one fern, i.e. exactly the same as happened with the F_1 plants. The full scheme can be represented as follows:

True-breeding Palm-leaf × True-breeding Fern-leaf

F_2 Palm

F_2 1 Palm 2 Palm 1 Fern

F_3 Palm 3 Palm 1 Fern Fern

The F_3 offspring of the *impure* F_2 palm plants show, by further breeding experiments, that, as in F_2, the ferns continue to breed true, one third of the palms breed true, and the remaining two-thirds produce dominants and recessives in the ratio 3 : 1.

These results are identical with those obtained by Mendel in studying the inheritance of two alternative characters (allelomorphic characters) in the edible pea. He explained the results by assuming that such characters were represented by definite independent factors in the gametes. Let P represent the factor responsible for the palm-leaf character, and *p* that responsible for the fern-leaf. Then if P is crossed with *p*, the F_1 palm-leaf plants may be shown as P*p*—i.e. they appear as palm-leaf because the dominant character masks the recessive which is also present. Mendel assumed that the gametes of such plants were pure for one character or the other, hence the " sorting out " or *segregation*, of the pure dominants and pure recessives in the F_2.

There is abundant evidence to the effect that hereditary characters are transmitted from parents to offspring mainly by means of the chromosomes. One of the most obvious facts pointing to this conclusion is that the male sperm which unites with the egg at fertilization is almost entirely nucleus, and yet, on the average, the characteristics of any individual are inherited equally from both parents.

Other strong support is obtained from the behaviour of the chromosomes at mitosis, which ensures that every cell of the adult body receives an equal amount of both maternal and paternal chromatin. It is now known that a few hereditary characters are controlled by genes in the cytoplasm. Closer study will show how well the chromosome theory fits in with meiosis.

Any zygote is formed by the union of male and female elements, each contributing a chromosome bearing a factor for (e.g.) leaf character. The pure-breeding palm-leaf plant will therefore have two chromosomes bearing palm and may be expressed PP. Similarly the pure-breeding fern is expressed pp for the pair of chromosomes concerning leaf character. Such plants are said to be homozygous for the character concerned. At meiosis the paired chromosomes separate from one another, and the resulting gametes will therefore bear only one P or one p chromosome. Thus:

Parents PP × pp

Gametes P (*Egg*) → p (*Egg*)

P (*Sperm*) → p (*Sperm*)

On cross-fertilization, all the resulting zygotes (F_2) are of the constitution Pp for leaf character and are called heterozygous for that character. After separation of the paired chromosomes at meiosis in F_1 the gametes will be:

P (both eggs and sperms)
p („ „ „ „).

Since any egg and any sperm may unite at fertilization, the following types of union will take place :

Eggs		*Sperms*		*Zygotes of* F_2
P	$\longrightarrow$	P		
	$\times$		=	PP Pp pP pp
p	$\longrightarrow$	p		

Thus in F_2 palm plants will appear to fern plants in the ratio 3 : 1. The fern-leaf plants, containing no P chromosome, will breed true to fern leaf, and one-third of the palm leaf will breed true to palm. The remaining two-thirds of the palms are heterozygous plants and on further breeding will continue to produce palm : fern as 3 : 1.

Mendel, without any of the present knowledge of chromosomes or gamete formation, formed an amazingly correct view of the laws of heredity.

Apart from their scientific interest, unit characters, dominance, and segregation, the fundamental "laws" of Mendelism, are of great practical economic importance. The plant (or animal) breeder is no longer groping in the dark ; in a few generations he can, by appropriate mating, combine together desirable qualities formerly existing only apart in separate breeds, and know that his product will breed true.

For example, Biffen's wheat was produced as follows. Professor Biffen wished to raise a wheat which had a high yield of grain and was immune to the destructive fungoid rust, the highest-yielding varieties of wheat being very susceptible to this disease and this susceptibility having been proved to be dominant to immunity. He therefore crossed a high-yielding variety with a less productive one which is immune to rust, and in the F_2 generation obtained plants which combine immunity to rust (extracted recessives) with the high-yielding quality of one of the original parents, and which breed true in subsequent generations ; their gametes are pure for the two unit characters, immunity to rust and high yield of grain.

Although for a large number of plant and animal characters the facts of heredity have been shown to be in accordance with Mendel's laws (Mendelian heredity), by no means all characters show such simple genetic behaviour. Dominance is not always complete ; the heterozygous F_1 may differ in appearance from either parent. An example is afforded by the primula. *P. sinesis* has large flowers with wavy petals ; *P. stellata* has much smaller flowers with flat petals. The F_1 offspring from crossing these two forms has flowers intermediate in size and appearance. When self-fertilized the plants behave in simple Mendelian fashion, giving *sinensis*, intermediates, and *stellata* in the ratio 1 : 2 : 1. This is a perfect example of the Mendelian principle of segregation, although there is no dominance of the character of either parent.

In the case of other characters, two or even more factors must be present to produce a given character. One of the most interesting examples in which such interaction between separate factors has been demonstrated is in the sweet pea. Two factors must be present for any colour to arise in the sweet pea ; if either factor is absent, the flower is white. It follows, then, that there should occur two kinds of white-flowered sweet peas which, though looking alike, differ genetically in that one carries one of the two colour-forming factors, the second carries the other. The wild sweet pea has, and breeds true to, purple flowers. When two genetically different white flowered plants are crossed, the factors responsible for colour production are brought together and a coloured offspring is produced.

This phenomenon of *reversion*, or the re-appearance of some ancient racial feature, may then take place if races bearing factors complementary to one another are mated. Representing the colour factors by the letters A and B, the scheme can be written thus :

Parents.	White	×	White
	AA*bb*		*aa*BB
Gametes.	A*b*		*a*B
F_1		A*a*B*b*	
		Coloured	

When the coloured F_1 plants are self-fertilized, the resulting F_2 generation consists of coloured : whites as 9 : 7. The heterozygous F_1 plants (AaBb) produce gametes of four kinds, AB, *a*B, A*b*, *ab*. By adopting the "chessboard" system (*see* Biology) and examining the types of zygotes produced, the student will see that nine out of the sixteen squares contain both A and B, while seven have either A or B alone or neither. The 9 : 7 ratio is really a modified form of the simple dihybrid ratio : 9 : 3 : 3 : 1. The deduction follows that the appearance of colour in the sweet pea depends upon the interaction of two factors which are independently transmitted according to the ordinary scheme of Mendelian heredity.

BOOK LIST

Introduction to Botany, J. H. Priestley and L. I. Scott (Longman) ; *Plant Form and Function*, F. E. Fritsch and E. Salisbury (Bell) ; *Botany of the Living Plant*, F. O. Bower (Macmillan) ; *Biology of Flowering Plants*, Macgregor Skene (Sidgwick and Jackson) ; *Introduction to Plant Anatomy*, A. J. Eames and L. H. MacDaniels (McGraw-Hill) ; *Morphology of Vascular Plants. Lower Groups*, A. J. Eames (McGraw-Hill) ; *Flowerless Plants*, D. H. Scott and F. T. Brooks (A. & C. Black) ; *Primitive Land Plants*, F. O. Bower (Macmillan) ; *The Advance of the Fungi*, E. C. Large (Jonathan Cape) ; *Morphogenesis in Plants*, C. W. Wardlaw (Methuen) ; *Plant Physiology*, W. O. James (O.U.P.) ; *Plant Life Through the Ages*, A. C. Seward (C.U.P.) ; *Fossil Plants*, J. Walton (A. & C. Black).

LAW

A [illegible] standing of the law and constitution of a country, and of developments [illegible] ing place in them is an essential part of good citizenship. This Course [illegible] consists of two parts: the first deals with English law as it affects the [illegible] citizen, the second with those special laws and customs which make up [illegible] tten and elastic constitution of the United Kingdom.

[illegible] ts played by parliament, by the courts of equity, by the judges, and by [illegible] stature in creating the present body of English and Scots law are explained, [illegible] with other important, interesting, and curious points, as well as the citizen's [illegible] es, and privileges in relation both to the law and to the constitution. The [illegible] should refer also to the Courses in BRITISH HISTORY, in Vol. 5; SOCIAL H[illegible]RY, in Vol. 4; and SOCIAL ANTHROPOLOGY, in Vol. 2.

18 LESSONS

LAW

12 LESSONS

BRITISH CONSTITUTIONAL LAW

LESSON 1

Fundamentals of Jurisprudence

WHAT is the average British citizen's attitude towards the law ? To-day there can be no possibility of his being unaware of at any rate one part of it in his ordinary life, whatever may have been the position a hundred years ago. Indeed, in the "welfare" state in which he now lives his awareness is probably sometimes excessive.

He has come to accept that almost all his business and private activities should be controlled even in peace-time by regulations of some government department or local council considered necessary for the public welfare, with the result that whenever he proposes to engage in any activity he realises that his first step must be to find out what regulations have been made controlling it and what are the do's and don'ts he must observe.

An unfortunate consequence of this awareness is that he comes to regard these regulations as being "the law." A part of the law they undoubtedly are, for they are laid down by or under the authority of parliament, but even to-day they must be regarded as a lesser part. Indeed, a famous judge has suggested, semi-jocularly, that we should invent for them some name such as "para-law" or "sub-law."

Vital Benefits Provided

The frequency with which the citizen comes into contact with these regulations tends to make him forget the other, and greater, part of the law, which, being based on principles developed through many centuries, is a system which forms a coherent whole and is not a mere collection of scattered and unconnected rules. This true law is administered not by officials in ministries and council offices but by judges in the courts, whose object it is to protect the citizen's fundamental rights of freedom, sometimes even against excessive encroachment by the welfare state. It is this part of the law which, by enforcing contracts, by preventing wrongs from being inflicted or requiring them to be compensated when prevention is too late, and by punishing crime, is still the most important factor in preserving order and stability in the citizen's life.

If anyone doubts the citizen's forgetfulness of this other law, let him ask himself when last he heard what was—not so long ago—the classic parting threat of two angry disputants: "I'll have the law on you. You'll be hearing from my solicitor." To-day the citizen seeking to enforce his rights is quite likely to turn first, not to his solicitor, but to some knowlegeable official of his local council.

This attitude, however understandable it may be, is unfortunate because it blinds the citizen to the real and very vital benefits which the law protecting his fundamental rights does provide for him.

Without it he could live only from day to day, and his life would be "nasty, brutish, and short." The punishment of crime by the law increases the probability that he can go about his normal business without being violently attacked by someone desirous of taking from him his note-case and loose cash. By enforcing contracts, the law enables him in his business to enter into engagements with others to be carried out at some future time, and to plan his affairs on the assumption that these engagements will be duly carried out.

Objects and Function of the Law

It may be said that the object and function of the law is to maintain an orderly state of society by compelling the individual in close association with other individuals to live in such a way that he pays some regard to the private wills and pleasures of the others, and does not act as if his own interest were everything and theirs nothing. Thus, in one direction the law deprives a man of his freedom by restricting his powers—e.g. his power to assault and rob his neighbours—but in another direction it increases his freedom, for he will in his turn be free from the untrammelled assault and robbery of his neighbours.

Man has always been an essentially social being, delighting in the companionship of his fellows, and has tended even in the most primitive times to form societies or groups, either because of this social instinct or for protection. This instinct still survives and operates to bring into existence the innumerable clubs and societies which are to be found in every community the world over.

Rules of Conduct

In early times the unit may at first have been very small, merely the family, but even in that unit some form of social order is required ; and as the unit becomes larger, in the village and finally the state, the need for some social order is recognized as a necessary condition of social intercourse. Social order can be obtained only by the observance by the members of the unit of some rules of conduct towards one another, for otherwise life together would be impossible. The larger the community, the more numerous and complex are these rules of conduct, but their object is the same.

If one examines the rules of conduct existing in any community, it will be found that the members of the community obey some rules for one reason and others for other reasons. In most communities a man is expected to greet a friend whom he meets in the street and not to ignore him ; and if he accepts an invitation to dinner he is expected to keep the appointment. In these instances the rule of conduct is founded on convention. If a man fails to observe these rules he will suffer in some way or other. His friends will cease to speak to him in the street or will not again ask him to dinner. He will find that "public opinion" is against him, and so it is by public opinion that such rules are enforced. Again, a man is expected to tell the truth, and a man who lies breaks a moral rule.

Law and Public Opinion

But there are some rules of conduct the observance of which is not enforced by public opinion or morality—or not by those only. If a man assaults his neighbour or drives his motor car in a dangerous manner he will find that he will have offended the state, which will attach certain penalties to his breach of the rule, over and above any that may be imposed by public opinion or morality, and may fine him or deprive him of his freedom by sending him to prison. It is this attribute of state enforcement which distinguishes rules of conduct that are "laws" from other rules of conduct, and so a law may be defined as "a rule of conduct enforced by the state."

It does not follow from this that laws are not also supported by public opinion and morality. Every law should have this support and in very many, if not the majority of, cases this will be more effective in enforcing the law than the fear of the punishment imposed by the state. Thus, the average man refrains from cheating at cards, not because he is afraid of being punished by the law, but because to cheat would either offend morally or would lead to his friends' refusal to play with him again. Conversely, when public opinion and morality do not require the observance of a certain rule of conduct which is law, the average citizen will not hesitate to break the rule. The definition of a law as a "rule of conduct enforced by the state" requires some elaboration. First, it should be noted that the definition relates only to rules of *conduct*. The state should not interfere with the thoughts or desires of its members until such thoughts or desires are expressed in conduct, for the very good reason that until that time arrives they can in no way affect other members of the community.

Enforcement of Laws

Law has not always confined itself to conduct, even in Britain. It has sought out a man's thoughts and punished them as crimes. As late as the days of Queen Elizabeth I anyone might be seized and brought before a tribunal, which then proceeded by a series of questions to find out his views on such matters as the Pope's right to excommunicate the Queen. Torture might be used to compel him to answer. But even in the 16th century the procedure was regarded with disfavour, the Queen herself declaring that she would make no window to look into a man's soul. These methods will at once recall those of the Spanish Inquisition and those revived along with other medieval practices by some of the dictatorships in Europe in the 20th century, and wherever "witch-hunts" are to be found.

The state does not enforce all its laws in the same way. In the case of those the breach of which is considered to be so serious in its effects as to endanger the community—i.e. the criminal laws—the state will itself intervene to have the offender punished.

In other cases—breaches of conduct which are merely civil wrongs, for example—the state will not interfere unless the member of the community who has been injured invokes its aid by suing the offender in the courts. In still other cases the state will in no circumstances inflict any punishment for the breach of the law, but will merely refuse its aid to any member who does not comply with the legal requirements. Here it directs but does not command. Thus, if any person makes his will without observing the required formalities, the state will refuse to recognize the validity of the will.

LESSON 2

The Laws of Man and the Law of Nature

To talk of the origin of law in a society is apt to be misleading, for it might be taken to imply that law is developed only after the society has come into existence. From what has already been said of the nature of law, it will be appreciated that this view is incorrect ; it is impossible for individuals to live together in a society without the rules of conduct which we call law. Law was not imposed artificially upon a primitive society after its formation, but was a natural growth and a condition of the formation itself. The existence of a society presupposes the existence of law.

It is necessary to stress this because the study

of the history and tradition of early communities might appear to lead to a contrary conclusion, since at a very early stage we find a record of the establishment of a code of law by some lawgiver. Some of the codes which have been preserved are of very great antiquity. One of the earliest known is the code of Hammurabi, which is older than 2000 B.C. Other examples are the Ten Commandments of Moses and the Twelve Tables of the Romans.

Judicial Decisions

In early times these codes were regarded as laying down new law, as is indicated by the word " lawgiver " applied to their compilers. But this view can no longer be accepted, for it is now clear that these codes were publications of laws previously existing in an unwritten and indefinite form.

Imagine an early judge deciding a dispute. There may be some tribal custom covering the point, and if so the judge will probably follow it. But if there is no custom, the judge will decide it according to his own ideas of right and wrong. He will probably not admit that he is doing so, for that would amount to an admission that he is making law, but he will state that he is " declaring " what has always been the law of the people. This attitude is that taken at the present day by the law of England with regard to judicial decisions.

When the judge has given his decision in the case imagined, the rule may be a guide to himself and later judges in later similar cases, handed on probably by oral tradition and teaching, and gradually a system of law will come into existence. This law will at first be purely traditional and a closely guarded secret reposed in a privileged class of judges. It is only at a later stage that some person will embody these rules in a written code, and this will usually happen as the result of some democratic revolt against the privileged class, when the democracy realizes the power that is given to the privileged class by having a monopoly of the knowledge of the law.

It has previously been said that the judge, in the absence of any custom or previous decision, will decide the case according to his own inherent ideas of right and wrong. It is a strange fact that men in all communites when required to decide as to right and wrong have arrived at the same decision on very many points. This explains why the ultimate principles of law are everywhere the same. Jurists have expressed this idea by saying that this common idea of right and wrong was the law of nature and that the law of nature was the source of all law—an unwritten law ingrained in the heart of man, which all human beings were everywhere bound to recognize, and endeavour to obey

The Law of Nature

This theory of the law of nature has been greatly developed by some jurisprudents, even to the extent of declaring that all laws must conform to the test of reason governing the law of nature, and that men have certain rights under the law of nature of which they have in certain cases been deprived by the law of the state. This view has had a very great influence on philosophy and politics, and it gave rise to the ideas contained in such expressions as the " natural rights of man " and in Rousseau's declaration on the first page of his *Social Contract* (1762) that " Man is born free, but is everywhere in chains "—which in due course became the maxim of the French Revolution.

In early English law some trace of this theory is found, and Sir Edward Coke (1552-1634), chief justice of the common pleas and later of the king's bench, went so far as to state in a case in 1610 that " when an act of parliament is against common right and reason, or repugnant . . . the common law will control it and adjudge such act to be void." But this view was not accepted even at that time, and to-day there is no room for it. If a law is in conflict with what someone may declare to be a part of the law of nature, the law will nevertheless be enforceable. It is by no means unknown for a judge in giving a decision to state that he recognizes that his decision will cause hardship, but that he regrets he is compelled to decide in this way because of previous cases which he must follow.

LESSON 3

How Laws Change

THE publication of a written code occurred at a comparatively early date in the history of most communities. While the advantage of publicity was no doubt great, the writing down of the law was also attended by disadvantages of equal importance. The chief of these disadvantages was a complete loss of elasticity in the law.

Even the best system, or code, of laws can provide only for the needs of the community at the time when it is formulated. As a society progresses, so its needs change. While the code specifically recognized certain rights, it denied, by implication, the existence of any rights not so specifically recognized, and put an end to the possibility of spontaneous development.

Even in cases where the code did establish a certain right, anyone who wished to enforce that right must do so in the manner prescribed, presenting his case in the approved form and carefully following a complicated procedure, a single departure from which would be fatal to his claim. To-day the courts in Great Britain will never allow a technical error in procedure to prevent justice from being done between the parties, but such elasticity is of recent growth. In the early law a man who alleged that his neighbour had struck him with his left hand when, in fact, the blow had been struck with his right hand, would have had his case dismissed without its merits ever being considered by the court.

Doe and Roe

The only communities that have progressed are those that have been able to adjust their laws to fresh circumstances and new demands. In such societies there have been three great instruments of change—legal fictions, equity, and legislation—and almost without exception they have made their appearance in that order of time. The method of change most familiar to us to-day—i.e. legislation—is almost unknown in the early state. Legislation involves a deliberate and conscious alteration of the law; in the early state alterations are brought about unconsciously, almost surreptitiously.

It is not easy to appreciate that until a very late period in the history of a state the law was regarded as perfect, sacrosanct, and incapable of improvement. It should further be borne in mind that to many codes a divine or quasi-divine origin is attributed, and their alteration by the hands of man is therefore unthinkable. Thus it is that the first changes came in disguise; and this will help to explain the devices and tricks to which lawyers of old resorted to achieve a result which in our day might be obtained by an act of parliament of ten lines.

The great weapon of the lawyer which enabled him to disguise the fact that he was changing the law was the legal fiction. These fictions are found in the legal systems of all countries which have developed a progressive civilization, and they usually involved the allegation by one party of a fact which, although untrue, the other party was not allowed to deny. An illustration from English law may be given.

This is connected with the names of John Doe and Richard Roe. Anyone who cares to look at the English law reports of a century and more ago will find many cases entitled "John Doe on the demise of . . . against Richard Roe," and this may engender surprise at the litigious disposition of the two. The fact is that neither John Doe nor Richard Roe ever existed in the flesh. They were merely the creations of some ingenious lawyer to enable the courts to recognize and protect rights in a manner in which, strictly, they were not entitled to act.

The old law provided no easy or simple method by which the title of two claimants to the *freehold* of a piece of land could be decided. The law did, however, provide a comparatively simple method, called the writ of ejectment, enabling the holder of a *lease* of land to establish his title to the lease if he were ejected. This writ of ejectment was made available for the freeholder by the following ingenious device.

Non-existent Lessee

The claimant to the freehold in the land went on to it with two friends. To one of them he granted then and there a lease of the land, whereupon the other, called the casual ejector, immediately turned the lessee off the premises. The lessee then brought the writ of ejectment against the casual ejector, and the person in possession of the land, i.e. the other claimant, was allowed to defend in the place of the casual ejector. The question immediately in issue was whether the lessee had a good lease, and as that depended on whether the first claimant had power to grant the lease or not, the decision of this point also decided the title to the freehold, because the claimant could have power only if he owned the freehold.

Soon the preliminary formalities were dispensed with. The real defendant, the person in possession, was allowed to defend in place of the imaginary Richard Roe, but only on condition that he admitted the (entirely fictitious) granting of the lease to the (non-existent) lessee, John Doe. Thus a new method of enforcing the right to the freehold of land had been created without any apparent change in the law. This legal fiction persisted until 1852, when it was abolished by the Common Law Procedure Act.

LESSON 4

The Origin and Meaning of Equity

Legal rules are necessarily general, whereas the circumstances of individual cases are particular, and so it is impossible to provide in advance for all situations which may arise. It is here that equity, the second great source of the law, appears. Where the established law is working an injustice because the particular case which has arisen was never contemplated when the rules were framed, there is some authority in the state which can intervene

and by the aid of equity prevent the injustice. The meaning of equity is most nearly defined as reasonableness or fairness. It would be a mistake to imagine that equity was willing to interfere in every case in which the law worked something less than justice, or that the equity of present-day English law plays any such part.

In all equitable systems it is only in the very earliest times that those administering equity are bound by no rules except their own ideas of right and wrong ; in such times the old taunt against the English equity administered by the lord chancellor—namely, that it varied with the length of the chancellor's toe, i.e. it was pure caprice—was no doubt true. But it is a strange feature of all equitable systems that in a short time they lose their original elasticity and develop into as rigid a code of rules as the old law, the harsh application of whose rules they were designed to moderate.

Equity in Rome

Equity should be regarded as a process for effecting changes in the law belonging to a middle period in legal history, later than the legal fiction, but earlier than legislation.

The Roman and English legal systems each developed a different but very vigorous type of equity. In Rome the growth of equity was due mainly to the growth of commerce and the consequent intercourse between Romans and non-Romans. For example, the old Roman contract was formal and cumbrous and unsuited to the needs of a trading community. Moreover, none but Roman citizens could enter into it because of its religious character. Thus intercourse with the outside world made necessary some form of contract of a simpler kind into which non-Romans could enter.

The Roman official whose duty it was to regulate the relationship between Romans and non-Romans was the praetor. He had no authority to make law, but he had to decide any disputes which came before him as best he could. By means of these decisions he built up a system of law outside the formal law applicable to Roman citizens, but suited to the needs of a trading community. At first this law applied only where a non-Roman was concerned ; but the advantages of the new informal rules over the old formal rules were so obvious that soon the new law was being applied to Roman citizens also.

Equity in England

It is in English law that equity is seen in its most vigorous state, for here it not only created legal principles in aid of the existing principles—those of the older common law—but also established a separate court—the chancery court, under the lord chancellor—distinct from the older common law courts, in which these principles were administered. This division had the peculiar result that a suitor who took his case to the common law courts would have judgment given against him if he wished to rely on equitable principles, and might thereafter have to apply to the chancery court to have his right established.

It is true that the common law courts in many cases added to the law by introducing new principles to adapt it to changed circumstances so far as they were not prevented from doing so by earlier decisions of their own, and that in so doing they were administering equity in its general sense. But in English law the word has acquired a special meaning, as those innovations and adaptions which were introduced and administered in the court of equity—the chancery court—only.

The Chancellor

In England equity arose in the following way. The king was recognized as the fount of all justice, and when he ceased to hear cases himself and delegated his duties to his judges, he was still regarded as retaining a jurisdiction to do justice in cases where these judges in applying the law had failed or were unable to do so. This residual jurisdiction—equity came in time to be exercised by the chancellor, who in early times was invariably a churchman. This fact is of the utmost importance.

First, the chancellors had behind them a training in the canon law (the law of the Church), and in Roman law, and thus by their decisions introduced into English law many principles of these legal systems ; secondly, the chancellors, as churchmen, tended to consider whether the conduct of the parties before them had been according to good faith or to conscience, and they would compel the parties to act as conscience dictated. The chancery court was named after them. The division between the common law courts and the courts of equity continued until the Judicature Act, 1873, when the courts were merged and it was provided that if the rules of common law and equity conflicted on any point, the rules of equity should prevail.

The Last Hundred Years

Legislation is always the last method by which changes are made in the law. In Great Britain, although many important acts of parliament were passed from the 13th century onwards, the last hundred years or so have been a period of legislative activity incomparably greater than that of any earlier age. The great landmarks of that period are the changes in the criminal law made in the first half of the 19th century, including the abolition of the death penalty for such offences as theft ; the changes in the courts made by the Judicature Acts, 1873 and 1875 ; the revolution—it is not too strong a

word made in 1925 in the law relating to land; and—greatest of all—the almost innumerable acts which have brought the welfare state into existence. In the last hundred years there is probably no branch of the law which has not been changed by act of parliament; and to-day, in place of the old attitude, upheld by very eminent lawyers as late as the end of the 18th century, that the law was perfect, we have nearly reached the attitude that change is desirable for change's own sake. In the early state antiquity was a merit of itself; to-day it is almost a fault.

LESSON 5

The Law of Contract

THE law of obligations is divided naturally into "contracts" and "civil wrongs." A person may by his own voluntary act, i.e. by entering into a contract, acquire rights and duties towards another person; such obligations are contractual. Without any voluntary act, however, every person owes duties to others arising not from contract but from the needs of society. Thus the driver of a motor car owes a duty to other road users to exercise reasonable care in his driving; and everyone owes a duty to others not to libel them. A breach of these duties is a civil wrong.

Forms of Agreement

A contract requires an agreement between at least two parties. The agreement must be one which is intended by the parties to have a legally binding effect. If A asks B to dinner on a certain date and B accepts, there is an agreement, but it cannot be said that A and B contemplated that if B should not keep the engagement he should be liable to any legal action. The parties to the agreement were making a social arrangement not intended to affect their legal positions. A similar result may be brought about by the express provision of the agreement. It may provide—as do the rules of some football pools—that it is to be binding in honour only, not legally.

Again, the agreement must be one which the law will enforce. The law may require the agreement to be in a certain form, e.g. in writing, or to have some special feature, e.g. consideration in English law. In other cases the law may refuse to enforce a contract, because it is against public policy, e.g. an agreement to commit a crime.

From Status to Contract

In early law, when trade was small, society managed to carry on its affairs without entering into relationships which would now be considered contractual. Persons had rights and duties not because of any contract into which they had entered, but because of the status they held in society. Thus rights which a master had over his servant did not arise out of any contract between them, but out of the relationship of master and servant itself. Sir Henry Maine expressed this in words which have become famous, by saying that the development of the law has been "from status to contract."

The high-water mark of freedom of contract was reached in the 19th century when under the doctrine of *laissez-faire* and the economic theories of Adam Smith (1723–90) it was considered essential to the well-being of the state that persons should be free to make such contracts as they pleased. Each individual, it was said, was the best judge of his own interests, and if everyone were left free to enter into such contracts as he pleased the result would be in the best interests of the community as a whole. To-day we do not put our faith in freedom of contract, regarding that freedom as illusory when one of the parties to the contract is in a much more advantageous bargaining position than the other.

As Lord Justice Denning, in *Freedom under the Law* (pub. 1949), has stated concerning the doctrine of freedom of contract in the 19th century as applied to the housing situation: "Have you not heard a landlord saying to a prospective tenant 'Take it or leave it,' which is the equivalent to saying 'Pay my price or go on the streets?' What freedom is there for the tenants there?"

Compulsory Legal Enactments

In recent times, therefore, freedom to make contracts has in a great many matters been restricted, and in many cases persons are not free to agree on whatever terms they please but must accept some code of rights and liabilities imposed by some act of parliament on them because of the relationship into which they have entered. For instance, the Rent Restriction Acts, which limit rents and prevent tenants from being evicted, apply to a large number of houses in this country, whatever landlord and tenant may have agreed; in 1938 an act empowered wages boards to award holidays with pay to employees whose wages they regulated; and so on. Of late years legislation has established definite control over many of the great industries, e.g. agriculture.

For our present purpose the importance of these various legal enactments lies in the fact that they are compulsory and cannot be altered by agreement of the persons to whom they

apply. Thus the legal relationship between a tenant farmer and his landlord depends to a very large extent on various Agricultural Holdings acts which apply notwithstanding any contract between the tenant and the landlord.

Limited Contractual Power

In early times we find neither the control with which we are now familiar nor the 19th-century conception of freedom of contract. On the rare occasions on which a contract was required, it assumed invariably a semi-religious form, with the parties making vows to perform their obligations, the duty being regarded as being owed to the Deity rather than to the other party to the contract. Here the *form* of the contract was the essential part, the parties having to use the precise formal words of the ceremony. Even when contract began to lose its religious character, the usual formalities remained, and the important question in any contract was not "What was the intention of the parties?" but "What forms did they go through?"

Furthermore, it was not open to the parties to contract on any matters that they pleased. There were certain recognized contracts—e.g. hire or sale—into which they could enter with certain well-defined consequences, reminiscent perhaps of the idea of status; but to contract on other matters was not possible. As trade expanded, this limited contractual power became increasingly inadequate, and gradually in each country a more extensive system of contracts was developed. In England the formal contract had been the contract under seal—the deed—to which the parties, in the days before writing was a common accomplishment, affixed their seals instead of their signatures.

Doctrine of Consideration

When the law expanded, the courts in England refused to hold that every promise given was binding as a contract, and would not recognize a contract unless the person who was seeking to enforce a promise by the other party had himself undertaken some obligation in the contract towards that other party. This mutuality was called consideration, and may be defined as some benefit given or promised by A, or loss or liability incurred by B in return for A's promise. Thus if A promises to lend B his horse for the harvest, and B in return promises to give A some part of his wheat, there is a good contract. But if A had promised to lend B his horse and B had promised nothing in return, B could not compel A to lend the horse.

This doctrine of consideration is so essential a part of the law of England that it is sometimes difficult to realize that it is not the law of all countries and that in many cases it works definite injustice. No similar rule exists in Scotland; and, on principle, it is not easy to see why a promise made and relied upon should not be enforceable. Lay opinion generally tends to regard such a promise as binding, and the courts—notably the Court of Appeal in a case decided in 1947—have of recent years done what they can to prevent the rule's causing injustice, but their powers are severely limited by earlier decisions. As a result of the doctrine, if A and B are negotiating for the sale of a house which A offers to sell for £2,000, and A agrees to keep the offer open for a week and meantime not to sell to anyone else, then as there is no consideration for this promise to keep the offer open, if A sells the house to C before the week has elapsed, B cannot legally complain. Yet if B gave A one shilling for his promise it would be binding.

Although this is the legal position, B will certainly think that he has been unfairly treated. In 1937 the Law Revision Committee, made up of judges and other eminent lawyers, recommended that at some date to come the doctrine should be mainly abolished.

When Contracts are Not Binding

Since the binding effect of a contract is now generally recognized as depending on the mutual consent of the parties, it is not surprising to find that, where this consent on one side is not free and genuine, the contract is not binding. In a limited number of circumstances mistake of the parties may have this effect if the mistake is mutual and relates to a material part of the contract.

Thus, if A agrees to sell to B a cargo of wheat on a ship called the *Peerless* and if there are two ships of that name, A thinking of one and B of the other, the parties are both mistaken as to the subject matter of the contract, which is thus of no effect. Mistake by one party only is not in general sufficient to upset a contract, unless the mistaken party is led into his mistake by the other party. Thus, if A buys a picture from B, mistakenly thinking it is by Rembrandt, and B does nothing to lead A to make the mistake, the contract is good; but if B tells A the picture is a Rembrandt, the contract may be set aside. Similarly, if a person is compelled to enter into a contract by duress, or induced to enter into it by an untrue statement made by the other party, his consent is vitiated.

Enforcement of Contractual Obligations

There are two ways in which the law may compel a person to observe his contractual obligations. First, it may compel the party to perform what he has promised to do; secondly, it may, instead, compel him to pay a sum of money as damages to the other party, the amount of the damages being what the other party lost through the non-performance of the

contract. English law is unwilling to adopt the first course in any case where damages can compensate the other party. In some cases they obviously cannot. Thus, if A agrees to sell to B an antique snuff-box used by Prince Charlie, no sum of money which A could pay B would enable him to buy another similar snuff-box, and the court would order A to hand over the snuff-box to B. In certain other cases, even where damages would not compensate completely, the courts will not order the carrying out of the contract. So if A agrees to work for B as his secretary and then refuses to do so, the court would award damages, but would not compel A to act as secretary.

As a general rule, the result of a breach of contract will be that damages will be awarded. The object of these damages will be to put the injured party in the same position as if the contract had been carried out. So if X agrees to sell to Y a motor car for £100 and fails to do so, and Y could have bought a similar car elsewhere for £120, the damages will normally be £20, the amount of the difference.

LESSON 6

Civil Wrongs

Civil wrongs are breaches of rights which arise not from contract but from the common law. The law of civil wrongs is at once of much greater antiquity and less fully developed than the law of contract. In primitive law the idea of a crime as an offence against the state is unknown, and in its place are acts which we should to-day consider criminal treated as we should to-day treat civil wrongs. Thus, a man assaulted by another is expected to exact revenge himself, or take the matter to the courts, where he would be awarded a money sum varying with the nature of his injury. This latter system has much more in common with the modern law of civil wrongs than with the law of crime, for the act is regarded as an offence against the individual injured and not against the state.

Acts which are crimes may also be civil wrongs—that is, if they infringe the rights of the individual as well as those of the state. When goods are stolen, the thief may be prosecuted by the state and also sued by the owner for damages. One great distinction between crimes and civil wrongs is that in a crime it is in most (though not all) cases essential that the criminal should have had in his mind a criminal motive.

No General Law of Tort

To constitute a civil wrong, no such motive is required. Thus, if A takes B's overcoat from his club without B's permission, the act will be a civil wrong, whether A knew it was B's coat or whether he took it in mistake for his own. But it will not be a crime unless A took it knowing it to be B's and intending to deprive B of it.

A breach of contract may also be a civil wrong, for persons may enter into a contract and specify under the contract rights which they have already at common law. Thus, A may hire out his motor car to B and stipulate in the contract that B shall return it whenever asked. If B fails to do this, he may be sued for either breach of contract or civil wrong.

As previously stated, the law of contract at one time recognized only certain specific contracts, such as sale and hire, and by a later development a general law of contract grew up. The law of civil wrongs has never succeeded in emerging from the earlier stage, and so in England to-day there is no general law of tort, as a civil wrong is called, but merely a series of acts recognised as civil wrongs without any general principle of liability. If a person has been injured by the act of another, he can recover damages only if he can show that the act constitutes one of the known civil wrongs.

No Distinguishing Principle

There is no principle which will enable anyone to distinguish between acts which are regarded as civil wrongs and those which are not. If A is employed by B, and C induces B to dismiss A without proper notice and, therefore, so as to break the contract between A and B, A will have an action against C; but if C merely induces B to dismiss A *with proper notice*, A has no remedy. Again, if A induces B to revoke a legacy in his will in favour of C, C has no remedy. If the question is asked, "Why has C a remedy in some cases but not in others?" the only answer that can be given is the very unsatisfactory one that English law has recognised some injurious acts as civil wrongs but not others.

Three Classifications

The civil wrongs known to English law can be classified roughly under three heads. First, there are certain acts, which a man does, as it is said, "at his peril": and if damage results from them he is liable, regardless of whether or not he has taken every step possible to prevent damage. Secondly, there are acts which cause injury for which a man is liable only if he has failed to take proper precautions and exercise care. Thirdly, there are acts for which a man is

liable only if he did them with the definite intention of injuring the person who has, in fact, been injured.

Examples of the first class arise where a man has upon his land something of a dangerous character, such as a wild animal or a reservoir of water. There is nothing illegal in keeping these things, but if by any chance the wild animal escapes or the reservoir bursts and damage is caused to any person, the owner will be liable, even though he can show he has taken every possible precaution against the event which has happened.

Dog's Free Bite

Animals which are not wild, e.g. dogs, are in a peculiar position. The owner is not liable if his dog bites some person, unless he (the owner) knew the dog was dangerous and likely to bite human beings; and in order to prove this knowledge it will usually be necessary to prove that the dog had previously bitten some person. Thus it is true to say, so far as human beings are concerned, that a dog is entitled to one free bite. Special provision dispensing with this free bite has been made by statute in cases where the dog has bitten an animal such as a sheep, and not a human being.

Liability for Negligence

The second class of acts is very wide, covering what is known as liability for negligence. By no means every act of "carelessness" in the popular sense is "negligence" in law. It must first be established that there is a duty to take care recognised by the law. Negligence has thus been defined as a failure to exercise in one's conduct the degree of care *required by the law*. This cannot be said to be a satisfactory definition, but it does make it clear that a person may be careless and so cause damage to others and yet not be liable, unless the law imposes on him a duty to exercise a certain degree of care to the persons injured.

The defect in the definition is that it still leaves to be answered the question, "In what cases does the law impose a duty to take care?" and the answer is not easy. The most satisfactory answer is in a speech delivered in a famous case in 1932:

> The rule that you are to love your neighbour becomes, in law, you must not injure your neighbour; and the lawyer's question, "Who is my neighbour?" receives a restricted reply. You must take reasonable care to avoid acts or omissions which you can reasonably foresee would be likely to injure your neighbour. Who, then, in law is my neighbour? The answer seems to be—persons who are so closely and directly affected by my act that I ought reasonably to have them in contemplation as being so affected when I am directing my mind to the acts or omissions which are called in question.

This statement of the circumstances in which a legal duty to take care arises is of outstanding importance. But its application is seriously restricted by earlier decisions of the courts, which are still binding.

Reasonable Care

In the case from which the foregoing extract is taken a woman had drunk part of a bottle of ginger beer which a friend had bought from a retailer. The bottle, it was alleged, contained a dead snail, and the woman became seriously ill. She sued the manufacturers of the ginger beer, who were held by the house of lords to be under a legal duty to take reasonable care that their ginger beer was not injurious to health. In other words, the woman was a "neighbour" within the meaning of the foregoing legal statement.

In a later case, a man sold a house to a Mr. A. It was so badly built that the ceiling of one of the rooms collapsed and injured A's mother. On general principles, one would suppose that a person putting on the market a dangerously built house owes a duty to those who may live in it, just as great as that which a person manufacturing ginger beer injurious to health owes to persons who later drink it. But the court had to hold that the builder was not liable and owed no such duty because the house of lords had decided 30 years earlier that no such legal duty existed with respect to a dangerous house.

Had it not been for this earlier decision, the courts might well have followed the general principle laid down in the "snail" case. At any rate, they were prepared to apply the general principle to a person who carelessly erected a tombstone (to which the "dangerous house" case decided by the house of lords did not, of course, apply) which fell and injured a person in the churchyard.

With Malice

The third class of acts is comparatively small, for the motive with which an act has been done is not usually material in considering whether or not it is a civil wrong. In some cases it is necessary that the act should have been done, as it is said, with "malice." Thus, to bring an unsuccessful criminal prosecution against a man may cause him much damage and mental suffering, but if the prosecutor had reasonable grounds for prosecuting, no action will lie. But he may be sued when his motive has been merely to cause injury to the person prosecuted.

It is not possible here to go in detail into various specific wrongs recognised by English law, and it must be enough to say that their general object is to protect property from being damaged or interfered with, and to protect a man's person from being injured and his reputation from being attacked.

LESSON 7

The Law of Crimes

THE average layman defines a crime with reference to its consequences as some act for which the person responsible may be punished by a criminal court. Any attempt to define a crime with reference to the nature of the act as distinct from its consequences is beset with great difficulties. The distinction between crimes and civil wrongs is found in every civilized state, but acts which are crimes in one community may be mere civil wrongs in another, and it is not easy to arrive at an exact definition of a crime which will explain the distinction.

At first sight the most obvious practical distinction is that the state, on behalf of the community at large, takes cognizance of a crime without waiting for the person injured to invoke its aid ; whereas in the case of civil wrongs it is left to the individual injured to set the law in motion against the person who injured him.

Setting the Law in Motion

For example, if A " holds up " B on the street and robs him of his watch, he commits a crime, and the state will take criminal proceedings against A to bring about his punishment, even though B does not invoke its aid. But if B, having borrowed A's watch, refuses to hand it back when asked to do so, he commits only a civil wrong, and the state will not initiate any proceedings. A may himself start civil proceedings against B for detaining the watch, and if he does so the state will order B to return the watch, but unless A brings the matter to the courts the state will not interfere. The reason is that it is a crime to rob a man of his watch ; but it is not a crime to refuse to hand back something you have borrowed.

This distinction is of value, but there are two objections to it. First, it is always open to a private individual to begin criminal proceedings, just as he may begin civil proceedings. Secondly, the distinction is not one which relates to the *nature* of criminal and non-criminal acts, and it still leaves to be answered the question *why* the state will interfere in certain cases and not in others.

Morally Offensive

Another distinction that may be suggested is that crimes are wrongful acts which violently offend our moral feelings. This again is of value as a rough test, but no more ; for a breach of trust, or a breach of contract, which are not crimes, may be much more offensive morally than many crimes, some of which, e.g. failure to take out a dog licence, can scarcely be said to involve any moral turpitude. The large number of regulations made necessary by the welfare state has made this test even less reliable, for although a breach of these regulations is a crime, it is in many cases very difficult to say the breach is a moral wrong. Again, one cannot say that every act which injures the community is a crime, for the civil wrongs previously mentioned may indeed do great injury to the community.

Punishment and Compensation

The idea of punishment also helps to another distinction. When a civil wrong has been committed and the law is invoked by the person injured, the law is concerned in general not to *punish* the wrongdoer, but to *compensate* the injured person by awarding him damages. In assessing the damages the court will normally be concerned, not with the degree of moral guilt involved in the defendant's conduct, but solely with the results of his conduct to the plaintiff. There are exceptions to this rule, for in some actions for civil wrongs—e.g. actions for libel—damages may be awarded to punish the person sued. In the case of a crime, the object of the court is to inflict on the criminal the punishment deserved by his conduct.

Another distinction is that a civil wrong may always be pardoned or settled out of court by the person injured, but it is of itself a crime to agree either not to institute or to withdraw a criminal prosecution on payment of a consideration. So if your dog is stolen you cannot advertise in the papers and say that if it is returned a reward will be given and " no questions asked." This illustrates the principle that a crime concerns not only the criminal and the injured person but also the state.

Characteristics of a Crime

It can be concluded that a crime, generally speaking, may be distinguished from other illegal acts by means of the following characteristics.

(1) It involves a moral wrong ; (2) it injures the community or a large part of it ; (3) the state on behalf of the community will interfere to punish the act ; (4) the state in punishing the act is concerned not with the loss caused by the crime to any person, but with the quality of the act done by the criminal.

The infliction of punishment causes hurt not only to the criminal but also indirectly to many other persons, and when it takes the form of imprisonment it is wasteful and expensive. Thus, before any act is made a crime, the state should be satisfied that the evil caused by the

act is greater than the evil necessarily attendant on punishment. As a general rule, mere lying is not a criminal offence, but lying when under oath is, because the evil to the community is much greater in the second case—where a man may be wrongfully convicted or acquitted as the result of lying evidence. For practical purposes there is one further and most desirable requirement: the act which is made a crime must be one which the state can detect.

Co-operation of Private Individuals

It has been said that it is impossible to make a nation good by means of legislation, and the statement is true to this extent, that it is useless for the state to attempt to prohibit an act by making it criminal unless the prohibition can be enforced. Two factors may make this enforcement difficult: first, the act may be one which is usually done in such circumstances that it is impossible to obtain evidence that it has been done; secondly, the act may be one of which the community as a whole does not disapprove. It is no doubt the function of the law to raise where it can the moral standards of the community, but the process must be carried out with care.

The state cannot rely entirely on the police for the detection of crime, but must have the co-operation of private individuals. The result of declaring to be a crime an act which the community does not regard as wrong may be that this co-operation is absent, the law will be brought into disrepute, and the very dangerous idea promoted that a man may commit a criminal offence without doing anything morally wrong.

This attitude was found frequently in connexion with rationing offences during and immediately after the Second World War. Many highly moral persons found it difficult to believe that it was morally wrong to use a clothing coupon given to one by a friend. This attitude of mind was well illustrated by a remark made to the present writer by a highly competent lay magistrate when a man known to both of them had been convicted of selling eggs illegally. "The real trouble," said this magistrate, "is not that he sold the eggs but that he charged more than the regular black-market price for them." To this magistrate—and to how many other people?—there was nothing morally wrong in selling black-market eggs—at the recognized black-market price.

LESSON 8

Punishment

Difficult though it may be to decide which wrongful acts should be made crimes and so prosecuted by the state, and which should be left as civil wrongs to the redress of the injured party, the further task of deciding what is the proper basis of punishment for crime is still more difficult.

"Hanging Not Enough"

The whole attitude of the law towards punishment has changed in the last hundred and fifty years or so to an extent that is difficult to credit. Lord Brampton (1817–1907), formerly Sir Henry Hawkins, a famous judge, recalled that in 1830, when he was at school in Bedford, he saw being carried away in a cart for burial the body of a youth of 17 who had been hanged for setting fire to a stack of corn. To-day a person of that age cannot be sentenced to death, and indeed cannot be sent to prison except in special circumstances, as explained in pages 1607 and 1608.

Death was then the penalty for a vast number of offences—so many that Dr. Radzinowicz, the author of *A History of English Criminal Law*, refused to attempt to estimate their number. One act, the Waltham Black Act, passed in 1723 and in force for about a century, itself created 350 new capital offences. Many of them were of the most trivial character, e.g. being in the company of gypsies. There had, indeed, been those who considered a mere death penalty insufficient, and advocated some form of torture or additional degradation, as did the author of a pamphlet in 1701 entitled *Hanging Not Enough*.

This principle was long adhered to in the punishment of treason. It was not until 1790 that burning alive ceased to be the legal punishment of women convicted of treason. And the horrors of the punishment of male traitors (which make us realize that when the sentence for treason was amended to provide that a man should be hanged "until he is dead," the added words were not mere surplusage but a merciful addition) were not legally removed until 1814. The barbarity of the quartering after death of the bodies of the hanged remained until 1870. But in practice these severities were mitigated many years earlier. Public execution in Britain was abolished in 1868.

The Parish Constable

The severity of the law in the 18th century and the early years of the 19th century frequently led to juries refusing to convict of crimes, with the remarkable result that sometimes the persons whom the law was designed to protect would

petition for its repeal, as in 1830 when 735 bankers and business men petitioned for the abolition of the death penalty for forgery.

The ferocity of the criminal law at this time was due not so much to lack of humane feelings in the community as to the absence of any efficient police force. As a result of this, very few criminals were detected, and the ferocity of the law was designed to deter persons from committing crimes by holding up to them the severest of punishments if they were caught. In most parishes the only official of the law charged with the detection of crime was the parish constable—often not very different (we may suppose) from Master Constable Dogberry in Shakespeare's *Much Ado About Nothing* who, having charged his watchmen "To bid any man stand in the prince's name," was asked by a watchman, "How if a' will not stand?" to which Dogberry replied, "Why then, take no note of him but let him go . . . and thank God you are rid of a knave!"

The Modern Approach

The decade 1827–37, which saw the abolition of the death sentence for many crimes, saw also provision made for the establishment of an efficient police force in London (1829) by Sir Robert Peel (1788–1850), and in every municipal borough (1835). It was realised that the greatest deterrent to crime was not the severity of the punishment but the probability that the crime would be detected.

The modern approach to punishment is that its main object is to protect the public from crime. It may endeavour to do this in three ways: (1) it may prevent the criminal himself from committing further crimes by detaining him in prison or putting him to death; (2) it may by the example it makes of the criminal deter other members of the community from committing crimes; (3) it may reform the criminal by giving him such treatment and training that he will refrain from committing further crimes when he is released. These may be described respectively as the preventive, deterrent, and reformatory elements in punishment. The reformatory element is of recent growth, dating from the prison reform movement which started in England in the early 19th century. Another very important aspect of the problem involved in the prevention of crime is the creation of good living conditions and the spread of education so as to dissuade persons from ever committing crimes and so becoming liable to punishment.

There is still present as an object of punishment something of the old retributive element of revenge. This was a very important object in early law, where punishment was measured by the resentment of the wronged party, and a thief was more severely punished if caught red-handed, because the wronged party might be presumed to have exacted a greater penalty himself than if his anger had had time to cool. This idea that a criminal should pay for his misdeeds is so deeply engrained in the feelings of a large part of nearly all communities that it is impossible to leave it out of account.

Relevant Considerations

The proposition that punishment should be relative to the offence and to the offender would probably meet with as much general approval as can be expected for any general proposition in so controversial a subject. As regards the offences, the relevant considerations would appear to be: (1) the greatness of the evil that might be caused by the act; (2) the ease with which the act can be committed—for example, postmen are severely punished for thefts of letters; (3) the prevalence of the particular crime at the time—for example, during the Second World War "black-market" and looting offences were severely punished.

The condition of the offender must next be considered. Was he subjected to great temptation? What are his antecedents? Has he ever "had a chance"? How will the punishment affect him personally?

Imprisonment will have a more serious effect on the social status of a man in a good position than on that of a labourer; while a labourer may find it much harder to pay a fine. Is the criminal capable of reformation, or has he already been several times in prison?

Habitual Criminals

A remarkable recent development is in the Criminal Justice Act, 1948, which revolutionised methods of punishment in England. Its most striking features are, first, the ending of the system previously in force of giving repeated short sentences to habitual criminals. Instead, the general principle is that so long as a convicted person is likely to be capable of being reformed he shall be given sentences which will result in his receiving training which may reform him; but as soon as it is apparent that he will not reform, then he is sentenced to a long period of preventive detention to prevent him preying further on the public. Long sentences of preventive detention have already had a great deterrent effect.

Young Criminals

Secondly, the principle of giving special attention to the treatment of young criminals, already set out in the Children and Young Persons Act, 1933, is extended. No one under 21 is to be sent to prison if there is any other appropriate method of dealing with him. Instead, a variety of courses is open. Such a person may be detained in a detention centre, usually for three

months; or, while left at liberty and allowed to continue his work or schooling, be compelled to attend attendance centres for a number of hours in his spare time; or be sent to an approved school; or be sent to Borstal for training. Any convicted person may be placed under the supervision of a probation officer for a minimum period of a year.

It is not proposed to discuss here the merits or demerits of capital punishment but merely to state the various recent occasions on which its abolition or limitation has been considered.

Capital Punishment To-day

On April 14, 1948, by a free vote of the house of commons a clause was introduced into the Criminal Justice Bill to suspend capital punishment for five years, and on April 16 the home secretary stated that in consequence he proposed to advise the crown to grant reprieves in every case, regardless of the merits of that case. On June 2 the clause was rejected by the house of lords, being strongly opposed by the lord chief justice, Lord Goddard; and on June 9 the home secretary announced that from that date each case for a reprieve would be considered on its merits. There was still much division of opinion in the house of commons, and in January, 1949, a Royal Commission was set up relating to capital punishment. This was concerned not with the question of abolishing capital punishment but with considering whether the liability to suffer death for murder should be limited or modified, i.e. whether there should be degrees of murder.

Extenuating Circumstances

It was realised that there are in England certain crimes where the accused is convicted of murder and therefore necessarily sentenced to death, but where a reprieve is nearly always granted, e.g. the survivor of a suicide pact or a person guilty of "mercy killing." Indeed, about one in every two persons convicted of murder in the whole of Great Britain is reprieved; the proportion is much higher in Scotland where, for this and other reasons, no one was executed between 1929 and 1946. It was felt, therefore, most undesirable that the death sentence should be pronounced where owing to the facts of the case there was little likelihood of it being carried out.

The commission reported in 1953, and recommended that the law of murder should be amended so as to confine the crime more to cases where the killing was intentional and exclude cases where the killing was in a sense accidental, e.g. where a burglar struggling to escape from the owner of the house inadvertently kills him. The commission did not favour degrees of murder, but favoured a proposal that it should be open to a jury to find extenuating circumstances and that if they did so, the judge could pass any sentence less than death. The commission realised that this proposal might not be accepted, and concluded that in that event the only issue remaining was "Should capital punishment be abolished?"

Very direct attention was called to the problem of capital punishment by the cases of Evans and Christie. In 1950 Evans was hanged for the murder of his child by strangulation, his wife having been found murdered by strangulation at the same time. In 1952, Christie, who had been living in the same house as Evans at the time of the murder of Mrs. Evans and the child, was charged with the murder of his (Christie's) wife by strangulation, and he then confessed that he had murdered several women at these premises by strangulation, including Mrs. Evans. Many persons felt that Evans had been wrongly convicted, but a Q.C. who held an inquiry reported that the evidence against Evans was overwhelming and that the conviction was just. The case strengthened the movement for abolition of capital punishment.

Recent indications (1956) show that the house of commons is more favourably disposed to the abolition of capital punishment than is the house of lords.

LESSON 9

The Law of Property

In most countries the state has allowed individual citizens to appropriate to themselves the things which make up property, and has passed laws to prevent others from interfering with them in the enjoyment of their property. The idea of the individual's owning property to the exclusion of others is of great antiquity in the laws of nearly all civilized states, though the idea is not found in the earliest period of which history has knowledge.

In those early days property was owned by the community as a whole and not by individual members. In England, land was for long cultivated and enjoyed in common by the inhabitants of a village. Socialist and communist movements have striven for the abandonment of the system of individual ownership in favour of communal ownership by the state.

Property falls into three classes: first, land, which is immovable and fixed in one place; secondly, chattels—corporeal things which can be moved from place to place; thirdly, rights of a

more vague character, such as shares in a company or sums of money owed by others. These last are not tangible things; for although we may handle a share *certificate*, the certificate is not in itself of value but merely evidence that the share represented by the certificate is owned by us, in the same way that title deeds to land are evidence that the land belongs to us. Property of this intangible character is usually referred to as an incorporeal movable.

Though this threefold division of property is recognized almost universally, for practical purposes the most important division is between property which is immovable—i.e. land—and property which is movable—i.e. corporeal and incorporeal movables, which are usually classed together.

Movables and Immovables

In English law, property is divided on these principles into real property and personal property. The division is not merely of theoretical, but also of great practical, importance (though less important in England since the passing of the Property Acts of 1925, explained later in this Lesson) for on it will depend the legal rules which govern the method by which the property may be transferred, and the persons entitled to succeed to it on the death of the owner intestate.

Thus, while the ownership of a motor car may pass from one person to another by a mere oral agreement for its sale, even though the *possession* remains in the seller, the ownership of land must be transferred by a written document, and the ownership of shares by a written transfer must be followed by registration in the company's books.

It is impossible to understand the English law of real property to-day without some considerable knowledge of its history, i.e. of what the law has been in the past.

The simple division into movables and immovables has not been maintained. Real property originally meant property which the owner could recover from any person who deprived him of it, whereas if a man were deprived of personal property he could not recover the actual property itself but must be satisfied with a sum of money paid to him as damages for his loss.

Systems of Tenure

Feudal law would not allow all the interests it created in land to be specifically recoverable; so it was that some interests in land were real property, while others were personal property. Estates for life or greater interests were real property, but leases, even if for 999 years, were personal property, because originally they had not been specifically recoverable.

No one in England could own land except the king. All other persons were tenants holding on some tenure some estate or interest in the land, either direct from the king or through some intermediate tenant holding from the king. This system of tenure was essential to the feudal system. The tenant rendered to his lord certain services for the estate he held. There were originally four kinds of free tenure distinguished by the different services which the tenants were required to render.

Estates and Service

A tenant in knight service was bound to do military service; a tenant in frankalmoign, religious services; a tenant in sergeanty, personal services; and a tenant in socage originally performed specified work on the lord's land, and later paid a money rent. With the decay of the feudal system the majority of these tenures became obsolete, and in 1660 all tenures were turned into socage, with the payment of rent. Side by side with these free tenures existed unfree tenure, or copyhold, the copyhold tenant holding by virtue of the court roll of some lord's manor.

There might be several tenants all holding different "estates" in one piece of land. Thus, A might be tenant in fee simple, B tenant in tail, and C tenant for life. A would hold an estate which entitled him to dispose of the land to anyone he wished, and on his death the land would pass to his heir, even if that heir was some collateral and not a direct descendant. B had an estate which could descend only to his descendant; if he had none, then the estate came to an end. C's estate must come to an end with his own life.

Interests in Land

Only one of these persons could be in possession of the land—the tenant for life, in the example cited—and the other tenants merely stood by ready to come into possession on the termination of the estate before them. Thus, on C's death, B would take possession, and if at any time B's heirs in the direct line failed, A or his successors in title would take possession. C was said to have an estate in possession, and B and A to have future estates. Other estates could be created, e.g. it might be provided that A should hold land until he married and that then the land should pass to C.

For many years before 1925 the only feature of tenure which remained was "escheat," that is, the rule that when a tenant died without any persons entitled to succeed to him his estate escheated to his lord. Since 1290 it has not been possible to create any new tenures, so that the result of escheat had for long been that the land passed to the Crown as the lord of the tenant. A series of statutes in 1925 revolutionised the law of real property in England.

These acts had two objects: (1) to get rid of artificial distinctions between land and movable property, and (2) to simplify dealings in land. Until 1925 real property had passed on intestacy under rules entirely different from those governing personal property. The law has assimilated the two, and has provided rules so that if a man dies intestate, his property will pass to the persons he would probably have wished to have it if he had made a will. The general principles of estates in land have also been greatly simplified, and future interests in land must now be created by means of a trust—thus further assimilating the law of land to that of movables. Again, before 1925 real property was liable in a different order from personal property for the debts of a deceased person; now both are governed by the same rules.

Chattels

Personal property included all movable objects, commonly called chattels, but it also included leases for a term of years—very valuable rights, but not recognized as "estates" by the common law.

The reason for this is of considerable interest. In the Middle Ages, when the lease for a term of years was first introduced, trade and industry in the form in which we have them to-day were unknown; a person who wished to invest money so that it might yield him an income could buy no stocks or shares in limited companies. A lease of land which could be sub-let to others at a money rent was the only form of investment. At this time the law imposed many inconvenient restrictions on real property, and the new moneyed class did not desire that this popular investment—a leasehold—should be regarded as real property and so be subject to these restrictions.

Accordingly it became personal property for a reason very important in law—the pressure of powerful and interested public opinion. Lawyers felt that this new accession to the ranks of personal property had little in common with the ordinary chattel property of the time, e.g. precious metals, cattle, and horses; so they invented for it a new name, a chattel real; a confusing name, for a leasehold was not a chattel nor was it real property.

Chattels Corporeal

As previously pointed out, the only person who could own land was the king; all other persons interested in the land were tenants. No such rule ever applied in the case of chattels, nor did the law ever recognize estates in chattels similar to the estates in land. The chattels known to the early law were all chattels corporeal—easily destructible and lacking the permanence of land. To a certain extent the law did recognize a division of interests in chattels; it was always possible for the ownership to be in one person and the possession in another. The person in possession—the bailee—was given extensive rights for the protection of his interests. He could sue any person who deprived him of the goods or who damaged them, and could recover damages irrespective of whether or not he was liable to his bailor. Thus, if a motor car is hired by one person from another and is damaged by the negligence of a third person, the bailee can sue for the damage even if he is not liable to the bailor.

Chattels Incorporeal

Another class of chattels known to the law to-day forms by far the greatest part of all wealth. This is chattels "incorporeal," and it includes such intangible things as debts, negotiable instruments, shares, stocks, etc.

It is not possible from their nature to separate the ownership of chattels incorporeal from the possession. Nor is it permissible to create "estates" in them as in real property, although very much the same result may be reached by the use of trusts. Thus shares may be given to trustees on trust to pay the dividends on them to A during his life, and after his death to hand over the shares to B. A and B will here be in much the same position as if A had an estate for life and B had the fee simple in reversion.

The ownership of these various classes of property is transferred from one person to another in different ways. Real property and chattels real can be transferred only by a written conveyance; chattels corporeal are usually transferred by delivery, and chattels incorporeal by a written instrument—in some cases requiring registration or intimation.

Buying and Selling

As is so often the case in law, there are exceptions to some of the foregoing rules. In England, on the sale of chattels corporeal the property may pass before delivery, while the goods are still in the hands of the seller. This is not universal, and in many countries the property does not pass until delivery.

The distinction leads us to a matter of great practical importance. It is a fact that the financial stability of any person is frequently judged by the chattels he possesses, and he may be given credit on the basis that because he possesses the chattels he is also the owner of them. Further, one person may buy goods from another under the impression that the goods belong to the seller because they are in his possession. Because possession is thus so frequently made the practical test of ownership, the law has endeavoured to some extent to protect those who give credit to a person in possession of goods or who buy from a person goods which he appears to be entitled to sell.

In some countries, e.g. Scotland, it is impossible for any person to give to another a charge on chattels in security for a debt unless the possession of the chattel is also transferred to the creditor; in other countries, e.g. England, a less complete protection is given by requiring that all such transactions shall be registered and so made public.

The problem of protecting innocent purchasers is a more difficult one; but both Scots law and English law provide that where goods are sold and are allowed to remain in the possession of the seller after the ownership has passed, or are allowed to pass into the possession of the buyer before the ownership passes, then delivery under a sale by the seller or buyer, as the case may be, to an innocent third person will convey a good title to that person. Protection is also given to persons who buy in good faith from "mercantile agents"—i.e. dealers—who have goods in their possession and who fraudulently sell them.

LESSON 10

Marriage

MARRIAGE as understood in Christendom is a bond between husband and wife which has as its object that they shall live together for life to the exclusion of all other men and women. It is a legal contract, for the parties by entering into it undertake certain obligations towards one another. In most countries marriage is also a religious sacrament. Even in its contractual aspect it differs from ordinary contracts.

When parties are induced to enter into an ordinary contract by some untrue statement made by the one to the other the contract can be set aside, but this does not apply to marriage. If a man induces a woman to marry him by making untrue statements as to his wealth, the marriage is nevertheless valid. Ordinary contracts can be dissolved by the consent of both parties, but in England this does not apply to the bond of marriage.

With Ceremony or Without

The modern wedding ceremony owes a great deal to the ceremony of Anglo-Saxon days. The word is Anglo-Saxon, "wed" being a pledge given to secure the carrying out of a bargain. In the Anglo-Saxon wedding there were as preliminaries a contract, and then a settlement by the bridegroom making provision for the bride; at the ceremony the bride was delivered to the bridegroom by her family. To-day the settlement is represented by the bridegroom's statement "with all my worldly goods I thee endow" (hence, probably, the word "dower"); the ring is the "wed," handed over in security for the carrying out of the settlement; the "giving away" of the bride is the delivery of her to the bridegroom by her family.

The law of marriage has varied greatly in different times and in different countries. In some, no marriage is recognised unless it is solemnised with a religious ceremony; in others, all that the state requires is that the parties should go through certain formalities of a non-religious character. In some countries some degree of publicity is required, e.g. by the publication of banns or the obtaining of a licence; in others there need be no publicity, nor indeed any ceremony of any kind. The canon law endeavoured to secure that marriage was always accompanied by a religious ceremony, but it acknowledged marriages to be valid without it. Under that law a marriage might be constituted by consent, or even by a mere engagement to marry if this were followed by intercourse on the faith of the marriage, and both these forms survived in Scotland until 1940. To this day in Scotland a marriage may take place without any ceremony "by habit and repute," i.e. by the living together of the parties as man and wife with the general repute of being married, and usually for a lengthy period.

In England, until Lord Hardwicke's Act, 1753, it was considered that by the common law marriages without any religious ceremony and by mere consent were valid. Certainly marriages performed by a clergyman without banns or licence—"clandestine marriages"—were valid and frequent, although the clergyman celebrating them was liable to a penalty. The first Lord Holland (1705–74), father of the statesman Charles James Fox (1749–1806), was secretly married to the daughter of the second Duke of Richmond in 1744 by a Fleet parson, the chapel of the Fleet prison (in London) being a favourite place for such marriages. Fleet parsons and the marriages they solemnised became a scandal and Lord Hardwicke's Act, 1753, by making void any marriage without banns or a licence, put an end to such marriages in England.

Gretna Green Marriages

This led to the famous "runaway" marriages at Gretna Green and other places in Scotland, since Scots law recognised as valid marriages by consent without banns or licence. Lord Brougham's Act, 1856, made void any such marriage in Scotland unless one of the parties was domiciled there or had lived there for the

immediately preceding three weeks. These marriages by consent, even after the necessary residence, ceased to be valid in Scotland in 1940; but the fact that in Scotland the consent of parents is not required to the marriage of a person under 21 still leads to "runaway" marriages in that country.

In England to-day marriages may take place either in a church of the Establishment or in a Roman Catholic or nonconformist church, or without any religious ceremony in a registrar's office.

Christian and non-Christian countries both recognise certain impediments to marriage. The parties must not be under a certain age, the required age varying very much from country to country. It is now 16 in England and Scotland, having been raised from 14 for males and 12 for females in 1929. The parties must not be within certain prohibited degrees of relationship. These degrees may include persons who are related merely by marriage; in England until special provision was made by statute a man could not marry the sister of his deceased wife. The rabbinical law of the Jews always allowed marriages between such persons.

Sometimes the consent of certain persons besides the parties is necessary before a marriage can take place. In England persons under 21 are required to have the consent of their parents or guardians (or, as an alternative, the consent of a court); but the absence of this consent does not now invalidate the marriage, it merely makes the parties liable to penalties. In some countries this consent is required even though the parties are over 21 years of age.

Maintenance and Inheritance

No step which a human being can take is more productive of legal consequences than that of becoming married. In England the common law rule (now very much modified) was that man and wife became one person, the wife's legal personality becoming merged in that of her husband. As a result all her property passed to him and he was liable for all her pre-nuptial debts and all wrongs subsequently committed by her, e.g. slander. The rule as to the wife's property was in practice much mitigated by means of marriage settlements and was finally swept away by the Married Woman's Property Act, 1882; but the husband's liability for her wrongs remained until 1935. Husband and wife cannot now be compelled to live together, nor is the husband entitled to chastise or imprison his wife.

The parties, if they live apart, unless there are special grounds justifying their separation, can be compelled to support one another, the duty being heavier on the husband than on the wife. If a husband deserts his wife or is persistently cruel to her or wilfully neglects to maintain her, she may obtain in a magistrates' court an order requiring him to pay her maintenance, and if he fails to do so he is in some considerable danger of going to prison. If either husband or wife persistently refuses or neglects to maintain the other, who in consequence has to claim national assistance, the husband or wife who neglects or refuses may be fined or imprisoned. Although English law thus imposes on the husband a duty of maintaining his wife during his life, it is remarkable that until 1938 that law made no similar provision after his death. By his will, and without any reason, a man could (on his death) deprive his wife of the house in which she had lived with him and the home surrounding her if, as was usual, they belonged to him.

The Inheritance (Family Provision) Act, 1938, amended in 1952, now imposes duty on both husband and wife. The act gives the court power, when as the result of the will of a deceased husband (or wife) or the laws of intestacy reasonable provision is not made for the widow (or widower), to order reasonable provision to be made out of the estate of the deceased. The act also gives a dependent child a similar right.

Scots law has always contained a hard and fast rule entitling a widow (or widower) to certain rights in the income of the deceased spouse's land and to half the spouse's movable property, (e.g. goods, furniture, shares in companies), if there are no children, and to one-third if there are children. The children have a right to one-third of the parents' movable property if one parent is still alive, and to one-half if the parent who died was a widow or widower. Neither spouse nor children can be deprived of these rights by the will of the deceased.

LESSON 11

Divorce

An object of the state is to secure the permanence of marriages, as is indicated by the terms of reference of the royal commission on marriage and divorce set up in 1951. The commission was required to carry out its inquiry "having in mind the need to promote and maintain healthy and happy married life and to safeguard the well-being of children." The state endeavours to achieve its object by religious and other teaching, the provision of good and sufficient housing, good economic conditions, and machinery for reconciliation.

Nevertheless there are unhappy marriages, and unless one adheres to the view that marriage is without exception indissoluble, a point must be reached at which a marriage is so utterly broken down that its continuance is contrary not only to the interests of the parties to it but to those of the state also. At what stage is that point reached? When is divorce permissible?

Pros and Cons

On these matters there is the widest divergence of opinion. This divergence is found in the reports of the royal commissions of 1910 and 1951. Nor is this a modern development. Similar divergence existed among the commissioners appointed by Edward VI in 1552 to consider the question of divorce.

Those who advocate easy divorce contend that if divorce is difficult, a husband and wife who are hopelessly estranged will either separate, possibly forming illicit unions in which they will hesitate to have children, or if they continue to live together will do so in enmity and rancour. As to any children there may be of the marriage, it is contended that however unfortunate it may be for them to have a divided home, this is less injurious to them than living with parents whose marriage has more or less completely broken down.

Of the opponents of easy divorce, some oppose any divorce, as being contrary to the lifelong monogamy which, they contend, is prescribed without exception by Christianity. Others, who would permit divorce in certain circumstances, argue that if easy divorce is available, persons will be encouraged to enter marriage without properly considering the responsibilities they are assuming; and that many married couples who, if divorce were difficult, would make the effort in adjustment necessary to render their marriage successful, will, if divorce is easy, tend to rush to divorce on the most trivial disagreement.

Annulments

Before the Reformation, marriage being indissoluble in the Catholic Church, divorce was, in effect, unknown. Among royal families and nobility its place was taken by annulment, granted by the ecclesiastical courts, very often for political reasons, e.g. in the hope that a new marriage might provide an heir to a throne when a first marriage had failed to do so. As annulments did not purport to end an existing marriage, but declared that what had been thought to be a valid marriage had for some reason never been a valid marriage at all, the doctrine of the indissolubility of marriage was preserved.

Annulments were freely granted, the most usual grounds being pre-contract, i.e. at the time of the marriage one of the parties was under contract to marry someone else; and relationship within the prohibited degrees, these degrees being very artificially and extensively enlarged so as to facilitate the grant of annulments. They included not only relationship by blood and by marriage to the 7th degree, but also what was called spiritual affinity, e.g. a man would be related to a woman if he was godfather to one of her relations.

Henry VIII's "divorce" from Catherine of Aragon was, in fact, an annulment, the grounds being that as Catherine was the widow of Henry's brother Arthur, she was related to Henry by affinity, being his deceased brother's widow, and that the dispensation which the Pope had granted to enable Henry to marry Catherine in spite of this relationship was perfectly invalid.

After the Reformation the ecclesiastical courts continued to exercise their jurisdiction in granting annulments, but as the prohibited degrees were narrowed by various acts of parliament to those prescribed by the Book of Leviticus, the artificialities being removed, the power of the ecclesiastical courts to grant annulments ceased to be of importance.

Divorce by Act of Parliament

As to true divorce—i.e. the ending of an existing marriage—a proposal in the reign of Edward VI to permit divorce for adultery, desertion, or cruelty was not proceeded with, and throughout the second half of the 16th century it was uncertain whether or not the ecclesiastical courts had power to grant divorce. In 1602 it was decided that they had not this power. It is interesting to notice that Scots law allowed from 1560 the obtaining of a divorce on the ground of adultery, and from 1573 on the ground of desertion. Milton's famous pamphlet published in 1643, advocating divorce for incompatibility, was probably inspired by his own matrimonial troubles. But divorce in England was not possible until after the Restoration, when there grew up the procedure of obtaining a divorce by act of parliament—parliament being unlimited in its powers of making any law or altering any rights. The first of these divorces (that of Lord de Ross) occurred in 1669.

In order to obtain a divorce in this way it was necessary to prove adultery, and except in very rare cases a divorce was available only to the husband. The procedure was very expensive: in 1830 a divorce in England cost £600 or £700, whereas in Scotland it cost only £10 or £15. Divorce was thus a privilege of the rich, and only about 200 divorces were obtained before 1857, in which year there was introduced an entirely new procedure. This allowed the granting of a divorce by a new court set up under the Matrimonial Causes Act of that year.

This court is now the Probate Divorce and Admiralty Division of the High Court.

Under that act a husband could obtain a divorce from this court if his wife committed adultery ; but a wife had to prove not only her husband's adultery but also that there were circumstances of aggravation, e.g. the adultery was bigamy or incest, or that he had, in addition, been guilty of cruelty or desertion for two years. The law remained unchanged until 1923, when the sexes were placed on equality in this respect, both husband and wife being able to obtain a divorce on proof of adultery only.

Matrimonial Causes Act, 1950

In 1937 an act enabled a divorce to be obtained by either husband or wife on grounds other than adultery. The new grounds introduced were cruelty, desertion for three years, and incurable insanity. All these grounds had been recommended by the majority report of a royal commission which reported in 1912. The same act introduced a rule that, apart from certain exceptional circumstances no petition for divorce could be presented in the first three years of the marriage. This act is re-enacted in the Matrimonial Causes Act, 1950.

The act of 1937 also made great changes in the law relating to annulment of marriages, which had hitherto played little part in solving the social problem of the unhappy marriage. Up to that time the main grounds for nullity had been (1) that one of the parties was already married ; (2) that the parties were within the prohibited degrees of relationship ; (3) that one of the parties was under the age of marriage [raised in 1929 from 14 for boys and 12 for girls to 16 for either sex] ; (4) that one of the parties was physically incapable of consummating the marriage [mere wilful refusal to consummate was not sufficient]; (5) that one of the parties was insane at the time of the marriage.

Position of Divorced Persons

To these were added in 1937 the following: (1) wilful refusal of the party against whom proceedings were taken to consummate the marriage ; (2) either party being at the time of the marriage a mental defective or subject to recurrent fits of insanity or epilepsy ; (3) the party against whom proceedings were taken suffering from venereal disease at time of marriage ; (4) the wife being at the time of marriage pregnant by some other man. In the year 1951, in which there were nearly 30,000 divorces, a new royal commission was set up to consider the whole problem.

The position of divorced persons in relation to the Church of England arises particularly in connexion with two matters—their right to be re-married in church, and their right, when they have re-married in any way, to communicate, i.e. take part in the communion service. The Church of England is an established church, and therefore the rights of divorced persons in these matters depend on the law of the land and not on any laws or views of the church, as these do not bind the laity.

Clergyman's Obligations

In general, everyone has a right to be married in his parish church if neither he nor his prospective wife is already married, if they are of proper age and not within the prohibited degrees of relationship, and if the necessary preliminary steps, e.g. the proclamation of banns, have been taken. When divorce by decree of the court was introduced by the Matrimonial Causes Act, 1857, it was expressly provided that no clergyman was compelled to solemnise the marriage of a person whose former marriage had been dissolved on the ground of his or her adultery, i.e. the guilty party. (Adultery was then the only ground for divorce.) But if the clergyman refused to solemnise the marriage, he had to allow it to be solemnised in his church by another clergyman if one was willing to officiate. The effect of this was that by the law of the land a clergyman was under a duty to marry the innocent party and was free (but not under a duty) to marry the guilty party.

This remained the law until 1937 when several new grounds of divorce were added by the Matrimonial Causes Act of that year. It was provided that no clergyman should be compelled to solemnise the marriage of any person whose former marriage had been dissolved on any ground, and whose former wife or husband was still alive. No clergyman was to be compelled to allow his church to be used by another clergyman for such a marriage. This is at present the law, and therefore a clergyman is not under a duty to marry either the innocent or guilty party to a divorce whose spouse is still alive, although he is legally free to do so if he wishes.

Ecclesiastical Law

It is not proposed to discuss here the ecclesiastical law of the Church of England on this matter. But it should be realized that ecclesiastical law is binding only on the clergy and not on the laity. While ecclesiastical law cannot forbid a clergyman to do an act that he is bound to do by the law of the land, it can forbid him to do an act which the law of the land leaves him free to do or not to do as he himself pleases.

An act of 1547 provides that a parish clergyman shall not " without lawful cause " deny the holy sacrament " to any person that would

devoutly and humbly desire it." The Book of Common Prayer (which is part of the law of the land) provides in the rubric to the order for the administration of the Lord's Supper or Holy Communion that if any person "be an open and notorious evil liver or have done any wrong to his neighbour by word or deed so that the congregation be thereby offended," he shall not be permitted to take part in Holy Communion until he "have openly declared himself to have truly repented and amended his former naughty life." No cases have come before the courts in which this law has been considered in relation to divorced persons.

LESSON 12

How a Lawyer Works: The Common Law—1

THE two main sources of our law are (1) the common law, depending on judicial precedent, i.e. previous decisions of the courts going back hundreds of years, and (2) enacted law. Thus, if you consult your lawyer as to your legal rights on any matter, he will usually find the law applicable to your problem by consulting either previous decisions of the courts—common law; or by consulting some act of parliament, or some regulation made with the authority of parliament by a government department or other body to which parliament has given authority to make the regulation—enacted law.

Very often he will have to consider both previous decisions of the courts and enacted law, for the law as declared in decided cases may have been changed by some act of parliament, or the law as set out in an act of parliament may have been considered and interpreted by a court.

Records of Cases

Let us first assume that you consult your lawyer about a common law matter. Common law matters include contracts (except where the common law has been codified by being put into statutory form, e.g. the law relating to the sale of goods and bills of exchange) and civil wrongs, e.g. trespass to the person (assault) or to land or goods, libel and slander, fraud, negligence, e.g. in driving a motor car. The law relating to crimes is to be found partly in the common law and partly in enacted law.

The principles of the common law are to be deduced from decisions of the courts, the facts of which illustrate the principles. Because of this, recording of these decisions in the form of reports goes back very far in the history of our law.

The earliest known records of cases are Year Books dating from the reign of Edward I (1274–1307), although these are not reports in the modern style. The Year Books ended in the 16th century, and from that time onwards there are reports more or less in the modern style, made by some barrister; only reports for which a barrister will vouch can usually to-day be considered by a court.

Bearing in mind that reports of decided cases are the very basis of our common law, one might expect that the reports would have been published by the courts or in some other official manner. But this is not so. Except that since 1865 there has been one series of reports of high authority which may be described as semi-official, all reports are published by various competing private publishers, who engage barristers to attend the courts and prepare reports. These are usually published in weekly parts which contain cases recently decided, and each case is known by the names of the parties.

Thus, Bardell v. Pickwick means legal proceedings brought by Bardell against (" versus ") Pickwick; and when it is referred to, there is also given a reference to the name and page of the volume in which it will be found. For example, "[1925] A.C. 500" means page 500 of the volume of reports of cases decided in the house of lords in 1925, A.C. standing for appeal cases.

A law report of a case differs very much from the report in a newspaper. The law report does not give the evidence as the parties or witnesses gave it, but merely the facts as found by the judge, probably the legal arguments put forward by counsel for each party, and also—most important—a word-for-word report of the judgment in which the judge will state the legal principle that he thought applicable to the facts of the case and the process of reasoning by which he arrived at his decision.

Digests and Text Books

The length of reports varies considerably. Some consist of only one page; others may (exceptionally) consist of 100 pages. One weekly part of the law report, selected at random, comprised 48 pages and dealt with six cases; 36 pages, each of about 600 words, were occupied with the judgment of the judges, the longest being over 6,000 words.

No lawyer, however learned, could have knowledge of all the cases decided by the courts since the 13th century. Fortunately he does not need to. In the first place, only a very small

LESSON 13

How a Lawyer Works: The Common Law—2

How a Lawyer Works: Statute Law

If the answer is contained in some act of parliament, his task in finding the relevant act will not be so difficult as was his task in finding the cases to answer your other problem, although decisions of the courts have been arrived at when no relevant act of parliament has been brought to the notice of the court. If he does not know the relevant statute, a set of the acts indexed under their subject matter, or a text book, should direct him to it. And although there are still in force acts of parliament that were passed in the 13th century, he will not need to consult musty tomes to find them, for several up-to-date editions of all the statutes in force have been published in the last hundred years.

It is not frequently that he will need to consult any act over 100 years old, although Magna Carta of 1215 was relied on by a judge in a case in 1954, and our main law of treason is still contained in the Treason Act passed in 1351 in the reign of Edward III.

Periodically, ancient statutes which have become obsolete are repealed by Statute Law Revision acts, and our statute book is in a much tidier state than it was 100 years ago. The Statute Law Revision acts passed in 1950 and 1953 repealed between them many old statutes, including one, passed in 1509, which made provision for the dowry of Catherine of Aragon, wife of Henry VIII.

Legislation by Reference

One irritation your lawyer will find is what is called "legislation by reference." If it is desired to amend an earlier act, that act is frequently not repealed, but a new act is passed repealing, say, part of the old act, e.g. one section, adding further words to another section, and adding new sections. Your lawyer may thus have some complicated jigsaw puzzle work to do before he can be sure he has altered the main act so as to give effect to all the amendments. Recently the much more helpful practice has been adopted, at any rate when a statute has been extensively amended, of setting out the statute in its amended form in a schedule to the amending act.

The number and size of the statutes passed vary considerably from year to year; usually about 60 or 70 new statutes running to some 1,200 pages are passed, but many of them deal with administrative matters (e.g. The Iron and Steel Act, 1953, which provided for the [illegible] of the industry) that are not likely to concern the individual. They are not, [illegible].

Your lawyer's [illegible] in tracking down the law will be [illegible] if your problem requires him to consider what is called delegated legislation, i.e. orders, regulations, and bye-laws made by some person or body under authority given to it by parliament. The [illegible]ctice of parliament in delegating its legis[illegible] [illegible]vers in this way has greatly increased [illegible] century, particularly since the First Wo[illegible] [illegible]r, and some degree of it is indeed almos[illegible] [illegible]tial for the proper working of the welfare [illegible]. Some of these provisions of delegated [illegible]on are to be found fairly easily, being pu[illegible] [illegible]y the stationery office as statutory [illegible]ents; others of them are much more diffic[illegible] [illegible]o trace.

Precise Meaning

When your lawyer has found [illegible] [illegible]acted law that concerns your problem [illegible] [illegible] it is an act of parliament, he will have [illegible] [illegible]certain what it means. That may appear [illegible] self-evident statement, but it is nothing [illegible] kind. The very fact that he is to find [illegible] [illegible] the statute means shows his task is ent[illegible] [illegible]erent from that which he was undert[illegible] [illegible]en he advised you on a matter gover[illegible] [illegible] the common law. Then he was not [illegible]ed to find out what the statements of a [illegible] [illegible] as words standing by themselves [illegible] to ascertain the principle illustrated [illegible] case, and while the words of the judge [illegible]ng that principle were very valuable t[illegible] [illegible] had to bear in mind the facts of the case [illegible] other cases illustrating the same principle.

He reads a statute with the object [illegible] from it the meaning expressed [illegible] [illegible]. The distinction is not unlike that [illegible] exists between the general way you read a [illegible]ce in a book or newspaper and the det[illegible] [illegible] you would read the same sentence if it [illegible] one of the clues in a crossword puzzle.

Rules to Help the Lawyer

Your lawyer's reading of the [illegible] has another quality in common with your [illegible] of a crossword clue. When you have worked out crosswords by the same compiler many times, you come to know how the compiler's mind works and the manner in which he says—or hides—what he means. In a similar way your lawyer, trained to read acts of parliament, can arrive at the meaning of the statute much more accurately than a person who is not trained.

There are certain rules to help him. These may be contained in the act itself, which often defines the words it uses, or in some general act such as the Interpretation Act, 1889. The latter, for example, tells him that "month" in an act passed after 1850 normally means "calendar month," and not four weeks. Or the rules may be contained in decided cases which have laid down general principles for the construction of acts. Your lawyer must also have regard to any decisions of the courts interpreting the act he is considering, or even similar words or expressions in other acts. It is impossible here to consider the many rules

There [illegible] two main methods of interpretation that [illegible] to conflict. Shall the words of the act [illegible] literally, or shall they be read having [illegible] the object of the act as a whole? Actu[illegible] is not this sharp division between these [illegible] because every lawyer would agree that t[illegible] of an act *must* be regarded in order [illegible] pret the words. But some acts have [illegible] with what appears to be more attent[illegible] express words used than to the intent[illegible] ct, and in others a reverse process has [illegible] owed. An excellent example of litera[illegible] retation is that which was applied to S[illegible] claim to a pound of Antonio's flesh [illegible] Portia (in *The Merchant of Venice*):

> [illegible] gives thee here no jot of blood
> [illegible] xpressly are "a pound of flesh."

Ins[illegible] where the courts have refused to adop[illegible] ly literal construction are many. In [illegible] 1953 a question arose whether certa[illegible] of an artist given at a theatre on a Sund[illegible] "musical entertainment" under the S[illegible] Entertainment Act, 1932, which defin[illegible] musical entertainment" as "a concert [illegible] entertainment consisting of the perfo[illegible] of music, with or without singing or recitation." The turn of one artist consisted of imitations, dialogue, and representations, including those of a wife nagging her husband, two old ladies discussing a visit to Paris, and a husband insisting on going out every night to a "pub." Throughout his turn a piano was being played on the stage.

Was his whole performance a "musical entertainment"? The local justices considered it was. Literally it may have been, but the divisional court on appeal held it was not, the lord chief justice stating: "One has to apply a certain amount of common sense in construing statutes, and to bear in mind the object of the act, and the object of the act of 1932 is clearly to permit of there being given on Sundays public entertainment of the nature of what anybody would call concerts. If a music-hall artist appears on the stage and gives an entertainment consisting of patter, that cannot by any stretch of imagination be called a musical entertainment in the nature of a concert merely because there is somebody at the back of the stage playing a piano, or even an orchestra which plays so softly that the artist's words can be heard above the music."

LESSON 15

Can the Law be Made Certain?—Codification

There have now been mentioned the two [illegible]in sources of the law—common law and [illegible]atute law; that is, the two methods by which it is endeavoured to make the law certain and clear. The merits and demerits of each have been much canvassed, and many persons have advocated that the common law rules should be embodied in statutes either to form a partial code containing all the law on any one subject, e.g. sale of goods, or (as it is contended would be the ideal) to form a complete code containing in one document all the law on virtually every subject.

Several claims are put forward for statutory law as opposed to the common law. It is said that with statutory law it is more easy to find where the law is set out. In Lesson 14 we discussed the degree of difficulty your lawyer would encounter in tracing statutes. With common law one can never be anything like certain that one has not failed to trace some decision of the court which may be produced by the lawyer of the other side and prove fatal to one's case.

Resolution of the Judges

As previously pointed out, not all the cases that come before the courts are reported in a series of law reports. Much depends on the views formed as to its legal merits by the private reporter attending the court in which it is heard. This would not be so important if the courts were limited to considering cases which had been recorded in some series of law reports; but this is not so. They may, and frequently do, consider cases that have been reported elsewhere, e.g. in newspapers, so long as the report is by a barrister. It was a resolution of the judges, treated as equivalent to a case, passed in 1707 in circumstances not now known, of which no original record exists, and which was first mentioned in a law book in 1762, that provided the legal justification for the conviction of William Joyce ("Lord Haw-Haw") on a charge of high treason in 1945.

Law Reporting

In 1939 a committee set up to consider how the present very disordered and unsatisfactory system of law reporting could be improved achieved very little. One suggestion was that shorthand notes should be taken of all proceedings in the high court of justice and higher courts, that the judgments in these proceedings should be approved by the judges, filed, and indexed, and made available on request. This has been adopted to the limited extent that official shorthand notes of all these cases are now taken, and those of the court of appeal are lodged in the bar library at the royal courts of justice.

Again, in favour of statute law it is said that it is more easy by means of a statute than through case law to bring the law up to date. But this tends to be more true in theory than in practice. It is true that in an emergency a statute can be, and often is, passed very quickly to meet new situations, e.g. on the outbreak of war. But when there is no obvious emergency, parliamentary time is so occupied that change of law by statute is a very slow process.

Adapting a Law to New Needs

It was not until 1937 (and then only through the energy of a private member, Sir Alan Herbert) that a statute was passed giving effect to recommendations made in a divorce commission in 1912. While the power of the courts to adapt or extend the law—they would not accept the word " alter "—can be exercised promptly whenever a case comes before them, this power is a very limited one, because very often there is in existence some previous decision which ties their hands, and they must apply the existing law however much they may disapprove of it.

An example of the ability of the courts to adapt the law to new needs is furnished by doctrines developed since the Second World War to protect a wife deserted by her husband from being put out of the house which has been the matrimonial home. The husband, let us say, is a tenant, protected by the Rent Restriction Acts. That is to say, he cannot easily be evicted. Having deserted his wife, he gives up the tenancy to the landlord for spite or some other reason. Can the landlord turn out the wife ? She is not the tenant, and at first sight one would have thought she had no right to stay. There was no decision on the point, and the courts were therefore free to decide, in accordance with such general legal principles as seem to effect justice (which, after all, is the main object of a judge in every case), and in general they did protect the wife. But if there had been some earlier decision made in different social and economic conditions, the courts would have been powerless to alter it.

Judge will Challenge the Arguments

When the courts are free to adapt or extend the law, it is true that the opportunity afforded for considering whether the extension or limitation is desirable is almost unparalleled and even exceeds that offered for considering the merits or demerits of a proposed bill in parliament. Indeed, it is questionable whether any decision made by the human race is arrived at after a fuller consideration of all its implications. Counsel on each side argues his client's case, the one foretelling the dire legal consequences that will follow in parallel cases if the principle is extended, and the other foretelling the equally dire consequences that will follow if it is not extended.

The judge will challenge the arguments, and counsel will meet the challenge as best he can. If the case goes to the court of appeal and house of lords, it will have been considered by probably nine judges in all, the process of argument will have taken place three times, and when all is over, every possible advantage and disadvantage will have been canvassed and considered many times.

Clear and Unambiguous

A fourth merit claimed for statutory law is that it is more clear than case law. No doubt the vast majority of statutes are reasonably clear, but they cannot overcome the fact that it is extremely difficult—perhaps particularly in the English language—always to express oneself in words so that what one says is clear and unambiguous. A hundred years ago (1854) what was perhaps the most courageous and most tragic exploit of British arms—the charge of the Light Brigade—was brought about by a misunderstanding of a written military order containing 36 words.

Ambiguity is therefore a defect which cannot be entirely avoided in statutes. Very often the word or expression that has caused the trouble is a simple one in ordinary use and at first sight seems perfectly clear. By way of illustration, consider the following cases, and decide what meaning you would give to the words. The answers as given by the courts appear at the end of this Lesson.

(1) Is a person who is aged 23 years and six months " 23 years of age or under " ?

(2) Is linoleum " furniture " ?

(3) Is a motor-assisted pedal cycle a " motor vehicle " when it is being pedalled along the road with the engine in working order but not switched on ?

(4) Is a motor-assisted pedal cycle a " motor vehicle " when, an essential part of the engine having been removed, it is being pedalled along the road ?

Attitude of Courts to Statutes

Many of the difficulties in construing statutes result, beyond doubt, from bad drafting ; but it is often suggested that the attitude of the courts towards statutes is unduly critical and adds to these difficulties. The courts sometimes refer to statutes in terms bordering on disrespect which they would never think of applying to judgments of the house of lords—some of which, it must be conceded, are by no means easy to comprehend.

It is said that the judges tend to consider the common law as the true law and statute law as being something inferior, although no doubt necessary occasionally to supplement alterations in the common law. A great jurist, Sir Fredrick Pollock (1845–1937), stated that the very elaborate rules by which the judges

interpret acts of parliament "cannot well be accounted for except on the theory that parliament always changes the law for the worse and that the business of judges is to keep the mischief of its interference within the narrowest possible limits."

It has been contended that it is this attitude of the courts that leads the drafter of a statute to adopt complex language in an endeavour to ensure that despite all subtleties and ingeniousness of argument his meaning may be clear.

It is difficult to resist the conclusion that statutes as at present drafted are often far from clear. And the common law is very frequently difficult to ascertain and is lacking in clarity. Indeed, any lawyer who was asked to foretell what the final decision could be when the courts were asked to extend any common law principle in the manner outlined earlier in this Lesson, would have to confess that his forecast could be little more than a guess.

The defects that exist both in our present methods of drafting and construing statutes and in the system of common law have led, from time to time, to movements for more extensive and even complete codification; that is, for a statement of the law on most, if not all, subjects in the form of an act of parliament.

Ideal Conception of a Code

The main English advocate of codification was Jeremy Bentham (1748–1832). His conception of a code was idealistic. It should be a complete statement of the law on all topics, containing also "moral sentences." And "it would speak a language familiar to everyone; each one might consult it at his need. It would be distinguished from all other books by its greater simplicity and clearness."

Although the ideal of Bentham has never been achieved, there are on the European continent many codes, e.g. the Code Napoléon, and there codification has served a purpose long ago achieved in England by the common law, i.e. the replacement, by one uniform law over the whole country, of a mass of customs varying in each district. One objection always put forward to codification is that the common law principles, being in general terms, cannot adequately be expressed in the precise form of a statute. An approach to complete codification in England was made in 1866 by Lord Westbury (1800–73), lord chancellor. A royal commission was set up; but although one report was issued, no steps towards codification were taken. Sir James FitzJames Stephen (1829–94), a famous lawyer who prepared a code for India on several branches of the law and was later a judge of the high court in England, prepared a code of the English law of evidence and criminal law, but this never became law.

Consolidation Acts

Considerable progress was made in the end of the 19th century and the beginning of the 20th century in setting out in acts of parliament the common law principles relating to certain branches of the law, and this activity produced the Bills of Exchange Act, 1882; the Partnership Act, 1890; the Sale of Goods Act, 1893; and the Marine Insurance Act, 1906.

Another process which is sometimes termed codification is the passing of a consolidation act, setting out in one act provisions previously contained in many acts. An example of this is the Income Tax Act, 1952, which contains the provisions of the Income Tax Act, 1918, as amended by later finance acts.

In America the law has been stated in the form of a code in what is called the Restatement of American Law. This is unofficial, and is of itself of no binding authority; but it has acquired a very high reputation, and it is not unknown for it to be quoted in English courts as a statement of common law principles.

Answers given by the courts to the questions in page 1620: 1. Yes. 2. Yes, if easily detached, otherwise no. 3. Yes. 4. No.

LESSON 16

Law of Evidence and Procedure

A branch of the law which to the layman appears unnecessary is that concerned with "evidence" and "procedure"; to him it seems designed not at all to aid the court in arriving at a just decision, but rather to hinder it by imposing on it technical and artificial restrictions.

The layman in his ordinary life has constantly to make decisions of considerable importance; to enable him to do so he has to weigh evidence and come to conclusions as to facts very much as a judge must do in trying a case. If he cares to visit a court he will find that facts which would have influenced him considerably in arriving at his decision will not be considered by the judge at all, but will be excluded on the ground that they are not "evidence." The court will insist on everything being proved "at first hand," whereas he has been quite content to rely on second- or even third-hand information, provided that the persons through whom the information has come are known to him to be

reliable. In a court of law "hearsay" information would normally not even be listened to, let alone weighed or considered on its merits.

English law has filtered so finely the facts which it allows the court to consider in deciding a case, because of the jury system, under which the jury are the judges of fact. A judge, trained and experienced, could no doubt have been trusted to consider the value of "hearsay" evidence and give it due weight in his mind; but in the olden days, with a jury inexperienced, uneducated, and very often unintelligent, the matter was very different.

Direct and Circumstantial Evidence

Evidence may be either "direct" or "circumstantial." When a man is charged with murder the evidence of witnesses who saw him strike the blow would be "direct" evidence. That he was arrested near the scene of the crime, that his clothes were stained with blood, and that a weapon with which the fatal wound might have been caused was found in his possession, would be "circumstantial." If the chain of events connecting the prisoner with the crime is strong and complete, circumstantial evidence can be relied on. Indeed, it may be safer than direct evidence, for it is a fact that some of the greatest miscarriages of justice have come about through honest but mistaken direct evidence wrongly identifying the prisoner with the person whom the witness saw commit the crime.

The writer recalls how once, when he was in the company of a much respected coloured lawyer in the main street of a West Indian town, two coloured taxi drivers approached and asked the lawyer to settle a dispute that had arisen between them. One had been inside his car cleaning it, and had opened the door, not noticing that the other's car was driving up alongside, and both cars had been slightly damaged. The coloured lawyer listened to each of them in turn, asked some questions, and then gave his decision, which both accepted without demur. There was not even a fee payable, and the whole [illegible] did not last five minutes. What [illegible]?

Normally, when a case is to be tried involving many facts as to which there is a controversy between the parties, it would be impossible for a judge to try a case in this simple way. First, it is necessary that the "issues" should be determined—that each party should state in writing the facts on which he relies in support of his case, and should further state whether he admits or denies each of the facts alleged by his opponent. These statements are called the "pleadings," and on reading them the judge will quickly and easily discover what is in dispute between the parties. Unfortunately, lawyers at all times have tended to delight in procedure, the rules of which become very complicated, adding to the cost of litigation. The problem has been many times considered, but a solution has yet to be found.

Heard in Public

All cases in English courts, with a few exceptions, must be heard in public. This rule has been of great importance in the development of the law, for it ensures that the enormous force of public opinion is brought to bear on the proceedings and that justice [illegible] be done but should also manifestly be seen to be done." The litigant who has lost his case may leave the court confident, it may be, that the judge has been wrong, but also conceding that the facts in his favour have been fully heard and considered by an impartial court. It is the absence of this publicity in the activities of many of the administrative tribunals that have multiplied in the "welfare" state that has led to so much criticism of their activities.

Another procedural rule of great importance is that the burden of proof is in almost every case on the plaintiff in civil cases, on the prosecution in criminal trials. It is for the prosecution to prove that the prisoner is guilty, not for the prisoner to establish his innocence. This is of great practical importance, for should the prosecution fail in this task the accused may, without taking the risk of giving evidence and exposing himself to the possible dangers of cross-examination, submit that there is "no case" to answer and demand an acquittal.

LESSON 17

The Legal Profession

W[illegible] the facts [illegible] matter of great difficulty, demanding considerable natural ability as well as experience, but also the intricacies of the rules of evidence and procedure [illegible] the litigant in person. Nevertheless, many private persons prefer to conduct their own cases, and some have done so with outstanding success, even when they were opposed by the

[illegible] bers of the professional bar. In [illegible] ministrative tribunals a party [illegible] represented by a lawyer, some-[illegible] sometimes unless the consent of the trib[illegible] obtained, the reason for this rule being [illegible] bly the fear that legal represent-ation will [illegible] en and complicate the proceed-[illegible]. In [illegible] he lord chancellor gave in the house of [illegible] list of 14 tribunals to which restrictions [illegible] his nature applied.

The profe[illegible]n of law constitutes a monopoly even more [illegible] ite than that of medicine, for, while a pe[illegible] ithout qualifications is not for-bidden to [illegible] doctor even if he receives fees for his w[illegible] o long as he does not describe himself as [illegible] ualified practitioner, any un-qualified [illegible] who for reward does certain legal work [illegible] draws a conveyance, is liable, with certa[illegible] eptions, to a fine. Further, so far as one [illegible] h of the work of a legal prac-titioner is [illegible] rned—namely, addressing the court on [illegible] of a litigant—no person is entitled t[illegible] ent a litigant unless he is (in the highe[illegible] ts) a barrister or (in the lower courts) a [illegible] tor

Barristers [illegible] Solicitors

The l[illegible] ession in England has two branches [illegible] risters and solicitors. This divi-sion alth[illegible] not unknown abroad, is there the excep[illegible] rather than the rule, and in this country [illegible] unded on historical reasons. A barrister [illegible] be approached direct by a litigant, but can only be " instructed " through a solicitor. It is frequently suggested that this duplication makes litigation costly and is not [illegible]. In defence of the system it is con-tended that it enables the litigant to obtain more highly skilled advice and assistance than any other system, since it encourages specialisation among barristers ; and a solicitor knows who are the specialists in each branch of the law, and can thus put his client in touch with the barrister best qualified to assist him in his case.

A barrister is characterised primarily by the fact that he presents his client's case in court to a judge or jury. A solicitor's normal duty is to make the case ready for presentation, usually after the advice of the barrister. It is prob-able that originally the barrister was merely some person who was in court when the liti-gant's case came on for hearing, and then volunteered to assist him. A class of persons came to be recognized as competent to be " of counsel " with litigants, and to be called " counsel."

To-day the authority entitled to appoint barristers by " calling to the bar " is not the court itself, but one of the four Inns of Court—Lincoln's Inn, the Middle and Inner Temples, and Gray's Inn. These are societies of bar-risters ruled each by their " benchers," who have very autocratic powers and are not controlled by the societies or indeed by any other body. In theory they have an almost absolute dis-cretion as to the persons whom they will call to the bar ; in practice they will, as a rule, call any British subject (sometimes any alien) who is of good character and has certain qualifications.

These qualifications are twofold. The candi-date must pass certain examinations in law, and must also " keep " 12 terms, spread over three years, by dining in the hall of his Inn on six occasions (or in the case of a member of certain universities, three occasions) in each term. The eating of these dinners is probably a survival from a time when the candidate was required not only to dine, but also to reside within the precincts of his Inn for a certain number of terms. No such residence for students is now possible, for nearly all accommodation at the Inns is used by practising barristers or solicitors.

When the candidate has complied with these regulations, he is formally called to the bar by the benchers on Call Night. After he has been called, a barrister is subject to his benchers so far as professional conduct is concerned, and if he offends, he may be " disbarred."

There is also a class of barristers known as " Queen's (or King's) Counsel." These are appointed by the lord chancellor for the Crown, from among the senior barristers. They confine themselves for the most part to the actual conduct of cases in court, where they occupy a front row " within the bar " of the court, distinct from the ordinary barristers, and they cannot appear unless an ordinary barrister is also briefed. Their cases are usually fewer, but their fees larger, than those of an ordinary barrister.

Solicitor's Training

Solicitors before they are qualified have a training more practical than that of a barrister. They must be apprenticed to a practising solicitor for a period varying from three to five years, and must also pass certain examinations in law. The Law Society controls solicitors, admitting them to practice, and it may remove them from the roll for professional misconduct. The solicitor may appear in inferior courts on behalf of his client, and will do any conveyancing work, i.e. the preparation of deeds, for his clients without assistance from the bar.

Further, he will advise his client on law, and will not take the " opinion " of a barrister unless the case is of unusual difficulty. It is only when litigation commences that it is customary, in the higher courts, to introduce the client to some barrister. A solicitor is entitled to sue a client for his fees, whereas a barrister is not, and must rely upon the honour of the solicitor who in-structs him, and whose duty it is to see that the fees are paid. By way of compensation, a barrister cannot be sued for negligence, whereas a solicitor is liable for negligence.

LESSON 18

The Courts

The main division of the courts is between those exercising civil jurisdiction and those exercising criminal jurisdiction. The civil courts consist of the county court and the high court of justice, with appeals to the court of appeal and house of lords.

The county court is the civil court with which the average citizen is most likely to be familiar. There are heard actions on contracts and most actions for torts, e.g. for negligence (but excluding actions for libel, slander, seduction, and breach of promise) where the amount claimed is not over £400. There also a creditor who has secured judgment may endeavour to compel the debtor to pay his debt by means of a judgment summons which, if the debtor does not pay and it is shown he has had the means to pay, will result in his being sent to prison.

Left to County Court to Decide

County courts have also jurisdiction to deal within certain limits with many other matters, e.g. landlord and tenant disputes, and winding-up of small companies. And it has become the practice of the legislature, whenever any new act makes it necessary for some fact to be determined speedily and cheaply in connection with the modern system of control of contracts, to impose the duty of deciding the fact on the county court. Thus when the Landlord and Tenant Act, 1954, entitled landlords to increase the rent of their tenants so long as (among other requirements) the house was in certain specified respects in good repair, it was left to the county court to decide whether these conditions had been satisfied. In one important matter—namely, the control of the rent of furnished premises—the legislature did not make use of the county court, but set up for that purpose new administrative tribunals, called the Rent Tribunals. Since the end of the Second World War county court judges have added to their duties in many districts by hearing divorce cases as " commissioners."

Cases in the county court are heard by the judge sitting alone, and although in theory a case may be heard by a judge and jury, in practice this is now almost unknown. The county courts deal with by far the greater part of the civil litigation of the country. The courts are not connected with the counties, but England and Wales are divided into districts in each of which a county court sits at various convenient points as often as business requires. The maximum number of county court judges is 80.

A county court judge must be a barrister of at least seven years' standing, although in this case, as in all cases where a minimum standing is required for a judicial appointment, the persons appointed have normally a standing much in excess of the minimum. The salary is £2,800 a year, there is a retiring age of 72, which may be extended to 75, and there are rights to a pension. These judges are appointed by the lord chancellor, and may be removed by him for incapacity or misbehaviour. For each court there is a registrar, who is a solicitor, and one person may be registrar of several courts. The registrar is in charge of the office staff of the court, and he has power to hear certain proceedings.

High Court of Justice

Cases which are not heard in the county court—usually because the matter is outside the jurisdiction of that court—are heard by the high court of justice. This consists of three divisions: queen's bench; chancery; and probate, admiralty, and divorce. Although in theory any division of the high court may hear any matter, in practice there is specialisation. The queen's bench division hears actions on contracts or for torts, e.g. negligent driving; the chancery division is concerned with matters such as partnership and trusts; the probate, admiralty, and divorce division, with the matters indicated by its name. In addition to the lord chancellor (who in practice never sits in the high court), the lord chief justice, and the president of the probate, admiralty, and divorce division, there must be at least 25 high court judges and not more than 39.

Appointments to High Court Bench

Although in theory any of them may be required to sit in any division, in practice each keeps to the division with the work of which he is familiar, not fewer than five being attached to the chancery division, not fewer than 17 to the queen's bench division, not fewer than three to the probate, admiralty, and divorce division. The head of the queen's bench division is the lord chief justice; of the probate, admiralty, and divorce division, the president of that division; and both of these normally sit as judges in their divisions. The head of the chancery division is the lord chancellor, but he normally sits in the house of lords, not in his division. A high court judge must be a barrister of at least ten years' standing.

One might expect that appointment to the high court bench would be by way of promotion from the county court, but although

there have been some instances of this in recent years, such appointments are exceptional and the vast majority of the appointments are made from barristers in practice. The salary of a high court judge is £8,000 a year, and there is a pension after 15 years' service or on retirement for ill-health. There is no retirement age, and in practice judges tend to sit until well into their seventies. Indeed, there is usually at any given time at least one judge who is 80 or more. The judges are appointed by the lord chancellor, but one of the most important aspects of their office is that they can be removed only by the Crown on an address presented by both houses of parliament. This provision dates from the Act of Settlement, 1701, and is designed to make the judges independent of the executive. They need not fear they will be removed should they, when trying a case in which the Crown is a party, give a decision against the Crown.

Queen's Bench

The high court sits in London, but queen's bench cases may be heard at the assizes which are held at various provincial towns. For each case the court consists of one judge. Formerly, most cases in the queen's bench division were tried by a judge and jury, but to-day juries are used only if accusations of fraud are made, and in certain actions, e.g. libel, slander, and breach of promise. Probate cases and defended divorce petitions may still be heard by a jury, but in practice rarely are.

The court of appeal sits in London, and hears appeals from the high court of justice and also from the county courts. The judges of the court are the master of the rolls and eight lord justices, but the court is divided into divisions in each of which three judges sit. Not infrequently judges of the high court sit temporarily in the court of appeal. The salary of the master of the rolls is £9,000 a year, of the lords justices £8,000 a year, and pensions are attached.

Final Court of Appeal

The house of lords is the final court of appeal. Cases are not heard by the house of lords itself but by the lord chancellor and lawyers—usually promoted from the court of appeal—who are life peers and are called "law lords." There are nine life peers. Any peer who holds or has held high judicial office may also sit. The court usually consists of three or five of these judges. The salary of the lord chancellor of England is £12,000 a year, and of each law lord £9,000 a year.

It should not be assumed in any appeal that the evidence in the case is heard again. The procedure is very different. The appeal court—whether court of appeal or house of lords—has before it shorthand notes of the evidence, on an appeal from the high court, or the judge's longhand notes of the main points in the evidence in an appeal from the county court.

The most usual ground of appeal is that the judge has erred in law, that is, has wrongly applied the law to the facts he has found. It is also possible in all cases in the high court and certain cases in the county court to appeal on the ground that the judge has made a mistake in fact, i.e. has found facts which the appeal court does not consider should have been found on the evidence. In some cases leave of the court is necessary before an appeal can be brought. In the case of appeals on fact from trials by jury, it is necessary to show that there was no evidence on which the jury could have come to their decision.

Lay Justices of the Peace

It follows from what has been stated above that the judge of a court which hears civil proceedings must be a trained lawyer. When one comes to criminal proceedings it is therefore surprising to find that all cases (with a few exceptions) must in the first place come before judges who, except in the large towns, are laymen without any legal training at all, and that the large majority of cases are also in fact decided by lay judges. This is due to our system of lay justices of the peace who sit in magistrates' courts, formerly called police courts, the change of name having been made to indicate that these courts are impartial as between the police, who normally prosecute, and the accused. There are some 25,000 of these justices, and they sit in over 1,000 courts, the maximum number who may sit at any court on any case being seven.

The justices, many of whom are women, are appointed for each county and borough by the Crown on the advice of the lord chancellor. They receive no remuneration but, since 1949, certain expenses are paid. They decide questions both of fact and of law that come before them. Although justices are now required to attend courses of instruction, it may well be wondered how persons without full legal training can carry out these duties; the answer is provided by the fact that the justices have a paid clerk who must, in general, be either a barrister or a solicitor with five years' standing. He advises the justices on points of law that arise.

Two Categories of Cases

Cases that come before justices can be somewhat loosely divided into two categories: (1) less serious cases which they decide themselves, in "petty sessions"; (2) more serious cases which they merely investigate to ascertain whether they think the prosecution has made out a *prima facie* case—if there is such a case, they "commit" the accused, i.e. send him for trial before a judge and jury at quarter sessions or, for the most serious cases, at the assizes or,

in London, the Central Criminal Court (commonly called the Old Bailey).

In London and in some of the larger provincial towns the judicial work of the justices is carried out by a trained lawyer who receives a salary, and in the city of London by the lord mayor or an alderman. Outside the city of London, except where there is a magistrate who is a lawyer, at least two justices sit to hear any case.

Methods of Appeal

The greater part of the work of the justices consists in hearing and themselves deciding cases. There are roughly three-quarters of a million persons convicted in such cases annually, about one half of the convictions being for motoring offences. The number of persons who are convicted on a trial by a judge and jury after having been committed by the justices is likely to be about two per cent. of that number. The justices also have an extensive jurisdiction in disputes between husbands and wives and in a large number of miscellaneous matters, e.g. affiliation cases.

There are two methods of appeal from decisions of the justices in petty sessions. An appeal may be made to quarter sessions, where the whole case is heard again by an appeals committee of quarter sessions consisting of from three to twelve justices, or by a recorder in a borough which has a separate court of quarter sessions. There is no jury. An appeal on a point of law may be made to the queen's bench divisional court, consisting of three high court judges, either from the decision of petty sessions or quarter sessions. There is no further appeal to the court of appeal or house of lords.

Trial by Jury

When justices do not themselves decide a case but merely decide whether a *prima facie* case has been made out, and then commit the accused for trial at quarter sessions or assizes or, in London, the Old Bailey, that trial takes place before a judge and jury. The accused is then said to be tried on indictment, i.e. on a bill of indictment containing the charges made against him. At boroughs which have a separate court of quarter sessions, cases are tried by a jury and the recorder, who is the sole judge. He is paid a small salary, and must be a barrister with at least five years' standing. He normally also continues to practise, so that the office is only part-time. At county quarter sessions, where again there is a jury, the justices are the judges, but not more than nine may sit at any court; in practice, they have an experienced chairman who is generally a trained lawyer in receipt of a salary, and this chairman really acts more or less as the sole judge.

On trials at assizes there is a jury, and the trial is conducted by a high court judge or on some occasions by a queen's counsel of seniority specially appointed as "commissioner"—except at the Old Bailey in London. Here, although a high court judge sits to try very serious cases, e.g. murder, most cases are tried by the recorder of London, or by the common serjeant, or by the judge of the city of London court.

From trials by jury, whether at quarter sessions assizes or the Old Bailey, there is an appeal to the court of criminal appeal against conviction and for sentence. Like most other appeals, this does not involve the re-hearing of the evidence. The appeal lies on any point of law, and, with leave, on fact or against sentence. On appeal, the sentence may be increased—and frequently is. The court of criminal appeal has no power to order a retrial, apart from a few exceptional cases, although it has for long been strongly advocated that this power should be given to it. As a result, although the court has power to allow a conviction to stand in spite of some technical fault in the proceedings, if the court considers there has been no miscarriage of justice, in many cases the court is compelled to quash, i.e. cancel, a conviction because of some irregularity which is too serious to be regarded as a mere technicality, even though there is no real doubt as to the guilt of the accused. As the court has no power to order a re-trial, the accused escapes entirely. There is no appeal against an acquittal.

Court of Criminal Appeal

The court of criminal appeal consists usually of three judges of the queen's bench division, of whom the lord chief justice is frequently one. There may be about 100 appeals a year, the conviction being quashed in about one-third, and the sentence altered in about one-half. From the court of criminal appeal an appeal lies to the house of lords (constituted as in the case of appeals in civil cases), but only in very exceptional cases. The attorney-general (or in some circumstances the solicitor-general) must certify that the appeal involves a point of law of exceptional public importance and is desirable in the public interest. There is on average perhaps one such appeal every year.

Coroner's Court

This brief summary of the courts of English justice has of necessity omitted several. The most important of these is the coroner's court. It is a court of great antiquity and was ancient in Shakespeare's day. The main duty of the coroner's court to-day is to inquire into unexplained deaths, by an inquest or post-mortem examination. The coroner receives a salary and must have either legal or medical qualifications.

Another court is the judicial committee of the privy council. Its main task is to hear appeals from the colonies and member-countries of the British Commonwealth, although most of the dominions have now abolished this appeal. The judicial committee consists of law lords and other judges who normally hear appeals in the house of lords, with the addition, where the appeal involves some matter of the local law of some colony or dominion, of lawyers who have special knowledge of that law. The judicial committee also hears appeals from the ecclesiastical courts, e.g. when a clergyman has been removed for immoral conduct.

BRITISH CONSTITUTIONAL LAW

LESSON 1

Constitutional Law and its Sources

Constitutional law can be defined as the law relating to the government of a state : the law defining the relationship between those who govern and those who are governed, provided it is remembered that those who govern are usually also subject to the law.

In the government of any state three functions will be seen in operation. There must be some persons to say what acts may be done, some persons who do acts purporting to be justified by the permission given, and some persons who decide whether acts done have in fact been justified by the permission. These three functions are performed by the legislature, the executive, and the judiciary, although there is much overlapping ; and a great part of British constitutional law consists of the rules governing the activities of these bodies.

There is no reason why all these functions should not be performed by one person, and in a state governed by an absolute monarch they would be so performed, as witness the remark of Louis XIV, " *L'état, c'est moi* " (the state, that is myself), which was not mere vainglory but correctly stated a constitutional fact.

British Constitutional Rules

In democracies, particularly in the U.S.A., these functions tend to be divided among several bodies (with considerable overlapping), and the rules by which they are divided in any state make up an important part of the constitutional law of that state. An American, when he talks of the constitution of his country, has no difficulty in knowing to what he is referring. He can buy a copy of a document entitled *The Constitution of the United States, established and ordained by the people of the United States*, and if any question arises as to whether any act is or is not constitutional, i.e. whether any body has exceeded the powers reposed in it under the division referred to, the answer will be found by a consideration of the terms of that document and its amendments.

An Englishman is in a very different position. There is no document, or collection of documents, containing the authoritative statement of the whole of the British constitution. The truth is that the constitutional rules do not exist in any one document. They must be deduced from many sources.

These sources are of two kinds : the laws of the constitution proper, and the conventions of the constitution. The first includes written statute law and also unwritten common law to be derived from the decisions of judges. Instances of statute law are Magna Carta, 1215 ; the Petition of Right, 1628 ; the Act of Settlement, 1701, which established the house of Hanover as successors to the throne of England ; and the Declaration of Abdication Act, 1936.

Conventions of the Constitution

An instance of common law is to be found in the maxim " the king never dies." As a result of this rule, when the person who is king dies the sovereignty is transferred instantly to the person next in succession without any proclamation or coronation. That is why we speak of the " demise " of the Crown ; the precise meaning of " demise " is not " death " but " transfer."

The laws of the constitution are part of the ordinary law ; the conventions of the constitution are not, in reality, laws at all, for they are not enforceable by the courts. They are rules developed from the habits and practices of political life, and a breach of them might have political, but could not have legal, consequences. There is no law, either in the statute book or in any reported case, which makes it a legal offence for a government that does not enjoy the support of a majority in the house of commons to refuse to resign, or which compels parliament to meet every year. These are conventions, but at the present time they are observed as carefully as though they were laws, although their force rests merely on general convenience.

The conventions of the constitution may change through alterations in political thought. Thus, it may be said to-day that it is a convention of the constitution that the sovereign

should not veto any bill which has been passed by both houses of parliament ; 200 years ago no such convention existed, and the change has been brought about without the passing of any law.

Even in countries with constitutions declared in one document, conventions grow up as they do in Britain, and with, in practice but not in theory, the force of law. Thus the intention of the framers of the American constitution was that the president should be elected by a body of electors, themselves elected by the people. The electors were intended to use their own discretion in their voting. To-day the electors are themselves elected on a ticket system, and a vote for any elector represents a vote for a particular presidential candidate whom he is pledged to support. And this change has been brought about without any amendment of the American constitution.

LESSON 2

The Supremacy of Parliament

ONE of the most fundamental principles of English constitutional law is the supremacy of parliament as the sole lawmaking authority, although it will be necessary to consider how far in the present welfare state some aspects of that principle came to be accepted as practical realities to-day.

The supremacy of parliament involves three propositions : (1) that no person or body in the state can declare to be invalid or refuse to enforce any law made by parliament ; (2) that parliament's power of lawmaking is unlimited, and even parliament itself cannot limit it, e.g. by declaring any law to be unalterable ; (3) that no person or body in the state other than parliament can make any law unless the power to do so has been delegated to it by parliament. In these Lessons it will be necessary to consider each of these propositions in turn.

The first proposition—that no person or body in the state can declare to be invalid or refuse to enforce any law made by parliament—may not appear a very surprising statement, but when our parliament is compared with the legislative bodies of other countries, the unusual character of the proposition is made clear.

Parliament of the United Kingdom

In the constitution of the U.S.A. the most striking feature is not the sovereignty of any legislature, either congress or a state legislature, but the supremacy of the constituion. Each of these legislatures derives its powers from the constitution, and can make no law which it is not authorised to make by the constitution. If the legislature passes any law, the courts may declare the law invalid because it is against the constitution. In the United Kingdom parliament derives its power from the law itself and not from any formal constitution. If the question were asked, " Why is a law made by the congress of the U.S.A. binding on the courts ? " the answer would be " Because the power to make laws *on this matter* is given to congress by the constitution of the United States." If a similar question were asked in respect of laws made by the parliament of the United Kingdom, the answer would be simply, " Because under the law of the United Kingdom parliament has the power to make laws

The Bill of Rights

This proposition of the supremacy of parliament has not always been accepted. In former times the Crown did claim the power, not, perhaps, to declare any law made by parliament to be invalid, but to " suspend " the law so that it had no effect (which came to much the same thing), or to " dispense " with it in the case of certain individuals so that it did not apply to them. This claim of the Crown raised a problem that became acute in the reign of James II, when that king, being a Catholic, endeavoured to " dispense " with the laws that excluded Catholics from office, and next, by declarations of indulgence in 1687 and 1688, declared it his will and pleasure that all penal laws in ecclesiastical matters should be " suspended."

It was left to the Bill of Rights, 1689, to condemn absolutely the suspending power and to declare illegal " the pretended power of dispensing with laws . . . *as it hath been assumed and exercised of late.*" This undoubtedly ended the Crown's suspending power, but the exact position of the dispensing power is not quite clear after that time. In 1948, and again in 1956, when a bill abolishing capital punishment had been passed by the house of commons but not by the house of lords, it was suggested that if during that time the home secretary automatically reprieved all persons convicted of murder, this might amount to an illegal exercise of the dispensing power contrary to the Bill of Rights.

The second proposition involved in the doctrine of the supremacy of parliament—that the power of parliament is unlimited and there is no law that it cannot alter—also may not appear surprising until you consider the position of the legislatures of some other countries.

In countries in which the legislature is not sovereign but is under a constitution, the legislature has no power to change that part of the

law which consists of the constitution. The power of changing the constitution in such states is placed in the hands of some body other than the legislature. For example, an amendment of the American constitution can be brought about with the consent of two-thirds of both houses of congress and with the ratification by the legislatures of three-quarters of the states. The obvious result of this is a strong tendency to conservatism on all constitutional questions.

Unable to Bind Future Parliaments

Some illustrations of the complete sovereignty of the English parliament will make the distinction clear. The Act of Settlement, 1701, fixed the succession to the Crown in the house of Hanover. When one recollects the fierce struggle that had taken place less than a century earlier to uphold the divine right of kings, the great significance of the change to a statutory right is made clear.

Yet parliament was able to alter the succession again by the Abdication Act, 1936, which resulted in the passing of the Crown from Edward VIII to his brother George VI and his heirs.

The third proposition involved in the doctrine of the supremacy of parliament is that parliament is unable to bind future parliaments or to control its own future acts. A significant illustration of this is provided by the acts fixing the maximum periods for which one parliament may sit. The Parliament Act of 1911, among other enactments, repealed the Septennial Act, and decreed that the maximum life of parliament should be five years. This was intended to be a protection for the electorate so that it might have the right at frequent intervals of electing a body to represent its views, and should not be compelled to be ruled by a body in which it had lost confidence.

Yet the same parliament which passed the five-year decree extended its own life to eight years, in order to avoid a general election during the First World War. Similar annual acts of prolongation were passed during the Second World War by the parliament elected in 1935; and it would be perfectly legal for any parliament to pass a law that its life should be extended to twenty years.

The "Offending Numeral"

The doctrine of the supremacy of parliament has been carried so far that even if parliament declares a certain law to be unalterable, that does not bind parliament; it can at any time ignore the self-imposed restriction and alter the law. It is not easy to see why such a rule is essential, or why it should not be possible to give effect to a rule that certain laws should be alterable only in a special manner, e.g. by a referendum—by a vote of all the electors in the country. While the rule is established beyond question, in theory it is certain that there are many laws which parliament would never in fact alter, and so great is this certainty that one feels that for practical purposes the rule must be regarded as subject to some qualification. This is illustrated by two examples in recent years, one in Scotland, the other in South Africa.

When on the death of King George VI his elder daughter succeeded to the throne as Queen Elizabeth II, resentment was felt by some persons in Scotland at the use in her title of the numeral II, described as the "offending numeral." There were two grounds of objection. First, it was said that the numeral was inconsistent with historical fact, since the Queen was the first and not the second of that name to be queen of the United Kingdom of Great Britain, the first Queen Elizabeth having been queen of England only (reigned 1558–1603).

Scots Law

Secondly, the use of the numeral was said to be an infringement of the Treaty of Union between England and Scotland, which became effective in 1707, and of the acts of the parliaments of England and Scotland which had affirmed the treaty, because it inferred that the kingdom of England still existed as a separate kingdom and had not, as provided by the Treaty of Union, been united with Scotland into "one kingdom by the name of Great Britain."

As that treaty states with reference to many of its provisions that they are fundamental and unalterable, it was contended that it was not possible for parliament to alter them. Proceedings were taken in the court of session in Edinburgh to prevent the further use of the numeral, those taking the proceedings making it clear that they were in no way criticising the Queen, or disloyal. The court, on various grounds, dismissed the proceedings, but during them there was much discussion as to the view of Scots Law as to the power of parliament to alter an act stated to be unalterable, and it was conceded by the lord advocate on behalf of the Crown that fundamental provisions of the treaty could not be altered by parliament. The lord president stated that the principle of the unlimited sovereignty of parliament was not part of the constitutional law of Scotland, and indicated that in his view the principle even in England was based more on academic logic than on practical reality.

Statute of Westminster

The Statute of Westminster, 1931, has also made it difficult to accord full force in practice to this aspect of the doctrine of parliamentary supremacy. That statute, by section 4, enacted as law a rule which had previously existed only as a convention—namely, that the parliament of

Great Britain should not make any law for a self-governing dominion except at the request and with the consent of the dominion. Does this limit the supremacy of parliament? Could parliament repeal this provision and make a law for a dominion even against the expressed will of that dominion?

One answer has been provided by a lord chancellor (Viscount Sankey) in the house of lords in 1935: "Indeed the Imperial parliament could as a matter of abstract law repeal or disregard section 4 of the statute. But that is theory and has no relation to realities."

The lack of reality of the doctrine is illustrated by a case in the Union of South Africa in 1952 in which it was stated: "The only legislature which is competent to pass laws binding on the Union is the Union legislature. The Union is an autonomous sovereign state in no way subordinate to any other country in the world." In an earlier South African case it had been said: "Freedom once conferred cannot be revoked."

LESSON 3

The Supremacy of Parliament, and the Welfare State

THE third proposition involved in the supremacy of parliament is that no person or body in the land except parliament can make any law unless the power to do so has been delegated to it by parliament. Parliament has no rivals. This proposition is literally true to-day, but it will be necessary to consider later to what extent in practice it has been affected by constitutional changes in recent years and particularly by the vast transformation caused in this country by the welfare state.

In modern times, in this as in some other matters, we have something resembling a return to the practice of Tudor times, although the powers of making laws are enjoyed not by the king or queen in person but by the government departments, i.e. the executive, and although they are exercised not in defiance of parliament but under powers delegated by parliament. The power of the executive to make laws arises mainly to-day first, in times of emergency, and secondly, for the purpose of operating the welfare state.

Emergency Powers

The Emergency Powers Act, 1920, confers on the executive power to proclaim an emergency whenever it appears that any action has been taken or is immediately threatened by any person or body of persons calculated to deprive a substantial portion of the community of the essentials of life by interfering with the supply and distribution of food, water, fuel, light, or the means of locomotion. This emergency remains in force for only one month, although it may be renewed, and parliament, if not sitting, must be at once recalled.

So long as a state of emergency exists, the executive may make regulations by order in council for securing the essentials of life to the community, and a breach of these regulations is a criminal offence. The regulations cease to be valid unless they are confirmed by parliament within seven days, and in any event they cannot impose compulsory military service or industrial conscription, or make it a crime to strike. A state of emergency was declared during the general strike of 1926 and also during the railway strike of 1955.

In the emergencies of war much wider powers were given to the executive in both world wars. In the Second World War the Emergency Powers (Defence) Act, 1939, empowered the executive to make laws by means of defence regulations for a very large number of purposes, covering almost all the war activities of the country but excluding industrial conscription. A similar act in 1940 conferred the power to make regulations imposing industrial conscription. By means of these regulations a vast control was created of industry, supplies, and essential services, and innumerable restrictions were imposed on freedom of every kind. Perhaps the most important restriction on freedom from the constitutional point of view was the famous—or notorious—Regulation 18B, which interfered with personal liberty to an extent recalling the most arbitrary powers of the Stuart kings.

This regulation empowered the home secretary to detain, i.e. imprison *indefinitely* without trial, anyone, whether an alien or a British subject, on certain grounds, one being that he was a person who the home secretary had "reasonable cause to believe" was of "hostile origin or association," e.g. a member of a fascist organisation.

In emergencies and in war-time, departures from normal practices are perhaps to be expected, but the extremely wide power of the executive to make laws by regulations under ordinary peace-time conditions for the purposes of the welfare state must be considered a striking invasion of the supremacy of parliament. To-day for these purposes a mass of laws far exceeding the legislative output of parliament is made by the government departments of the executive under powers delegated by

parliament and in very many cases the power of making laws by regulations is not confined to government departments but extends to various bodies to which the power has been sub-delegated by some government department.

In order to appreciate the wide extent of this power of legislating by regulations and to be able to consider whether any check should be imposed on it, it is necessary to realise what is meant by the welfare state, and to be aware of the great changes that have taken place in the last hundred years or so in the legal and political conception of the duties of the state to the community.

In the early 19th century the activities which the state was expected to carry out were few in number in accordance with the accepted ideal that state interference could be only injurious.

Child-welfare was unknown. Unemployment relief was unknown. Wages were uncontrolled, and until 1824 trade unions enabling workmen to combine were illegal. There were virtually no building restrictions or town planning.

Freedom of Contract

As to services and goods, it was no concern of the state if water was impure, if coaches were rickety or their drivers incompetent, or if food was tainted. The only apparent exception arose where the state's own interest was affected. Thus it was a crime to adulterate tea, because there was a duty on tea and the revenue would suffer if material which had not borne the tea duty because it was not tea at all was sold as tea. There was the doctrine of freedom of contract, under which it was considered that the interests of the community would best be served by leaving everyone free to make such contracts as he pleased, it being conceived that as each individual knew his own interests best, he could safely be left to enter into such contracts as would forward his interests and so forward also the interests of the community. But what freedom of contract could there be between an employer and a workman in times of unemployment, or between a landlord and a tenant when houses were scarce ?

The fact that the state did not interfere in any of these matters was not due to mere heedlessness or lack of humane feelings. It was in part caused by a deep conviction that interference was bound to be injurious to the community, a conviction illustrated by an incident in the house of commons in 1833. The house was considering a proposal to reduce the hours of work of children in textile mills, and it was contended that this would result in the collapse of the industries of the country and cause inevitable famine, an argument which produced from William Cobbett (1726–1835) the retort that it appeared that the "great stay and bulwark" of England was not, as he had thought, her navy, her commerce, her colonies, or her bank, but the labour of a few little girls.

There was a very strong feeling in favour of individual liberty—that a man should be entitled to do what he liked with his own—a feeling that still persists in this country. But in the 19th century this right was recognized to an extent which ignored almost completely any conception of a duty not to injure the community, as two examples will show. The first of these is afforded by the almost inconceivable difficulties encountered by the state in bringing to an end the scandal of the "climbing boys"—the miserable boy chimney sweeps, such as Tom in *The Water Babies*, by Charles Kingsley (1819–75). In spite of a campaign originated in 1773 by Joseph Hanway (1712–86), in spite of the early activities of Lord Shaftesbury (1801–55) which led to an act of 1840 making it a criminal offence to sweep chimneys by setting boys to climb them, the practice continued until 1875, and in the later years even increased.

It was known that the practice was illegal, it was accepted that chimneys could be properly swept by mechanical means, it was realized that the use of boys led to frequent deaths by suffocation and also to the onset of cancer, and yet persons otherwise humane continued to have their chimneys swept in this way. In the rare cases where there was a prosecution the magistrates imposed a merely nominal fine, e.g. 2s. 6d. It was shown that it is useless to make laws unless one can ensure that they will be properly enforced. When the evil was ended by an act of 1875, the method which succeeded was one now very familiar to us in the welfare state—a system of control by licensing. Every chimney sweep who had employees was required to obtain an annual certificate from the police.

Landowner's Revenge

An example of the extreme lengths to which, in the 19th century, the right to do what one liked with one's own was carried is provided by a case in the house of lords in 1895. A landowner was annoyed with the Bradford corporation because they had refused to purchase part of his land for their water-supply scheme, the water for which percolated underground through his land. In revenge he sank a shaft on his land which reduced the amount of water for the scheme and also discoloured the water. The corporation brought an action against him, which ultimately went to the house of lords, but that court held he was entitled to do what he had done. One of the lords stated, "He prefers his own interest to the public good. He may be churlish, selfish, and grasping"—but he was entitled to use his own property as he pleased.

LESSON 4

The Welfare State: Legislation by Government Departments

When we turn from the picture of England in the 19th century and consider what the position is in the welfare state, we see a very great change and can appreciate the vast increase in state interference—administrative activity by the executive—that is necessary to-day. In the 19th century it was considered that the duty of the state could be summed up in the words "laissez faire"—leave people alone. Now it is considered to be the state's duty to regulate the day-to-day affairs of virtually everyone.

It is now almost the exception to find any activity that is not controlled by some law. To take only a few examples, we have laws of a most detailed nature regulating factories, mines, and shops, the employment of women, children and young persons, and the quality of the food we buy. We find a vast number of activities and occupations which can be carried on only with the approval of, or licence from, some authority, or by persons who possess certain qualifications—doctors, veterinary surgeons, bus drivers and conductors, dealers in stocks and shares, to mention only a few. We find extensive schemes administered by the state conferring rights on certain persons, e.g. the national health, national insurance, and pensions schemes.

Private Rights and Public Interest

We find the state setting up bodies to control certain industries, e.g. agriculture. We find the state not leaving persons in certain relationships to fix their own terms under freedom of contract, but imposing certain rights and duties on them whether they agree or not, e.g. the rent at which houses may be let. And we find the state with very large powers of overriding private rights in the public interest, e.g. powers of compulsory purchase of buildings unfit for human habitation, or of depriving a farmer of his land if he does not farm it properly.

So far as it is possible to derive some general principles from these vast state activities, one could say that it is now recognized that it is the duty of the state (1) to protect members of the community against disease and want; (2) to ensure that working and living conditions are safe and healthy; (3) to ensure that persons who supply certain goods and services are competent to do so and in fact do so properly; (4) to secure that the resources of the country are used to the best advantage of the community, thus recognising that the owner of property is not entitled to do whatever he likes with his own but has a duty not to use it contrary to the interests of the community, and in some cases even a positive duty to use it for the benefit of the community.

It is obvious that not only are many new laws necessary to enable the state to carry out these various activities, but the laws are different in their nature from those made in the 19th century. New laws were then mainly concerned with creating such new private rights or prohibiting by the criminal law such acts considered injurious, as might be made desirable by changes in conditions of life. Thus when the development of railways led to many deaths in railway accidents, the Fatal Accidents Act, 1846, remedied a defect of the common law and gave to the relatives of a person killed a right to recover damages against the railway company in respect of his death. When betting was very prevalent, the Betting Act, 1853, was passed to check it.

Administrative System

Laws to give effect to state activities in the welfare state must do much more than confer certain rights on persons or prohibit certain acts. They must provide a whole administrative system for the working of some scheme. If one introduces a scheme for old age pensions, it is not enough merely to say that everyone of a certain age who complies with certain conditions shall be entitled to a pension of so much a week. Someone must be appointed to decide whether a claimant is entitled to a pension; provision must be made for the compilation and issue of pension books, the payment of pensions, and a host of minor matters, e.g. how are claimants to prove their age? Must they claim their pension in person, or if a pensioner is ill can someone claim it on the pensioner's behalf?

Delegated Legislation

The administrative laws needed for the new welfare state are so detailed that it is impossible for parliament to devote time to including them in acts of parliament. Thus the practice has grown up by which in such matters parliament merely lays down in an act the general outline of the scheme, and delegates to some government department of the executive the task of making laws by regulations to provide for its detailed working. That government department may even be authorised by parliament to delegate the powers to make laws to some subordinate body, e.g. a local council.

There are other reasons which are said to justify this delegation. In the first place, many of the schemes involve matters of a highly technical nature, which parliament has not the technical skill to decide. For example, suppose an agricultural marketing scheme is desired to provide for the grading of eggs. Can parliament be expected to devote its time to debating such a matter, or to have the technical skill necessary to decide it ? Is it not preferable that the matter should be left to be decided by the experts of an agricultural marketing board or the ministry of agriculture ?

Again, many of the schemes are experimental. Let us imagine that it is found by experience that the system of grading eggs originally introduced could be improved and should be altered. If the original provisions have been made by act of parliament, the alterations will have to be made in that way also, and there is bound to be long delay. Regulations can be quickly made and altered

Safeguard Against Dictatorship

All this may seem so sensible that it may be wondered why anyone should object to the practice of parliament in delegating to government departments of the executive the power to make laws, particularly as this procedure has to some extent been adopted for several centuries. The practice has, however, very greatly increased in the last hundred years, and the principle of our constitution that our laws should be made by parliament is not to be lightly ignored. It is rightly regarded as a valuable safeguard against dictatorship that laws should be made in parliament after full and open discussion reported in the press, and should not be prepared in secret by some official in a government department. Legislation by the executive is the mark of a dictatorship, and was the method adopted by Hitler and Mussolini. It is ridiculous to suggest that because parliament authorises some government body to make laws about the grading of eggs then dictatorship is upon us, but if legislation by the executive ever became so general as to be accepted as the normal procedure, a frame of mind could be created in the community which would make it much easier for a dictatorship to be imposed.

Tradition of Freedom

Sir Stafford Cripps (1889–1952) in a book, *Problems of a Socialist Government*, written between the First and Second World Wars, advocated that on the coming to power of a socialist government an act of parliament should be immediately passed delegating to the executive the power to introduce socialist measures by orders and regulations, although he did not exclude some control by parliament. It is true that this policy was not adopted, but it could be used by some future government if the tradition of freedom in the country ever became weakened through the constant use of delegated legislation by the executive. As was once stated by a famous Irish orator and judge (John Philpot Curran, 1750–1817), "the condition upon which God hath given liberty to man is eternal vigilance."

"Henry VIII Clause"

The growth of delegated legislation and other increases in the power of the executive were attacked with violence by Lord Chief Justice Sir Gordon Hewart (1870–1943) in a book, *The New Despotism*, published in 1929, and this led to the setting up of a committee on ministers' powers, to consider the whole question. On the subject of delegated legislation, the main recommendation of the committee was that a stop should be put to the practice that had grown up of enabling a government department not only to make regulations for the purpose of working out the details of an act but also to alter the act itself—a power to which the sinister name of "Henry VIII clause" had been given to indicate the arbitrary power it conferred.

Further Safeguards

It must be accepted to-day that extensive delegated legislation by government departments is essential, but it is generally agreed that certain safeguards must be observed. (1) The power of the government department to legislate should be confined to matters of detail and should not extend to any matters of principle. (2) Whenever an act of parliament gives to a government department power to make regulations which may seriously infringe the freedom of the individual, the act should require the regulations to be laid before parliament, so that parliament may have an opportunity of taking action should it disapprove. Since 1944 there has been a scrutiny committee in the house of commons whose duty is to consider all delegated legislation which is laid before parliament, and to bring to the notice of the house any matters which they may consider require the action of the house. Unfortunately there is much delegated legislation that is not laid before parliament. (3) All regulations and other delegated legislation should be promptly printed, so that members of the public may quickly become aware of the law. (4) The act granting the power should always leave it open to anyone to challenge any delegated legislation in the courts on the ground that it goes beyond the power to legislate given by the act.

LESSON 5

The Rule of Law

A VERY important principle of our constitutional law is called the rule of law, and the most vital constitutional problem of the present day arises from the conflict between the rule of law and administrative law—the law by which the administration of the country, and particularly of the welfare state, is carried on.

This problem is not one of mere abstract interest, or a matter only for lawyers or politicians ; it affects every member of the community, not excluding the writer and probably also the reader of these Lessons, and on whether or not it is satisfactorily solved may depend their protection from unmerited ruin.

The rule of law means primarily protection from *arbitrary* or capricious power or, in more legal language, that no one can lose his liberty or be otherwise punished in any way except for the breach of some law, the fact that he has broken the law having to be established before one of the ordinary courts of the land. These courts have also certain characteristics designed to secure that the trial is fair.

Impartiality Between Parties

It is necessary also that the court, in arriving at its decision, should be guided solely by legal principles and not by considerations of policy. It would be a breach of the rule of law for a court to decide a case in a certain way because it considered that was desirable in the interests of the government or of a political party or creed. One recalls that in fascist Germany it was considered to be the duty of the judges to decide in the interests of fascism. Thus the duty imposed on a court, *Fiat justitia, et ruant coeli* (let justice be done though the heavens fall), requires not only an impartiality between the parties to the case but also a complete disregard of all considerations other than those of justice.

In order that this may be achieved it is also necessary that the judges of the court should be independent and should not be subject to pressure from the government. This is secured in the case of high court judges by the provisions of the Act of Settlement, 1701, that a judge holds office "during good behaviour," subject to the right of the Crown to remove him from that office on presentation of an address by both houses of parliament.

Magna Carta

Some conception of the rule of law is found very early in British history. We read in Magna Carta : "No freeman is to be arrested, imprisoned, put out of his freehold, outlawed, exiled, destroyed, or put upon in any way except by the lawful judgment of his peers or the law of the land." It would be quite wrong to imagine that this provision of Magna Carta meant what is understood by the rule of law to-day ; but for centuries it has been of immense effect, because of the interpretation in favour of liberty put upon it by the courts and more particularly by public opinion. The same principle is embodied in the Petition of Right, 1628, and the Bill of Rights, 1689

Despotic Power

When the rule of law was stated in the 19th century by the great constitutional lawyer A. V. Dicey (1835–1922), he used 18th-century France as an example of a country which lived under a despotism and where the rule of law was unknown. He pointed out that what was remarkable about the English system of government at that time was not that it was particularly good or lenient—many countries under despotic rule were well governed and not oppressed, whereas the criminal laws in England were extremely harsh—but that it was based on *law*. He recalled Voltaire (who was compelled by his misfortunes to leave France and take refuge in England), and continued : "When Voltaire came to England . . . his predominant sentiment clearly was that he had passed out of the realm of despotism to a land where the laws might be harsh but where men were ruled by law and not by caprice."

Voltaire (1694–1778) had indeed reason to know despotic power, for in 1717 he had been sent to the Bastille without trial, for a poem he had not written, and in 1725 had been sent to the Bastille again because, on being brutally thrashed by the servants of a nobleman, he had dared to complain. We do not need to go back to the 18th century to find examples of despotic power. We can recall Hitler's Germany when countless persons disappeared for ever, without trial and without trace, and where—and this perhaps is more immediately relevant to this Lesson—professors, teachers, doctors, and others were deprived of their right to carry on their occupations because they were thought to be unfavourable to the Fascist regime. Even in England we can recall the notorious Regulation 18B of the Second World War, which enabled the executive to deprive anyone of liberty without trial. (*See* Lesson 3.)

In peace-time there is still protection to-day in England for liberty of the person in accordance with the rule of law. No arbitrary power can deprive anyone of his liberty, for the

protection of the writ of habeas corpus is available. Anyone who considers he—or anyone else—is detained without legal authority may, if he can show any grounds for his contention, obtain a writ of habeas corpus from a judge of the queen's bench division. This writ requires whoever is detaining him to produce him to the court and there justify the detention. The writ of habeas corpus, although to-day "the principal bulwark of English liberty," was not of great constitutional importance until the passing of the Habeas Corpus Act, 1679, for until that time it was not fully effective against imprisonment by the king's command.

Writ of Habeas Corpus

The act was amended and the protection of the writ of habeas corpus extended to illegal civil detention by the Habeas Corpus Act, 1816. The writ has been used in a great variety of cases by parents to claim a child of theirs in an orphanage, by a married woman who was kept locked up by her husband, and in an outstanding case in 1771 when a slave, James Sommersett, was lying in irons in a ship in the Thames bound for Jamaica, where slavery was legal. In this last case the judge, Lord Mansfield (1705–93), ordered that Sommersett should be set at liberty, stating (or so tradition has it) : "The air of England is too pure for any slave to breathe. Let the black go free." How Voltaire might have envied the slave ! In connexion with this writ, it will be appreciated that the detention may well be justified, e.g. the detention of a child by his father, or of a person of unsound mind under an order, or of a convicted prisoner and in such a case the court will not interfere. The point to note is that justification *according to law* must be available.

In times of emergency, acts of parliament were formerly passed suspending the Habeas Corpus Acts, as happened, for example, at the time of the rebellions of 1715 and 1745. This practice appears now to be obsolete for during the world wars the acts were not suspended, but instead the executive was given wide powers of detaining persons under regulations ; and where a person had been so detained, an order for that detention was an answer to any writ of habeas corpus, as the detention was lawful. The Emergency Powers Act, 1920 (*see* Lesson 3), although it may result in the imprisonment of persons for offences against the regulations made under the act, requires that they should be tried and sentenced by the ordinary courts, and prohibits imprisonment without trial.

Among other protections for personal liberty are the restrictions on the powers of arrest, and the provision for bail and for a speedy trial. A police constable may arrest without a warrant any person on reasonable suspicion that that person has committed a felony, and indeed a private person may arrest without a warrant anyone who has in fact committed a felony. These rights arise under the common law, but for offences under many acts of parliament, e.g. a person driving a motor vehicle while under the influence of drink, a police constable may also arrest without a warrant. Where a constable has not this power, a warrant must be obtained from a magistrate. A person arrested may be allowed bail by the magistrates, and if they refuse he may apply to a judge of the queen's bench division.

Damages for False Imprisonment

Although the writ of habeas corpus provides the most immediate protection for personal liberty, because it can secure the release of the detained person, personal liberty is also protected by the right of a person who has been improperly detained to bring criminal proceedings for assault against the person who detained him, and to sue for damages for false imprisonment or malicious prosecution. An action for false imprisonment may be brought if a person is improperly detained even for only a few minutes, as when he is ordered by a policeman to accompany him to a police station. It will be noticed that proceedings may be brought against the police if they act in a manner not authorised by the law in the same way as against anyone else. The fact that the police are officials of the state does not place them in any privileged position if they do not observe the law ; actions against them are not infrequent.

LESSON 6

The Welfare State: The Right to a Livelihood and the Right to Property

In the preceding Lesson it was seen that the rule of law as enunciated in the 19th century still adequately protects personal liberty in England to-day. In this Lesson we consider how far that rule still protects two other very important rights.

It was pointed out in Lesson 4 that one of the functions of the welfare state is to ensure that persons who provide services or certain goods are competent to do so, and in fact do so satisfactorily. One of the methods in which this function is carried out is by the state's pro-

hibiting a person from carrying on certain occupations unless he first obtains from some official authority some permission which may be granted by the inclusion of his name in a list kept by the authority, or by the issue to him of some licence.

Official Permission to Work

In these cases, just as there are provisions for granting the permission, so also there are provisions for cancelling it if the person to whom it is granted does not carry on the occupation in a satisfactory way. Refusal of the permission in the first place may well be a serious matter, but the cancellation of a permission that has been once granted will almost certainly cause enormous hardship, because the person from whom the permission is withdrawn will be unable to earn his livelihood and may be completely ruined. It is impossible to give here anything like a complete list of the occupations for the carrying on of which some official permission is required, but the following random selection will give some idea of the many occupations involved: doctors, dentists, and chemists, who require to be on official lists before they can supply their services under the National Health Act; dairy farmers, stock and share dealers, commissionaires, bus and coach drivers and conductors, taxi-cab drivers, ice-cream and sausage manufacturers and sellers, pawnbrokers, keepers of agencies for domestic servants or nurses, dance-hall proprietors, keepers of pet shops. There is also the example of the ordinary driving licence which involves a test, particularly in the case of disabled persons.

Criminal Courts Rules

As might be expected from the varied nature of the occupations, there is a large number of persons or bodies on whom lies the duty of granting or cancelling the permission. Very often the duty falls on the local council, but in many cases it falls on other bodies, e.g. the police, some government official, or some person or body directly appointed by a government department.

When the question arises whether or not the permission should be cancelled, the ground will usually be because it is alleged that the person to whom permission has been granted has misconducted himself in some way that shows he is unsuited to carry on the occupation. Some person or body will have to decide whether or not the allegations against him are satisfactorily proved. This process is identical with that which takes place in a court whenever a person is charged with a criminal offence, and whatever person or body has to decide whether or not to cancel the permission goes through much the same procedure as a criminal court adopts when it is trying a case.

In criminal courts certain rules are observed. (1) The judge is a person who is impartial and not connected with either of the parties to the dispute. (2) The accused knows beforehand exactly what is alleged against him and what he is charged with. (3) He hears the evidence against him, and is entitled to give evidence himself and to call others to give their evidence on his behalf. (4) The judge decides on the evidence called before him and not on any private information of his own. (5) Either the judge is a trained lawyer or there is a trained lawyer present to advise the court as to the law, as does the clerk of a magistrates' court. (6) The proceedings are in public. (7) There is a right of appeal. All these requirements are observed in even the most trivial criminal proceedings—even when you are charged with a parking offence and fined £1.

National Health Service Tribunal

In the cases we are now considering—that is, where it is the state or some official person or body on whom lies the duty of granting or cancelling the permission—the procedure followed in most cases (although by no means in all) corresponds fairly closely with the procedure just outlined as being followed in a criminal court. Let us take two examples, first that of the persons, e.g. doctors, dentists, or chemists on the national health service lists kept by the local executive council. Let us also remember that in the case of all these persons removal from the national health service list may mean complete ruin, as although it does not of itself prevent them from carrying on their profession, they must confine themselves to private practice, which to-day is unlikely to yield much of a livelihood.

All complaints are heard by a body called the National Health Service Tribunal. There are three members: a chairman, who is a practising barrister or solicitor of not less than ten years' standing appointed by the lord chancellor, and two other members, one appointed by the minister of health, after consulting the bodies responsible for the administration of the service, the other appointed by the minister from a panel consisting of persons appointed by the minister, who are of the same profession as the practitioner against whom a complaint has been made, e.g. if that person is a doctor, the third member will be a doctor. The hearing of the complaint is in public if the person against whom complaint is made so desires.

Both the person complaining and the practitioner against whom complaint has been made may be legally represented, and may give evidence and call the evidence of others. The tribunal in giving their decision must state their findings of fact and their conclusions; and if they consider that the continued inclusion of the

name of the practitioner in the list would prejudice the efficiency of the service, they must direct that his name be removed from the list. The practitioner may appeal to the minister of health. On the face of it, the minister may not appear to be a very satisfactory appeal tribunal, as he is connected with the proceedings to the extent that they relate to a member of his medical service, but great care has been taken to secure that the appeal is properly considered and determined.

Advisory Committees

The appeal is heard by some person appointed by the minister, who has to assist him an assessor of the same branch of the medical profession as the person appealing, and the assessor also is appointed by the minister. The person hearing the appeal reports to the minister. In the case of doctors and dentists, the minister must then consult an advisory committee consisting of six doctors (or dentists if the complaint relates to a dentist), three of them being on the staff of the ministry, the other three being taken from a panel of general practitioners nominated by the doctors' (or dentists') professional body. The decisions both of the tribunal and of the minister must be published.

The conclusion as to this procedure would appear to be that apart from the prominent part played by the minister in a matter affecting a service administered by him—so that he does not *appear on the face of it* to be in such a position of impartiality as a judge in a criminal case—the procedure does not differ very substantially from that of a criminal court.

Area Licensing Authorities

Let us take a bus driver as our second example. He must obtain a licence from the licensing authority for public service vehicles, and the same authority may at any time revoke or suspend his licence on the ground that "by reason of his conduct or physical disability" he is not a fit person to hold a licence. The licensing authority consists in each area of three persons, all appointed by the minister. Anyone the minister pleases may be appointed chairman, but the two other members are drawn from panels nominated by local authorities. The bus driver, if his licence is cancelled or suspended, may require the authority to "reconsider the matter" and may then be heard personally or by a representative. His main protection is that if the licensing authority are still against him he may appeal to an ordinary court of law—the local magistrates' court. Thus, in this case, the bus driver is given full protection for his livelihood.

The procedure used in the case of the bus driver, namely, an appeal to the local magistrates' court, is also made available in many other cases where permission is withdrawn. It is available, for example, when a disabled person is refused a driving licence because of his disability, and when manufacturers and sellers of ice-cream and sausages have the registration of their premises cancelled.

It is unfortunate that even in this class of cases, i.e. where the permission required is an official one, there are cases where a man may be deprived of his livelihood without the protection which exists in the foregoing examples. In 1953 it was decided by the courts that a London taxi-cab driver, who had been deprived of his licence by the commissioner of police, could not complain on the ground that he had not been given an opportunity of calling certain evidence he wished to call in his defence.

The commissioner of police is entitled to revoke a taxi-driver's licence if he is *satisfied* the taxi-driver is not a fit person to hold a licence; and although in the case in question the driver had been allowed to appear, it would seem that the commissioner can act quite arbitrarily—capriciously—if he wishes to do so. No doubt a person in the position of the commissioner would always endeavour to act fairly, but as a judge has said, "Justice should not only be done but should manifestly be seen to be done." There should be no room for any possible doubt.

It would seem desirable that some right of appeal to a court of law should be given in every case where permission to carry on an occupation is refused or withdrawn by some official person or body.

Domestic Tribunals

In the next class of cases to be considered, the right to a livelihood is sometimes less adequately protected. These arise where the granting or cancelling of permission is in the hands of what is called a domestic tribunal of some organization, usually a voluntary body of persons who themselves carry on the occupation in question. Some few of these domestic tribunals are set up under an act of parliament to control the conduct of members of some profession, e.g. doctors and architects, and in these cases there is now usually an appeal to a court of law; but the vast majority of the domestic tribunals are entirely private organizations, with which the law can rarely interfere. A few random examples of the persons whose right to a livelihood depends on the decision of an entirely private domestic tribunal are members of the stock exchange, who are subject to their own rules; jockeys and trainers, who are subject to the Jockey Club and National Hunt Committee; members of trade associations and trade unions, who are subject to the rules of their associations and unions.

The domestic tribunals coming within this wide class are no doubt composed of persons honestly endeavouring to decide fairly in matters that come before them, and very often they have a procedure designed to prevent the making of any mistake, and a system of appeal. But this all depends on the rules of the particular body, and it would certainly be incorrect to assume that the procedure of all domestic tribunals contains all the requirements present at a trial in a criminal court, or gives adequate protection against the possibility of a mistake. So the protection of your right to a livelihood may be less complete than your protection against being fined £1 for a parking offence.

Requirements of " Natural Justice "

Although the courts recognise that, as stated by a judge, " A man's right to work is just as important to him as, if not more important than, his rights of property," the courts cannot normally interfere with the decision of a domestic tribunal.

There is some indication that the courts will interfere in some cases if the requirements of " natural justice " are not observed. Unfortunately there is little agreement as to what the requirements of " natural justice " are. It is thought that they make it necessary at least that a man shall be made aware of the allegations against him and be given an opportunity of answering them—but that is a very small protection. The courts will also interfere where the domestic tribunal has made a serious mistake in procedure by not following its own rules. Recently also the courts have interfered where the domestic tribunal has made a mistake in law, e.g. has given to its rules a meaning they could not reasonably have.

Unfair Competition

In one case a tribunal held that one of the members of its association had been guilty of " unfair competition," which was forbidden by its rules, and fined him. The member refused to pay. The rules of the association entitled the tribunal to expel a member for failure to pay a fine, so the tribunal expelled the member. The court held that what the member had done *could not* be " unfair competition " within the rules, and that his expulsion from the association was void and of no effect.

The court will not, however, inquire whether the allegations against a man have been proved to its satisfaction or whether he has been given a trial in any way resembling the trial he would have received in an ordinary criminal court.

It is not suggested that domestic tribunals do not endeavour to act fairly, nor that some of them do not afford the fullest safeguards. But it is felt that as all of them are exercising such wide powers, their procedure should reduce the possibility of a mistake to the minimum, and, as indicated by the words of the judge previously quoted, not only should justice be done but it should be clearly obvious to everyone that it has been done.

Compulsory Acquisition

The problem is a very serious one, and all the more serious because it is very difficult to think of any solution. The law will not normally interefere with the rules on which persons engaged in an occupation choose to associate together. Perhaps the best that can be hoped for is that all such associations should provide in their rules for an appeal to some independent tribunal including a lawyer, adopting procedure like that of a magistrate's court.

The protection given to property by the rule of law has been much weakened by powers given to the executive and local authorities relating to land and houses. Various acts entitle a government department or other authority to acquire land and houses compulsorily, e.g. for a new town. Sometimes a public inquiry has to be held, but the courts have held that in some cases at any rate the minister is not bound to act in accordance with the report of the inquiry. There are in many cases provisions for appeals, but very often the appeal lies to the government department itself.

Another matter in connection with property which is removed from the ordinary courts is the fixing of rents of furnished premises. This task is carried out by local rent tribunals, which in many respects differ from ordinary courts. The chairman often is, but need not be, a lawyer. He is appointed (and may be removed) by the minister of health. There is no appeal. These tribunals have been much attacked by some constitutional lawyers.

Not Adequately Protected

The conclusion is that at present neither the right to a livelihood nor the right to property is in all cases adequately protected. It has been suggested that an administrative court of appeal should be set up to hear appeals in cases such as those which involve compulsory acquisition of land. In 1955 a committee was appointed to consider (a) tribunals other than the ordinary courts constituted under an act of parliament by a minister, and (b) the procedure relating to holding inquiries or hearing by or on behalf of a minister on an appeal, and the procedure for the compulsory purchase of land.

This committee cannot consider the activities of private domestic tribunals, so it appears that however welcome its appointment may be, it nevertheless leaves untouched one of the very serious problems mentioned in this Lesson, affecting the lives of nearly all of us to-day.

LESSON 7

Short History of Our Legislature

THE most striking feature of the legislature of this country, and the attribute which has made it the model for democratic countries all over the world, is that it is a representative body, consisting of members chosen to represent, so far as possible, all interests and opinions. Of recent years representative government has reached in this country what may perhaps be regarded as its full extension in manhood and womanhood suffrage; but the principle of legislation "by the people for the people" is to be found, though in a rudimentary form, very early in English history.

Origins of Representative Legislature

While it was the custom of the Saxons and early Norman kings to hold councils of the great men of the land for the purpose of discussion—hence the word "parliament"—and obtaining advice, those councils can in no sense be regarded as representative bodies. The king called to him such persons as he chose and, at any rate in Norman days, the council consisted of great landowners, who held land of the king as his feudal tenants; and the interests they served were their own and not those of the country as a whole. The first germs of a representative legislature are to be found in the parliaments of the 13th century, notably in the famous parliament of 1265, to which Simon de Montfort summoned representatives of the counties, cities, and boroughs, and in the Model Parliament of 1295.

The striking feature of these parliaments is that they contained, side by side with the great men specially summoned by the king, persons selected by and sitting as representatives of others. Thus the electors exercised the right to choose some at least of those who assisted in the government, if they did not yet actually govern the land.

Formation of a National Assembly

To the Model Parliament, Edward I summoned separately the two archbishops, the bishops, the greater abbots, seven earls, and 41 barons, and the clerics were directed to bring with them certain of the clergy in their dioceses. Also each sheriff was directed to cause two knights of each shire, two citizens of each city, and two burgesses of each borough to be elected to parliament.

These representatives of the *communitates* or *communes*, i.e. communities of shires, cities, and boroughs, became the house of commons (which does not, as is generally supposed, derive its name from the "commons" or common people). In this parliament is seen for the first time a definite departure from a feudal council and the formation of a national assembly.

The writ requiring the attendance of the minor clergy was not long obeyed. The higher clergy met with the other great landowners in the body which has become the house of lords. Another important feature of the Model Parliament is that the knights of the shire, who were drawn from much the same class as the barons, sat with the representatives of the cities and boroughs, and so there never arose in England that sharp cleavage between the nobility and the commons which developed in other countries.

The Middle Ages were not a time of great legislative activity, and so the main task of parliament was the voting of money to the king. The king always enjoyed a large revenue from his own estates and the feudal dues, but if he wished to raise any further sums he could do so only with the consent of parliament.

Parliament had its grievances from time to time, and when called together for the purpose of voting supplies it usually endeavoured to strike a bargain with the king, demanding legislation to remove injustices—or further its own interests—in exchange for the vote of money.

Statutes framed by the King

By the end of the 14th century, parliament had established the right to impose taxes of all kinds, and in order that the king might be compelled to call parliament at frequent intervals, parliament usually refrained from imposing more taxes than were required to meet the immediate needs. At first the lords and commons each used to impose separate taxes on their own orders, but before 1400 the practice had grown up of making a common grant, and gradually, since the largest burden of taxation fell upon those whom the commons represented, the very important constitutional principle became established that money grants had to be initiated in the house of commons, and that the king could not consent to them unless both houses agreed.

In other matters the commons were at first petitioners for legislation rather than legislators themselves. Some early statutes are expressed to be made by the king with the *assent* of the earls, prelates, and barons, and *at the request* of the knights of the shire.

All statutes were at first framed by the king in his council in answer to some petition of the lords or commons, with the result that not infrequently the statute as framed did not meet the grievance of the petition; but by the reign

of Henry VI (king from 1422 to 1461) the principle became established that the lords and commons should themselves frame the legislation they wanted in the form of a "bill," which they would present to the king. He could then accept or refuse it, but he could not alter it.

This change is marked by an alteration of the phraseology in the preamble to a statute, for thereafter statutes are stated to be made not only with the "advice and consent of the lords and commons in parliament," but "by the authority of" the same. Parliament no longer merely advised legislation, but actually initiated it.

King by Virtue of Act of Parliament

When we leave the Middle Ages and come to the time of the Tudors, we find that parliament is subjected to the will of strong monarchs such as Henry VIII (reigned 1509–47) and Elizabeth I (reigned 1558–1603), who did not overrule parliament, but ruled as they pleased through parliament. When James I came to the throne, he lacked the ability to manage his parliaments in this way, and many constitutional questions which had been evaded by his more tactful predecessors came into sharp prominence. The quarrels that arose out of these under the Stuarts resulted in the victory of parliament, and the king became king and held his throne only by virtue of an act of parliament.

The 18th century saw the rise of the great Whig families and the development of the "pocket boroughs." The latter were finally swept away in the great wave of reform in the 19th century, by the Reform Act of 1832. From that time the changes have been rather in the electorate than in parliament itself, and in the growth and development of the cabinet system, linking legislature and executive. The most important changes in parliament itself have been the increased supremacy of the house of commons over the house of lords—culminating in the Parliament Acts of 1911 and 1949; the admission of women to sit in the house of commons (1918); and the payment of an annual salary to members of the lower house, first introduced in 1911.

Parliament's Duration Limited

Parliament is to-day a permanent institution, sitting continuously with only short vacations. Formerly, parliaments were both infrequent and irregular in duration. If parliament favoured the king, he might retain it for many years; if it opposed him, he could dissolve it at once, and call no other. Now, the maximum life of parliament is limited.

In 1694 the Triennial Act required a new parliament to be summoned every three years, and limited the duration of parliament to three years. This was extended in 1716 to seven, and reduced in 1911 to five years. There is still no statute which requires the king to call parliament every year, but if he did not do so he could not govern, for he could have no army and no taxes, because the Mutiny Act, which authorises the army, and the Finance Acts, are passed to last for one year only, and have to be renewed annually.

LESSON 8

The Privileges of Parliament

FROM what we have hitherto said it might perhaps appear that the house of commons has stood for freedom against the tyranny of the Crown. This would not be a true representation of the facts, for the commons, in their struggle with the Crown, were not consciously asserting the rights of the people or of freedom, but were striving to establish the power of their own house against king and people alike.

Ancient Rights

From these struggles the house has emerged with certain privileges, which it asserts against all those outside itself. At the beginning of every parliament the speaker, on behalf of the commons, lays claim to their ancient and undoubted rights and privileges and especially to freedom from arrest, freedom of speech, and free access to the sovereign, and to have the most favourable construction on their proceedings; and the lord chancellor, on behalf of the Crown, most readily grants and confirms them. The freedom from arrest is now of less importance than formerly, since it does not apply to arrest for crimes; and arrest for civil process, e.g. for debt, is now virtually abolished.

Freedom of Speech

A much more important privilege is that of freedom of speech. This is a question upon which much dispute was likely to arise between the commons and the Crown, and so early as 1397 we find the king, Richard II, rebuking the commons for adopting a bill to reduce the charges of the royal household. Under the Tudors and early Stuarts, members whose speeches were considered to be obnoxious to the Crown were summoned before the council and sent to prison.

A general privilege of freedom of speech was recognized, but it was interpreted by the Crown to mean a privilege "not to speak everyone what

he listeth or what cometh in his brain to utter; but your privilege is 'aye' or 'no'." A privilege of this extent was not of great value, and the struggle went on until the Bill of Rights after the 1689 Revolution declared that the freedom of speech in parliament should not be questioned in any court or place out of parliament.

The wide extent of this privilege will be recognized when it is remembered that no action for defamation of character can be brought on any words spoken in parliament, and this explains the invitation occasionally made by a member whose character has been attacked by another to "repeat the statement outside the house," when proceedings could be taken for slander. A privilege so great could be abused, and so the house itself imposes a restraint upon its members, who may be expelled, fined, or committed to prison in serious cases, or, in minor cases, asked to withdraw from the house, or suspended.

Reports of Debates

In early times the right claimed by the members to say what they pleased had a not unnatural corollary in the claim to exclude from the house all persons who were not members, and who, if admitted, might carry reports of what was said to the king. Secrecy of debate was as necessary as freedom of speech. Until 1875, if any member called the Speaker's attention to the fact that strangers were present, the Speaker was obliged to cause them to withdraw; but as newspaper reporters were (and are) technically strangers, the rule was somewhat inconvenient, and in 1875 it was altered so that the strangers are not required to withdraw unless the house so resolves. Secret sessions are held for security reasons during war-time; but in normal times any person can obtain admission to the Strangers' Gallery on making application.

This policy of secrecy as to how members spoke has had a considerable bearing on the publication of reports of debates. At present, except when the house is in secret session, full reports of all debates are published, and members are called to answer to all sections of a critical electorate for the most minute deeds—or misdeeds, according to the point of view—of their time in the house.

Lord Mayor Committed to the Tower

In the 18th century the country as a whole, and London in particular, became much interested in the doings of parliament; and as parliament afforded no facilities to enable accurate reports to be made, reporters relied on their inventive faculties, their reports gaining in scurrility what they lost in accuracy. The great Dr. Johnson (1709–84) was on one occasion the "reporter" of an oration which gained temporary fame for a member, though the words in it had never been uttered by him.

In 1771 the house committed the lord mayor and two aldermen to the Tower for having refused to obey the orders of the commons to arrest a printer of debates, but thereafter the house admitted reporters on sufferance, probably to save itself from the inaccuracies of inventive journalists.

Hansard

Although whatever a member might say in the house is privileged so far as proceedings for defamation are concerned, it is still technically a breach of the privileges of the house to publish outside the house unofficial reports of speeches, although such reports have for long been permitted. Reports of the proceedings of the house, taken from the newspapers, began to be printed in the 18th century by the family firm of Hansard, whose name was for long colloquially and is now officially applied to the official parliamentary reports published daily when the house is sitting.

On one occasion, in 1837, Hansard was sued for publishing a defamatory libel contained in a report published by order of the house. The courts held him liable, but the house was very indignant at what it considered an infringement of its privileges, and an act was passed in 1840 preventing proceedings in such cases. Freedom of report is obviously in the interests of the large public which depends on statements in the house for its knowledge of affairs. A report of the actual proceedings of either house is free from attack on the grounds of libel, even though it is not authorised by the house, so long as it is fair and accurate and published without malice, and this it is which protects the reports of parliamentary proceedings that appear in the daily press.

Punishments

These privileges of the house would be of little value if some method did not exist of punishing those who fail to observe them. The punishment of members has already been considered. The house punishes strangers by admonition, reprimand, or commitment, i.e. imprisonment. A person admonished may escape further punishment by an apology to the house. If a person is to be reprimanded he is first arrested by the sergeant-at-arms—the "policeman" of the house—and then brought before the house. The house cannot imprison for longer than the duration of the session.

The house is jealous of its privileges, and complaints that they have been infringed are not infrequent. The matter does not come before the ordinary courts. If the Speaker decides that a *prima facie* case has been made, the matter may be considered by a committee of the

house, which in due course reports to the house. The matters which most frequently give rise to complaints are: any attack on the Speaker or chairman of ways and means, particularly any suggestion that they are not impartial; and any libel on the house as a whole or any libel on any individual member, so long as it reflects on him in respect of his conduct in the transaction of the business of the house.

Constituencies

Parliament consists of the sovereign, the lords spiritual and temporal (house of lords), and the commons (house of commons). Of the two houses, the house of commons is to-day by far the more important, not only in practice but also in theory.

It is now provided that there shall be not substantially greater or less than 613 constituencies in England, Scotland, and Wales, of which there must be not fewer than 71 in Scotland, and not fewer than 35 in Wales. In addition, there are 12 constituencies in Northern Ireland. The exact number of constituencies depends on reports of four boundary commissions, one for each country, and these commissions report at intervals of from three to seven years. From 1955 there were 630 constituencies, the number having been increased by five in England as a result of a boundary commission report. Each constituency has a single member, and the number of electors varies from 40,000 to 80,000, but the majority of constituencies have between 50,000 and 60,000 electors. A member receives a salary of £1,000 a year, plus an expense allowance of £2 for each parliamentary day, excluding Fridays. There is no doubt that this system of single-member constituencies does not provide an adequate representation of minorities, and a party which, in the country, has secured 40 per cent of the total votes may find that in the house of commons it is represented by 10 per cent., or even less, of the seats.

Misrepresentation

Thus even if a party had only a small majority of the electorate in its favour it would probably find itself represented in the house by a large majority of the members; and if there were three or more candidates in most of the constituencies, a party which had polled less than 50 per cent. of the total votes might gain the majority of seats.

There is a division of opinion as to whether this system of representation is desirable or deplorable, for many persons consider that it ensures that the country shall have a strong goevrnment, with sufficient power in the house to carry out its programme, and prevents the development of government by a coalition of small groups, which is so frequent on the Continent. The remedy most usually advocated by those who consider the system undesirable is proportional representation. This involves the creation of large constituencies, each returning three or more members. The voters in each constituency do not vote merely for one candidate, but place the several candidates in order of preference. The various preferences are then counted, and the required number of members declared elected. By this means, it is contended, the house of commons would more accurately represent the general state of feeling in the country.

Disqualifications

To-day almost any man or woman who is a British subject of full age is qualified to be a member of the commons. The principal exceptions are bankrupts; clergy of the Churches of England, Scotland, or Ireland, and the Roman Catholic Church; peers (except certain Irish peers); and the holders of certain offices under the Crown. The last of these disqualifications is of great historical importance. At one time the house of commons, when engaged upon its struggle with the Crown, was much concerned to prevent the Crown's gaining favour among its members by gifts of pensions and profitable offices, and accordingly a law was passed declaring that on the acceptance of such office a member lost his seat. As cabinet government developed, the danger from the Crown grew less, and the rule caused great inconvenience, since it required that on every appointment of a member as a minister of the Crown there should be a by-election. In 1919 this rule was relaxed to the extent that any minister appointed within nine months of a new parliament could retain his office without re-election. In 1926 the rule was swept away so far as ministers were concerned. Other servants of the Crown, such as judges, civil servants, and members of the forces on the active list, are still incapable of sitting. During war-time provision was made to enable members of the forces to sit. The law on this topic is obscure and confused, and from time to time it is found that a member of parliament is in fact disqualified because he is the holder of some office. In these cases a special act is passed, relieving the member of the penalties he may have incurred by sitting while disqualified.

Stewardship of the Chiltern Hundreds

Once a member has been elected to the house of commons, it is not possible for him to resign. This rule owes its existence to the fact that in early days membership of the house was a duty rather than a privilege, and a member called on to sit was required to do so. To-day a member is able to vacate his seat by an ingenious use of the rule imposing disqualification upon Crown servants. Any member wishing to resign now applies for an office under the Crown of purely

nominal importance, with no duties. The office usually selected is the stewardship either of the Chiltern Hundreds or of the Manor of Northstead. The chancellor of the exchequer makes the appointment as a matter of course, the member automatically vacates his seat, and, having achieved his object, resigns his newly-acquired office.

Every man and woman over 21 years old who is a British subject or citizen of the Republic of Ireland is, with a few exceptions, entitled to be registered as an elector if he or she has qualification by residence. Peers of the realm have no vote. The register is prepared annually.

Constitution of House of Lords

The house of lords is presided over by the lord chancellor, and consists of about 870 members including (1) the princes of the blood royal who are peers (e.g. the royal dukes); (2) hereditary peers of the United Kingdom; (3) the two archbishops, the bishops of London, Winchester, and Durham, and the 21 most senior of the other bishops; (4) sixteen peers, who are peers of Scotland, but not of the United Kingdom, elected by all the peers of Scotland, for the duration of each parliament; (5) Irish peers, who are not peers of the United Kingdom, elected by the body of Irish peers for life (originally 28 peers were elected, but after the creation of the Irish Free State in 1922 no new elections were made); (6) nine "law lords," who are salaried judges, and are entitled to the rank of baron for life. The peers are entitled to individual summonses from the sovereign, and it is now recognized that if any person can prove he is the direct descendant and heir of a person who received such an individual summons, he is entitled to the rank of peer.

Powers of House of Lords

To-day peers are created by letters patent under the Great Seal, and the sovereign still exercises the power of appointment. This power has occasionally been used, as a threat, by ministers of the Crown in order to induce the house of lords to accept a measure which has been passed in the house of commons. Thus, in 1911, H. H. Asquith, then prime minister, secured the passage through the lords of the Parliament Act of that year, which definitely established the supremacy of the house of commons over the house of lords, by threatening to ask the king to create a sufficient number of new peers favourable to his policy to ensure a majority in the lords. The lords submitted.

The reform of the house of lords has frequently been a live topic in politics; but, like many other threatened institutions, the house of lords has preserved its constitution unchanged, although it has lost some of its powers. On paper it would appear to be about the most cumbrous and inefficient legislative body imaginable, but in practice it works very much better than might be expected. The vast majority of the peers rarely take their seats, but those who do attend constitute a workable assembly, including men who have proved their ability and worth in all branches of life.

LESSON 9

The Making of Our Laws

In general, acts of parliament do not become law until they have received the approval of both houses of parliament and the sovereign's assent. Exceptions arise from the Parliament Act, 1911 (as amended by the Parliament Act, 1949), which secured the undivided authority of the house of commons over finance, and its predominance in all legislation.

In the case of bills dealing with finance—money bills—if the bill is passed by the commons and on being sent to the lords, is not passed by that house, within one month it becomes law on receiving the royal assent, and the approval of the lords is dispensed with. No money bill can be initiated in the lords. If a bill other than a money bill or a bill to extend the maximum duration of parliament beyond five years is passed by the commons in two successive sessions and twice rejected by the lords, it will nevertheless become law on receiving the royal assent, provided that one year has elapsed between the second reading of the bill in the commons in the first session and its final passing in the second session.

It would be a mistake to imagine that the lords have lost all effective legislative control by reason of the Parliament Act. In fact, no house of commons would venture to rely on the provisions of that act—at any rate in the case of bills other than money bills—unless they were assured of a large majority in the country in favour of their policy, and in such a case would probably prefer to go to the country on the issue of the disputed bill.

If the country then supported the policy of the bill, it is improbable that the lords would continue their opposition.

Question Time

Let us picture the ordinary working of the house of commons, and trace a bill from the time it is introduced until it receives the royal assent and becomes an act of parliament. At the beginning of the sitting the Speaker

enters the house, preceded by the sergeant-at-arms with the mace. The mace is laid on the table, prayers are said, and the Speaker takes the chair. Then comes "question time." Inquiries are addressed to ministers of the Crown, designed to inform the members (and, through the medium of the press, the whole country) as to how the ministers have been exercising their powers as the executive of the Crown. The answers may be given orally if desired. When question time has expired, questions that have not been answered orally are answered in writing.

The importance of questions becomes greater every year, for so heavy is the pressure of work to be carried out by the government that the time of the house available for private members is very small, and questions are the only method by which matters of importance outside the ordinary business may be brought to the notice of the house. Any member may move at the end of question time that the house be adjourned "for the purpose of discussing a definite matter of urgent public importance," in which the conduct of a minister is involved.

If the Speaker decides that the matter is definite and urgent, and the motion is supported by not fewer than 40 members standing up from their seats, it is allowed to be proposed. If it is carried, the discussion takes place later on the same day. The next business is giving notices of motion, in which way new bills are introduced, and thereafter the house proceeds to orders of the day.

Progress of a Bill

When a member desires to introduce a bill, he first gives notice of his intention to do so. This is usually formal, but if it is opposed, a discussion may take place and, on a division, leave to introduce the bill may be refused. If not, the bill is immediately presented by the member introducing it, and the clerk of the house reads its title aloud. The questions that the bill be now read a first time and be printed are then put, no debate or amendment taking place, and it is ordered that the bill be read a second time on a later day.

The bill then comes on as part of the orders of the day, and a general discussion ensues on the motion that the bill be read a second time. If it passes the second reading, it is committed to one of the standing committees of the house. Certain bills, e.g. those imposing taxes, are referred to a committee of the whole house. On such occasions the Speaker leaves the chair without question put on each day when the bill is one of the orders of the day, and the chairman of committees takes his place. To whatever committee the bill is referred, it is there discussed in great detail, clause by clause, by the members forming the committee.

When the bill has been fully considered, the chairman of the committee reports it to the house, and it is then considered on the report stage. Amendments may be made, but if they are complicated the bill will be sent back to the committee to be discussed and embodied in the bill. At the conclusion of the report stage the motion is made that the bill be read a third time, and when this is carried, the house orders the clerk to carry the bill to the lords and "desire their concurrence."

The Speaker has very wide powers of control over the house. It is his duty to certify whether a bill is or is not a money bill for the purposes of the Parliament Act. In the house itself he can request a member to withdraw if he is disorderly, and if the offending member refuses to do so, he may be named and a motion will thereafter be made for his suspension. The Speaker must necessarily be completely impartial, and though he is selected by the government in power when a vacancy occurs, all parties are consulted on the appointment. During his tenure of the chair the Speaker is free from all party ties and he takes no part in the discussions.

Closure and Guillotine

It is convenient here to notice the special procedure known as the "closure" and the "guillotine." The former was introduced in 1882 to combat the obstructive methods of the Irish Nationalist members who, by prolonging the discussion of bills, were deliberately bringing business to a standstill. Under the closure the debate can be cut short at any time if a member rises and proposes that "the question be now put." Unless the Chair decides that the motion is an abuse of the rules of the house or an infringement of the rights of the minority, the question is put without amendment or debate.

The guillotine is a far more drastic means of curtailing debate. By this method certain times are by resolution of the house devoted to certain stages of a bill, and at the end of the time allotted for each stage discussion is closed, whether concluded or not. This procedure is now often made necessary by pressure of work.

When a bill is sent from the commons to the lords, the procedure in the lords is very much the same as in the commons. If the lords accept the bill, it is sent at once to the sovereign for assent; if they propose any amendment, the bill is returned to the commons, where the house may accept the amendments. If it does not, negotiations take place between the party leaders in the two houses until agreement is reached. For all practical purposes the bill is then law, for it is almost inconceivable that the sovereign would to-day refuse assent to a bill which has passed through both houses of parliament.

LESSON 10

The Executive in the British State

PARLIAMENT, apart from the delegated legislation previously considered, is the body by which laws are proposed and made; it is necessary, however, not only to make laws authorising acts to be done, but also to do the acts so authorised; and this is one of the duties carried out by the executive.

In this country the executive consists of the various ministers of the Crown who are at the head of the different government departments. These ministers are in normal times all members of parliament, and in modern practice the chief of them—the prime minister—is a member of the house of commons. Policy is determined by the cabinet, a body of upwards of 20 ministers, selected by the prime minister. In time of war a still smaller group—a war cabinet—may be formed. If the policy of the ministry does not meet with the approval of the houses of parliament, a vote against them will be registered there and, according to constitutional practice, they will be required to tender their resignations to the sovereign.

Another group of ministers may then take office; but if they cannot command a majority in the parliament (to-day one must say, in the house of commons), they must also resign. It is always open to ministers, instead of resigning, to "go to the country," i.e. to hold a general election, and this they will do when they consider that parliament, in rejecting them, is not acting in accordance with the wishes of the electorate. Thus, while the ministers are in theory the queen's (king's) ministers, and do all acts in her name, they are in fact answerable not to her but to the houses of parliament. An executive of this kind is known as a parliamentary executive.

Right of Impeachment

In some countries—notably the United States of America—the executive is "non-parliamentary," that is, it is not answerable to the legislature for its acts, and may continue in office even though its policy conflicts with the views of the legislature. The president of the United States is elected at a special election quite apart from those which appoint the congress, and members of the cabinet cannot be members of either house of congress, nor are they allowed to speak in congress.

A parliamentary executive in England is of comparatively recent growth, for until the Revolution of 1689 the ministers of the crown were not members of the house of commons, and such control as the house enjoyed over them was by the right of impeachment, which entitled parliament to try ministers of whose policy they disapproved. This right was exercised frequently during the 17th century, and Stuart kings on several occasions dissolved parliament in order to save a minister. When the principle became established that the sovereign's ministers must command a majority in the house of commons, the procedure of impeachment fell into disuse.

Cabinet Meetings

Another striking feature of the executive in England is that the sovereign leaves all the acts of government in the hands of the ministers, and all questions of policy are decided upon by the prime minister and his cabinet. At first, even after it had become understood that the executive should enjoy a majority in the house of commons, there was no reason why the sovereign should not meet with ministers and discuss matters of state. That the system of cabinet government ever grew up is said to have been due to the fact that as George I could speak no English and Sir Robert Walpole could speak no German, the king never attended the meetings of his ministers. From his accession the practice of the sovereign's attending cabinet meetings fell into disuse.

This led to the necessity for having some person to preside at cabinet meetings, and the person presiding came in the course of time to be known as the "prime minister." There was no such office known to the law until 1937, when the Ministers of the Crown Act was passed providing (inter alia) for payment to "the person who is prime minister and first lord of the treasury" of an annual salary of £10,000. The act provides for payment of salaries to other ministers, and also of a salary of £2,000 a year to the leader of the opposition. As a result of the system of cabinet government, the powers which the king used formerly to exercise himself "with the advice of his ministers" have been, in effect, transferred to a committee of ministers.

Collective Responsibility

One of the most important principles of the cabinet is what is known as "collective responsibility." The members of the cabinet may not agree on some question of policy, but this disagreement is not disclosed to the public, and the decisions of the cabinet are those of it as a body, for which all members must accept responsibility, even though they may have been opposed to the decision when the question of policy was discussed.

for their proposed acts rather than to rely on their prerogative rights. But one great constitutional writer has pointed out that the Crown could, if it chose, disband the army and navy; sell all battleships and stores; make every citizen, male or female, a peer; dismiss most of the civil servants; and empty all the prisons by conferring free pardons.

Fortunately, it may be thought, it does not choose to do any of these acts; but disputes still do arise, and it is for the courts in the last resort to say whether the Crown is or is not entitled to do any act by virtue of its prerogative. In 1856 the Crown tried unsuccessfully to create peers for life, but was unable to do so until it acquired statutory authority 20 years later. During the First World War the power of the Crown to impose taxes by right of the prerogative was denied. The food controller, being empowered by statute as a servant of the Crown to issue licences to control the supply of milk, made it a condition of the grant of a licence that the licensee should pay twopence for every gallon of milk purchased. The courts held this to be illegal, because it amounted to the imposition of a tax without the consent of parliament, which was beyond the power of the Crown.

Statutory Authority

At the present time, when parliament is so much under the control of the ministers of the Crown, it is easier for the Crown to obtain statutory authority for the acts it wishes to do than to rely upon its prerogative rights, which may be ill-defined. Thus it is that all the principal acts of government are carried on under the authority of an act of parliament, and it is under this authority that the great government departments, such as the treasury and the various ministries, carry out their work. These departments of state are too numerous to be noticed here in detail.

Permanent Officials

Their staffs of permanent officials are civil servants, and these continue to function independently of any changes in the government. At the head of each department is a minister of the Crown, who is a member of one of the houses of parliament, and who holds office until he resigns, is dismissed, or moved to some other appointment, or the ministry as a whole resigns. Whether or not he is in the cabinet will mainly depend on the department of which he is head. These ministers are assisted and, if necessary, represented by other non-permanent officials known by names differing according to the departments to which they belong. The treasury is by far the most important department, for to it belongs the duty of controlling the expenditure of public money.

Of the civil service itself, little can here be said. Its chief characteristic, perhaps, is the principle that its members must take no part in politics. In view of the fact that they must serve under governments of all parties, the reason and good sense of this rule are obvious.

LESSON 12

Freedom of the Press and of the Air

A COUNTRY enjoys freedom of the press when it is not necessary for a person in it to obtain permission from any authority before he publishes anything he wants to publish—that is, where there is no censorship. In a country where there is freedom of the press it does not follow that a person who has published something which infringes the law may not be punished afterwards for having done so. Censorship is arbitrary, depending on the will—it may be mere caprice—of some body or official, whereas punishment after publication can be imposed only when it has been established before a court of law in the ordinary way that the law has been broken.

It is impossible to conceive of a democracy *without* freedom of the press, or to conceive of a dictatorship *with* freedom of the press. It was not without reason that one of Hitler's first steps to complete dictatorship was the abolition of the freedom of the press in Germany in 1933. In a democracy the members of the community would not be able to carry out their functions of government unless the press made them aware of the issues affecting them, and of the activities of the organs of government, and made them able to form their views on those issues and to express their approval or disapproval of the government's activities.

Freedom of the press would be of little value unless there was also in the community a tradition of publicity in the carrying on of the affairs of government. Parliament, the courts, and most local government bodies sit in public, and as it would be impossible for all members of the community to be present, the press attends on their behalf and reports what is of importance. It is this presence of the press that makes the least significant court or local government body in the most remote part of the country conscious that its conduct is all the time under the scrutiny of the whole community. For that reason a judge has said that in every case the court itself is on trial.

Freedom of the press has not been achieved in England without many bitter struggles. At first printing was much feared by the government, and was restricted both by a censorship and by limiting the number of presses. The main fear was the spread of heresy, and an act of 1533 prohibited the importation of books to keep out those containing the doctrines of Luther. Control of the press continued after the Reformation, and in 1585 Elizabeth I ordered that the only presses should be in London, Oxford, and Cambridge, and that all books should be approved before they were printed. In 1644 John Milton wrote his *Areopagitica*, the classic exposition in the English language of the case against censorship.

Government Control

Both Charles I and the parliamentarian government which followed him were opposed to press freedom, and after the Restoration the Licensing Act, 1662, continued censorship and other severe restrictions. This act was passed to remain in force for a number of years only, and had to be renewed. It was renewed several times, but in 1692 it was allowed to lapse, and so, in a rather unspectacular way, the freedom of the press was born. But the government continued to exercise considerable control over the press, partly by subsidising newspapers, partly by imposing a stamp duty which made newspapers dear and so kept the circulation low, and partly by prosecutions for seditious libel. The stamp duty was abolished in 1855, and at once there was a vast increase in the numbers of newspapers and their circulations. Fox's Libel Act, 1792, had given a great safeguard against prosecutions for seditious libel, by providing that whether or not what had been published was libellous should be decided not by the judge but by the jury, juries being then less inclined than judges to hold publications of that nature to be libellous.

The Press Council

Until the 19th century no right of criticising government action was recognized, and as late as 1808, when a newspaper proprietor was prosecuted for attacking flogging in the army, the judge directed the jury that "it was not to be permitted to any man to make the people dissatisfied with the government under which he lives."

Proceedings for seditious libel are now rare. A modern problem arises in connexion with the laws making it an offence to publish obscene books. Opinions may differ very widely as to what is "obscene," and reputable publishers may be deterred from publishing books of high literary merit because of their uncertainty as to whether some court considering the matter might not hold the work to be obscene.

A most interesting modern development was the setting up of a press council in 1953 as a result of the royal commission on the press which reported in 1949. The press council is a private body composed of a number of newspaper editors and representatives of the managements of newspapers and of journalists' trade unions. Its objects include preserving the freedom of the press and maintaining its character in accordance with the highest professional and commercial standards. It investigates complaints against newspapers, and although the press council can impose no penalty it can express its disapproval of the conduct of a newspaper.

The main legal restrictions on the press are the law of libel and the law of contempt of court. Proceedings for criminal libel are now infrequent, but there are many civil libel actions for damages. Proceedings for contempt of court may be brought when something has been published which is calculated to prejudice some pending trial; a newspaper editor was sent to prison for contempt of court in 1949.

Privileged Reports

In order that the press should be able to carry out its function of informing the community of matters of importance, it is entitled to report a large number of matters without fear of a libel action, even though what is published is libellous of someone. This applies to such matters as the proceedings in public of a court, or of parliament, or of a meeting of a local authority. Reports of such proceedings are said to be privileged. The press (like anyone else) is also entitled to comment and express its views on matters of public interest without fear of a libel action so long as the comment is fair. Of recent years certain restrictions have been imposed on the press in reporting certain court proceedings, e.g. those in the divorce court and in juvenile courts.

There have been proposals to prevent press reports of proceedings in magistrates' courts preliminary to trial of an accused person at quarter sessions or assizes, on the ground that such reports are prejudicial to the accused, as the case for the prosecution only and not the defence of the accused is given at these proceedings. There is strong opposition to these proposals, on the ground that the benefits derived from publicity outweigh the disadvantages. The Defamation Act, 1952, provided some protection to the press against having to pay damages for accidental libels.

One form of censorship still exists—that of theatrical performances. Under the Theatres Regulation Act, 1843, every new play must be approved by the lord chamberlain before it is acted, and he has power to forbid the acting of

any play (whether new or old) if he is of the opinion that this is fitting "for the preservation of good manners, decorum, and of the public peace." Prosecutions for departing from the approved script are not infrequent.

Sound and Television

Two immensely important new methods of disseminating news and views are broadcasting by means of sound and television. All wireless transmission and receiving is strictly controlled by the government, no one being entitled to have a set either for transmitting or receiving without holding a licence from the postmaster general. The British Broadcasting Corporation is the sole licensee for sound broadcasting. It is an independent authority set up under royal charter, the members of its board of governors being appointed by the government. The corporation is also licensed to broadcast television. The postmaster general has an extensive control of its activities.

The Television Act, 1954, introduced an entirely new factor—commercial television. It created a new authority, the Independent Television Authority (I.T.A.), whose members are appointed by the postmaster general, and the function of the authority is to provide for ten years from July 1954 additional television broadcasting services. The I.T.A. does not itself in general provide programmes, but broadcasts programmes provided by persons known as programme contractors with whom it makes contracts for that purpose. The programmes may contain advertisements.

Both the I.T.A. and the postmaster general have great powers of controlling the programmes. The act imposes on the I.T.A. the duty of seeing that the programmes comply with a number of requirements, including the following. (1) Nothing must offend against good taste or decency, or be likely to encourage crime, or lead to disorder, or be offensive to public feeling, or contain any offensive representation of, or reference to, any living person. (2) The programmes must maintain a proper balance in their subject matter, and a general high standard of quality. (3) News must be accurate and be presented impartially. (4) Those presenting the programmes (i.e. the programme contractors) must be impartial on matters of political and/or industrial controversy or relating to current public policy.

Advertisements are permitted, but there are restrictions on the total time that can be taken up by advertisements and the occasions when they can be shown, e.g. they cannot be shown except at the beginning or end of any programme or in natural breaks. In addition to these specific restrictions, there is also a complete power of censorship. The I.T.A. can forbid the broadcasting of any matter and can also require that nothing can be broadcast without their prior approval. The I.T.A. can require the programme contractor to supply them in advance with scripts and particulars of all programmes, including advertisements.

The postmaster general also has wide powers of control. He may require any announcement to be broadcast, and he may prohibit the broadcasting of any matter. A power of this nature was exercised in 1955, when the postmaster general forbade either the British Broadcasting Corporation or the I.T.A. to broadcast any discussion on matters coming before parliament within the next 14 days. This became known as the "14-day rule." The rule, although in principle it had been in force for some years, was strongly criticised in the house of commons. The postmaster general may also fix the maximum and minimum time for broadcasting, and the hours.

BOOK LIST

Common Sense in Law, Vinogradoff and Hanbury (Home University Library) ; *Elements of English Law*, Geldart (Home University Library) ; *Parliament*, Ilbert and Carr (Home University Library) ; *English Courts of Law*, Hanbury (Home University Library) ; *English Constitutional History*, Chrimes (Home University Library) ; *Modern Constitutions*, Wheare (Home University Library) ; *Crime and Abnormality*, Binney (Home University Library) ; *Sociology*, Ginsberg (Home University Library) ; *Local Government*, Maud (Home University Library) ; *Juvenile Delinquency and the Law*, Jones (Pelican Books) ; *The Queen's Government*, Jennings (Pelican Books) ; *John Citizen and the Law*, Rubinstein (Pelican Books) ; *Local Government in England and Wales*, Jackson (Pelican Books) ; *The Magistrates' Courts*, Giles (Pelican Books) ; *Freedom under the Law*, Denning (Stevens and Sons) ; *The Inheritance of the Common Law*, O'Sullivan (Stevens and Sons) ; *The Rational Strength of English Law*, Lawson (Stevens and Sons) ; *English Law and the Moral Law*, Goodhart (Stevens and Sons) ; *The Queen's Peace*, Allen (Stevens and Sons) ; *Executive Discretion and Judicial Control*, Hamson (Stevens and Sons) ; *The Proof of Guilt*, Glanville Williams (Stevens and Sons) ; *The Government of Britain*, Harrison (Hutchinson's Universal Library) ; *Our Parliament*, Strathearn Gordon (The Hansard Society) ; *Everybody's Guide to Parliament*, W. J. Brown (George Allen & Unwin) ; *Your Local Authority*, Charles Barratt (Pitman) ; *The English Heritage: Our Laws and Central Government*, Maurice W. Thomas (Thomas Nelson & Sons) ; *The British Constitution*, Ivor Jennings (Cambridge University Press) ; *Report of the Royal Commission on the Press 1947–1949* (H. M. Stationery Office) ; *The Press and the People*, Annual reports of the General Council of the Press (General Council of the Press) ; *The Rule of Law*, Inns of Court Conservative and Unionist Society (Conservative Political Centre) ; *The New Despotism*, Lord Hewart (Ernest Benn) ; *The Passing of Parliament*, Keeton (Ernest Benn) ; *This is the Law Series*, popular handbooks on various branches of the law (Stevens and Sons) ; *The Law in Action*, a series of broadcast talks (Stevens and Sons) ; *The Changing Law*, Denning (Stevens and Sons).

GREEK

Although both are "dead" languages, some knowledge of Greek is but little less valuable than a knowledge of Latin. The English language has borrowed much from Greek, if less widely than from Latin, but both Greek literature and Latin have had great influence on modern thought. In this Course, Greek is so presented that the student can get a good preliminary grasp of the language, sufficient to enable him, with more advanced study, eventually to read the treasures of Greek literature in the original.

Having completed this Course, as well as that on LATIN *in Vol. 2, the student will arrive at a keener appreciation of the Course on* CLASSICAL LITERATURE *in Vol. 1.*

12 LESSONS

LESSON 1

Alphabet, Breathings, Stops, and Pronunciation

Why should one study Greek at all? There are two "practical" reasons. One is that Greek is not really a dead language, but an ancient language which is still very much alive in its own land; so much so that if an ancient Greek could be given a modern Greek newspaper to read he would probably find it easier to understand than *The Daily Telegraph* would be to Chaucer. The second reason is that our own language, like most European languages, has borrowed so many words from Greek that unless one has some knowledge of Greek, one is apt to misunderstand one's own tongue.

But there are reasons of no less importance than these, though they are not usually reckoned as "practical." Greek thought and Greek literature have been an incalculable inspiration and influence in all modern European thought and literature. In the Middle Ages, Greek touched western Europe only through Latin; but with the revived study of it some four and a half centuries ago came the release of Europe from the intellectual fetters which had bound it for a thousand years. In every branch of literature the ancient Greeks left monumental works which have never been surpassed; and the Scripture on which Christianity rests—the New Testament—came to us in Greek, though for centuries it was known only through Latin versions. And still in those works, in their original language, there lives an inspiration which no translations can give. He who can read them for himself has "a possession for ever" which is denied to him who cannot.

It is the aim of this Course to help students through the early stages of teaching themselves enough Greek to be able to study Greek literature on their own account.

The Greek Dialects

The Greek language has different forms which were spoken and written at different times and in different parts of what the Greeks themselves called not Greece but Hellas, so that in works of scholarship the term Hellenic is often used in preference to Greek. But the two principal dialects used by the great writers are the early Ionic of Homer and the later form of Ionic which was developed at Athens and perfected there by the great men of the fifth and fourth centuries B.C., and is called Attic or Classical Greek. When Attic Greek has been mastered, it is easy to learn the variations from its forms which will be found in the works in other dialects; and therefore these pages are confined to the Attic, in which the great bulk of the works which have descended to us were written.

The Alphabet

The Greek alphabet (the word alphabet, as you will see, is derived from the names of the first two Greek letters) has 24 letters. It leaves out the English c, h, j, q, v, w, and y; and puts in some letters for which there are no separate equivalents in English.

Name		Form		English Equivalent
Alpha	*Ἄλφα*	*Α*	*α*	a
Beta	*Βῆτα*	*Β*	*β*	b
Gamma	*Γάμμα*	*Γ*	*γ*	g (*ha*
Delta	*Δέλτα*	*Δ*	*δ*	d
Epsilon	*Ἒψιλόν*	*Ε*	*ε*	e (*sho*
Zeta	*Ζῆτα*	*Ζ*	*ζ*	z, dz
Eta	*Ἦτα*	*Η*	*η*	e (*long*
Theta	*Θῆτα*	*Θ*	*θ*	th
Iota	*Ἰῶτα*	*Ι*	*ι*	i
Kappa	*Κάππα*	*Κ*	*κ*	k, *or hard c*
Lambda	*Λάμβδα*	*Λ*	*λ*	l
Mu	*Μῦ*	*Μ*	*μ*	m
Nu	*Νῦ*	*Ν*	*ν*	n
Xi	*Ξῖ*	*Ξ*	*ξ*	x
Omicron	*Ὂμικρόν*	*Ο*	*ο*	o (*sho*
Pi	*Πῖ*	*Π*	*π*	p
Rho	*Ῥῶ*	*Ρ*	*ρ*	r
Sigma	*Σίγμα*	*Σ*	*σ, ς*	s
Tau	*Ταῦ*	*Τ*	*τ*	t
Upsilon	*Υ̓͂ψιλόν*	*Υ*	*υ*	u *or* y (see note 4)
Phi	*Φῖ*	*Φ*	*φ*	ph, f
Chi	*Χῖ*	*Χ*	*χ*	kh *or hard* ch
Psi	*Ψῖ*	*Ψ*	*ψ*	ps
Omega	*Ὦμέγα*	*Ω*	*ω*	o (*long*)

1. The labial, dental, and guttural sounds are respectively *sibilated* in three of the consonants (ψ, ζ, ξ) and *aspirated* in three others (φ, θ, and χ).

2. The short and the long e and o are represented by different letters; though this, as always in English, is not so with any other vowels. The meaning of the word O-mega is O (great), and of O-micron, O (little).

3. Sigma and theta both have two forms; but in the latter the variation is only a matter of taste, whereas we *always* have σ unless it is the last letter of a word, when it is always ς. The Greek r has the form of the English P, and the English X stands in Greek not for Xi but for Chi.

4. In English words taken from Greek words (e.g. hypocrite, physic, psychic) the *y* stands for υ, and the *c* for κ. Greek φ is always *ph*, not f, and χ becomes *ch*, not kh.

Breathings

Instead of *h*, the Greek has a mark called a "rough breathing," like a comma turned round (ʽ). All words beginning with a vowel have a rough breathing if an *h* is to be sounded, but a smooth breathing, with the comma *not* turned round (ʼ), if there is no *h*. If there is a double vowel or diphthong, the second vowel takes the breathing. Thus ὁδός = hodos, οὖν = oun. Also ρ always has a rough breathing when it begins a word (e.g. ῥόδον = rhodon), while ρρ in the middle of a word is always ῤῥ = rrh. Otherwise no consonants have breathings, aspirated mutes being represented by θ, χ, and φ.

Punc[illegible]ation Marks

Gr[illegible]ek uses the *comma* and the *full-stop*, like Engl[illegible]h. It uses also a *colon*, a single point above the line (·), the equivalent of the English colon and semi-colon. The mark of interrogation in Greek (;) is [illegible]al in form with the English semi-colon.

Ac[illegible]

[illegible]tudent will have noticed that every Greek w[illegible]erto mentioned has another kind of mark o[illegible] ὁδός, βῆτα. This is called an Accent, an[illegible]tended to show the correct intonation. [illegible]ses no such marks. The Greek system of ac[illegible] however, too complicated to be explained in [illegible]urse. Here it is enough to notice that the ac[illegible]ent (´) may be found on any of the last t[illegible]bles of a word, the grave (`) only on the la[illegible] when another word follows immediately, th[illegible]mflex (῀) only over a long vowel or diphth[illegible] only on the last syllable or last but one. [illegible] breathing, the accent on a diphthong is a[illegible]ver the second vowel. And there is a very s[illegible]up of words which have no accent at all.

Pr[illegible]nciation

[illegible] Greek is read aloud or quoted in England, it [illegible] custom to sound all the consonants as sh[illegible]n the preceding alphabet-table; except t[illegible]hen followed by a second γ or other g[illegible] is sounded like the *n* in *hang*, *blanket*, etc. Single vowels are given their most common English sounds—short *a* as in *fat*, long as in *fate*; short *e* (ε) as in *met*, long (η) as in *meet*; short *i* as in *fin*, long as in *fine*; short *o* (ο) as in *not*, long (ω) as in *note*; while υ is pronounced like the *u* in *stupid*. In diphthongs, αι and ει are sounded like the *i* in *fine*, αυ and ευ as in *August* and *Europe*, οι like *oy* in *boy*, ου like *ou* in *round*, υι like *wi*. But in English words taken from the Greek κ becomes c, not k, and follows the ordinary rules for pronouncing an English c; γ is softened as in English (e.g. genealogy) before e, i, or y; and υ becomes an English y. Which is doubtless rather confusing, but is the established practice.

EXERCISE

Write the following Greek words in English letters: ῥοδόδενδρον, διάγνωσις, ἄγκυρα, μεταμόρφωσις, ἄνθος, καταστροφή, παντόμιμος, Ξέρξης, Σωκράτης, ψαλτήριον.

Later Lessons will sometimes contain exercises, the solutions of which (where necessary) are given in the following Lesson. The student should make and keep a separate list of the English words without the Greek, and the Greek words without the English, in each successive exercise, and from time to time test his memory of those words. When he begins to read Greek in other books, he should supply himself with the abridged edition of Liddell and Scott's *Greek-English Lexicon*, and with Abbott and Mansfield's *Greek Grammar*.

LESSON 2

First and Second Declensions

S[illegible] proceeds with nouns and their declensions, taking in this Lesson only the first and second.

There are three declensions of nouns and adjectives, corresponding roughly to the first three declensions in Latin. The first is called the *A* declension, the second the *O* declension. These two are the vowel declensions, as opposed to the third, which contains all other nouns and is sometimes called the consonant declension.

Nouns are declined for number and case; adjectives for number, case, and gender. The numbers are three; singular, dual, and plural. The dual is used to denote two objects, but its use is not frequent, and it is generally replaced by the plural.

The genders are three; masculine, feminine, and neuter. In English inanimate things are almost without exception neuter. In Greek, as in Latin and most other languages, they may be of any gender. Thus οἶκος (*m.*) and οἰκία (*f.*) both mean *house*. But whatever is masculine or feminine in English is generally the same in Greek. Most names of rivers, winds, and months are masculine; of countries, towns, trees, and islands, feminine, as also are most nouns denoting qualities or conditions.

The cases, which number five, are as follows:

	Answers the question:
1. Nominative	Who *or* what?
2. Vocative	(Case of the person addressed.)
3. Accusative	Whom *or* what?
4 Genitive	Of *or* from whom?
5. Dative	To, for, by, with whom *or* what?

NOTES. 1. The nominative and vocative plural are always alike.

2. In neuter nouns, the nominative, vocative, and accusative are alike in all numbers; and in the plural of neuter nouns these three cases always end in α (sounded short).

3. The nominative, vocative, and accusative dual are always alike; and so are the genitive and dative dual.

4. Students of Latin will note that the functions of the Latin ablative are divided between the Greek genitive and dative.

5. Cases other than the nominative are spoken of collectively as "oblique" cases.

First Declension Nouns: A Stems

Masculine nouns of this declension end in ας or ης; and feminine nouns end in α or η. One knows the gender by the termination. There are no neuters.

The masculine nouns are declined like the two following: νεανίας (young man), πολίτης (citizen).

	Singular	
Nom.	νεανίας	πολίτης
Voc.	νεανία	πολῖτα
Acc.	νεανίαν	πολίτην
Gen.	νεανίου	πολίτου
Dat.	νεανίᾳ	πολίτῃ
	Dual	
Nom., Voc., Acc.	νεανία	πολίτα
Gen., Dat.	νεανίαιν	πολίταιν

	Plural	
Nom., Voc.	νεανίαι	πολῖται
Acc.	νεανίας	πολίτας
Gen.	νεανιῶν	πολιτῶν
Dat.	νεανίαις	πολίταις

NOTE. The little ι written under the last letter of the dative singular is called *iota subscript*. The dative originally ended in ι, as νεανίαι, πολίτηι; but the ι is now always written below the other vowel. It is not sounded, but it shows that the other vowel is long.

EXERCISE I

Like one or other of the two nouns given above, decline ταμίας, steward; κριτής, judge; ναύτης, sailor; δεσπότης, master; Πέρσης, a Persian; γεωμέτρης, geometer; and στρατιώτης, soldier.

The feminine nouns of this declension are declined like the four following: ψυχή (soul), οἰκία (house), χώρα (land), θάλασσα (sea).

	Singular			
Nom. Voc.	ψυχή	οἰκία	χώρα	θάλασσα
Acc.	ψυχήν	οἰκίαν	χώραν	θάλασσαν
Gen.	ψυχῆς	οἰκίας	χώρας	θαλάσσης
Dat.	ψυχῇ	οἰκίᾳ	χώρᾳ	θαλάσσῃ
	Dual			
N., V., A.	ψυχά	οἰκία	χώρα	θαλάσσα
G., D.	ψυχαῖν	οἰκίαιν	χώραιν	θαλάσσαιν
	Plural			
N., V.	ψυχαί	οἰκίαι	χῶραι	θάλασσαι
Acc.	ψυχάς	οἰκίας	χώρας	θαλάσσας
Gen.	ψυχῶν	οἰκιῶν	χωρῶν	θαλασσῶν
Dat.	ψυχαῖς	οἰκίαις	χώραις	θαλάσσαις

NOTE. Nouns ending in α preceded by ε, ι, or ρ, retain the α throughout the singular, and are declined like οἰκία or χώρα; in these the terminal α is long. Other nouns ending in α are declined like θάλασσα (with a short α), unless the α is a contracted form of αα, as μνᾶ (mina), contracted from μνάα (gen. μνᾶς, dat. μνᾷ).

Second Declension Nouns: O Stems

The masculine nouns of this declension, and also the feminine, of which there are few, mostly (see below) end in ος; the neuter nouns end in ον.

	Singular	
Nom.	λόγος (word)	δῶρον (gift)
Voc.	λόγε	δῶρον
Acc.	λόγον	δῶρον
Gen.	λόγου	δώρου
Dat.	λόγῳ	δώρῳ
	Dual	
Nom., Voc., Acc.	λόγω	δώρω
Gen., Dat.	λόγοιν	δώροιν
	Plural	
Nom., Voc.	λόγοι	δῶρα
Acc.	λόγους	δῶρα
Gen.	λόγων	δώρων
Dat.	λόγοις	δώροις

There are, however, several nouns of this declension originally ending in οος, εος, οον, εον, which have been contracted into ους and ουν. Thus, νόος, mind (νοῦς); ὀστέον bone (ὀστοῦν)

	Singular	
Nom.	νοῦς	ὀστοῦ
Voc.	νοῦ	ὀστοῦ
Acc.	νοῦν	ὀστο
Gen.	νοῦ	ὀστ
Dat.	νῷ	ὀστ
	Dua.	
Nom., Voc., Acc.	νώ	ὀστ
Gen., Dat.	νοῖν	ὀστ
	Plural	
Nom., Voc.	νοῖ	ὀ
Acc.	νοῦς	ὀσ
Gen.	νῶν	ὀσ
Dat.	νοῖς	ὀσ

A few nouns of this declension end in ως (... line and feminine) and ων (neuter). Thus, νεώς, temple (m.), and ἀνώγεων, upper room (n).

	Singular	
Nom., Voc.	νεώς	ἀνώγεω
Acc.	νεών	ἀνώγεω
Gen.	νεώ	ἀνώγε
Dat.	νεῴ	ἀ
	Dual	
Nom., Voc., Acc.	νεώ	ἀ
Gen., Dat.	νεῴν	ἀ
	Plural	
Nom., Voc.	νεῴ	ἀνώγ
Acc.	νεώς	ἀνώγ
Gen.	νεών	ἀνώ
Dat.	νεῴς	ἀνώγεως

EXERCISE II

Decline ἄνθρωπος, man; νόμος, law; κίνδυνος, danger; πόταμος, river; βίος, life; θάνατος, death; δοῦλος, slave; ἵππος, horse; πόλεμος, war; στρατηγός, general; διδάσκαλος, teacher; ὁδός (f.), road; νῆσος (f.), island; σῦκον, fig; ἱμάτιον, outer garment; ζῷον, animal; τόξον, bow; and δένδρον, tree, which has an irregular dative plural (δένδρεσι); θεός, god (voc. θεός).

Definite Article

Proceed now with the definite article (= the) and those adjectives which follow in the main the declension of the nouns already given. There is no indefinite article in Greek, but there is a definite article, which is declined as follows:

	Singular			Dual			Plural		
	Masc.	Fem.	Neut.	Masc.	Fem.	Neut.	Masc.	Fem.	Neut.
N.	ὁ	ἡ	τό	τώ	(τά)	τώ	οἱ	αἱ	τά
A.	τόν	τήν	τό	,,	,,	,,	τούς	τάς	τά
G.	τοῦ	τῆς	τοῦ	τοῖν	(ταῖν)	τοῖν	τῶν	τῶν	τῶν
D.	τῷ	τῇ	τῷ	,,	,,	,,	τοῖς	ταῖς	τοῖς

NOTE. The feminine dual forms τά and ταῖν (especially τά) are rare; τώ and τοῖν are generally used for all genders.

RULE. The definite article agrees with its noun in number, gender, and case, as: ἡ κόρη, the girl; τὰ ὀστᾶ, the bones; ταῖς οἰκίαις, for the houses. Adjectives and pronouns follow the same rule.

Adjectives: First and Second Declensions

The adjectival endings of the first and second declensions are usually ος, η, ον (masc., fem., neut.). The feminine is declined like a feminine noun of the first declension, and the masculine and neuter like nouns of the second declension. If a vowel or ρ precedes the termination, ος, the feminine ends in α, not in η. Thus, *σοφός, σοφή, σοφόν* (wise); *ἐχθρός, ἐχθρά, ἐχθρόν* (hostile).

Singular

N.	σοφός	σοφή	σοφόν	ἐχθρός	ἐχθρά	ἐχθρόν
V.	σοφέ	σοφή	σοφόν	ἐχθρέ	ἐχθρά	ἐχθρόν
A.	σοφόν	σοφήν	σοφόν	ἐχθρόν	ἐχθράν	ἐχθρόν
G.	σοφοῦ	σοφῆς	σοφοῦ	ἐχθροῦ	ἐχθρᾶς	ἐχθροῦ
D.	σοφῷ	σοφῇ	σοφῷ	ἐχθρῷ	ἐχθρᾷ	ἐχθρῷ

Dual

N. V. A.	σοφώ	σοφά	σοφώ	ἐχθρώ	ἐχθρά	ἐχθρώ
G. D.	σοφοῖν	σοφαῖν	σοφοῖν	ἐχθροῖν	ἐχθραῖν	ἐχθροῖν

Plural

N. V.	σοφοί	σοφαί	σοφά	ἐχθροί	ἐχθραί	ἐχθρά
A.	σοφούς	σοφάς	σοφά	ἐχθρούς	ἐχθράς	ἐχθρά
G.	σοφῶν	σοφῶν	σοφῶν	ἐχθρῶν	ἐχθρῶν	ἐχθρῶν
D.	σοφοῖς	σοφαῖς	σοφοῖς	ἐχθροῖς	ἐχθραῖς	ἐχθροῖς

NOTE. The masculine form of the dual is often used for the feminine, and forms like *σοφά* and *σοφαῖν* are comparatively seldom met with.

Some adjectives in ος, however, have only two endings, ος and ον, the feminine ending in ος like the masculine—e.g. *ἄδικος, ἄδικον*, unjust ; *ἄλογος, ἄλογον*, unreasonable.

Most adjectives ending in εος and οος are contracted thus : *χρύσεος, χρυσέα, χρύσεον*, golden, becomes *χρυσοῦς, χρυσῆ, χρυσοῦν* ; *ἀργύρεος, ἀργυρέα, ἀργυρέον*, silver, becomes *ἀργυροῦς, ἀργυρᾶ, ἀργυροῦν*, and *ἁπλόος, ἁπλόη, ἁπλόον*, simple, becomes *ἁπλοῦς, ἁπλῆ, ἁπλοῦν*.

There are also a few adjectives of the second declension ending in ως (masculine and feminine), ων (neuter), corresponding to the nouns of the Attic declension (see *νεώς*). These two groups of adjectives are declined as follows :

Singular

	Masc.	Fem.	Neuter	Masc., Fem.	Neuter
Nom.	ἁπλοῦς	ἁπλῆ	ἁπλοῦν	ἵλεως	ἵλεων
Acc.	ἁπλοῦν	ἁπλῆν	ἁπλοῦν	ἵλεων	ἵλεων
Gen.	ἁπλοῦ	ἁπλῆς	ἁπλοῦ	ἵλεω	ἵλεω
Dat.	ἁπλῷ	ἁπλῇ	ἁπλῷ	ἵλεῳ	ἵλεῳ

Dual

N., V., A.	ἁπλώ	ἁπλά	ἁπλώ	ἵλεω	ἵλεω
G., D.	ἁπλοῖν	ἁπλαῖν	ἁπλοῖν	ἵλεῳν	ἵλεῳν

Plural

Nom.	ἁπλοῖ	ἁπλαῖ	ἁπλᾶ	ἵλεῳ	ἵλεω
Acc.	ἁπλοῦς	ἁπλᾶς	ἁπλᾶ	ἵλεως	ἵλεω
Gen.	ἁπλῶν	ἁπλῶν	ἁπλῶν	ἵλεων	ἵλεων
Dat.	ἁπλοῖς	ἁπλαῖς	ἁπλοῖς	ἵλεῳς	ἵλεῳς

VOCABULARY. *σοφός*, wise ; *ἐχθρός*, hostile ; *ἄδικος*, unjust ; *ἄλογος*, unreasonable ; *χρυσοῦς*, golden ; *ἀργυροῦς*, silver ; *ἁπλοῦς*, simple ; *ἵλεως*, gracious.

EXERCISE III

Turn the following into Greek, remembering that adjective and article must agree with the noun in number, gender, and case :

1. Of the simple girl. 2. To the hostile soldier. 3. Golden gifts. 4. Of the wise young men. 5. The unjust girl (accusative). 6. To the silver sea. 7. Unreasonable laws. 8. Of the wise mind. 9. Of the golden temple. 10. The simple judges (accusative).

LESSON 3

The Third Declension

THE third declension includes all nouns not belonging to one or other of the two previous declensions. Some of these have vowel stems, but most are consonantal. The stem is generally found by dropping the termination of the genitive singular ος or ως : thus, *λέων, λέοντος*, stem *λεοντ*——.

The cases are formed by adding the endings below to the stem or a modification of it :

	Singular M. & F.	Singular N.	Dual M. & F., N.	Plural M. & F.	Plural N.
N.			ε	ες	α
A.	α or ν	none	ε	ας	α
G.	ος or ως		οιν	ων	
D.	ι		οιν	σι	

NOTE. In the nominative singular and dative plural the terminations ς and σι respectively often become united with the last letter of the stem, as *φύλαξ*, guard, for *φυλακ-ς*; and dative plural *φύλαξι*, for *φυλακ-σι*.

The following are examples of the commonest forms of nouns of this declension with mute or liquid stems :

MASCULINES AND FEMININES

ὁ κόραξ, raven ; *ἡ μάστιξ*, whip ; *ὁ λέων*, lion.

Singular

Stem	κορακ-	μαστιγ-	λεοντ-
Nom.	κόραξ	μάστιξ	λέων
Voc.	κόραξ	μάστιξ	λέον
Acc.	κόρακα	μάστιγα	λέοντα
Gen.	κόρακος	μάστιγος	λέοντος
Dat.	κόρακι	μάστιγι	λέοντι

Dual

N., V., A.	κόρακε	μάστιγε	λέοντε
Gen., Dat.	κοράκοιν	μαστίγοιν	λεόντοιν

Plural

Nom., Voc.	κόρακες	μάστιγες	λέοντες
Acc.	κόρακας	μάστιγας	λέοντας
Gen.	κοράκων	μαστίγων	λεόντων
Dat.	κόραξι	μάστιξι	λέουσι

ὁ παῖς, boy ; ὁ or ἡ ὄρνις, bird ; ὁ γίγας, giant.

Singular

Stem	παιδ-	ὀρνιθ-	γιγαντ-
Nom.	παῖς	ὄρνις	γίγας
Voc.	παῖ	ὄρνι	γίγαν
Acc.	παῖδα	ὄρνιν or ὄρνιθα	γίγαντα
Gen.	παιδός	ὄρνιθος	γίγαντος
Dat.	παιδί	ὄρνιθι	γίγαντι
		Dual	
N., V., A.	παῖδε	ὄρνιθε	γίγαντε
Gen., Dat.	παιδοῖν	ὀρνίθοιν	γιγάντοιν
		Plural	
Nom., Voc.	παῖδες	ὄρνιθες	γίγαντες
Acc.	παῖδας	ὄρνιθας	γίγαντας
Gen.	παίδων	ὀρνίθων	γιγάντων
Dat.	παισί	ὄρνισι	γίγασι

NEUTERS

τὸ σῶμα, body ; τὸ πέρας, end ; τὸ ἧπαρ, liver.

Singular

N., V., A.	σῶμα	πέρας	ἧπαρ
Gen.	σώματος	πέρατος	ἥπατος
Dat.	σώματι	πέρατι	ἥπατι
		Dual	
N., V., A.	σώματε	πέρατε	ἥπατε
Gen., Dat.	σωμάτοιν	περάτοιν	ἡπάτοιν
		Plural	
N., V., A.	σώματα	πέρατα	ἥπατα
Gen.	σωμάτων	περάτων	ἡπάτων
Dat.	σώμασι	πέρασι	ἥπασι

Decline the following nouns :

	Stem	*Dat. pl.*
ἡ φλέψ, vein	φλεβ-	φλεψί
ὁ γύψ, vulture	γυπ-	γυψί
ὁ φύλαξ, guard	φυλακ-	φύλαξι
ὁ κῆρυξ, herald	κηρυκ-	κήρυξι
ἡ πτέρυξ, wing	πτερυγ-	πτέρυξι
ἡ σάλπιγξ, trumpet (pronounced *salpinx*)	σαλπιγγ-	σάλπιγξι
ὁ ὄνυξ, nail, claw	ὀνυχ-	ὄνυξι
ὁ ἄναξ, chief, prince	ἀνακτ-	ἄναξι
ἡ νύξ, night	νυκτ-	νυξί
ὁ γέλως, laughter	γελωτ-	—
ἡ Ἑλλάς, Greece	Ἑλλαδ-	—
ἡ κόρυς, helmet	κορυθ-	κόρυσι
ἡ ἐλπίς, hope (voc. ἐλπί)	ἐλπιδ-	ἐλπίσι
ἡ λαμπάς, torch	λαμπαδ-	λαμπάσι
ἡ πατρίς, native land	πατριδ-	πατρίσι
ὁ ἐλέφας, elephant	ἐλεφαντ-	ἐλέφασι
ὁ γέρων, old man (voc. γέρον)	γεροντ-	γέρουσι
ὁ ὀδούς, tooth	ὀδοντ-	ὀδοῦσι
ὁ ποιμήν, shepherd	ποιμεν-	ποιμέσι
ὁ λιμήν, harbour	λιμεν-	λιμέσι
ὁ Ἕλλην, Greek	Ἑλλην-	Ἕλλησι
ὁ μήν, month	μην-	μησί
ὁ αἰών, age	αἰων-	αἰῶσι
ὁ χειμών, winter, storm	χειμων-	χειμῶσι
ὁ ἡγεμών, leader	ἡγεμον-	ἡγεμόσι
ἡ χιών, snow	χιον-	—
ὁ δαίμων, divinity (voc. δαῖμον)	δαιμον-	δαίμοσι
ὁ δελφίς, dolphin	δελφιν-	δελφῖσι
ὁ ἅλς, salt	ἁλ-	ἁλσί
ὁ θήρ, wild beast	θηρ-	θηρσί
ὁ θής, hired man	θητ-	θησί
ἡ ῥίς, nose (cf. "rhinoceros")	ῥιν-	ῥισί

ὁ ῥήτωρ, orator (voc. ῥῆτορ)	ῥητορ-	ῥήτορσι
τὸ πρᾶγμα, thing	πραγματ-	πρᾶγμα

Many nouns of the third declension end in ης and ος ; those which end in ος are neuter. They are declined in the manner shown below. It should be noted that contraction takes place whenever the ε of the stem precedes a vowel.

ἡ τριήρης, trireme ; τὸ γένος, race, stock.

Singular

Stem	τριηρε-	γενε-
Nom.	τριήρης	γένος
Voc.	τριῆρες	γένος
Acc.	τριήρη (τριήρεα)	γένος
Gen.	τριήρους (-εος)	γένους (-εος)
Dat.	τριήρει (-εϊ)	γένει (-εϊ)
	Dual	
N., V., A.	τριήρη	γένη
Gen., Dat.	τριήροιν	γενοῖν
	Plural	
Nom., Voc.	τριήρεις (-εες)	γένη (γένεα)
Acc.	τριήρεις (-εας)	γένη
Gen.	τριήρων	γενῶν or γενέων
Dat.	τριήρεσι	γένεσι

Like τριήρης decline Σωκράτης, Socrates, Δημοσθένης, Demosthenes (in the singular). Like γένος decline τὸ κάλλος, beauty ; τὸ mountain ; and τὸ θέρος, summer.

Nouns ending in ις, ι, υ, υς, and ευς are declined ; ι and υ are neuter endings ;

ἡ πόλις, city ; τὸ ἄστυ, town ;
ὁ πῆχυς, fore-arm, cubit ; ὁ βασιλεύς, king.

Singular

Stem	πολε-	ἀστε-	πηχε-	βασιλε-
N.	πόλις	ἄστυ	πῆχυς	βασιλεύς
V.	πόλι	ἄστυ	πῆχυ	βασιλεῦ
A.	πόλιν	ἄστυ	πῆχυν	βασιλέα
G.	πόλεως	ἄστεος or -εως	πήχεως	βασιλέως
D.	πόλει	ἄστει	πήχει	βασιλεῖ
		Dual		
N., V., A.	πόλεε	ἄστεε	πήχεε	βασιλέε
G., D.	πολέοιν	ἀστέοιν	πηχέοιν	βασιλέοιν
		Plural		
N., V.	πόλεις (-εες)	ἄστη (-εα)	πήχεις	βασιλεῖς
A.	πόλεις (-εας)	ἄστη (-εα)	πήχεις	βασιλέας
G.	πόλεων	ἀστέων	πήχεων	βασιλέων
D.	πόλεσι	ἄστεσι	πήχεσι	βασιλεῦσι

NOTE. Nouns ending in ι (as τὸ σίναπι, mustard) are declined like ἄστυ. Most nouns in υς retain υ and are regular e.g. ἰχθύς, fish ; *gen.*, ἰχθύος; *dat.*, ἰχθύι ; *nom. pl.*, ἰχθύες ; *dat. pl.* ἰχθύσι.

Nouns ending in ους and αυς are thus declined : ὁ, ἡ βοῦς, ox, cow ; ἡ γραῦς, old woman ; ἡ ναῦς, ship.

Singular

Stem	βο-	γρα-	να-, νε-
Nom.	βοῦς	γραῦς	ναῦς
Voc.	βοῦ	γραῦ	ναῦ
Acc.	βοῦν	γραῦν	ναῦν
Gen.	βοός	γραός	νεώς
Dat.	βοΐ	γραΐ	νηΐ
		Dual	
N., V., A.	βόε	γρᾶε	νῆε
Gen., Dat.	βοοῖν	γραοῖν	νεοῖν

	Plural		
N., V.	βόες	γρᾶες	νῆες
Acc.	βοῦς	γραῦς	ναῦς
Gen.	βοῶν	γραῶν	νεῶν
Dat.	βουσί	γραυσί	ναυσί

NOTE. ναῦς is very irregular, and has also acc. sing., νῆα or νέα ; gen., νηός or νεός ; etc.

Nouns ending in ας, genitive αος (neuter), are declined in the following manner, contraction taking place when the α of the stem is followed by a vowel :

	Singular	Dual	Plural
Nom. Voc. Acc.	γέρας, prize	(γέρα)	γέρα
Gen.	γέρως (-αος)	(γερῷν)	γερῶν
Dat.	γέρᾳ	(γερῷν)	γέρασι

κέρας, κέρατος, horn, is sometimes declined regularly (κέρας, κερατος, κεράτι, etc.), sometimes like γέρας (κέρως, κέρᾳ, etc.).

There are a few nouns in ω and ως, declined as follows (the dual and plural are rare) :

Singular		Dual		Plural	
N.	ἠχώ, echo	N. V. A.	ἠχώ	N. V.	ἠχοί
V.	ἠχοῖ				
A.	ἠχώ			A.	ἠχούς
G.	ἠχοῦς	G. D.	ἠχοῖν	G.	ἠχῶν
D.	ἠχοῖ			D.	ἠχοῖς

ἡ αἰδώς, shame ; accusative, αἰδῶ ; genitive, αἰδοῦς ; dative, αἰδοῖ. This has neither dual nor plural.

ὁ ἥρως, hero, has genitive ἥρωος, and is regular (accusative, ἥρωα ; dative, ἥρωϊ, etc.).

Some nouns in ηρ, as ὁ πατήρ, father ; ἡ μήτηρ, mother ; ἡ θυγάτηρ, daughter (stem πατερ-, etc.), are shortened by dropping the ε before the ρ in the genitive and dative singular ; also in the dative plural they change ερ into ρα before σι. Thus :

Singular		Dual		Plural	
N.	πατήρ	N. V. A.	πατέρε	N. V.	πατέρες
V.	πάτερ				
A.	πατέρα			A.	πατέρας
G.	πατρός	G. D.	πατέροιν	G.	πατέρων
D.	πατρί			D.	πατράσι

Irregular Nouns

There are a few nouns in common use the declension of which is irregular ; that is, the oblique cases are formed on a stem which is not what we should expect from the nominative. The following list comprises some of these most frequently used nouns :

1. ἀνήρ (ὁ) man ; *st.* ἀνδρ- ; *voc.* ἄνερ, *acc.* ἄνδρα, *gen.* ἀνδρός ; *dat. pl.* ἀνδράσι.
2. γάλα (τό), milk ; *st.* γαλακτ- ; *gen.* γάλακτος, etc.
3. γόνυ (τό), knee ; *st.* γονατ- ; *gen.* γόνατος, etc.
4. γυνή (ἡ), wife, woman ; *st.* γυναικ- ; *voc.* γύναι, *acc.* γυναῖκα, *gen.* γυναικός ; *dat. pl.* γυναιξί.
5. δόρυ (τό), spear ; *st.* δορατ- ; δόρατος, δόρατι or δορί ; *pl.* δόρατα, δοράτων, δόρασι.
6. Ζεύς, Zeus, Jupiter ; *st.* Δι- ; *voc.* Ζεῦ ; but *acc.* Δία, *gen.* Διός, *dat.* Διί.
7. θρίξ (ἡ), hair ; *st.* τριχ- ; *gen.* τριχός ; but *dat. pl.* θριξί.
8. κύων (ὁ, ἡ), dog ; *st.* κυν- ; *gen.* κυνός ; *dat. pl.* κυσί.
9. μάρτυς (ὁ, ἡ), witness ; *st.* μαρτυρ- ; *gen.* μάρτυρος ; *dat. pl.* μάρτυσι.
10. οὖς (τό), ear ; *st.* ὠτ- ; ὠτός, ὠτί ; *pl.* ὦτα, ὤτων, ὠσί.
11. πῦρ (τό), fire; πυρός, πυρί ; *pl.* πυρά (watch-fires), πυρῶν, πυροῖς.
12. ὕδωρ (τό), water ; *st.* ὑδατ- ; *gen.* ὕδατος ; *dat. pl.* ὕδασι.
13. υἱός (ὁ), son, is declined both regularly like λόγος, and as a noun of the third declension, υἱέος, υἱεῖ ; *pl.* υἱεῖς, υἱεῖς, υἱέων, υἱέσι.
14. χείρ (ἡ), hand ; regular, exc. *dat. pl.* χερσί.

Third Declension Adjectives

Adjectives of the third declension have usually two endings, one for masculine and feminine, the other for neuter. Most of these end in ης (neuter ες) and ων (neuter ον), as ἀληθής, true ; ψευδής, false ; πλήρης, full ; εὐδαίμων, happy ; πέπων, ripe ; σώφρων, prudent. They are thus declined :

	Singular			Singular	
	M. & F.	Neuter		M. & F.	Neuter
N.	ἀληθής	ἀληθές	N.	εὐδαίμων	εὔδαιμον
V.	ἀληθές		V.	εὔδαιμον	
A.	ἀληθῆ	ἀληθές	A.	εὐδαίμονα	εὔδαιμον
G.	ἀληθοῦς (for ἀληθέος)		G.	εὐδαίμονος	
D.	ἀληθεῖ (for ἀληθέϊ)		D.	εὐδαίμονι	
	Dual			Dual	
N., V., A.	ἀληθῆ		N., V., A.	εὐδαίμονε	
G., D.	ἀληθοῖν		G., D.	εὐδαιμόνοιν	
	Plural			Plural	
N. V.	ἀληθεῖς (έες)	ἀληθῆ (έα)	N. V.	εὐδαίμονες	εὐδαίμονα
A.	ἀληθεῖς (έας)	ἀληθῆ	A.	εὐδαίμονας	εὐδαίμονα
G.	ἀληθῶν		G.	εὐδαιμόνων	
D.	ἀληθέσι		D.	εὐδαίμοσι	

Some adjectives of the third declension, however, have only one termination for the nominative singular of all genders, as : πένης, genitive πένητος, poor ; ἄπαις, ἄπαιδος, childless ; ἀγνώς, ἀγνῶτος, unknown ; ἅρπαξ, ἅρπαγος, rapacious ; φυγάς, φυγάδος, fugitive.

Adjectives Combining the First and Third Declensions

These have, besides the masculine and neuter, third declension forms, a feminine form which follows the first declension. Most of these end either in υς, εια, υ, or in εις, εσσα, εν—as, ταχύς, ταχεῖα, ταχύ, swift ; χαρίεις, χαρίεσσα, χαρίεν, graceful.

	Singular		
	Masculine	Feminine	Neuter
Nom.	ταχύς	ταχεῖα	ταχύ
Voc.	ταχύ	ταχεῖα	ταχύ
Acc.	ταχύν	ταχεῖαν	ταχύ
Gen.	ταχέος	ταχείας	ταχέος
Dat.	ταχεῖ	ταχείᾳ	ταχεῖ

	Masculine	Feminine	Neuter
		Dual	
N., V., A.	ταχέε	ταχεία	ταχέε
Gen., Dat.	ταχέοιν	ταχείαιν	ταχέοιν
		Plural	
N., V.	ταχεῖς	ταχεῖαι	ταχέα
Acc.	ταχεῖς	ταχείας	ταχέα
Gen.	ταχέων	ταχειῶν	ταχέων
Dat.	ταχέσι	ταχείαις	ταχέσι

		Singular	
Nom.	χαρίεις	χαρίεσσα	χαρίεν
Voc.	χαρίεν	χαρίεσσα	χαρίεν
Acc.	χαρίεντα	χαρίεσσαν	χαρίεν
Gen.	χαρίεντος	χαριέσσης	χαρίεντος
Dat.	χαρίεντι	χαριέσσῃ	χαρίεντι
		Dual	
N., V., A.	χαρίεντε	χαριέσσα	χαρίεντε
Gen., Dat.	χαριέντοιν	χαριέσσαιν	χαριέντοιν
		Plural	
N., V.	χαρίεντες	χαρίεσσαι	χαρίεντα
Acc.	χαρίεντας	χαριέσσας	χαρίεντα
Gen.	χαριέντων	χαριεσσῶν	χαριέντων
Dat.	χαρίεσι	χαριέσσαις	χαρίεσι

NOTE. The feminine of ταχύς is declined like οἰκία, and the feminine of χαρίεις like θάλασσα.

There are three adjectives ending in ας—πᾶς, πᾶσα, πᾶν, all; μέλας, μέλαινα, μέλαν, black; and τάλας, τάλαινα, τάλαν, wretched. They are thus declined, τάλας being like μέλας:

	Masculine	Feminine	Neuter
		Singular	
Nom.	πᾶς	πᾶσα	πᾶν
Acc.	πάντα	πᾶσαν	πᾶν
Gen.	παντός	πάσης	παντός
Dat.	παντί	πάσῃ	παντί
		Plural	
Nom.	πάντες	πᾶσαι	πάντα
Acc.	πάντας	πάσας	πάντα
Gen.	πάντων	πασῶν	πάντων
Dat.	πᾶσι	πάσαις	πᾶσι
		Singular	
Nom.	μέλας	μέλαινα	μέλαν
Voc.	μέλαν	μέλαινα	μέλαν
Acc.	μέλανα	μέλαιναν	μέλαν
Gen.	μέλανος	μελαίνης	μέλανος
Dat.	μέλανι	μελαίνῃ	μέλανι
		Dual	
N., V., A.	μέλανε	μελαίνα	μέλανε
Gen., Dat.	μελάνοιν	μελαίναιν	μελάνοιν
		Plural	
N., V.	μέλανες	μέλαιναι	[illegible]
Acc.	μέλανας	μελαίνας	[illegible]
Gen.	μελάνων	μελαινῶν	[illegible]
Dat.	μέλασι	μελαίναις	[illegible]

Two Irregular Adjectives

μέγας, great, and πολύς, much, are declined irregularly:

	Masculine	Feminine	Neuter
		Singular	
Nom.	μέγας	μεγάλη	μέγα
Voc.	(μεγάλε)	μεγάλη	μέγα
Acc.	μέγαν	μεγάλην	μέγα
Gen.	μεγάλου	μεγάλης	μεγάλου
Dat.	μεγάλῳ	μεγάλῃ	[illegible]
		Dual	
N., V., A.	μεγάλω	μεγάλα	[illegible]
Gen., Dat.	μεγάλοιν	μεγάλαιν	μεγάλοιν
		Plural	
N., V.	μεγάλοι	μεγάλαι	[illegible]
Acc.	μεγάλους	μεγάλας	[illegible]
Gen.	μεγάλων	μεγάλων	[illegible]
Dat.	μεγάλοις	μεγάλαις	μεγάλοις
		Singular	
Nom	πολύς	πολλή	πολύ
Acc.	πολύν	πολλήν	πολύ
Gen.	πολλοῦ	πολλῆς	πολλοῦ
Dat.	πολλῷ	πολλῇ	πολλῷ
		Plural	
N., V.	πολλοί	πολλαί	πολλά
Acc.	πολλούς	πολλάς	πολλά
Gen.	πολλῶν	πολλῶν	πολλῶν
Dat.	πολλοῖς	πολλαῖς	πολλοῖς

Key to Exercise III in Lesson 2

1. τῆς ἁπλῆς κόρης. 2. τῷ ἐχθρῷ στρατιώτῃ. 3. δῶρα χρυσᾶ. 4. τῶν σόφων νεανιῶν. 5. τὴν ἄδικον κόρην. 6. τῇ ἀργυρᾷ θαλασσῃ. 7. νόμοι ἄλογοι. 8. τοῦ νοῦ σόφου. 9. τοῦ χρυσοῦ νεῶ. 10. τοὺς ἁπλοῦς κριτάς.

LESSON 4

Pronouns and Their Inflexions

THE next subject is the pronouns, here considered in detail. As will be seen, the declension of the pronouns of the first and second persons is highly irregular.

Personal Pronouns

The pronoun of the first person is ἐγώ, I; and of the second person σύ, thou; in the plural, ἡμεῖς and ὑμεῖς.

	Singular	
Nom.	ἐγώ	σύ
Acc.	ἐμέ, μέ	σέ
Gen.	ἐμοῦ, μοῦ	σοῦ
Dat.	ἐμοί, μοί	σοί
	Dual	
Nom., Acc.	νώ, we (us) two	σφώ, you two
Gen., Dat.	νῷν	σφῷν
	Plural	
Nom.	ἡμεῖς, we	ὑμεῖς, ye *or* you
Acc.	ἡμᾶς	ὑμᾶς
Gen.	ἡμῶν	ὑμῶν
Dat.	ἡμῖν	ὑμῖν

The nominative of the personal pronouns is seldom used, except for emphasis, e.g. I save is σώζω, not ἐγὼ σώζω. The forms ἐμέ, ἐμοῦ, ἐμοί are more emphatic than μέ, μοῦ, μοί.

The third personal pronoun, in all the oblique cases, but *not* in the nominative, is represented by

αὐτόν, -ή, -ό. When the nominative is required for emphasis, we must fall back on the demonstrative *ἐκεῖνος* that (man), *οὗτος* or *ὅδε*, this (man).

Other Uses of *αὐτός*. On the other hand, in all its cases, including the nominative, *αὐτός* is used for *self* (*ὁ ἄνηρ αὐτός* = the man himself). But when *αὐτός* has the article (*ὁ αὐτός, ἡ αὐτή τὸ αὐτό*, generally contracted as *αὑτός, αὑτή, ταυτό*) it means *same* : *αὐτὸς ἄνηρ, τὸν αὐτὸν ἄνδρα* = the same man. *αὐτός* is thus declined :

	Masc.	Fem.	Neuter
		Singular	
Nom.	*αὐτός*	*αὐτή*	*αὐτό*
Acc.	*αὐτόν*	*αὐτήν*	*αὐτό*
Gen.	*αὐτοῦ*	*αὐτῆς*	*αὐτοῦ*
Dat.	*αὐτῷ*	*αὐτῇ*	*αὐτῷ*
		Dual	
Nom. Acc.	*αὐτώ*	*αὐτά*	*αὐτώ*
Gen., Dat.	*αὐτοῖν*	*αὐταῖν*	*αὐτοῖν*
		Plural	
Nom.	*αὐτοί*	*αὐταί*	*αὐτά*
Acc.	*αὐτούς*	*αὐτάς*	*αὐτά*
Gen.	*αὐτῶν*	*αὐτῶν*	*αὐτῶν*
Dat.	*αὐτοῖς*	*αὐταῖς*	*αὐτοῖς*

Reflexive Pronouns

The reflexive pronouns have, of course, no nominative. They are three in number, one for each person : *ἐμαυτόν* (*ἐμέ αὐτόν*), myself ; *σεαυτόν* (*σέ αυτόν*), thyself ; *ἑαυτόν*, usually contracted as *αὑτόν*, himself. They are thus declined :

1st Person—

	Singular Masc.	Fem.	*Plural* Masc.	Fem.
A.	*ἐμαυτόν*	*ἐμαυτήν*	*ἡμᾶς αὐτούς*	*ἡμᾶς αὐτάς*
G.	*ἐμαυτοῦ*	*ἐμαυτῆς*	*ἡμῶν αὐτῶν*	*ἡμῶν αὐτῶν*
D.	*ἐμαυτῷ*	*ἐμαυτῇ*	*ἡμῖν αὐτοῖς*	*ἡμῖν αὐταῖς*

Note that in the plural the two words (the personal pronoun and *αὐτός*) appear separately. *σεαυτόν* is usually contracted into *σαυτόν*, thus:

2nd Person—

	Singular Masc.	Fem.	*Plural* Masc.	Fem.
A.	*σαυτόν*	*σαυτήν*	*ὑμᾶς αὐτούς*	*ὑμᾶς αὐτάς*
G.	*σαυτοῦ*	*σαυτῆς*	*ὑμῶν αὐτῶν*	*ὑμῶν αὐτῶν*
D.	*σαυτῷ*	*σαυτῇ*	*ὑμῖν αὐτοῖς*	*ὑμῖν αὐταῖς*

3rd Person. *ἑαυτόν*, or *αὑτόν* (note the rough breathing), is declined like *αὐτός*. It must, however, be carefully distinguished from the oblique cases of *αὐτός*, which has the smooth breathing.

In addition to these three reflexive pronouns there is one which is sometimes called personal, but seems more properly to come under reflexive, as it is generally used as an indirect reflexive. It has no nominative singular, but the accusative is *ἕ*. (This compounded with *αὐτόν* gives *ἑαυτόν* above.)

Singular		*Plural*	
		Nom.	*σφεῖς*
Acc.	*ἕ*	Acc.	*σφᾶς*
Gen.	*οὗ*	Gen.	*σφῶν*
Dat.	*οἷ*	Dat.	*σφίσι*

Example : *καθορῶσι τὰς τῶν Κερκυραίων ναῦς ἐπὶ σφᾶς πλεούσας*, They see the ships of the Corcyreans sailing against *them* (i.e. themselves ; *ἐπὶ αὐτούς*, or *ἐφ' αὐτούς*, would have referred to "Corcyreans").

Reciprocal Pronoun

The reciprocal pronoun is *ἀλλήλω* (accusative), one another ; it has no nominative and is used only in dual and plural, in which numbers it is declined regularly, like *σοφός*.

Possessive Pronouns

These are declined like adjectives in *ος*. They are :

ἐμός, ἐμή, ἐμόν, my	*ὑμέτερος, -α, -ον*, your
σός, σή, σόν, thy	*σφέτερος, -α, -ον*, their
ἡμέτερος, -α, -ον, our	(always reflexive)

For *his*, the genitive of *αὐτός* is used, as *ὁ πατήρ αὐτοῦ*, his father (lit., the father of him).

The article is regularly used with these possessive pronouns, as *ὁ ὑμέτερος ταμίας*, your steward.

It is quite as good Greek to say *ὁ πατήρ μου* (my father) as to say *ὁ ἐμὸς πατήρ*, and so with all the others.

Demonstrative Pronouns

ἐκεῖνος, that, is declined quite regularly like *σοφός*, except that the neuter nominative and accusative singular are *ἐκεῖνο*, not *ἐκεῖνον*.

ὅδε, this, is simply the article *ὁ* with the particle *δε* added ; it is therefore declined like the article : *ὅδε, ἥδε, τόδε* ; genitive : *τοῦδε, τῆσδε, τοῦδε, κ. τ. λ.* (*κ. τ. λ.* stands for *καὶ τὰ λοιπά*, and the remaining things, or et cetera.)

οὗτος, this, is declined as follows :

		Singular	
Nom.	*οὗτος*	*αὕτη*	*τοῦτο*
Acc.	*τοῦτον*	*ταύτην*	*τοῦτο*
Gen.	*τούτου*	*ταύτης*	*τούτου*
Dat.	*τούτῳ*	*ταύτῃ*	*τούτῳ*
		Dual	
Nom., Acc.	*τούτω*	*τούτω*	*τούτω*
Gen., Dat.	*τούτοιν*	*τούτοιν*	*τούτοιν*
		Plural	
Nom.	*οὗτοι*	*αὗται*	*ταῦτα*
Acc.	*τούτους*	*ταύτας*	*ταῦτα*
Gen.	*τούτων*	*τούτων*	*τούτων*
Dat.	*τούτοις*	*ταύταις*	*τούτοις*

When these demonstrative pronouns are used as adjectives with a noun, the noun takes the article—as, *οὗτος ὁ ἀνήρ*, this man ; *ἐκείνη ἡ μήτηρ*, that mother.

Interrogative and Indefinite Pronouns

The interrogative pronoun is *τίς, τί*, who ? what ? The indefinite pronoun is *τις, τι*, anyone, anything, or someone, something. The only distinguishing mark between them is the accent : the former always takes the accent (on the first syllable), while the latter usually has no accent at all : when there is one, it falls on the second syllable. They are thus declined :

	Interrogative M. & F.	Neuter	Indefinite M. & F.	Neuter
		Singular		
Nom.	*τίς*	*τί*	*τις*	*τι*
Acc.	*τίνα*	*τι*	*τινά*	*τι*
Gen.	*τίνος, τοῦ*		*τινός, του*	
Dat.	*τίνι, τῷ*		*τινί, τῳ*	
		Dual		
Nom., Acc.	*τίνε*		*τινέ*	
Gen. Dat.	*τίνοιν*		*τινοῖν*	

Plural

	M. & F.	Neuter	M. & F.	Neuter
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]		[illegible]
Dat.		[illegible]		[illegible]

[illegible] (interrogative) can mean "why ?" as well as "what ?"

[illegible]

Relative Pronouns

[illegible]

	Singular			*Dual*			*Plural*		
N.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
A.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
G.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
D.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible] whoever

Singular

Nom.	[illegible]	[illegible]	[illegible]
Acc.	[illegible]	[illegible]	[illegible]
Gen.	[illegible]	[illegible]	[illegible]
Dat.	[illegible]	[illegible]	[illegible]

Dual

Nom., Acc.	[illegible]	[illegible]	[illegible]
Gen., Dat.	[illegible]	[illegible]	[illegible]

Plural

Nom.	[illegible]	[illegible]	[illegible]
Acc.	[illegible]	[illegible]	[illegible]
Gen.	[illegible]	[illegible]	[illegible]
Dat.	[illegible]	[illegible]	[illegible]

[illegible] that ὅ τι is written as two words to distinguish it [illegible]

[illegible] of quantity or number, and of quality [illegible]

LESSON 5

Degrees of Comparison, Adverbs, Numerals

T[illegible]

Usual Comparisons

[illegible]

Less Usual Comparisons

[illegible]

	Masc. and Fem.	Neuter
N. V.	[illegible] or [illegible]	[illegible] or [illegible]
	[illegible]	[illegible]
D.	[illegible]	[illegible]

Irregular Comparison

The following adjectives are irregular:

	Comparative	Superlative
[illegible] good	[illegible]	[illegible]
	[illegible]	[illegible]
	[illegible]	[illegible]
	[illegible]	—
[illegible] painful	[illegible]	[illegible]
[illegible] robbing	[illegible]	[illegible]
[illegible] old	[illegible]	[illegible]
[illegible] bad	[illegible], worse	[illegible]
	[illegible]	[illegible]
	[illegible]	([illegible])
	(in less)	
[illegible] beautiful	[illegible]	[illegible]
[illegible] happy	[illegible]	[illegible]
[illegible] long	[illegible]	[illegible]
[illegible] great	[illegible]	[illegible]
[illegible] small	[illegible]	[illegible]
	[illegible], less	[illegible]
	[illegible]	([illegible])
	[illegible]	[illegible]
[illegible] little, few	—	[illegible]
[illegible] old	[illegible]	[illegible]
[illegible] poor	[illegible]	[illegible]
[illegible] ripe	[illegible]	[illegible]
[illegible] much	[illegible] / [illegible]	[illegible]
[illegible] easy	[illegible]	[illegible]
[illegible] dear	[illegible]	[illegible]
	[illegible]	[illegible]
	[illegible]	[illegible]
[illegible] false	—	[illegible]

NOTES. Comparatives and superlatives [illegible]

[illegible]

Adverbs

[illegible]

The Comparison of Adverbs

[illegible]

[illegible]

Numerals

[illegible]

	Cardinals	Ordinals	Adverbs
1	[illegible]	[illegible]	[illegible]
2	[illegible], two	[illegible]	[illegible] twice
3	[illegible]	[illegible]	[illegible], thrice
4	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]
11	[illegible]	[illegible]	[illegible]
12	[illegible]	[illegible]	[illegible]
13	[illegible]	[illegible]	—
14	[illegible]	[illegible]	—
15	[illegible]	[illegible]	—
16	[illegible]	[illegible]	—
17	[illegible]	[illegible]	—
18	[illegible]	[illegible]	
19	[illegible]	[illegible]	
20	[illegible]	[illegible]	[illegible]
21	[illegible] or [illegible]	[illegible]	
25	[illegible] or [illegible]	[illegible]	—
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
50	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

The first four cardinals are thus declined:

	M.	F.	N.		
N.	[illegible]	[illegible]	[illegible]	N. A.	[illegible]
A.	[illegible]	[illegible]	[illegible]	G. D.	[illegible]
G.	[illegible]	[illegible]	[illegible]		
D.	[illegible]	[illegible]	[illegible]		

	M. & F.	N.	M. & F.	N.
N.	[illegible]	[illegible]	[illegible]	[illegible]
A.		[illegible]	[illegible]	[illegible]
G.	[illegible]		[illegible]	
D.	[illegible]		[illegible]	

[illegible]

LESSON 6

Introduction to the Verb

The student now comes to the study of the Greek verb, which is more complicated than that of most European languages, because it has more voices, more moods and more tenses than any of them, besides having a dual as well as a singular and plural. It will readily be understood, therefore, that Greek marks for us, in the various forms many more shades of meaning even than Latin.

In speaking of Greek, as of Latin, verbs, the custom is to name them not by the infinitive, but by the 1st person singular indicative active : *εἰμί*, I am; *λύω*, I loose, instead of *εἶναι*, to be ; *λύειν*, to loose.

Voices

There are two voices, active (I loose, etc.) and passive (I am loosed). Greek adds a middle or reflexive voice (I loose myself, *or* I loose for myself). The middle, however, is distinguished in form from the passive only in two tenses, the future and aorist.

Moods

Greek has five moods—indicative, subjunctive, optative, imperative, infinitive, with forms differing from each other. Latin has only four, combining the optative (I would) with the subjunctive (I may).

Tenses

Greek has seven tenses : one for the present ; one for the future ; five for the past, expressing different shades of meaning : these are the imperfect (I was loosing), the perfect (I have loosed), the aorist, which means " indefinite " (I loosed), the pluperfect (I had loosed), and the future perfect (I was about to loose). Latin has no separate aorist, including it in the perfect. English has only a present and a past tense, the shades of the other tenses being expressed (even in the future, I shall loose) by using an auxiliary verb. Further, Greek forms the aorist in two different ways, called respectively the first or weak aorist and the second or strong aorist ; sometimes also there is a second or strong perfect. But, in either case, very few verbs have both a strong and a weak form, and in neither is there any difference of meaning.

The numbers are three—singular, dual, and plural ; and the persons are three, except in the imperative, where there are only second and third persons. The first person dual is the same as the first person plural, and being very rarely used is usually omitted in the schemes or paradigms of verbs.

Notes. 1. The principal parts of a Greek verb are the first person singular, present, future, aorist, and perfect indicative active ; the perfect and aorist indicative passive ; and the second aorist (when it occurs). From these parts can be formed all the other tenses of the verb, e.g. the principal parts of [illegible] (active), λέλυμαι and ἐλύθην (passive).

2. There are two principal forms of conjugation of Greek verbs, that of verbs ending in ω (as λύω) and that of verbs ending in μι, as δίδωμι, I give: τίθημι, I put; ἵστημι, I set. All verbs have eith the ω or the μι ending, but there are many mo verbs in ω than in μι.

3. Conveniently, but not quite accurately, on may say that the future is formed by adding to the stem ; the weak aorist by adding σα, besid s prefixing what is called the augment (see below The perfect adds κα, and, where the word begi with a consonant, prefixes a reduplication (λ perfect λε-λύκα). Observe that where the v begins with θ, φ, or χ, the reduplication is, resp tively, τεθ-, πεφ-, or κεχ- ; as, θνήσκω, τέθνη φιλέω, πεφίληκα; χαίρω, κεχάρηκα. The imperfect and pluperfect also prefix the augment (that is, initial ε), which, when the word begins with a vo is contracted with it (as, λύω, imperfect ἔ ἔχω, I have, imperfect εἶχον; ἄγω, I lead, imperfe ἦγον). The infinitives and participles of th imperfect and pluperfect are respectively the sa as those of the present and perfect.

4. Deponent is the name given to a numbe verbs which have no active voice ; as, βούλο I wish; γίγνομαι, I become ; ἔρχομαι, I come.

5. There are also verbs which for some tens especially the strong aorist—use the tense taken from a kindred verb which has otherwise beco obsolete, or even of an entirely different verb whi has the same meaning. Thus the aorist of τυγχάν (happen) is ἔτυχον, while for ἔρχομαι the aorist is ἦλθον

Conjugation of Verbs

One can now begin the necessary study of th conjugation of Greek verbs with λύω, a regula and typical example of the verbs in ω, taking first only the tenses as they shape in the indicative mood, infinitive and participles, in the active voice. It will be noted that, throughout, the two persons of the dual (which rarely has a first person) are placed between the three persons of the singular and the three persons of the plural.

λύω, I loose

Tenses of the Indicative Active

	Present	Imperfect
1. Sing.	λύω, I loose	ἔλυον, I was loosing,
2. „	λύεις	ἔλυες or
3. „	λύει	ἔλυε(ν) used to loose
2. Dual	λύετον	ἐλύετον
3. „	λύετον	ἐλυέτην
1. Plur.	λύομεν	ἐλύομεν
2. „	λύετε	ἐλύετε
3. „	λύουσι(ν)	ἔλυον

	Perfect	Pluperfect
1. Sing.	λέλυκα, I have	ἐλελύκειν, I had
2. „	λέλυκας loosed	ἐλελύκεις loosed
3. „	λέλυκε(ν)	ἐλελύκει
2. Dual	λελύκατον	ἐλελύκειτον
3. „	λελύκατον	ἐλελυκείτην
1. Plur.	λελύκαμεν	ἐλελύκειμεν
2. „	λελύκατε	ἐλελύκειτε
3. „	λελύκασι(ν)	ἐλελύκεσαν

		Future	Aorist
1	Sing.	λύσω, I shall	ἔλυσα, I loosed
2	,,	λύσεις loose	ἔλυσας
3	,,	λύσει	ἔλυσε(ν)
1	Dual	λύσετον	ἐλύσατον
2	,,	λύσετον	ἐλυσάτην
1	Plur.	λύσομεν	ἐλύσαμεν
2	,,	λύσετε	ἐλύσατε
3	,,	λύσουσι(ν)	ἔλυσαν

NOTES. 1. λύω has no strong or second aorist. A strong aorist is usually conjugated like an imperfect, though having a different form—as, ἔβαλον from βάλλω, I throw; ἔτυπον from τύπτω, I strike.

2. Remark the augment in imperfect, pluperfect, and aorist. Before a consonant, as here, it is called the syllabic augment. When contracted with a vowel, as in εἶχον, ἦγον, it is called the temporal augment.

3. The imperfect denotes continued or repeated action; the aorist expresses a completed action in indefinite past time: ἔλυον = I was in the habit of loosing, *or* I was loosing; ἔλυσα = I loosed. The perfect denotes a definite time in the past—I have loosed.

4. (ν) When the third person, singular or plural, ends with σι or ε, and the following word begins with a vowel, the letter ν is added; just as in English the indefinite article *a* becomes *an* before a vowel. (The ν is similarly added to dative plurals in σι.)

Infinitives and Participles

Each tense has an infinitive mood which does not change, and a participle which is declined like a three-gender adjective.

λύω: Present infin., λύειν; participle, λύων, -ουσα, -ον; m. and n. stem, λυοντ-; as λέων (Lesson 3).

Perfect infin., λελυκέναι; participle, λελυκώς, -υῖα, -ός; m. and n. stem, λελυκοτ-.

Future infin., λύσειν; participle, λύσων, -ουσα, -ον; as λύων.

Aorist infin., λῦσαι; participle, λύσας -ασα, -αν; as πᾶς (Lesson 3).

The present and perfect serve also for imperfect and pluperfect respectively.

The Verb "To Be"

The student will now learn—still not going beyond the indicative, infinitive, and participle—the verb which is in most universal use in all languages, *to be*. He will absorb some rules about nouns and verbs, and add some useful words to his vocabulary; and, having already studied a model verb in the indicative active, he will be equipped for exercises in translating not merely words but sentences from one language into the other. Then in the two Lessons immediately following he can complete the study of verb conjugation.

εἰμί, I am

INDICATIVE

		Present	Imperfect	Future
1.	Sing.	εἰμί	ἦν or ἦ	ἔσομαι
2.	,,	εἶ, εἶς	ἦσθα	ἔσει or ἔσῃ
3	,,	ἐστί(ν)	ἦν	ἔσται
2.	Dual	ἐστόν	ἦτον	ἔσεσθον
3.	,,	ἐστόν	ἤτην	ἔσεσθον
1.	Plur.	ἐσμέν	ἦμεν	ἐσόμεθα
2.	,,	ἐστέ	ἦτε	ἔσεσθε
3.	,,	εἰσί(ν)	ἦσαν	ἔσονται

INFINITIVE

Pres. εἶναι. Fut. ἔσεσθαι.

Participle Pres. ὤν, οὖσα, ὄν (st. ὀντ-). Fut. ἐσόμενος, -α, -ον.

NOTES. 1. εἰμί, I am, must be carefully distinguished from εἶμι, I go. The complete conjugation of both is given in Lesson 8.

2. In present or primary tenses the second and third persons of the dual are alike, each ending in ον; but in past or historic tenses the third dual ends in ην (cf. ἤτην above).

RULES. 1. A verb agrees with its subject in number and person and a participle with its noun in gender, number and case.

2. When any part of εἰμί, am, connects the subject with a following noun or adjective the verb is called the *copula*, and the noun or adjective following is called the predicate. If in such sentences the predicate is a noun it will be in the same *case* as the subject; if it is an adjective, it will agree with the subject in *number*, *gender*, and *case*—e.g. Οἱ γέροντές εἰσι ῥήτορες, The old men are orators; Οἱ ἡγεμόνες ἦσαν σώφρονες. The leaders were prudent.

3. In Greek a nominative in the *neuter plural* takes a *singular* verb—as, Τὰ δῶρά ἐστι χρήσιμα, The gifts are useful; Τὰ ἄστρα ἦν φίλια, The stars were friendly. (The Greeks regarded neuter plurals as forming a class, and as practically equalling a collective noun; hence the singular verb.)

4. Several subjects connected by *and* (καί) usually take a plural verb, and if the subjects are of different persons, the verb is in the first person rather than the second, and in the second rather than the third—as, Ἐχθροὶ ἐγὼ καὶ σὺ ἐσμέν = I and you are enemies (literally, "Hostile I and you, we are," the emphatic word coming first).

5. An adjective, with the article prefixed, is often used as a noun, the person or thing being understood—as, ὁ σοφός = the wise man; οἱ δίκαιοι = the just; ἡ σοφή = the wise woman; τὰ ἄδικα = unjust things, what is unjust (and so very often = injustice). In English, this practice is limited to plurals (as, None but the brave deserve the fair).

6. (Very important) The article is not used with the predicate, only with the subject—as, Ὁ ὄρνις ἦν τοῦ γέροντος δῶρον = The bird was the gift of the old man (not τὸ δῶρον). In this way it is easy to distinguish the subject from the predicate, whatever the order of the words in the sentence. For example: Διδάσκαλος τῆς κόρης ἦν ὁ ποιμήν, The shepherd was the teacher of the girl. This rule will be found not to hold good in New Testament Greek which is slightly different from Classical Greek

7. A relative pronoun agrees with its antecedent in number, person, and gender, but not in case. Its case depends on the construction of the clause to which it belongs—as, Ὑμεῖς οἳ ταῦτα γράφετε, You who write this; οἱ στρατιῶται οὓς σέσωκα εἰσι δειλοί The soldiers whom I have saved are cowardly

8. *Duration of time* is expressed by the accusative case—the answer to the question "How long?"—as, They were a long time in the island, [illegible]. He stayed for three days, [illegible]; I remain [illegible].

9. *Time when* (definite) is expressed by the dative case—the answer to the question "When?" "At what time?"—as, [illegible], on the first day; [illegible], on the same night.

10. *Time when* (indefinite) or *Time within which* is expressed by the **genitive—the answer to the question "Within what time?" "During the course** of what time?"—as, [illegible], in winter time; [illegible], by night; [illegible], within a short time. Time may also be expressed by prepositions, as [illegible], in the night.

11. The participle, with the article prefixed, may be used as a substantive—as, [illegible], the speakers. But most commonly the participle with article represents an English relative clause—as, [illegible], the things that are being said; [illegible], those who live well; [illegible], he who abases himself; [illegible]. He that loveth his life (soul) shall lose it.

VOCABULARY

basket, [illegible]	delight, [illegible]
egg, [illegible]	time, ὁ χρόνος

dear, [illegible]	ancient, [illegible]
long, [illegible]	not, [illegible] before a consonant, [illegible] before unaspirated vowel ([illegible]), [illegible] before aspirated vowel ([illegible])
pleasant, [illegible]	
unkind, [illegible]	
sad, [illegible]	
wicked, [illegible]	
wise, [illegible]	
I admire, [illegible]	and, [illegible]
I dance, [illegible]	but, [illegible], δέ
I fall, πίπτω	that, ὅτι (conjunction)
I hope for, ἐλπίζω (tr.)	in, ἐν (taking a dative)
I pursue, διώκω	out from, ἐκ (genitive)
I run, τρέχω	
I say, λέγω	

EXERCISE

TRANSLATE INTO GREEK: 1. There are a b[illegible] and a girl in the house (say, a boy and a girl in the house). 2. The house is full of bask[illegible] and in the baskets are eggs. 3. The eggs w[illegible] the gift of the kind old man. 4. The boy adm[illegible] the baskets: he loosens his (say *the*) outer garm[illegible] 5. He will soon loosen a basket. 6. The gir[illegible] wicked, and dances *with delight* (simple d[illegible] without a preposition). 7. She hopes for the eg[illegible] she says "The time is long." 8. But the d[illegible] unkind: he pursues the boy and the girl. 9. T[illegible] basket falls: the eggs are ancient, and the hou[illegible] not pleasant. 10. The boy and the girl run [illegible] of the house, sad and wise.

LESSON 7

Further Study of the Verb

AFTER the *εἰμί* interlude we revert to the other three moods in the active of λύω, and complete the conjugation of our verb of the first regular formation with its middle and passive in all moods. In all voices the imperfect and pluperfect tenses are confined to the indicative mood—they do not occur in any of the other moods; while the future has no subjunctive or imperative. There is no first person in the imperative.

λύω, I loose

ACTIVE VOICE—Remaining Moods

Subjunctive Mood

	Present	Aorist	Perfect (rare)
1.	[illegible]	[illegible]	[illegible]
2.	[illegible]	[illegible]	[illegible]
3.	[illegible]	[illegible]	[illegible]
2.	λύητον	λύσητον	λελύκητον
3.	λύητον	λύσητον	λελύκητον
1.	[illegible]	[illegible]	[illegible]
2.	λύητε	λύσητε	λελύκητε
3.	λύωσι	λύσωσι	λελύκωσι

Optative Mood

	Present	Aorist	Perfect (rare)
1.	λύοιμι	λύσαιμι	λελύκοιμι
2.	λύοις	λύσαις, λύσειας	λελύκοις
3.	λύοι	λύσαι, λύσειε	λελύκοι
2.	λύοιτον	λύσαιτον	λελύκοιτον
3.	λυοίτην	λυσαίτην	λελυκοίτην
1.	λύοιμεν	λύσαιμεν	λελύκοιμεν
2.	λύοιτε	λύσαιτε	λελύκοιτε
3.	λύοιεν	λύσαιεν, λύσειαν	λελύκοιεν

[illegible]: λύσοιμι, which is [illegible]

[illegible]

Imperative Mood

	Present	Aorist	Perfect (rare)
2.	λῦε	λῦσον	λέλυκε
3.	λυέτω	λυσάτω	λελυκέτω
2.	λύετον	λύσατον	λελύκετον
3.	λυέτων	λυσάτων	λελυκέτων
2.	λύετε	λύσατε	λελύκετε
3.	λυέτωσαν or λυόντων	λυσάτωσαν or λυσάντων	λελυκέτωσαν

MIDDLE VOICE

Indicative Mood

Pres.	Imperf.	Fut.	Aorist	Perf.	Pluperf.
λύομαι	ἐλυόμην	λύσομαι	ἐλυσάμην	λέλυμαι	ἐλελύμην
λύει	ἐλύου	λύσει	ἐλύσω	λέλυσαι	ἐλέλυσο
λύεται	ἐλύετο	λύσεται	ἐλύσατο	λέλυται	ἐλέλυτο
λύεσθον	ἐλύεσθον	λύσεσθον	ἐλύσασθον	λέλυσθον	ἐλέλυσθον
λύεσθον	ἐλυέσθην	λύσεσθον	ἐλυσάσθην	λέλυσθον	ἐλελύσθην
[illegible]	ἐλυόμεθα	λυσόμεθα	ἐλυσάμεθα	λελύμεθα	ἐλελύμεθα
λύεσθε	ἐλύεσθε	λύσεσθε	ἐλύσασθε	λέλυσθε	ἐλέλυσθε
λύονται	ἐλύοντο	λύσονται	ἐλύσαντο	λέλυνται	ἐλέλυντο

Subjunctive Mood

Present	Aorist	Perfect	
λύωμαι	λύσωμαι	λελυμένος	ὦ
[illegible]	λύσῃ	—	[illegible]
λύηται	λύσηται	—	[illegible]
λύησθον	λύσησθον	λελυμένω	[illegible]
λύησθον	λύσησθον	—	[illegible]

Present	Aorist	Perfect	
λυώμεθα	λυσώμεθα	λελυμένοι	ὦμεν
λύησθε	λύσησθε	—	ἦτε
λύωνται	λύσωνται	—	ὦσι

The perfect is made up of the perfect participle middle of λύω and the present subjunctive of *εἰμί*; as [illegible] we use the auxiliary verb *have* with a participle to make the perfect and pluperfect: I have loosed, I had loosed.

Optative Mood

Present	Aorist	Perfect	
λυοίμην	λυσαίμην	λελυμένος	εἴην
λύοιο	λύσαιο	—	εἴης
λύοιτο	λύσαιτο	—	εἴη
λύοισθον	λύσαισθον	λελυμένω	εἴητον
λυοίσθην	λυσαίσθην	—	εἰήτην
λυοίμεθα	λυσαίμεθα	λελυμένοι	εἴημεν
λύοισθε	λύσαισθε	—	εἴητε
λύοιντο	λύσαιντο	—	εἴησαν

The perfect optative is made up of the perfect participle middle of λύω and the optative of *εἰμί*. The dual and plural forms of the latter are usually shortened into *εἶτον, εἴτην, εἶμεν, εἶτε, εἶεν.*

Imperative Mood

Present	Aorist	Perfect
λύου	λῦσαι	λέλυσο
λυέσθω	λυσάσθω	λελύσθω
λύεσθον	λύσασθον	λέλυσθον
λυέσθων	λυσάσθων	λελύσθων
λύεσθε	λύσασθε	λέλυσθε
λυέσθωσαν or λυέσθων	λυσάσθωσαν or λυσάσθων	λελύσθωσαν or λελύσθων

Infinitive Mood

Present	Future	Aorist	Perfect
λύεσθαι	λύσεσθαι	λύσασθαι	λελύσθαι

Participles

Present	Future
λυόμενος, -η, -ον	λυσόμενος, -η, -ον
Aorist	**Perfect**
λυσάμενος, -η, -ον	λελυμένος, -η, -ον

Middle participles are declined regularly.

Passive Voice

This is identical with the middle voice, excepting the future and aorist, which are different, and the future perfect, which is additional. These are conjugated as follows.

Indicative Mood

Future	Aorist	Future Perfect
λυθήσομαι	ἐλύθην	λελύσομαι
λυθήσει	ἐλύθης	λελύσει
λυθήσεται	ἐλύθη	λελύσεται
λυθήσεσθον	ἐλύθητον	λελύσεσθον
λυθήσεσθον	ἐλυθήτην	λελύσεσθον
λυθησόμεθα	ἐλύθημεν	λελυσόμεθα
λυθήσεσθε	ἐλύθητε	λελύσεσθε
λυθήσονται	ἐλύθησαν	λελύσονται

Subjunctive Mood	Imperative Mood
Aorist	**Aorist**
λυθῶ	
λυθῇς	2. λύθητι
λυθῇ	3. λυθήτω
λυθῆτον	2. λύθητον
λυθῆτον	3. λυθήτων
λυθῶμεν	
λυθῆτε	2. λύθητε
λυθῶσι	3. λυθήτωσαν or λυθέντων

Optative Mood

Future	Aorist	Future Perfect
λυθησοίμην	λυθείην	λελυσοίμην
λυθήσοιο	λυθείης	λελύσοιο
λυθήσοιτο	λυθείη	λελύσοιτο
λυθήσοισθον	λυθεῖτον	λελύσοισθον
λυθησοίσθην	λυθείτην	λελυσοίσθην
λυθησοίμεθα	λυθεῖμεν	λελυσοίμεθα
λυθήσοισθε	λυθεῖτε	λελύσοισθε
λυθήσοιντο	λυθεῖεν	λελύσοιντο

Infinitive

Future	Aorist	Future Perfect
λυθήσεσθαι	λυθῆναι	λελύσεσθαι

Paradigm of Verbs in -ω.

	Indic.	Subjunc.	Optative	Imper.	Infin.	Participle
			Active Voice			
Present	λύω	λύω	λύοιμι	λῦε	λύειν	λύων
Imperfect	ἔλυον					
Future	λύσω	—	λύσοιμι		λύσειν	λύσων
Aorist	ἔλυσα	λύσω	λύσαιμι	λῦσον	λῦσαι	λύσας
Perfect	λέλυκα	λελύκω	λελύκοιμι	(λέλυκε)	λελυκέναι	λελυκώς
Pluperfect	ἐλελύκειν					
			Middle Voice			
Present	λύομαι	λύωμαι	λυοίμην	λύου	λύεσθαι	λυόμενος
Imperfect	ἐλυόμην					
Future	λύσομαι		λυσοίμην	—	λύσεσθαι	λυσόμενος
Aorist	ἐλυσάμην	λύσωμαι	λυσαίμην	λῦσαι	λύσασθαι	λυσάμενος
Perfect	λέλυμαι	λελυμένος ὦ	λελυμένος εἴην	λέλυσο	λελύσθαι	λελυμένος
Pluperfect	ἐλελύμην					
			Passive Voice			
Present	Same as middle					
Imperfect						
Perfect						
Pluperfect						
Future	λυθήσομαι	—	λυθησοίμην		λυθήσεσθαι	λυθησόμενος
Aorist	ἐλύθην	λυθῶ	λυθείην	λύθητι	λυθῆναι	λυθείς
Future Perfect	λελύσομαι	—	λελυσοίμην		λελύσεσθαι	λελυσόμενος

Participles

λυθησόμενος, -η, -ον	λυθείς, -εῖσα, -έν (stem, λυθεντ-)	λελυσόμενος, -η, -ον

The scheme or diagram, called a paradigm (pattern), given in page 1665, displays at a glance the first person singular of every tense in each mood of each of the three voices, as well as the imperative, infinitive and participles. The student should thoroughly visualize and memorize this, both across and down.

Contracted Verbs

There are now to be considered verbs that undergo contraction in certain tenses. Verbs in αω, εω, and οω are usually contracted in the present and imperfect tenses. The following are the contracted tenses of τιμάω, I honour; φιλέω, I love; and δηλόω, I manifest.

Active Voice

Present Indicative

τιμῶ (for τιμάω)	φιλῶ	δηλῶ
τιμᾷς (for τιμάεις)	φιλεῖς	δηλοῖς
τιμᾷ (for τιμάει)	φιλεῖ	δηλοῖ
τιμᾶτον, etc.	φιλεῖτον	δηλοῦτον
τιμᾶτον	φιλεῖτον	δηλοῦτον
τιμῶμεν	φιλοῦμεν	δηλοῦμεν
τιμᾶτε	φιλεῖτε	δηλοῦτε
τιμῶσι(ν)	φιλοῦσι(ν)	δηλοῦσι(ν)

Imperfect Indicative

ἐτίμων (ἐτίμαον)	ἐφίλουν	ἐδήλουν
ἐτίμας	ἐφίλεις	ἐδήλους
ἐτίμα	ἐφίλει	ἐδήλου
ἐτιμᾶτον	ἐφιλεῖτον	ἐδηλοῦτον
ἐτιμάτην	ἐφιλείτην	ἐδηλούτην
ἐτιμῶμεν	ἐφιλοῦμεν	ἐδηλοῦμεν
ἐτιμᾶτε	ἐφιλεῖτε	ἐδηλοῦτε
ἐτίμων	ἐφίλουν	ἐδήλουν

Present Subjunctive

Same as present indicative	φιλῶ	δηλῶ
	φιλῇς	δηλοῖς
	φιλῇ	δηλοῖ
	φιλῆτον	δηλῶτον
	φιλῆτον	δηλῶτον
	φιλῶμεν	δηλῶμεν
	φιλῆτε	δηλῶτε
	φιλῶσι	δηλῶσι

Present Optative

τιμῷμι (τιμάοιμι)	φιλοῖμι	δηλοῖμι
τιμῷς	φιλοῖς	δηλοῖς
τιμῷ	φιλοῖ	δηλοῖ
τιμῷτον	φιλοῖτον	δηλοῖτον
τιμῴτην	φιλοίτην	δηλοίτην
τιμῷμεν	φιλοῖμεν	δηλοῖμεν
τιμῷτε	φιλοῖτε	δηλοῖτε
τιμῷεν	φιλοῖεν	δηλοῖεν
or	or	or
τιμῴην (τιμαοίην)	φιλοίην	δηλοίην
τιμῴης	φιλοίης	δηλοίης
τιμῴη	φιλοίη	δηλοίη
τιμῴητον	φιλοίητον	δηλοίητον
τιμῳήτην	φιλοιήτην	δηλοιήτην
τιμῴημεν	φιλοίημεν	δηλοίημεν
τιμῴητε	φιλοίητε	δηλοίητε
τιμῴησαν	φιλοίησαν	δηλοίησαν

Present Imperative

τίμα	φίλει	δήλου
τιμάτω	φιλείτω	δηλούτω
τιμᾶτον	φιλεῖτον	δηλοῦτον
τιμάτων	φιλείτων	δηλούτων
τιμᾶτε	φιλεῖτε	δηλοῦτε
τιμάτωσαν or τιμώντων	φιλείτωσαν or φιλούντων	δηλούτωσαν or δηλούντων

Present Infinitive

τιμᾶν (τιμάειν)	φιλεῖν	δηλοῦν

Present Participle

τιμῶν,-ῶσα,-ῶν	φιλῶν,-οῦσα,-οῦν	δηλῶν,-οῦσα

Passive and Middle Voices

Present Indicative

τιμῶμαι	φιλοῦμαι	δηλοῦμαι
τιμᾷ	φιλεῖ	δηλοῖ
τιμᾶται	φιλεῖται	δηλοῦται
τιμᾶσθον	φιλεῖσθον	δηλοῦσθον
τιμᾶσθον	φιλεῖσθον	δηλοῦσθον
τιμώμεθα	φιλούμεθα	δηλούμεθα
τιμᾶσθε	φιλεῖσθε	δηλοῦσθε
τιμῶνται	φιλοῦνται	δηλοῦνται

Imperfect Indicative

ἐτιμώμην	ἐφιλούμην	ἐδηλούμην
ἐτιμῶ	ἐφιλοῦ	ἐδηλοῦ
ἐτιμᾶτο	ἐφιλεῖτο	ἐδηλοῦτο
ἐτιμᾶσθον	ἐφιλεῖσθον	ἐδηλοῦσθον
ἐτιμάσθην	ἐφιλείσθην	ἐδηλούσθην
ἐτιμώμεθα	ἐφιλούμεθα	ἐδηλούμεθα
ἐτιμᾶσθε	ἐφιλεῖσθε	ἐδηλοῦσθε
ἐτιμῶντο	ἐφιλοῦντο	ἐδηλοῦντο

Present Subjunctive

Same as present indicative	φιλῶμαι	δηλῶμαι
	φιλῇ	δηλοῖ
	φιλῆται	δηλῶται
	φιλῆσθον	δηλῶσθον
	φιλῆσθον	δηλῶσθον
	φιλώμεθα	δηλώμεθα
	φιλῆσθε	δηλῶσθε
	φιλῶνται	δηλῶνται

Present Optative

τιμῴμην	φιλοίμην	δηλοίμην
τιμῷο	φιλοῖο	δηλοῖο
τιμῷτο	φιλοῖτο	δηλοῖτο
τιμῷσθον	φιλοῖσθον	δηλοῖσθον
τιμῴσθην	φιλοίσθην	δηλοίσθην
τιμῴμεθα	φιλοίμεθα	δηλοίμεθα
τιμῷσθε	φιλοῖσθε	δηλοῖσθε
τιμῷντο	φιλοῖντο	δηλοῖντο

Present Imperative

τιμῶ (τιμάου)	φιλοῦ	δηλοῦ
τιμάσθω	φιλείσθω	δηλούσθω
τιμᾶσθον	φιλεῖσθον	δηλοῦσθον
τιμάσθων	φιλείσθων	δηλούσθων
τιμᾶσθε	φιλεῖσθε	δηλοῦσθε
τιμάσθωσαν or τιμάσθων	φιλείσθωσαν or φιλείσθων	δηλούσθωσαν or δηλούσθων

Present Infinitive

τιμᾶσθαι	φιλεῖσθαι	δηλοῦσθαι

Present Participle

τιμώμενος,-η,-ον	φιλούμενος,-η,-ον	δηλούμενος,-η,-ον

NOTE. A few verbs in αω take η for α in their contracted forms; of these the commonest are ζάω, live; διψάω, thirst; πεινάω, hunger; κνάω, scrape; ψάω, rub; χράω, give oracles; and χράομαι, use. Infinitives ζῆν, ψῆν, χρῆσθαι, etc.

The following rules will be found helpful for the exercise that follows.

Rules of Syntax

1. A substantive may have another substantive added to explain or describe it; the latter is then

said to be in *apposition* to the former, and agrees with it in case—as, *Νικίας ὁ στρατηγός*, Nicias the general ; *ὁ υἱὸς τοῦ Δαρείου τοῦ Πέρσου*, the son of Darius the Persian.

2. Words which qualify or describe a noun are usually placed between that noun and its article—as, the vulture's claws, *οἱ τοῦ γυπὸς ὄνυχες* ; the trees in the [illegible], *τὰ ἐν τῇ νήσῳ δένδρα.*

3. [illegible] without the article means *every*—as, *πᾶσα πόλις*, every city ; but with the article *πᾶς* means *all* or *the whole*—as, *πᾶσα ἡ πόλις* all the city ; *ἡ πᾶσα πόλις*, the whole city.

4. When *μέγας* or *πολύς* is used with another adjective, the two adjectives are generally connected by *καί* as, The great black bird, *ὁ μέγας καὶ μέλας ὄρνις* ; He saved many poor citizens, *ἔσωσε πολλοὺς καὶ πένητας πολίτας.*

5. The article is frequently used with an adverb—as, *οἱ πάλαι*, the men of old ; *οἱ ἐκεῖ*, the people there ; *οἱ νῦν*, the men of the present day. The noun has here been omitted, the full phrase being *οἱ πάλαι ἄνθρωποι*, and so on.

6. [illegible] Greek the article is required with nouns, den[illegible] whole class of things as, *Οἱ ἄνθρωποι εἰσι* [illegible], Men are mortal ; *Οἱ λέοντές εἰσι δεινοί*, Lions are terrible.

VOCABULARY

here, *ἐνταῦθα*	I make, *ποιέω* (aorist *ἐποίησα*)
Cyrus, *Κῦρος*	citadel, *ἡ ἀκρόπολις*
pal[illegible] *[illegible]λεια* (plural)	soul, *ἡ ψυχή*
park, *παράδεισος*	I magnify, *μεγαλύνω*
I hunt, *θηρεύω*	Lord, *Κύριος*
from [illegible] (governs gen.)	I rejoice, *ἀγαλλιάω* (aorist *ἠγαλλίασα*)
through, *διά* „	I flow, *ῥέω*
source, *ἡ πηγή*	Saviour, *σωτήρ, -ῆρος*
Apollo, *Ἀπόλλων*	good, *ἀγαθός, -η, -ον*
I kill, *κτείνω* (aorist *ἔκτεινα*)	corrupt, *σαπρός, -α, -ον*
	fruit, *ὁ καρπός*
	evil, *πονηρός, -α, -ον*

EXERCISE

TRANSLATE INTO GREEK : 1. Here Cyrus had a palace (say : there was to Cyrus a palace) and a great park, full of wild beasts. 2. The king used to hunt the wild beasts on horseback (say : from a horse). 3. Through the park flows the river Maeander (*Μαίανδρος*), and the sources of the river start from (say : are out of) the palace. 4. It flows through the whole city. 5. The men of old used to say that (*ὅτι*) Apollo killed Marsyas (*Μαρσύας*) here. 6. There Xerxes, the king of the Persians, made a citadel and a palace. 7. My soul (say : the soul of *me*, *μου*) doth magnify (magnifies) the Lord and my spirit hath rejoiced (aorist) in (*ἐπί*, governing dative) God my Saviour. 8. Every good tree bringeth forth (maketh, *ποιέω*) good fruit (plural), but the corrupt tree bringeth forth evil fruit.

KEY TO EXERCISE IN LESSON 6

1. *ἐν τῇ οἰκίᾳ εἰσὶ παῖς καὶ κόρη.* 2. *ἡ οἰκία ἐστὶ πλήρης κανῶν, καὶ ἐν τοῖς κανοῖς ἐστιν ᾠά.* 3. *τὰ ᾠὰ ἦν τοῦ ἵλεω γέροντος δῶρον.* 4. *ὁ παῖν θαυμάζει τὰ κανᾶ· λύει τὸ ἱμάτιον.* 5. *ταχέως λύσει κανοῦν.* 6. *ἡ κόρη ἐστὶ κακὴ καὶ χορεύει χαρᾷ.* 7. *ελπίζει τὰ ᾠά λέγει ὅτι ὁ χρόνος ἐστὶ μακρός.* 8. *ἀλλ' ὁ δαίμων ἐστὶν ἄγριος· διώκει τὸν παῖδα καὶ τὴν κόρην.* 9. *τὸ κανοῦν πίπτει· τὰ ᾠά ἐστι παλαιὰ καὶ ἡ οἰκία οὐκ ἔστι τερπνή.* 10. *ὁ παῖς καὶ ἡ κόρη τρέχουσι ἐκ τῆς οἰκίας, οἰκτροὶ καὶ σοφοί.*

LESSON 8

Verbs of the Second Regular Formation, Irregular Verbs

THE verbs of the second " regular " formation are those in which the first person indicative active ends in *μι*. Their inflexion differs from that of the *ω* verbs almost exclusively in the tenses formed from the present and second (or strong) aorist stems. In these tenses—the present, imperfect and second aorist—the four typical verbs, *ἵστημι*, set ; *τίθημι*, put ; *δίδωμι*, give; *δείκνυμι*, show, are conjugated as shown in the following tables.

ACTIVE VOICE

Indicative

Present

ἵστημι	*τίθημι*	*δίδωμι*	*δείκνυμι*
ἵστης	*τίθης*	*δίδως*	*δείκνυς*
ἵστησι	*τίθησι*	*δίδωσι*	*δείκνυσι*
ἵστατον	*τίθετον*	*δίδοτον*	*δείκνυτον*
ἵστατον	*τίθετον*	*δίδοτον*	*δείκνυτον*
ἵσταμεν	*τίθεμεν*	*δίδομεν*	*δείκνυμεν*
ἵστατε	*τίθετε*	*δίδοτε*	*δείκνυτε*
ἱστᾶσι	*τιθέασι*	*διδόασι*	*δεικνύασι*

Imperfect

ἵστην	*ἐτίθην*	*ἐδίδουν*	*ἐδείκνυν*
ἵστης	*ἐτίθεις*	*ἐδίδους*	*ἐδείκνυς*
ἵστη	*ἐτίθει*	*ἐδίδου*	*ἐδείκνυ*
ἵστατον	*ἐτίθετον*	*ἐδίδοτον*	*ἐδείκνυτον*
ἱστάτην	*ἐτιθέτην*	*ἐδιδότην*	*ἐδεικνύτην*
ἵσταμεν	*ἐτίθεμεν*	*ἐδίδομεν*	*ἐδείκνυμεν*
ἵστατε	*ἐτίθετε*	*ἐδίδοτε*	*ἐδείκνυτε*
ἵστασαν	*ἐτίθεσαν*	*ἐδίδοσαν*	*ἐδείκνυσαν*

Second Aorist

ἔστην	[*ἔθην*]	[*ἔδων*]	None
ἔστης	[*ἔθης*]	[*ἔδως*]	
ἔστη	[*ἔθη*]	[*ἔδω*]	
ἔστητον	*ἔθετον*	*ἔδοτον*	
ἐστήτην	*ἐθέτην*	*ἐδότην*	
ἔστημεν	*ἔθεμεν*	*ἔδομεν*	
ἔστητε	*ἔθετε*	*ἔδοτε*	
ἔστησαν	*ἔθεσαν*	*ἔδοσαν*	

The future, perfect, and first aorist are respectively: *στήσω, θήσω, δώσω, δείξω; ἕστηκα, τέθεικα, δέδωκα, δέδειχα; ἔστησα, ἔθηκα, ἔδωκα, ἔδειξα*—all following the ordinary conjugation through the other voices and moods.

NOTE. The words in square brackets, *ἔθην*, *ἔδων*, etc., are never used.

Subjunctive

Present

ἱστῶ	τιθῶ	διδῶ	δεικνύω
ἱστῇς	τιθῇς	διδῷς	δεικνύῃς
ἱστῇ	τιθῇ	διδῷ	δεικνύῃ
ἱστῆτον	τιθῆτον	διδῶτον	δεικνύητον
ἱστῆτον	τιθῆτον	διδῶτον	δεικνύητον
ἱστῶμεν	τιθῶμεν	διδῶμεν	δεικνύωμεν
ἱστῆτε	τιθῆτε	διδῶτε	δεικνύητε
ἱστῶσι	τιθῶσι	διδῶσι	δεικνύωσι

Second Aorist

στῶ	θῶ	δῶ	None
στῇς	θῇς	δῷς	
etc.	etc.	etc.	

Optative

Present

ἱσταίην	τιθείην	διδοίην	δεικνύοιμι
ἱσταίης	τιθείης	διδοίης	δεικνύοις
ἱσταίη	τιθείη	διδοίη	δεικνύοι
ἱσταίητον	τιθείητον	διδοίητον	etc.,
or ἱσταῖτον	or τιθεῖτον	or διδοῖτον	like verb
ἱσταιήτην	τιθειήτην	διδοιήτην	in ω
or ἱσταίτην	or τιθείτην	or διδοίτην	
ἱσταίημεν	τιθείημεν	διδοίημεν	
or ἱσταῖμεν	or τιθεῖμεν	or διδοῖμεν	
ἱσταίητε	τιθείητε	διδοίητε	
or ἱσταῖτε	or τιθεῖτε	or διδοῖτε	
ἱσταίησαν	τιθείησαν	διδοίησαν	
or ἱσταῖεν	or τιθεῖεν	or διδοῖεν	

Second Aorist

σταίην	θείην	δοίην	None
σταίης	θείης	δοίης	
etc.	etc.	etc.	

Imperative : Present

ἵστη	τίθει	δίδου	δείκνυ
ἱστάτω	τιθέτω	διδότω	δεικνύτω
etc.	etc.	etc.	etc.

Imperative : Second Aorist

στῆθι	θές	δός	None
στήτω	θέτω	δότω	
στῆτον	θέτον	δότον	
στήτων	θέτων	δότων	
στῆτε	θέτε	δότε	
στήτωσαν	θέτωσαν	δότωσαν	
or στάντων	or θέντων	or δόντων	

Present Infinitive

ἱστάναι	τιθέναι	διδόναι	δεικνύναι

Second Aorist Infinitive

στῆναι	θεῖναι	δοῦναι	—

Present Participle

ἱστάς	τιθείς	διδούς	δεικνύς

Second Aorist Participle

στάς	θείς	δούς	—

NOTE. ἱστάς is declined like λύσας, and τιθείς like λυθείς. διδούς is declined like λύων, except in nominative and vocative singular :

Nom., Voc.	διδούς	διδοῦσα	διδόν
Gen.	διδόντος	διδούσης	διδόντος
	etc.	etc.	etc.

δεικνύς is thus declined :

Nom., Voc.	δεικνύς	δεικνῦσα	δεικνύν
Acc.	δεικνύντα	δεικνῦσαν	δεικνύν
Gen.	δεικνύντος	δεικνύσης	δεικνύντος
	etc.	etc.	etc.

PASSIVE AND MIDDLE VOICES

Indicative

Present

ἵσταμαι	τίθεμαι	δίδομαι	δείκ[illegible]
ἵστασαι	τίθεσαι	δίδοσαι	δείκ[illegible]
ἵσταται	τίθεται	δίδοται	δείκ[illegible]
ἵστασθον	τίθεσθον	δίδοσθον	δείκνυσθον
ἵστασθον	τίθεσθον	δίδοσθον	δείκνυσθον
ἱστάμεθα	τιθέμεθα	διδόμεθα	δεικνύμεθα
ἵστασθε	τίθεσθε	δίδοσθε	δείκνυσθε
ἵστανται	τίθενται	δίδονται	δείκνυνται

Imperfect

ἱστάμην	ἐτιθέμην	ἐδιδόμην	ἐδεικνύμην
ἵστασο	ἐτίθεσο	ἐδίδοσο	ἐδεί[illegible]
ἵστατο	ἐτίθετο	ἐδίδοτο	ἐδε[illegible]
etc.	etc.	etc.	etc.

Second Aorist (middle only)

None	ἐθέμην	ἐδόμην	None
	ἔθου	ἔδου	
	ἔθετο	ἔδοτο	
	etc.	etc.	

Subjunctive

Present

ἱστῶμαι	τιθῶμαι	διδῶμαι	δεικνύωμαι
ἱστῇ	τιθῇ	διδῷ	δεικνύ[illegible]
ἱστῆται	τιθῆται	διδῶται	δεικνύ[illegible]
etc.	etc.	etc.	etc.

Second Aorist (middle only)

None	θῶμαι	δῶμαι	None
	θῇ	δῷ	
	θῆται	δῶται	

Optative

Present

ἱσταίμην	τιθείμην	διδοίμην	δεικ[illegible]
ἱσταῖο	τιθεῖο	διδοῖο	δεικνύοιο
ἱσταῖτο	τιθεῖτο	διδοῖτο	δεικνύοιτο
etc.	etc.	etc.	etc.

Second Aorist (middle only)

None	θείμην	δοίμην	None
	θεῖο	δοῖο	
	θεῖτο	δοῖτο	
	etc.	etc.	

Imperative : Present

ἵστασο	τίθεσο	δίδοσο	δείκνυσο
or ἵστω	or τίθου	or δίδου	
ἱστάσθω	τιθέσθω	διδόσθω	δεικνύσθω
etc.	etc.	etc.	etc.

Imperative : Second Aorist (middle only)

None	θοῦ	δοῦ	None
	θέσθω	δόσθω	
	etc.	etc.	

Present Infinitive

ἵστασθαι	τίθεσθαι	δίδοσθαι	δείκνυσθαι

Second Aorist Infinitive (middle only)

None	θέσθαι	δόσθαι	None

Present Participle

ἱστάμενος	τιθέμενος	διδόμενος	δεικνύμενος

Second Aorist Participle (middle only)

None	θέμενος	δόμενος	None

ἵστημι has also a second perfect form: infinitive ἑστάναι, participle ἑστώς. This is used only in dual and plural in the indicative : dual ἕστατον, ἕστατον ; plural ἕσταμεν, ἕστατε, ἑστᾶσι.

Irregul… Verbs

Amo… the irregular verbs *εἰμί*, am, and *εἶμι*, go, must b… very carefully distinguished. The indicative, in…itive, and participles of *εἰμί*, am, have already …en given in Lesson 6 ; they are included here in order that the verb in all its inflexions may be com…red with *εἶμι*, go.

εἰμί (stem *ἐσ-*, Latin *es-se*), am.

Pres. Indicative.	Imperf.	Fut.	Pres. Subj.	Pres. Opta.	Pres. Imperf.
εἰμί	ἦν or ἦ	ἔσομαι	ὦ	εἴην	—
εἶ	ἦσθα	ἔσει or ἔσῃ	ᾖς	εἴης	ἴσθι
ἐστί	ἦν	ἔσται	ᾖ	εἴη	ἔστω
ἐστόν	ἦτον	ἔσεσθον	ἦτον	εἴητον, εἶτον	ἔστον
ἐστόν	ἤτην	ἔσεσθον	ἦτον	εἰήτην, εἴτην	ἔστων
ἐσμέν	ἦμεν	ἐσόμεθα	ὦμεν	εἴημεν, εἶμεν	—
ἐστέ	ἦτε	ἔσεσθε	ἦτε	εἴητε, εἶτε	ἔστε
εἰσί	ἦσαν	ἔσονται	ὦσι	εἴησαν, εἶεν	ἔστωσαν ἔστων, ὄντων

Present Infinitive : *εἶναι*, to be.

Present Participle : *ὤν, οὖσα, ὄν* (gen. *ὄντος οὔσης, ὄντος* ; nom. pl. *ὄντες, οὖσαι, ὄντα*) being.

Future Infinitive : *ἔσεσθαι*.

Future Participle : *ἐσόμενος*.

Verbal Adjective : *ἐστέον*.

There is also a future optative, *ἐσοίμην, ἔσοιο, ἔσ…ο*, etc., quite regular.

εἶμι (stem *ἰ-*,, Latin *i-re*), go.

Pres. Indic.	Imperf. Indic.	Pres. Subj.	Pres. Opta.	Pres. Imper.
εἶμι	ᾖειν or ᾖα	ἴω	ἰοίην	—
εἶ	ᾖεις or ᾔεισθα	ἴῃς	ἴοις	ἴθι
εἶσι	ᾖει or ᾔειν	ἴῃ	ἴοι	ἴτω
ἴτον	ᾔειτον or ᾖτον	ἴητον	ἴοιτον	ἴτον
ἴτον	ᾐείτην or ᾔτην	ἴητον	ἰοίτην	ἴτων
ἴμεν	ᾔειμεν or ᾖμεν	ἴωμεν	ἴοιμεν	—
ἴτε	ᾔειτε or ᾖτε	ἴητε	ἴοιτε	ἴτε
ἴασι	ᾔεσαν or ᾖσαν	ἴωσι	ἴοιεν	ἴτωσαν or ἰόντων

Infinitive : *ἰέναι*, to go.

Participle : *ἰών, ἰοῦσα, ἰόν* (gen. *ἰόντος*, etc.).

Verbal Adjective : *ἰτός* and *ἰτέον*.

NOTE. The present indicative of *εἶμι* has usually a future sense, namely I shall go, and consequently is used as the future of *ἔρχομαι*.

ἵημι, send, is conjugated like *τίθημι*, but the third plural present indicative is *ἱᾶσι*. It has future *ἥσω*, aorist *ἧκα*, perfect *εἷκα*, perfect passive *εἷμαι*, aorist passive *εἵθην*.

φημί, say, is conjugated almost like *ἵστημι*. It has imperfect *ἔφην*, and future *φήσω*. The second singular present indicative is *φῄς*—*φημί, φῄς, φησί* ; *φατόν, φατόν*; *φαμέν, φατέ, φασί*.

οἶδα, I know

This is really the surviving second perfect of a verb otherwise obsolete except in the second aorist, *εἶδον*, I saw, from the stem *ἰδ-*, see. From meaning *I have seen*, *οἶδα* came to mean *I know* ; whereas *εἶδον* always means *I saw*, taking the place of the aorist for *ὁράω*, *I see*.

Perf. Indic.	Pluperf. Indic.	Perf. Imper.
οἶδα	ᾔδειν or ᾔδη	—
οἶσθα	ᾔδεισθα or ᾔδεις	ἴσθι
οἶδε	ᾔδει or ᾔδη	ἴστω
ἴστον	ᾔδειτον or ᾖστον	ἴστον
ἴστον	ᾐδείτην or ᾔστην	ἴστων
ἴσμεν	ᾔδειμεν or ᾖσμεν	—
ἴστε	ᾔδειτε or ᾖστε	ἴστε
ἴσασι	ᾔδεσαν or ᾖσαν	ἴστωσαν

Perfect Infinitive : *εἰδέναι*, to know.

Perfect Participle : *εἰδώς, εἰδυῖα, εἰδός* knowing

Verbal Adjective : *ἰστέον*.

Subjunctive : *εἰδῶ, εἰδῇς, εἰδῇ*, etc.

Optative : *εἰδείην, εἰδείης, εἰδείη*, etc.

Future Indicative : *εἴσομαι, εἴσει, εἴσεται*, etc

VOCABULARY

frog, *ὁ βάτραχος, -ου*
once, *ποτέ*
I send, *πέμπω*
ambassadors, *οἱ πρέσβεις*
Jupiter *ὁ Ζεύς* (acc. *Δία*)
marsh, *ἡ λίμνη*
depth, *τὸ βάθος, -ους*
as, *ὡς*
motionless, *ἀκίνητος*
they despised, *κατεφρόνουν* (imperfect of *καταφρονέω*—governs genitive)
I think, *νομίζω*
to be, *εἶναι*
I ask, *αἰτέω* (aor. *ᾔτη…α*)
to provide, *παρασχεῖν*
I throw, *ῥίπτω* (aor. *ἔρριψα*)
log of wood, *τὸ ξύλον*
unworthy, *ἀνάξιος*
fool, *μωρός*
water-serpent, *ἡ ὕδρα*
by, *ὑπό* (governs genitive ; *ὑφ'* before an aspirated vowel)
they were eaten up, *κατησθίοντο* (imperfect passive of *κατεσθίω*, I eat up)

EXERCISE

TRANSLATE INTO GREEK : 1. The frogs once sent ambassadors to (*ἐπί* with accusative) Jupiter and asked (him) to provide a king for them. 2. Jupiter threw a log of wood into the marsh. 3. The frogs in fear threw themselves into the depths of the marsh. 4. But as the log was motionless, they soon despised it, and thought this king to be unworthy. 5. They asked Jupiter (for) another king. 6. He said " Ye are fools," and sent them a water-serpent, by whom they were eaten up.

KEY TO EXERCISE IN LESSON 7

1. *ἐνταῦθα Κύρῳ βασίλεια ἦν καὶ παράδεισος μέγας θηρῶν πλήρης.* 2. *ὁ βασιλεὺς ἐθήρευε τοὺς θῆρας ἀπὸ ἵππου* (generally written *ἀφ' ἵππου*). 3. *διὰ τοῦ παραδείσου ῥεῖ ὁ Μαίανδρος ποταμός* (note the order), *καὶ αἱ τοῦ ποταμοῦ πηγαί εἰσιν ἐκ τῶν βασιλείων.* 4. *ῥεῖ διὰ τῆς πάσης πόλεως.* 5. *οἱ πάλαι ἔλεγον ὅτι ἐνταῦθα Ἀπόλλων ἔκτεινε Μαρσύαν.* 6. *ἐνταῦθα Ξέρξης, ὁ τῶν Περσῶν βασιλεύς, ἐποίησεν ἀκρόπολιν καὶ βασίλεια.* 7. *μεγαλύνει ἡ ψυχή μου τὸν κύριον, καὶ ἠγαλλίασε τὸ πνεῦμά μου ἐπὶ τῷ Θεῷ τῷ σωτῆρί μου.* 8. *πᾶν δένδρον ἀγαθὸν καρποὺς καλοὺς ποιεῖ· τὸ δὲ σαπρὸν δένδρον καρποὺς πονηροὺς ποιεῖ.* (Literally, Every good tree bringeth forth *beautiful* fruit.)

NOTE. *δέ* is one of the commonest of the Greek particles. In any continuous passage of Greek each sentence is generally connected with the preceding by some connecting particle. Some of these particles, such as *δέ*, are never the first word of a sentence, but generally the second. The particles will be discussed later.

LESSON 9

The Accusative, Genitive, and Dative

THE accusative always depends on a verb, or on a verbal notion in a noun. It may denote: 1, external object; 2, internal object (including the accusative of result, and the cognate accusative); 3, aim; 4, extent; 5, relation (including the accusative of the part affected, and the limiting accusative).

1. EXTERNAL OBJECT. This is the ordinary use of the accusative, to limit or direct the action of the verb—as: *τοῦτο ποιοῦμεν*, we do this; *ἡ πίστις σου σέσωκέ σε*, thy faith hath saved thee.

An intransitive verb, i.e. one whose meaning is complete in itself, may be used transitively by an extension of its meaning—e.g. *λανθάνω*, I lie hid; *φεύγω*, I flee; *φθάνω*, I am first. Thus, *λανθάνουσι τοὺς φύλακας*, they escape the notice of the guards; *φεύγει τὸν πατέρα*, he flees from (shuns) his father; *ἔφθησαν* (from *φθάνω*) *τὸν χειμῶνα*, they anticipated the storm; *ὄμνυμι τοὺς θεούς*, I swear by the gods.

This accusative is frequent in expressions of emotion, a verb being understood—as *νὴ τὸν Δία*, Yes, by Zeus; *μὰ τὸν Δία*, No, by Zeus; *οὐ τὸν ἥλιον*, No, by the sun (I swear).

2. INTERNAL OBJECT. The accusative repeats with more or less modification of meaning the idea given by the verb. Under this we have:

(*a*) Accusative of result—as: *πημονὰς ἐρεῖς*, you will give pain by speaking (lit., you will speak pains); *βοθρὸν ὤρυξε*, he dug a hole. With this may be reckoned the accusative in apposition to the whole sentence, denoting the result of the action of the verb—as: *Ἑλένην κτάνωμεν . . . Μενελέῳ λύπην πικράν*, let us kill Helen (which will be) bitter grief to Menelaus.

(*b*) The cognate accusative, noun and verb having a common stem—as: *βουλὰς βουλεύειν*, to plan plans; *ἀποστήσεσθαι διπλῆν ἀποστάσιν*, to revolt a double revolt. Under this heading comes what is often called the adverbial accusative, where a noun, pronoun, or adjective is used in the accusative case with an adverbial meaning—as *προφάσιν*, ostensibly; *καιρόν*, opportunely; *πᾶσαν ἰδέαν*, in every way; *δίκην*, like; *χάριν*, for the sake of; *τοῦτον τὸν τρόπον*, in this way; *τέλος*, finally; *τί*, in what respect? *πρῶτον*, at first; *τὸ λοιπόν*, for the rest; *οὐδέν*, not at all.

Under the head of internal object must be placed the *Accusative Absolute*, usual with the participles of certain impersonal verbs—as *δέον*, it being a duty; *ἐξόν*, it being allowed; *προσῆκον*, it being fitting; *δόξαν*, it having been decided; *μετόν, παρόν, ὑπαρχόν, τυχόν, δεδογμένον, εἰρημένον*, though it had been stated, *κ. τ. λ.* Example: *οὐδεὶς* (*ἐξὸν εἰρήνην ἄγειν*) *πόλεμον αἱρήσεται*, no one (when it is possible to have peace) will choose war.

3. ACCUSATIVE OF AIM, or "terminus ad quem," after verbs of motion. In prose this is preceded by a preposition, such as *πρός, εἰς, ἐπί*, to, towards, against, etc. But in poetry it is found without a preposition—as: *ἔβης Θήβας*, thou didst go to Thebes (Sophocles); *χρεία τίς σε Θεσσαλῶν χθόνα πέμπει*, some necessity sends thee to the land of the Thessalians.

4. ACCUSATIVE OF EXTENT. (*a*) Extent [illegible]ace—as: *ἀπέχει δ' ἡ Πλάταια τῶν Θηβῶ[illegible]αδίους ἑβδομήκοντα*, Plataea is distant from T[illegible]es 70 stades. (*b*) Duration of time—as: *ἡμ*[illegible] *ἔχουν* (= *ἔχεον*, from *χέω*) *ἑβδομήκοντα*, they w[illegible]ked at the mound for 70 days.

5. ACCUSATIVE OF RELATION, RESPECT, R[illegible]RENCE, OR DEFINITION. (*a*) Of the part affected as: *ἀλγεῖ τοὺς ὀδόντας*, he has a pain in his teeth; *πόδας ὠκύς*, swift of foot (lit., swift as to his feet). (*b*) Accusative of limitation, used in certain common words—as: *μέγεθος* (in size), *ὄνομα* (by name), *γένος, φύσιν*—e.g. *διαφέρει τὴν φύσιν*, he differs in nature; *Ἕλληνές εἰσι τὸ γένος*, they are Greeks by race.

DOUBLE ACCUSATIVE: 1. After verbs of m[illegible]king, thinking, finding, naming, appointing, [illegible]-as: *στρατηγὸν αὐτὸν ἀπέδειξε* (*ἀποδείκνυμι*), he ap[illegible]nted him general.

2. After verbs of asking, teaching, con[illegible]ing, taking away, clothing, unclothing, re[illegible]ing, depriving, etc.—as: *Θηβαίους χρήματα* [illegible] (from *αἰτέω*), they asked the Thebans for [illegible]y; *μή με κρύψῃς ταῦτα*, do not hide this fr[illegible] me; *ἀναμιμνήσκειν τοὺς Ἀθηναίους τὴν συμμα*[illegible], to remind the Athenians of the alliance.

3. After verbs of doing anything to, o[illegible]ing anything of a person—as: *δρᾶν τινά τι*, to do something to someone; *εὖ ποιεῖν τινά*, to benefit someone; *πλεῖστα κακὰ τὴν πόλιν ποιοῦσιν*, they do the most evils to the state. (NOTE. *εὖ πράσσω* means: I fare well, *not* I do good to someone.)

The Genitive

The Greek genitive is a mixed case, standing both for the genitive proper and for the Latin ablative. In Sanskrit there were eight cases—nominative, vocative, accusative, genitive, dative, ablative, locative and instrumental. Latin has lost the last two as separate cases, having absorbed both into the ablative. Greek has lost the ablative as well, and makes the genitive include the Sanskrit ablative, but transfers the locative and instrumental to the dative.

The Genitive Proper

The genitive proper most often qualifies a noun, less often an adjective, adverb or verb. It denotes: 1. Origin. 2. Possession. 3. Object. 4. Quality. 5. Material. 6. Relation of whole and part.

1. Origin, generally of paternity, as *Διὸς Ἄρτεμις*, Artemis (daughter) of Zeus. Under this head may be classed the genitive of cause and the genitive of exclamation. The genitive of cause is common after verbs expressing emotions, as *wonder, pity, envy, revenge*, etc.—as: *ζηλῶ σε τοῦ νοῦ*, I envy you for your mind. It is also frequent in legal language after verbs of accusing, acquitting, condemning, convicting, etc.—as: *διώκει με φόνου*, he prosecutes me for murder: *φεύγειν φόνου*, to be charged with murder. The genitive of exclamation gives the cause of the astonishment or grief—as: *ὤμοι ἐμῆς ἄτης*, Alas, for my infatuation!

2. Possession, the chief noun being often omitted —as: τὰ τῶν Ἑλλήνων, the possessions of the Greeks. It may be used predicatively after a verb—as: ἐστιν ἀνδρὸς σοφοῦ κοσμίως βιοῦν, it is the part of a wise man to live discreetly. With this may be classed the subjective genitive, denoting the subject of an action or feeling—as: ἡ τοῦ δήμου εὔνοια, the goodwill felt by the people. This is sharply contrasted with the next use.

3. Object, often called the objective genitive, denoting the direction or object of the action, or feeling. After a noun—as: ἕρκος πολέμου, a protection against war; πεῖρα ἐχθρῶν, an attack on one's enemies; τὸ Μεγαρέων ψήφισμα, the decree concerning the Megareans (*not* proposed by them); ἡσυχία ἐχθρῶν, peace from foes. After an adverb—as: λάθρη Λαομέδοντος, unknown to Laomedon. After an adjective or participle—as: ἐπίτροφος ἀνθρώπων, conversant with men; τόξων εἰδώς, well skilled in the bow. After a verb, to express: (*a*) Direction of physical effort—e.g. after verbs like *begin, rule, try, touch, aim at, desire, guide, reach, fail of*—as: ἔτυχε (τυγχάνω) οἴκτου, he gained pity; ἅψασθαι (ἅπτομαι) χειρός, to cling to someone's hand; οὐ πολέμου ἄρχομεν, we do not begin war. (*b*) Direction of mental effort, after verbs like *hear, know of, speak of, remember, remind, forget, perceive*—as: φωνῆς ἀκούω, I hear a voice; ἀλκῆς μνήσασθαι, to remember one's strength.

4. Quality or description—as: ὄμμα τόλμης ὠμᾶς, an eye of cruel daring; τινὲς τῆς αὐτῆς γνώμης, some men of the same opinion. With this we may class the genitive of price—as: πέντε μνῶν τιμᾶται, he is valued at five minae; ἄξιός ἐστι θανάτου, he is worthy of death; ποσοῦ διδάσκει; for what price does he teach? Also the genitive of definition—as: ἕρκος ὀδόντων, a fence formed by the teeth.

5. Material—as: εὐνὴ ἐσθῆτος μαλακῆς, a bed of soft clothes; αἱ δὲ βόες χρυσοῖο (Homeric genitive form) τετεύχατο κασσιτέρου τε, the cows were wrought in gold and tin.

6. Partitive genitive—i.e. the genitive of a noun which in the nominative denotes a divisible whole—as: δέπας οἴνου, a measure of wine; θεῶν Διί, to Zeus among the gods; πολὺ τῆς γῆς, much of the land. It is often used after an abverb—as: ἄλλοθι γαίης, in another land (lit. elsewhere of land).

But the most important use is after a verb: (*a*) Predicatively after "to be"—as: ἤθελε τῶν μενόντων εἶναι, he wished to be one of those who remained. (*b*) The genitive denoting that of which part, not the whole, is given or taken—as: τῆς ἑαυτοῦ γῆς ἔδωκε, he gave some of his own land. Similarly, after verbs like *fill, lack, enjoy, taste, smell, spare, share,* etc.—as: κρομμύων ὀσφραίνομαι, I smell onions; μέλιτος γεύεσθαι, to taste honey; μετέχω τῆς λείας, I share the spoil. Sometimes after a negative adjective—as: ἀδωρότατος χρημάτων, taking (of) no bribes. (*c*) A genitive of *place* or *time* may be used in loose dependence on a verb to denote the limits within which the action is confined, as the accusative is used to denote its extent. *Place*: ἔρχονται πεδίοιο (Homeric genitive) they go through the plain; ποῦ γῆς; where on earth? In prose this use is found in a genitive neuter of an adjective used adverbially with a verb of "wanting"—as: δέω πολλοῦ, I fail by much: ἐλαχίστου ἐδέησε διαφθεῖραι, it came within an inch of destroying (lit., it lacked very little to destroy). It may even be used as a plain adverb, as μικροῦ, nearly. *Time*: νυκτός, by night; δραχμὴν τῆς ἡμέρας, a drachma a day.

The Ablative Genitive

This denotes motion or separation from a person, thing or place. It is used: (*a*) After a verb, to denote motion from—as: Ὀλύμποιο (Homeric genitive) κατήλθομεν, we came down from Olympus; or to denote, "hearing from" a person, after ἀκούω, κλύω, κ.τ.λ. It may also denote separation in general—as: Τρῶας ἄμυνε νεῶν, he kept the Trojans off from the ships. (*b*) After a comparative—as: μείζων ἐκείνου, greater than he; the idea being "the greater, starting from him, or as viewed from him." This genitive is used after verbs, adjectives or adverbs implying comparison—as: ἡσσᾶσθαί τινος, to be weaker than someone, to be beaten by him; ἕτεροι τούτων, others than these; ὕστεροι τῆς μάχης, too late for the battle; ἐμοῦ σοφώτερος, wiser than I; κάλλιστον τῶν προτέρων, fairest of those before (i.e. fairer than all predecessors); μεγίστην τῶν πρὸ αὐτῆς, greatest of those before it (i.e. greater than any before it).

The Genitive Absolute

A noun and a participle, not connected with the main construction of the sentence, may stand by themselves in the genitive. This is called the genitive absolute (corresponding to the ablative absolute in Latin)—e.g. δύοντος τοῦ ἡλίου ἀφίκετο, as the sun was setting, he arrived; ἐμοῦ καθεύδοντος ταῦτα ἐγένετο, this happened while I was asleep; τούτων οὐκ ἀφικομένων ἀπήλθομεν, as they had not come, we departed; ταῦτα ἐπράχθη Κόνωνος στρατηγοῦντος, this was done when Conon was general; μηδενὸς ἐπαρκοῦντος ἀπόλωλα, if no one aids, I am ruined. The participle sometimes stands alone, the noun being understood—as: οὕτω δ'ἐχόντων, εἰκός ἐστιν, κ.τ.λ., and this being so (understand πραγμάτων, lit. things being so) it is likely, etc. This is the regular construction, the accusative absolute being confined to certain impersonal verbs.

The Dative

The dative actually stands for three distinct cases; these are: (1) the Locative; (2) the Instrumental (both of which are included in the Latin ablative); and (3) the Dative Proper.

The Locative

The locative survives in a few forms as a distinct case, the ending being ι short; but, generally, the locative coincides in form with the dative. It denotes (*a*) place where (*b*) time at which.

Place Where. With names of towns—as: Πλαταιᾶσι, at Plataea (locative form); or Πλαταιαῖς (dative form). With names of countries—as: Ἑλλάδι, in Greece. It often denotes the sphere of action, whether in a locative form (as οἴκοι, at home) or in a dative form (as δόμῳ, in the house; οὐρανῷ, in heaven). From the locative denoting "place where" is derived the use of the locative to denote direction, or the place of arrival, after a verb of motion—as: χαμαὶ πεσεῖν (from πίπτω),

to fall to the ground ; *γῇ ἔκειτο*, he lay on the earth ; *ἐδέξατο χειρὶ κύπελλον*, he took a cup in his hand.

Time at Which. This is found in Greek only in dative forms ; there are no locative forms for time—e.g. *τρίτῳ ἤματι*, on the third day; *γήρᾳ*, in old age ; *νουμηνίᾳ*, at the new moon, on the first of the month ; *ὑστέρῳ χρόνῳ*, in after time.

The Instrumental

This case primarily denotes association with, and hence attendant circumstances and relation. The name instrumental is misleading, as the idea of instrument is subordinate.

(*a*) The associative instrumental denotes association with a person, common after *ἕπομαι*, follow; *μίγνυμι*, mix ; *ὁμιλέω*, consort with ; *μάχομαι*, fight ; *διαλέγομαι*, converse ; *κοινός*, common ; *ἴσος*, equal ; *ὅμοιος*, like ; *ὁ αὐτός*, the same—as : *οὗτός ἐστιν ὁ αὐτὸς ἐκείνῳ*, this man is the same as that.

(*b*) Of circumstances attending the action—as : *ἦλθον πλήθει οὐκ ὀλίγῳ*, they came with no small crowd ; *ἔπλεον δεξίῳ κέρᾳ ἡγουμένῳ*, they sailed with the right wing leading ; *μίαν ναῦν αὐτοῖς ἀνδράσιν εἷλον*, they took one ship, men and all (lit. with the men themselves). This has three special uses :

1. To denote the agent of a person—as : *θεοῖς σεσωσμένος*, saved by the gods (lit. with the help of the gods) ; *ὠφελητέα σοι ἡ πόλις ἐστιν*, the city must be benefited (verbal adjective) by you ; *παρεσκεύαστο τοῖς Κορινθίοις*, preparations had been made by the Corinthians (usually after a verb in the perfect, or the verbal adjective ; this is generally rendered by *ὑπό* and the genitive).

2. To denote the means or instrument—as : *ὁρῶμεν τοῖς ὀφθαλμοῖς*, we see with the eyes ; and after *χρῆσθαι*, to use (which always takes the dative), meaning literally to serve oneself with—as : *χρῆσθαι ἱματίῳ*, to use (*or* wear) a garment.

3. To denote reason, cause or ground—as : *τοῖς πεπραγμένοις φοβούμενος*, fearing on account of what had been done ; *οὔτε πενίᾳ οὔτε ἀσθενείᾳ τετίμηται οὐδείς*, no one is honoured on account of poverty or weakness ; *ἀποθνήσκει νόσῳ*, he dies of disease.

(*c*) Of relation, defining the action of a verb or the application of a noun. Of this there are two special uses to denote manner and space :

1. Manner : as *σιγῇ*, in silence, silently ; *βίᾳ*, forcibly ; *τῷ ὄντι*, in reality ; *ταύτῃ*, thus ; *μανίαις ψαύων*, tempting (the god) in his madness. Under this head may be classed the instrumental of measure, denoting difference of extent—as : *πολλῷ μεῖζον*, much greater (lit. greater by much) ; *τῇ κεφαλῇ ἐλάσσων*, a head shorter; *τοσούτῳ ἄμεινον*, so much the better.

2. Space ; either of space over which motion takes place, or of time through which anything lasts—as : *τῷ χρόνῳ*, in course of time ; *ἀγήρῳ χρόνῳ δυνάστης*, ruler through endless (lit. ageless) time.

Note on the instrumental case : Homer has many archaic forms in *-φι* or *-φιν*, which are properly instrumental—as : *βίηφι*, by force, but are also used as locatives—e.g. *ὄρεσφι*, on the mountains. More loosely still, they are used as genitives or datives, as *Ἰλιόφι κλυτὰ τείχεα*, for *Ἰλίου* [illegible]famed walls of Troy).

The Dative Proper

This is used in Greek only of a person, the p[illegible]son affected by the action, although in Latin it is used of a thing as well. It may either depend on a verb or not.

1. Depending on a verb : The verb may be either transitive, as *δίδωμι*, give—e.g. *δός μοι τοῦτο*, give me this ; or intransitive, as : *ἀρέσκω*, please ; *εἴκω*, yield ; *πιστεύω*, trust ; *πείθομαι*, obey ; *ὀνειδίζω*, reproach ; *βοηθῶ*, help ; *πρέπει*, it is becoming ; *δοκεῖ*, it seems, etc., all of which are followed by the dative—e.g. *πιστεύω τοῖς φίλοις*, I trust my friends ; *πρέπει μοι λέγειν*, it befits me to speak. This dative is especially used of the possessor after the verb *to be* and similar verbs—as : *τρεῖς δέ μοί εἰσι θύγατερες*, I have three daughters.

2. Not depending on a verb : (*a*) Dative of interest—as : *πᾶς ἀνὴρ αὑτῷ πονεῖ*, every man labours for himself ; *ἐμῇ κεφαλῇ περιδείδια*, I fear for my head. (*b*) Dative of respect—as : [illegible] *τέθνηκεν*, his death is the gods' concern. This dative is sometimes indistinguishable from the possessive genitive, except by some slight ide[illegible] of the person being *concerned* in the situation—as : *ἦρχον τοῦ ναυτικοῦ τοῖς Συρακοσίοις*, they [illegible]manded (governs genitive) the navy of the [illegible]cusans. (*c*) Participial dative, either limiting [illegible] action—as : *ἐν δεξιᾷ ἐσπλέοντι*, on the [illegible] one sails in (lit. with respect to one sailing [illegible]) ; *συνελόντι*, or *ὡς συνελόντι εἰπεῖν* (particip[illegible] of *συναιρέω*), to speak concisely (lit. for one [illegible]ing it concise) ; *ὡς ἐμοί*, in my opinion ; [illegible] *ὡς γέροντι προυστάλης ὁδόν*, for thou didst [illegible]ertake a long journey for an old man ; or p[illegible]atively, after a verb like *to be*—as : *αὐτῷ* [illegible] *ἐστίν*, he wishes it (lit. it is to him wishing [illegible]). (*d*) Ethic dative, of the person sympath[illegible] or affected (personal pronouns only) as : *πῶς* [illegible] *ἔχεις ;* how are you (we wish to know) ?

Notes. There is a peculiar use of the dative found in statements of time—e.g. *ἡμέραι ἦσαν τῇ Μυτιλήνῃ ἑαλωκυίᾳ* (perfect participle of *ἁλίσκομαι*) *ἑπτά*, it was (lit. there were) seven days since Mytilene had been captured. The dative follows many verbs compounded with prepositions—as : *ὑπόκειται τὸ πεδίον τῷ ἱερῷ*, the plain lies below the temple.

Vocabulary

δεινός, -ή, -όν, terrible	*τὸ πνεῦμα,-ατος*, breath, spirit (cf. pneumatic)
φίλιος, -α, -ον, friendly	*ὅτι*, because (*ὅτι* also =that, conjunction)
ἔχω, I have	*ἡ βασιλεία*, kingdom
ἡ ἀρχή, beginning	*ὁ οὐρανός* heaven (cf. Uranus)
πρός, near, with (governs accusative)	*σώζω*, I save future *σώσω*)
μακάριος, -α, -ον, blessed, happy	
πτωχός, cringing, poor	

Exercise I

Translate into English : 1. *ὁ θανατός ἐστι δεινὸς τοῖς κακοῖς*. 2. *οἱ δίκαιοι ἦσαν φιλιοι τοῖς πένησιν*. 3. *οἱ Ἕλληνες οὐκ ἔχουσι σόφους ἡγεμόνας*. 4. *ἐν ἀρχῇ ἦν ὁ λόγος, καὶ ὁ λόγος ἦν πρὸς*

ν Θεόν, καὶ Θεὸς ἦν ὁ λόγος. 5. ὁ Ξενοφῶν ἦν ...ρατηγὸς τῶν Ἑλλήνων. 6. μακάριοί (εἰσι) οἱ ...ωχοὶ τῷ πνεύματι· ὅτι αὐτῶν (= of them—i.e. ...eirs) ἐστιν ἡ βασιλεία τῶν οὐρανῶν. 7. οἱ ...ιμένες θαυμάζουσι τὴν χιόνα ἐν τῷ χειμῶνι. ...σώσομεν τὴν πατρίδα ἐκ πολέμου. 9. εἰσὶν ...λίγοι ποταμοὶ ἐν τῇ Ἑλλάδι. 10. ὦ Πέρσα, ...εις τὰ τοῦ ἀγγέλου ὀστᾶ ἐν τῷ κανῷ.

(Note the order of the words: τὰ agrees with ὀστᾶ, ...nd τοῦ with ἀγγέλου; but Greek says "the ...f the messenger bones," not "the bones of the ...essenger.")

Note that Greek uses the article with abstract ...ouns—e.g. ὁ θάνατος, death, in sentence No. 1 ...bove; also frequently with proper names—as, Ξενοφῶν, Xenophon, in sentence No. 5.

VOCABULARY

ὅτε, then, at that time	ἀθροός, collected, in heaps
... καί, both . . . and (τε always follows its word; cf. μεγάλοι τε καὶ θολεροί, below)	ἡ γῆ, land
	ἔνθενπερ, whence
	αὔξω, I increase
...λερός, turbid	καθαρός, clear
...ύς, -εῖα, -ύ, swift	ἰδεῖν, to see
...εῦμα, -ατος, current	περάσιμος, passable, able to be crossed
ὥρα, season	
ὁ ἔτος, -ους, year	πλήν, except (governs genitive)
ὁ ὕδωρ, -ατος, water	

EXERCISE II

TRANSLATE INTO ENGLISH: καὶ τότε οἱ ποταμοὶ πάντες οἱ Ἰνδοὶ μεγάλοι τε καὶ θολεροὶ ἔρρεον (imperfect of ῥέω) καὶ ὀξεῖ τῷ ῥεύματι· ἦν γὰρ (= *for*, another connecting particle like δέ, and never put first in a sentence) ὥρα ἔτους. ταύτῃ δὲ τῇ ὥρᾳ (= and at this season) ὕδατά τε ἐξ οὐρανοῦ ἀθρόα καταφέρεται (verb = are carried down) ἐς (= εἰς) τὴν γῆν τὴν Ἰνδικήν, καὶ αἱ χιόνες αἱ τοῦ Καυκάσου, ἔνθενπερ τῶν πολλῶν ποταμῶν αἱ πηγαί εἰσι, αὔξουσιν αὐτοῖς (= their) τὸ ὕδωρ· χειμῶνος (= in winter) δὲ ὀλίγοι τε γίγνονται (= they become) καὶ καθαροὶ ἰδεῖν καὶ περάσιμοι πάντες, πλήν γε δὴ (two more connective particles, γε generally meaning *at least*, and δή being almost untranslatable in English) τοῦ Ἰνδοῦ καὶ Γάγγου.

KEY TO EXERCISE IN LESSON 8

1. οἱ βάτραχοί ποτε πρέσβεις ἔπεμψαν ἐπὶ τὸν Δία καὶ ᾔτησαν βασιλέα αὐτοῖς παρασχεῖν. 2. ὁ Ζεὺς ξύλον εἰς τὴν λίμνην ἔρριψεν. 3. οἱ βάτραχοι φόβῳ ἑαυτοὺς εἰς τὰ τῆς λίμνης βάθη ἔρριπτον. 4. ἀλλ' ὡς ἀκίνητον ἦν τὸ ξύλον, ταχέως αὐτοῦ κατεφρόνουν καὶ ἐνόμιζον τοῦτον τὸν βασιλέα ἀνάξιον εἶναι. 5. ᾔτησαν τὸν Δία ἄλλον βασιλέα. 6. ἔλεγε "ὑμεῖς ἔστε μωροί," καὶ ὕδραν (literally a hydra) αὐτοῖς ἔπεμψεν ὑφ' ἧς κατησθίοντο.

LESSON 10

Prepositions and Their Meanings

THE prepositions in Greek, as in English, were originally adverbs, and this adverbial use is clearly seen when prepositions appear in composition with verbs—e.g. κατα-βαίνω, I go down. The Greek prepositions govern various cases, as will be seen from the following scheme:

Genitive only: ἀντί, ἀπό, ἐκ (ἐξ before a vowel) and πρό.

ἀντί, instead of, for the sake of—as: ἀντί σου, instead of you, *or* for your sake. In composition with a verb it means *against* (ἀντειπεῖν, to speak against); *instead* (ἀντιλαμβάνειν, to receive instead); *in return* (ἀντιδίδωμι, I give in return).

ἀπό (Latin, *a*, *ab*; English *off*), from, away from—as: ἀπὸ τούτου τοῦ χρόνου, from this time; ἀφ' ἵππου μάχεσθαι, to fight from a horse, i.e. on horseback. In composition with a verb it means *from*, *away*, *in return*—as: ἀποστέλλω, I send away (cf. "apostle").

ἐκ (Latin, *e*, *ex*), from, out of—as: ἐκ ταύτης τῆς πόλεως φεύγει, he is banished from this city; ἐκ τοῦ νῦν, from the present (time); ὄναρ ἐκ Διός ἐστίν the dream is from Zeus. In composition it means *out*, *away*—as: ἐκπίπτω, I am driven out (lit. I fall out, used as the passive of ἐκβάλλω, I expel).

πρό (Latin *pro*), before, on behalf of—as: πρὸ τοῦ νεώ, before the temple; πρὸ τῆς μάχης, before the battle; πρὸ πατρίδος μάχεσθαι, to fight for fatherland; πρὸ τούτων, in preference to this. In composition it means *before*, *forth*—as: προκαλέω, I call forth.

Dative only: ἐν and σύν.

ἐν (Latin *in* with ablative), in, within, among—as: ἐν τῇ νήσῳ, in the island; ἐν τούτῳ, meanwhile (τῷ χρόνῳ understood); ἐν δήμῳ λέγειν, to speak among the people. In composition it means *in*, *on* (ἔγκειμαι, I lie on).

σύν or ξύν (Latin *cum*), together with, with the help of—as: σὺν τοῖς θεοῖς, with the help of the gods; σὺν Ξενοφῶντι ἐπορεύετο, he went with Xenophon. In composition it means *with*, *together*—as: συμμάχομαι, I fight along with; συμπαθεῖν, to feel with. (This preposition appears in English words beginning with syn, sym, syl, sys., etc.)

Accusative only: εἰς or ἐς, into, to, towards (Latin *in* with accusative)—as: εἰς Ἀθήνας ἔφυγον, they fled to Athens; εἰς νύκτα, till night; εἰς ἕκατον, up to a hundred; χρήσιμος εἰς πόλεμον, useful for war.

The form ὡς is sometimes found, but only with persons: ὡς Κῦρον εἰσιέναι, to go to visit Cyrus.

Accusative and Genitive: διά, κατά, ὑπέρ.

διά, through. 1. With accusative *on account of*: διὰ τοῦτο, on this account; διὰ τὴν νόσον, on account of the disease. 2. With genitive, *through*: διὰ νυκτός, through the night; διὰ τῆς χώρας, through the country; δι' οὐδένος ποιεῖσθαι, to value at nothing; δι' ὀργῆς ἔχειν, to be in anger (ἔχειν is often used intransitively = to be). In composition it means *through*, *in different directions*, *mutually*—as: διαρρίπτω, I throw in different directions.

κατά, down. 1. With accusative *down along*, *at* (of time), *according to*, *by* (distributively)—as: κατὰ ῥοῦν, down stream; οἱ καθ' ἡμᾶς, those

at our time; κατ' ἐμέ, according to me; καθ' ἡμέραν, day by day; κατὰ τρεῖς, by threes. 2. With genitive *down from, below, against*—as: κατὰ γῆς, below ground; κατὰ τῆς πέτρας ἅλλεσθαι, to leap down from the rock; κατά τινός λέγειν, to speak against someone. In composition *down, against*: καταβαίνω I go down; καταψεύδομαι, I tell lies against.

ὑπέρ, over (Latin *super*; note that a rough breathing in Greek is often represented by *s* in Latin—as: ἑπτά, septem; ἅλς, sal). 1. With accusative *beyond*—as: ὑπὲρ δύναμιν, beyond one's strength. 2. With genitive *above, on behalf of*—as: ὑπὲρ τῆς κεφαλῆς, over his head; μάχεσθαι ὑπέρ τινος, to fight for someone. In composition *over, exceedingly, on behalf of*. (English *hyper*, as in hypercritical.)

Accusative, Genitive and Dative: ἀμφί, ἐπί, μετά, παρά, περί, πρός, ὑπό and, very rarely, ἀνά.

ἀμφί, about (Latin *amb-*, both), on two sides of. 1. With accusative *about* (time, space, number or circumstances)—as: οἱ ἀμφὶ Πλάτωνα, those about Plato (i.e. the Platonists); ἀμφί τι ἔχειν, to be busy about a thing. 2. With genitive *about* (of subject matter), rare in prose; usually περί. 3. With dative *about* (of place), *for the sake of, owing to*—as: ἀμφ' Ἑλένῃ μάχεσθαι, to fight for Helen; ἀμφὶ φόβῳ, for very fear.

ἐπί, on, upon. 1. With accusative *on to, to, towards, against*—as: προελθεῖν ἐπὶ τὸ βῆμα, to come forward on to the platform; πλεῖν ἐπὶ τοὺς Ἀθηναίους, to sail against the Athenians. Also in phrases like τὸ ἐπ' ἐμε, as far as I am concerned; ἐπὶ τούτο, for this purpose; ἐπὶ τὸ πολύ, for the most part. 2. With genitive *on, towards, in the time of*—as: ἐπὶ βήματος, on a platform; ἐπὶ Σάμου πλεῖν, to sail towards Samos; ἐφ' ἡμῶν, in our time. 3. With dative *on, at, on condition of, in the power of, in addition to*—as: τὸ ἐπ' ἐμοί, so far as is in my power; ἐπὶ τούτοις, on these terms, *or*, in addition to this; μέγα φρονεῖν ἐπί τινι, to be proud of a thing.

μετά, among, with (akin perhaps to Latin *medius*). 1. With accusative *after*—as: μετὰ τὸν πόλεμον, after the war; μετὰ ταῦτα, thereafter. Also rarely (in poetry) = into the midst of—as: μετὰ στρατόν, into the midst of the host. 2. With genitive *with, on the side of* (implying closer union than σύν)—as: μετά τινος πάσχειν, to suffer with someone; μετ' Ἀθηναίης, with the help of Athena. 3. With dative *among* (only used in poetry)—as: μετὰ κύμασι, among the waves (Homer). In composition μετά often denotes *change*—as: μεταμόρφωσις, transformation; μετανοέω, I change my mind, repent.

παρά, near, alongside. 1. With accusative *to the side of, along, during, because of, contrary to, compared with*—as: καταφυγὴ παρὰ φίλους, a flight to one's friends; παρὰ τὸν βίον ἅπαντα, during the whole of life; παρὰ τοῦτο γέγονεν, it has happened because of this; παρὰ φύσιν, contrary to nature; παρὰ πολύ, by far. 2. With genitive *from the side of, from*—as: ἄγγελος παρά τινος, a messenger from someone. 3. With dative *at the side of, near*—as: παρά τινι παρεστάναι, to stand by someone. In composition παρά often means *beyond, over, wrongly*—as: παρανομέω, I transgress the law.

περί, around, about. 1. With accusative *about, near* (practically the same as ἀμφί)—as: οἱ περὶ Κῦρον, Cyrus and his attendants. 2. With genitive *about, concerning* (Latin *de*), *above* (in poetry)—as: περί τινος λέγειν, to speak about someone; also the common phrase: περὶ πολλοῦ ποιεῖσθαί τι, to reckon a thing worth much. 3. With dative *around, about, by reason of* (rare in Attic prose)—as: θώρακα περὶ τοῖς στέρνοις ἔχειν, to have a breast-plate round the breast; περὶ δείματι, for fear (Latin *prae*). In composition περί often denotes excess—as: περικαλλής, very beautiful; περιδέω, I fear exceedingly (cf. Latin *permagnus*, exceeding great).

πρός, in front of, at, by. 1. With accusative *to, towards, against, in reference to*—as: βοᾶν πρὸς οὐρανόν, to cry to heaven; πρὸς θεὸν ἐρίζειν, to strive against God; πρὸς ἑσπέραν, towards evening; πρὸς τὸν λόγον, in reference to the argument; and often adverbially—as: πρὸς βίαν, = βιαίως, forcibly; also, πρὸς ἀνάγκην, = ἀναγκαίως necessarily. 2. With genitive *from* (rare), *by* (of agent), *in the eyes of*, and in oaths—as: τετιμῆσθαι πρός τινος, to have been honoured by someone; πρὸς ἀνθρώπων, in the eyes of men; πρὸς θεῶν, by the gods (I swear); also in sentences like οὐ πρὸς ἰατροῦ σοφοῦ θρηνεῖν, it is not the part of (or it is not fitting) a wise physician to wail. 3. With dative *at, near, in addition to*—as: τεῖχος πρὸς τῇ θαλάσσῃ, a wall near the sea; πρὸς τούτοις, in addition to this, furthermore.

ὑπό (Latin *sub*), under, by. 1. With accusative *to under, towards* (of time)—as: ἰέναι ὑπὸ γαῖαν, to go under the earth (i.e. to die); ὑπὸ νύκτα towards night (Latin *sub noctem*). 2. With genitive *from under, through* (causal), *by* (of agent, after passive verbs; Latin *ab*)—as: λύειν τοὺς ἵππους ὑπὸ τοῦ ζυγοῦ, to loosen the horses from under the yoke; ὑπὸ δέους, through fear; τιμᾶσθαι ὑπὸ τῶν πολιτῶν, to be honoured by the citizens; also in such phrases as: ὑπ' αὐλοῦ, to the accompaniment of the flute. 3. With dative *at, under* (place and circumstances)—as: εὕδειν ὑπὸ πέτρῃ, to sleep beneath a rock. In composition ὑπό often means *secretly*, or *slightly*, or *gradually*—as: ὑποθέω, I make a secret attack; ὑπόλευκος, somewhat white.

ἀνά, up (opposite of κατά). 1. With accusative *up, along*—as: ἀνὰ τὸν ποταμόν, up the river; ἀνὰ νύκτα, all night through. Distributively—as: ἔστησαν ἀνὰ ἑκατόν, they stood in hundreds; ἀνὰ πεντήκοντα, by fifties. 2. With genitive, only in the phrase ἀνὰ νηὸς βαίνειν, to go on board ship (Homer). 3. With dative *on, upon* (only in poetry)—as: ἀν' ὤμῳ, upon the shoulder. In composition ἀνά denotes *up, back, repetition, strengthening*—as: ἀναχωρέω, I retreat.

In addition to the above, which are the prepositions proper, there are some adverbs used as prepositions. These cannot be compounded with verbs, and are called improper prepositions. They are: ἄνευ and ἄτερ, without; ἄχρι and μέχρι, as far as, until; μεταξύ, between; ἕνεκα, for the sake of; πλήν, except. These all take the genitive.

Prepositions in Composition

In the case of a verb compounded with a preposition, the augment usually follows the preposition—as: προσλαμβάνω, προσέλαβον; ἐκλείπω, ἐξέλιπον; εἰσακούω, εἰσήκουσα; συγκρίνω, συνεκρίθην;

συλλαμβάνω, συνέλαβον; *συμβάλλω, συνέβαλον*; *συστέλλω, συνέσταλκα.*

Prepositions that end in a vowel drop the final vowel before the augment *ε*, except *περί* and *πρό*—as: *ἀποβάλλω, ἀπέβαλλον*; but *περιέβαλλον* and *προέβαλλον*. (*πρό* is usually contracted with the augment—as: *προὔβαλλον* for *προέβαλλον*.)

A few verbs take the augment before the preposition—as: *καθίζω, ἐκάθιζον*; while some have both augments—as: *ἀνέχω, ἠνειχόμην*.

ὑποπτεύω, suspect, makes *ὑπώπτευον*; and *παρανομέω*, transgress, makes *παρηνόμουν* (very irregular).

General Note

In English a few words are formed by composition with a prefix, often a preposition, which gives them their specialized meaning. Such are: forethought and afterthought, overthrow, overrun, override, outcome, income (what comes in), undertake, underpin, undermine, etc. We have a very much larger number of words taken from Latin which were formed in the same way—precede, intercede, prefix, antediluvian, postpone, exit; similarly we sometimes compound an English word with a Latin or even a Greek preposition, as in pre-war, post-war, antifat; but there is no comparison between such usage in English, or even in Latin, and that of the Greeks who prefixed a variety of prepositions, perhaps two at a time, to an immense number of words, especially verbs, to give them a special significance. Thus one will find in the lexicon a vast number of words which begin with *ἀπο-, παρα-, περι-, προ-, ἐν-, ἐκ-* or *ἐξ-, συν-*, etc., giving to the word a remarkable precision of meaning.

Key to Exercise I in Lesson 9

1. Death is terrible to the wicked. 2. The just were friendly to the poor. 3. The Greeks have not wise leaders. 4. In (the) beginning was the Word, and the Word was with God, and the Word was God. 5. Xenophon was general of the Greeks. 6. Blessed are the poor in spirit: because theirs is the kingdom of heaven (literally of the heavens). 7. The shepherds wonder at the snow in winter. 8. We shall save (our) native land from war. 9. There are few rivers in Greece. 10. O Persian, you have the bones of the messenger in the basket.

Key to Exercise II in Lesson 9

And at that time all the Indian rivers were flowing both great and turbid and with a swift current; for it was the season of the year. And at this season the waters from heaven are carried down in heaps to the Indian land, and the snows of the Caucasus (whence are the sources of many of the rivers) increase their water. But in winter they become small and clear to see and all passable, except, at least, the Indus and the Ganges.

For "the Indian rivers" Greek says either *οἱ Ἰνδοὶ ποταμοί* (the simple and natural order), or *οἱ ποταμοὶ οἱ Ἰνδοί* (as here), or even *ποταμοὶ οἱ Ἰνδοί*; but *not οἱ ποταμοὶ Ἰνδοί*, for that would mean "the rivers (are) Indian." This latter is the *predicative* use of the adjective, the former being the attributive.

LESSON 11

Case Attraction, Negatives, Particles

The relative pronoun in Greek is especially marked by what is called attraction of case—i.e. it either attracts its antecedent to its own case (inverse attraction), or is itself attracted to the case of its antecedent (Attic attraction, as being in Attic the normal construction). The latter is by far the more frequent. Something analogous takes place in the English "I know who you mean" for "I know whom you mean."

Inverse attraction is seen in such a sentence as the following—*οἴχεται φεύγων ὃν ἦγες μάρτυρα*, the witness whom you brought has gone fleeing away (instead of *ὁ μάρτυς ὃν ἦγες*). It is especially frequent when the principal sentence is completed subsequently—as: *θῆκαι ὅσαι ἦσαν τῶν τεθνεώτων πάσας ἀνεῖλον*, they removed (*ἀναιρέω*) all the sepulchres of those who had died (*θῆκαι* instead of *θήκας*). Another instance of this attraction, common in prose, is the phrase *οὐδεὶς ὅστις οὐ*, there is no one who not—i.e. everybody (*ἐστί* being understood). In the nominative there is no attraction, but in the oblique cases, whenever *ὅστις* is governed by a verb or some other word, *οὐδείς* is attracted into the case of *ὅστις*—as: *οὐδένα κίνδυνον ὅντινα οὐχ ὑπέμειναν*, there was no danger that they did not endure (for *οὐδεὶς κίνδυνος ἦν ὅντινα, κ.τ.λ.*); *οὐδενὸς ὅτου οὐκ ἂν πατὴρ εἴην*, there is no one whose father I might not be (for *οὐδείς ἐστι, κ.τ.λ.*).

Attic attraction usually occurs only when the relative would naturally have been accusative—as: *ἀπὸ τῶν πόλεων ὧν ἔπεισε*, from the cities which he persuaded (*ὧν* instead of *ἅς*); *τοῖς ἀγαθοῖς οἷς ἔχομεν*, with the good things which we have (*οἷς* for *ἅ*). Especially when the antecedent is omitted—as: *ἥμισυ οὗ διενοεῖτο*, half of what he intended (*οὗ* for *ἐκείνου ὅ*, of that thing which); *σὺν οἷς ἔχω τὰ ἄκρα καταλήψομαι*, I will seize the heights with the men whom I have (*οἷς* for *τοῖς ἀνδράσι οὕς*); *πρὸς οἷς ἐκτήσαντο*, in addition to what they gained (*οἷς* for *τούτοις ἅ*). More rarely, and always with antecedent omitted, a relative is attracted from genitive to accusative, or from dative to genitive—as: *ἀφ' ἧς ὠμόσατε ἡμέρας*, from the day on which you swore (for *ἀπὸ τῆς ἡμέρας ᾗ ὠμόσατε*); and hardly ever from nominative to genitive—as: *ὧν ἂν δόξῃ πέρι* (for *περὶ ἐκείνων ἃ ἂν δόξῃ*), concerning those things which may seem good to us. (Note the difference in accent when a preposition *follows* its noun; *πέρι* and *περί*.)

A similar construction is found with relative adverbs—as: *διεκομίζοντο ὅθεν ὑπεξέθεντο παῖδας καὶ γυναῖκας*, they brought over their wives and children from the places in which they had deposited them for safety (*ὅθεν*, from which, standing for *ἐκεῖθεν ὅπου* from there where, *or* from the places in which.)

Note. A peculiar attraction takes place in

certain phrases with οἷος, such as: πιστεύω οἵῳ σοι ἀνδρί, I trust such a man as you (for τοιούτῳ οἷος σύ); τοῖς οἵοις ἡμῖν χαλεπόν ἐστι, it is hard for people like us (for τοιούτοις οἷοι ἡμεῖς, for such as we are).

Negatives

There are two negatives in Greek, οὐ and μή. It is rather hard to understand thoroughly the difference between them; only by noticing their usage carefully whenever they occur will the student become quite conversant with their use. The following remarks give the broad outlines.

οὐ is specific, μή is general; οὐ denies, μή forbids or deprecates. In other words, οὐ is used in negative *statements* (including interrogative sentences), μή in negative *conceptions* (including purpose, consequence, prohibition and condition). What is said of οὐ and μή applies also to their compounds, οὐδείς, μηδείς, οὐδέ (not even), μηδέ, οὔτε (nor), μήτε, οὔποτε (never), μήποτε, etc.

In principal clauses οὐ is used in assertions, μή in commands and wishes. In subordinate clauses οὐ is used in causal and temporal sentences, μή in final (i.e. expressing purpose) and conditional sentences. In indirect or reported speech—i.e. oratio obliqua—the regular negative is οὐ, though exceptions occur. (Oratio obliqua is dealt with in Lesson 12, page 1678.)

Examples: πυρὸς ἑκὼν οὐχ ἅπτομαι, I do not willingly touch fire; εἰ ἐκεῖσε ἀπῆλθον, οὐκ ἂν εὗρον αὐτήν, if I had gone there, I should not have found her; οὐ βούλεσθε ἐλθεῖν; don't you wish to go? οἰόμενοι τὴν βουλὴν οὐ ψηφιεῖσθαι, thinking that the senate would not vote; ἐπειδὴ οὐχ εἷλον, ἀπεχώρησαν, as they could not take it, they went away; ἀπῆλθον ἵνα μὴ ἴδοιμι αὐτόν, I went away in order that I might not see him; μὴ τοῦτο δράσῃς, do not do this; μηκέτι ζῴην ἐγώ, O that I may live no longer! εἰ μὴ τοῦτο δρῴην, μαινοίμην ἄν, if I were not to do this, I should be mad; ᾔτησα μὴ δρᾶν, I asked him not to do it.

When used with single words, οὐ is definite, μή indefinite. Thus: οἱ οὐκ ἀγαθοὶ πολῖται, certain definite citizens who are not good; οἱ μὴ ἀγαθοὶ πολῖται, any citizens who are not good; οἱ οὐ νοσοῦντες, certain special not diseased persons; οἱ μὴ νοσοῦντες, those who are not ill, the class of not diseased persons; τὸ μὴ καλόν, whatever is dishonourable; τὸ μὴ δύνασθαι, inability; τὸ μὴ ὄν, non-existence; οὔ φημι, I say not, I deny. In direct questions οὐ expects the answer *Yes* (Latin *nonne*), μή expects *No* (Latin *num*). Sometimes the fuller form ἆρα οὐ or ἆρα μή is found. Thus, ἆρα μή (or simply μή) βούλεσθε ἐλθεῖν; you don't wish to go, do you?

There seems very little difference between the use of οὐδέν and μηδέν as nouns (= nothing), but after the article the forms beginning with μή are alone used—as: οὐ γὰρ ἠξίου τοὺς μηδένας, for he used not to value men that were nobodies.

Double Negative

In Greek, as a rule, two negatives do not nullify each other; they rather strengthen the negation. Thus, "I never heard anything anywhere from anybody," would be οὔποτε ἤκουσα οὐδὲν οὐδαμοῦ οὐδενός, lit. "I never heard nothing nowhere from nobody." μὴ δίδου μηδὲν μηδενί, do not give anything to anybody. So with οὐδέ, not even—as: οὐδεμία γυνὴ οὐδὲ πειρᾶται, no wo[man] even tries. In English the double negative [wa]s still common in Shakespeare's time, but was not the rule. In Greek it is the rule.

After a word of negative meaning Greek often has a negative expression where other lang[uages] would have an affirmative. 1. After a compa[rative] —as: πόλιν ὅλην διαφθεῖραι μᾶλλον ἢ οὐ[…] αἰτίους, to destroy the whole city rather tha[n …] guilty; the idea being that in all comparison[s the] very notion of preference also implies rejecti[on or] denial. 2. After a verb of negative meaning [(…] deny, dispute, hinder, distrust), where μή with [the] infinitive is used for the infinitive alone—as: [they] hindered me from coming, ἐκώλυσάν με μὴ ἐλ[θεῖν]; I deny that he did it, ἀπαρνοῦμαι μὴ ἐκεῖνον δρᾶ[σαι]; and even after a noun—as: ἡ ἀπορία τοῦ μὴ ἡσυχά[ζειν], the impossibility of keeping quiet.

μὴ οὐ. When such a verb of negative mean[ing] as we have just considered is itself negatived b[y οὐ], Greek requires the negative οὐ to be put in [the] subordinate clause as well, so that in the lat[ter] clause we get the combination μὴ οὐ—as[:] ἐκώλυσάν με μὴ οὐκ ἐλθεῖν, they did not hin[der] me from coming; οὐκ ἀπαρνοῦμαι μὴ οὐκ ἐκεῖνο[ν] δρᾶσαι, I do not deny, etc.; τί μέλλει μὴ οὐ παρο[…] ἔχειν; what hinders him from being present? (i.e. nothing hinders).

οὐ μή. This is used (1) interrogatively with the future indicative to express a strong [pro]hibition—as: οὐ μὴ ἐξεγερεῖ; will you not not-rouse?—i.e. do not rouse; οὐ μὴ φλυα[ρή]σεις; will you not stop talking nonsense? —i.e. stop talking nonsense. (2) With the subjunctive, mostly with the aorist—as: οὐ [μὴ] ἔλθῃ, there is no chance of his coming; οὐ μὴ ληφθῶ, there is no chance of my being caught. This expression is probably the negative of μὴ ληφθῶ in its Platonic sense of "I shall probably be caught," which was a revival of the Hom[eric] use of the construction as meaning "I fear I shall be caught." The original meaning of μὴ ληφ[θῶ] was "may I not be caught!" whereas in Plato [the] same construction becomes a cautious assertion (μὴ φαῦλον ᾖ, it will probably prove bad); hence οὐ μὴ ληφθῶ, there is no reason to suspect I shall be caught.

Particles

Greek has several words which cannot begin a sentence or clause, but must always follow another word. Thus, *but Cyrus* may be rendered ἀλλ' (for ἀλλὰ) ὁ Κῦρος, or ὁ δὲ Κῦρος, always in that order. So *a certain man* is ἀνήρ τις, not τις ἀνηρ, but τίς interrogative may begin, as τίς πόθεν εἶς ἀνδρῶν; Who of men are you?

Such particles are constantly used in Greek for emphasis, or to convey shades of meaning which in English can be expressed only by a clumsy periphrasis, or by tone of voice or emphasis, or by a careful arrangement of the words.

The first of these invaluable aids of expression to be noted are the particles of contrast, μέν and δέ, on the one hand . . . on the other hand. ὁ μὲν Κροῖσος πλούσιος ἦν, ὁ δὲ Σόλων σόφος, Croesus indeed was rich, whereas Solon was wise. This English is so awkward that we are usually content to say: Croesus was a rich man, Solon a wise

e; which is the right way of emphasizing the contrast marked by μέν and δέ, without any clumsiness at all. To translate the word μέν is almost impossible.

δέ joined to the negatives οὐ and μή (οὐδέ, μηδέ) makes both equivalent to *but not*, or to *neither . . . nor*; as: οὐδὲ τόδε οὐδὲ τἄλλο (τὸ ἄλλο), neither this nor the other.

δή gives emphasis; sometimes it is properly represented by *indeed*, but is more often better omitted. Sometimes it is sarcastic. In speech we can express it by a tone; in writing, *forsooth* expresses it, but is an obsolete word, which has an artificial effect.

δῆθεν is ironical = *as he pretends*. When people write ironically in English, readers as often do not think that they are expressing their own serious opinion. We have no particle to save us from that, as δῆθεν does, though sometimes we can mark the irony by using quotation marks or a mark of exclamation.

γε is an enclitic; that is, it throws back its accent on the preceding word, which it emphasizes. Sometimes it may be translated by *at least*: ὡς ἐμοί γε, I at least think so; but, *speaking for myself*, or *in my own opinion*, would be better.

καὶ δὴ καί is a combination emphasizing something additional—*and what is more . . .* It is a particular favourite with Demosthenes and other orators.

τε, *and* (τε . . . τε, both . . . and) is an enclitic; thus, Zeus is πατὴρ ἀνδρῶν τε θεῶν τε, father of men and gods.

We may conclude this Lesson with the intensive particle περ, *exactly, just*, an enclitic which, in Attic Greek, is generally a suffix—as: ὅσπερ, ἔνθαπερ, ὥσπερ.

Exercises in Translation

(1) In Lesson 8 we have the fable of the frogs asking for a king. let us now render into Greek John Milton's reflections thereon:

"Nor are you happier in the relating or the moralizing your fable. 'The frogs' (being once a free nation, saith the fable) 'petitioned Jupiter for a king: he tumbled among them a log: they found it insensible; they petitioned then for a king that should be active; he sent them a crane' (a stork, saith the fable), 'which straight fell to pecking them up.' This you apply to the reproof of them who desire change: whereas indeed the true moral shows rather the folly of those who being free seek a king; which for the most part either as a log lies heavy on his subjects, without doing aught worthy of his dignity and the charge to maintain him, or as a stork is ever pecking them up and devouring them."

Before attempting to turn this into Greek we should do well to re-write it in English that is simpler and more adaptable to the Greek idiom, somewhat as follows:

"You appear neither to tell nor to understand the myth aright. For the frogs (having become free, says the story of old) asked Zeus for a king: therefore he threw down a log, which to them indeed seemed something senseless, so that they asked for an active king. Therefore he sent a crane, or, which the myth says, a stork, and he immediately ate them up. And this you reproach to those desiring change; but rather it wishes to tell the folly of those being free and desiring a king, who for the most part, either oppresses his subjects as a log, not doing anything in accordance with his dignity and the expense of his keep, or, as a stork, always seizes them and eats them up."

(2) The student is now set a piece for translation taken directly from the work of a Greek author, the *Anabasis* (i.e. going up) of Xenophon, for which the student will have to use his Liddell & Scott's (small) Greek-English Lexicon. Later, he will find it advisable to use the larger Liddell & Scott, of which this is an abridgement.

(The *Anabasis* is so called because it is the account of a military expedition ,which "went up" from the coast of Asia Minor to Persia in 401 B.C. to help the "younger Cyrus" (not Cyrus the Great, who had been dead for more than a hundred years) to wrest the Persian crown from his elder brother, Artaxerxes II. The death of Cyrus in the hour of victory wrecked the purpose of the expedition, and Xenophon had led the famous Retreat of the Ten Thousand Greeks through a difficult and hostile country to the Black Sea. This passage tells of the joy with which the retreating force caught the first sight of its goal.)

Ἐντεῦθεν ἐπορεύθησαν διὰ Χαλύβων καὶ ἦλθον (1) πρὸς πόλιν μεγάλην, ἣ ἐκαλεῖτο Γυμνίας. ἐκ ταύτης τῆς πόλεως ὁ τῆς χώρας ἄρχων πέμπει ἡγεμόνα τοῖς Ἕλλησιν· ἐλθὼν δὲ ἐκεῖνος εἶπεν ὅτι ἄξοι (2) αὐτοὺς εἰς χωρίον ὅθεν ὄψοιντο (3) θάλασσαν. ὁ δὲ ἡγεῖτο (4) αὐτοῖς· καὶ ἀφικνοῦνται ἐπὶ τὸ ὄρος τῇ πέμπτῃ ἡμέρᾳ· ἐπεὶ δὲ οἱ πρῶτοι ἐγένοντο (5) ἐπὶ τοῦ ὄρους καὶ κατεῖδον τὴν θάλασσαν, κραυγὴ πολλὴ ἐγένετο. ὁ δὲ Ξενοφῶν καὶ οἱ ὀπισθοφύλακες ἀκούσαντες ᾠήθησαν (6) πολεμίους ἐπιτίθεσθαι τοῖς ἔμπροσθεν. ἐπειδὴ δὲ βοὴ πλείων ἐγίγνετο, καὶ οἱ ἀεὶ ἐπιόντες (7) ἔθεον δρόμῳ ἐπὶ τοὺς ἀεὶ βοῶντας, ἐδόκει δὴ τῷ Ξενοφῶντι μεῖζόν τι εἶναι καὶ ἀναβὰς ἐφ' ἵππον καὶ ἀναλαβὼν τοὺς ἱππέας, παρεβοήθει.(8) καὶ τάχα δὴ ἀκούουσι τῶν στρατιωτῶν βοώντων 'Θάλασσα, θάλασσα,' καὶ παρακελευομένων ἀλλήλοις. ἔνθα δὴ πάντες ἔθεον· ἐπεὶ δὲ ἀφίκοντο ἐπὶ τὸ ἄκρον, ἐνταῦθα δὴ περιέβαλλον (9) ἀλλήλους καὶ στρατηγοὺς καὶ λοχαγοὺς δακρύοντες· καὶ εὐθὺς οἱ στρατιῶται φέρουσι λίθους καὶ ποιοῦσι κολωνὸν μέγαν. μετὰ ταῦτα οἱ Ἕλληνες ἀποπέμπουσι τὸν ἡγεμόνα δῶρα δόντες (10) αὐτῷ, ἵππον καὶ φιάλην ἀργυρᾶν καὶ σκευὴν Περσικὴν καὶ χρήματα· ὁ δὲ δείξας (11) αὐτοῖς κώμην, οὗ ἔδει σκηνεῖν, καὶ τὴν ὁδὸν ἣν πορεύσοιντο, ᾤχετο τῆς νυκτός.

NOTES (1) Irregular aorist of ἔρχομαι (participle ἐλθών). (2) Future optative of ἄγω. (3) Future optative of ὁράω. (4) Governs the dative (from ἡγέομαι). (5) Second aorist of γίγνομαι. (6) Aorist of οἴομαι. (Note that verbs beginning with a diphthong take the temporal augment on the first vowel of the diphthong, αι becoming ῃ, and οι becoming ῳ—e.g. αἰτέω, I ask, ᾔτησα; οἴχομαι, ᾠχόμην, as below: ᾤχετο τῆς νυκτός.) (7) Present participle of ἔπειμι, come near. (8) and (9) Imperfect of παραβοηθέω and περιβάλλω respectively (10) Aorist participle of δίδωμι. (11) Aorist participle of δείκνυμι.

LESSON 12

Indirect Statement

STUDENTS of Latin have the advantage of understanding this construction; they have the disadvantage, however, of bringing to the study of the Greek construction certain preconceived ideas which, though excellent from the point of view of Latin, are inconsistent with the Greek idiom. Everyone knows what direct speech is: "I am well," "who are you?" "do this!" — these are examples respectively of direct statement, direct question, and direct command. Now if we wish to quote someone as using these expressions, we may either quote them *directly*—as: he said, "I am well"; he asks, "Who are you?" he commands, "Do this!"; or *indirectly*—as: "he said that he was well"; "he asks who you are"; "he orders you to do this."

In Latin the natural way of expressing an indirect statement is by means of the accusative with the infinitive; in Greek this is only *one* of the natural ways.

The accusative and infinitive construction is used after verbs of *saying* and *thinking* to express "that . . ."—as: he said that the laws were unjust, *ἔλεγε τοὺς νόμους εἶναι ἀδίκους*; thinkest thou that I am unhappy? *νομίζεις δυστυχεῖν ἐμέ*; Also after impersonal verbs, as: "it was agreed that," "it happened that," "it was proposed that," etc. The negative used here is *οὐ* not *μή*.

In two points the accusative and infinitive construction in Greek differs from Latin.

After verbs of *feeling* and *knowing* (perception as opposed to statement proper) the accusative with the participle, and not with the infinitive, is the rule, as: *οἶδα αὐτὸν λυπηρὸν ὄντα*, I know that he is troublesome; *τυράννους ἐκπεσόντας ᾔσθημην*, I perceived that the tyrants had been expelled; *οἶδα σε ἀγαθὸν ὄντα*, I know that you are good.

If the subject of the infinitive (or participle) is the same as the subject of the principal verb, it is put in the *nominative*, instead of in the accusative, by a kind of attraction, as: *Ἀλέξανδρος ἔφασκεν εἶναι Διὸς υἱός*, Alexander used to assert that he was the son of Zeus; [illegible] we think that we shall be rich.

Notice that in these examples the pronouns "that *he*," "that *we*," etc., are not expressed. There is no need to express them, unless there is an especial emphasis laid on the pronoun, as for example, *οὐκ ἔφη αὐτός, ἀλλ' ἐκεῖνον* [illegible], he said "Not I, but you, are general" (lit. he said that not himself but that man was general); here we have the two constructions (nominative with infinitive, and accusative with infinitive): [illegible], he said that she was beautiful, but that he himself was ugly. This construction makes Greek beautifully clear; it avoids all the confusedness of the English reported speech, in which it is often difficult to make out the persons to whom the various pronouns refer.

Instead of the accusative with the infinitive it is equally good Greek to translate the English "that" by *ὅτι* or *ὡς* followed by the verb in a *finite* mood. The negative here too is *οὐ*. Examples: *λέγει ὅτι γράφει*, he says that he is writing; *εἶπεν ὅτι γράφοι*, he said that he was writing.

From these examples it will be noticed that after a main verb in a primary tense (i.e. present, future, perfect), the verb in the *ὅτι* clause remains in the same mood and tense as in the corresponding direct statement; **but after a historic tense verb (i.e. aorist, imperfect, pluperfect) the verb in the *ὅτι* clause becomes *optative*. This is the strict rule,** but for the sake of vividness, it is not unusual to retain the indicative, as with the primary tenses, as: *εἰπόντες ὅτι πρέσβεις πέμψουσι*, [illegible], they went away, saying that they would send ambassadors ("they will send," instead of [illegible], as strictly required). Here are two examples showing the strictly regular use with optative: *ἐπειρώμην αὐτῷ δεικνύναι ὅτι οἴοιτο μὲν εἶναι σοφός, εἴη δ' οὔ*, I tried to show him that he thought he was wise, but was not so; *ὑπειπὼν ὅτι αὐτὸς* [illegible] *ἐκεῖ πράξοι, ᾤχετο*, hinting that he would himself manage things there, he departed.

After verbs expressing *emotion*—e.g. *θαυμάζω*, wonder; *μέμφομαι*, blame; *ἀγανακτῶ*, am indignant; *αἰσχύνομαι*, am ashamed; the Greeks used *εἰ* (if) instead of *ὡς* or *ὅτι*, as: *θαυμάζω εἰ μὴ ἀσμένοις ὑμῖν ἀφῖγμαι*, I am astonished that my arrival should be unwelcome to you. The negative is *μή*.

Let us see what happens in a dependent or subordinate clause. Here, again, the Latin student must beware; for whereas in Latin the verb in a dependent clause in oratio obliqua is subjunctive, in Greek the subjunctive is never due to oratio obliqua. In Greek the rule is much simpler: if the principal verb is primary the dependent clauses stand exactly as they would if the statement were direct; if the principal verb is historic, all dependent verbs in the indicative or subjunctive should strictly become optative, though for the sake of vividness they are frequently retained in the indicative or subjunctive.

Examples: [illegible] *φησιν ὁ μήτ' αἰσχύνην μήτ'* [illegible] *αὐτῷ φέρει*, he says he will do whatever brings neither shame nor discredit to him ([illegible] is exactly what it would be in direct speech); [illegible] *ὅτι μανθάνοιεν ἃ οὐκ ἐπίσταιντο*, he replied that they were learning what they did not understand (where *ἐπίστασθε*, indicative, you do not understand, becomes optative, though it might have remained indicative); *ἔλεγον* [illegible] *οὓς ἔχοιεν ζῶντας*, they said they would kill the men whom they had living (*ἔχοιεν* indicative retained rather than changed into *ἔχοιεν*; their actual words were [illegible] *οὓς ἔχομεν*).

NOTE 1. The word "saying" or "thinking" need not be actually expressed in Greek in order to throw what follows into the accusative and infinitive construction. It is enough if the idea of saying or thinking is suggested only—as: [illegible] *καὶ οὐκ εἴα τοῖς γεγενημένοις* [illegible] *γνώμῃ ἡσσῆσθαι* [illegible] (Thucydides, VI. 72), he encouraged them and would not allow them to yield in face of what had happened; for he said, it was not their spirit that was broken (the idea of "saying" is suggested by *θαρσύνων*).

Note 2. The accusative and infinitive is often employed after the article τό, making the sentence into a kind of substantive as τὸ ἁμαρτάνειν ἀνθρώπους οὐδὲν (ἐστι) θαυμαστόν, the (fact) that mortals err is not surprising ; διὰ τὸ ξένος εἶναι οὐκ ἂν οἴει ἀδικηθῆναι ; do you think you would not be wronged on account of your being a stranger?

Indirect Question (*Interrogatio Obliqua*)

This is very simple, the usage following the rule of the indirect statement with ὅτι—the verb in the indirect question being indicative or optative according as the main verb is primary or historic. And here, as there, the indicative may remain after a historic main verb for vividness. Examples : ... δράσει, I ask what he will do ; ἠπόρουν τί ... λέγει, I was at a loss to know what he meant (λέγει should strictly be optative, but the words of the actual direct question are allowed to remain, τί ..., what does he say ?). Similarly, if the direct question is a deliberative subjunctive, the subjunctive can remain after a historic main verb, instead of becoming optative—as : ἐβουλεύοντο εἴτε κατακαύσωσιν εἴτε τι ἄλλο χρήσωνται, they were deliberating whether they should burn them or dispose of them in some other way.

Indirect Command or Petition (*Petitio Obliqua*)

The same rules apply here. Examples : κελεύει ... δρᾶσαι τόδε, he orders you to do this (negative μή) ; ἐκέλευσεν ὅ τι δύναιντο λαβόντας μεταδιώκειν, he ordered them, taking what they could, to pursue (his actual words were "take what you can," ... ἂν δύνησθε, and the subjunctive δύνωνται might have been retained here for vividness instead of optative). προεῖπον αὐτοῖς μὴ ναυμαχεῖν Κορινθίοις, ... μὴ ἐπὶ Κέρκυραν πλέωσι, they instructed them not to fight the Corinthians at sea, unless these should be sailing against Corcyra (subjunctive πλέωσι here retained instead of optative πλέοιεν).

Books to Read

1. The Greek Testament, especially the Gospels. This is valuable, not because it is the best model of Greek prose—it was written long after the best of the Greek literature—but by reading it you will learn to see your way in Greek, as the meaning is probably already familiar ; and also you have ready to hand a version in the very best of English.

2. Begin with prose, not poetry, because the structure of poetry is very rarely simple. When you are at home with what is written simply, you can go on to what is written intricately. Homer, indeed, is simple, but because the Homeric dialect differs in many ways from Attic Greek, you will find it less confusing to acquire a reasonable mastery of Attic Greek before trying Homer.

3. For the same reason take first the easiest writers of Greek prose. Begin with Xenophon. The *Anabasis*, from which you have had an extract, is a fascinating story in itself. So is his *Memorabilia* (the Latin name by which the book is generally called), his account of Socrates, whose friend he was.

4. A little more difficult is Plato's *Apology*, which also tells of Socrates. But though some day you may find Plato the most fascinating of teachers, his other works are not suited to beginners.

5. When you have made sufficient progress, you can try the Sixth Book of Herodotus, "the Father of History," in which he tells the story of Marathon. He is easy reading, and the variations of his dialect from the Attic should not be confusing. And you should now be ready to face

6. Thucydides, who tells the history of the wars between Athens and Sparta. But he is often difficult, and sometimes very difficult. But by this time you will have advanced far beyond the point for which this Course is intended to prepare you, and should already have begun to read Homer's *Odyssey* and to make acquaintance with some of the easier tragedies of Euripides and Sophocles, such as the *Alcestis* and *Antigone*.

The best of all ways of learning Greek composition for oneself is : (1) to translate a passage from the Greek author one is reading into English ; (2) after an interval, translate the translation back into Greek without referring to the original ; (3) to compare one's own completed version with the original. That is the best possible practice ; but it is wise also to make use of the best composition text-books.

Greek Version of Exercise in Lesson 11

τὸν γὰρ μῦθον οὔτε ὀρθῶς λέγειν οὔτε συνιέναι φαίνεσθε. οἱ γὰρ βάτραχοι (λέγει τὸ πρίν, ἐλεύθεροι γενόμενοι) τὸν Δία βασιλέα ᾐτήσαντο· ξύλον οὖν κατέβαλεν, ὃ αὐτοῖς ἀναίσθητον δή τι ἔδοξεν, ὥστε βασιλέα αἰτεῖσθαι καρτερόν. γέρανον οὖν ἔπεμψεν (ἣ ὃν ὁ μῦθος λέγει πελαργόν), ὁ δὲ εὐθὺς κατήσθιεν αὐτούς. καὶ ταῦτα τοῖς μεταβολῆς ἐπιθυμοῦσι (dative plural of present participle of ἐπιθυμέω, governs genitive) ὀνειδίζετε. ἀλλὰ μᾶλλον βούλεται μωρίαν λέγειν τὴν τῶν ἐλευθέρων ὄντων (present participle of εἰμί) καὶ βασιλέως ἐπιθυμούντων, ὅσπερ ἐπὶ τὸ πολὺ ἢ ὡς ξύλον βαρύνει τοὺς ὑπηκόους οὔτι πρὸς τὸ ἀξίωμα καὶ τὴν τῆς τροφῆς δαπάνην ποιῶν, ἢ ὡς πελαργὸς ἀεὶ καταλαμβάνει αὐτοὺς καὶ κατεσθίει.

Note. The above might be translated back again after the lapse of a few days.

Key to Passage from Xenophon in Lesson 11

Thence they marched through the Chalybes and came to a great city which was called Gymnias. From this city the ruler of the country sends a guide for the Greeks, and he coming said that he would bring them to a spot whence they could see the sea. So he led them, and on the fifth day they reach the mountain. When the vanguard were on (the summit of) the mountain and caught sight of the sea, there arose a great shout. Xenophon and the rearguard, hearing it, thought that an enemy was attacking those in front. But when the shouting became more, and those who kept coming up ran at full speed to those who kept on shouting, it seemed to Xenophon to be something more serious (lit. greater) ; so, mounting his horse and taking the cavalry, he went to the rescue. Soon, however, they hear the soldiers shouting, "The sea ! the sea !" and encouraging one another. Then they all ran ; and when they got to the top, there they embraced one another, generals and captains alike, weeping. Immediately the soldiers carry stones and make a great heap. After this the Greeks dismiss the guide, giving him gifts ; a horse, a silver vessel, a Persian dress, and money ; and he, having shown them a village where they ought to encamp, and the way which they should go went away in the night.